A Level · AQA · Physics

31 questions

Hard Physics Questions

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Total Marks **/315**

1 (a) The Tesla, symbol T, is the unit of magnetic flux density.

Use the equation *Force* = *Magnetic flux density* × *Current* × *Length* write 1 Tesla in SI units.

Answer

- *Magnetic flux density* [**T**] = $\frac{Force[N]}{Current[A] \times Length[m]}$ [1 mark]
- N = kg m s^{-2} (from $F = ma$) [1 mark]
- [T] = $\frac{[kg\ m\ s^{-2}]}{[A][m]}$
- 1 T = kg s^{-2} A^{-1} [1 mark]

[Total: 3 marks]

This equation is not introduced until the second year of the course; however, you might be expected to deduce the units of any unfamiliar equation given some additional information and your knowledge of AS quantities and equations.

(3 marks)

(b) Complete **Table 1** by converting the following measurements into their appropriate SI unit and express to an appropriate number of significant figures.

Table 1

	Value	**SI Unit**
7 kWh		
81.6 hours		

Answer

	Value	SI Unit
7 kWh	*30 000 000* *or 3 × 10^7*	*J* *or Joule*
81.6 hours	*294 000* *or 2.94 × 10^5*	*s* *or second*

Example of calculation:

- 1 kWh = 1000 W × 3600 s = 3.6 × 10^6 J [1 mark]
- 7 kWh = 7 × (3.6 × 10^6) = 2.52 × 10^7 J = 3 × 10^7 J (1 s. f) [1 mark]

Example of calculation:

- 1 hour = 3600 s [1 mark]
- 6 hours = 81.6 × 3600 = 293760 s = 2.94 × 10^5 s(3 s .f) [1 mark]

[Total: 4 marks]

(4 marks)

(c) 1 eV is the amount of kinetic energy gained by a single electron accelerating from rest through a potential difference of 1 V.

1 eV = 1.60 × 10^{-19} J.

Convert 0.01 kWh into TeV.

Answer

List the known quantities:

- 1 kWh = 3.6×10^6 J (from part b)
- 01 kWh = 0.01 × (3.6×10^6) = 36000 J (**or** 36 kJ) [1 mark]

Convert from J to eV:

- $= \dfrac{36000}{1.60 \times 10^{-19}}$ = 2.25×10^{23} eV [1 mark]

Convert from eV to TeV:

- 2.25×10^{23} eV = 2.25×10^{11} TeV [1 mark]

[Total: 3 marks]

(3 marks)

2 (a) A muon and an anti-muon annihilate leading to the emission of two photons.

Calculate the wavelength of the photons.

Answer

From the data sheet:

- Rest energy of a muon = 105.659 MeV
- Planck constant, $h = 6.63 \times 10^{-34}$ J s
- Speed of light, $c = 3.0 \times 10^{8}$ m s^{-1}
- Photon energy $E = hf = \frac{hc}{\lambda}$

Calculate the minimum energy of each photon:

- Energy of photon pair = 2 × rest mass – energy of muon
- Energy of photon pair = 2 ×105. 659 MeV = 211.318 MeV
- 211.318 MeV = 211.318 × (1.6×10^{-13}) = 3.38×10^{-11} J
- One photon, $E = \frac{3.38 \times 10^{-11}}{2}$ = 1.69×10^{-11} J [1 mark]

Calculate the wavelength of one photon:

- One photon has energy $E = hf = \frac{hc}{\lambda}$
- So, one photon has wavelength $\lambda = \frac{hc}{E}$
- $\lambda = \frac{(6.63 \times 10^{-34}) \times (3.0 \pm 10^{8})}{1.69 \times 10^{-11}}$ = 1.2×10^{-14} m [1 mark]

[Total: 2 marks]

At some point in your calculation you will need to account for the fact that there are two photons emitted. You can either do this explicitly by calculating the energy of the photons × 2 then dividing by 2 to obtain the energy of one photon, or you can omit the 2 entirely.

Since you multiply the energy by 2 and then divide it by 2, it would be valid to not include that 2 in your multiplication or division as you end up with the same answer.

However, if it helps you understand the concept better, there is nothing wrong with multiplying the energy by 2 and then dividing it by 2 to get the energy of one photon. Whichever method you use, you will end up with the same answer!

(2 marks)

(b) Using your answer to part (a):

(i) Deduce the type of electromagnetic radiation emitted after the annihilation.

(ii) Explain why this is the longest possible wavelength of the photons produced.

Answer

(i) Type of EM radiation emitted:

- Gamma; [1 mark]
    - This is because gamma radiation is any EM wave with a wavelength smaller than 10^{-12} m

(ii) The calculated wavelength is the longest possible because:

- The energy calculated is based on the mass of the muon / anti–muon only, hence this represents the minimum possible energy
- Kinetic energy of the muon / anti–muon not taken into account; [1 mark]
- Energy is inversely proportional to wavelength (hence) the minimum possible energy is equivalent to the longest possible wavelength; [1 mark]

[Total: 3 marks]

(3 marks)

(c) Recombining the photons can produce a muon and an anti–muon in a process called pair production. Alternatively, one of the photons could spontaneously convert into an electron and positron.

Show that the energy required to produce a positron is about 200 times less than the energy required to produce an anti–muon and hence deduce the mass of a muon in kg.

Answer

From data sheet:

- Rest energy of an electron / positron = 0.510999 MeV
- Rest energy of a muon / anti–muon = 105.659 MeV
- Mass of an electron / positron, = 9.11×10^{-31} kg

Calculate the ratio of the rest energies of the positron and anti–muon:

- The energy needed to produce an electron–positron (or muon–anti–muon) pair is equal to 2 × the rest energy of an electron (or muon)
- Therefore, the energy needed to produce **one** of the leptons is simply equal to its rest energy
- Ratio: $\frac{anti - muon\ rest\ energy}{positron\ rest\ energy} = \frac{105.659}{0.510999}$ = 206.8 ≈ 200 [1 mark]
- Therefore, the energy to produce an anti–muon ≈ 200 × the energy needed to produce a positron

Calculate the mass of a muon in kg:

- Since mass and energy are equivalent according to $E = mc^2$, the mass of a muon must be ~200 times larger than that of an electron
- Mass of a muon = $206.8 \times (9.11 \times 10^{-31}) = 1.88 \times 10^{-28}$ kg [1 mark]

[Total: 2 marks]

'Show that' questions normally require a calculation, make sure this is included in your answer for full marks.

(2 marks)

(d) In 1995, scientists at CERN created atoms of anti-hydrogen for the first time. An atom of anti-hydrogen consists of a positron in orbit around an antiproton. Each atom of anti-hydrogen survived for only around forty nanoseconds.

Describe the properties of an atom of anti-hydrogen and explain why they only survived for around forty nanoseconds.

Answer

An atom of anti-hydrogen has:

- The same mass and overall charge (as an atom of hydrogen); **AND**
- But opposite charges at the centre and in orbit / antiprotons are negatively charged, positrons are positively charged; [1 mark]

Atoms of anti-hydrogen were short-lived because:

- Annihilation occurred; [1 mark]
- When a matter particle met an antimatter particle / when an antiproton met a proton and a positron met an electron; [1 mark]

[Total: 3 marks]

When a particle meets an antiparticle, it will annihilate (all its mass will turn into energy). Since there is much more matter in the known universe, which is all around us, there is much more matter than there is anti-matter and therefore is a greater chance of annihilation for any antiparticles. This striking imbalance of matter and anti-matter in the universe is called 'matter - anti-matter asymmetry' and is still an active area of research at CERN to this day.

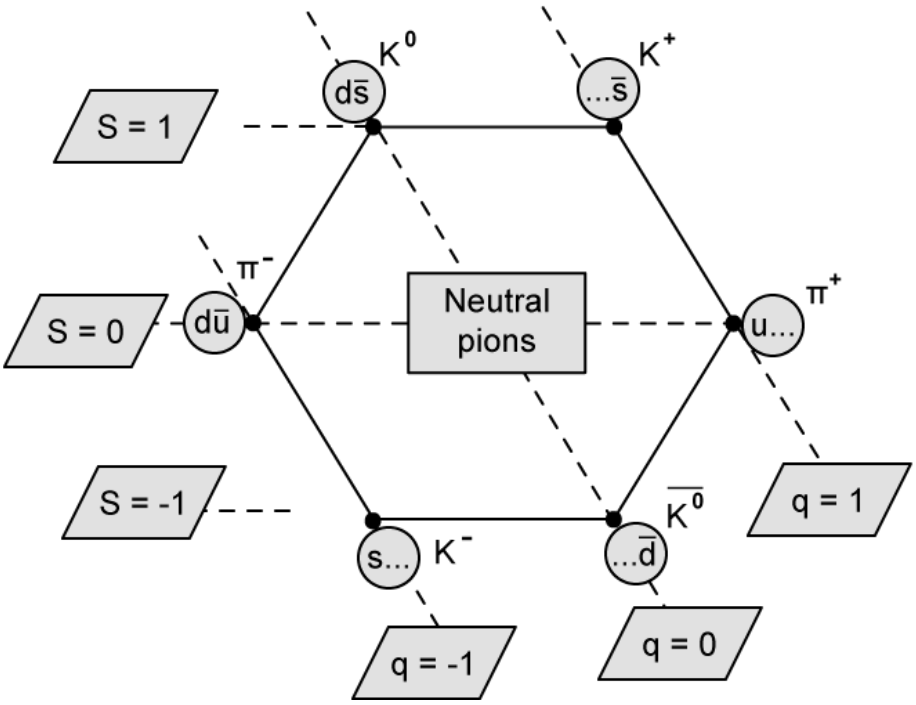
(3 marks)

3 (a) The 'eightfold way' is an organisational scheme for subatomic particles. This question uses the eightfold way to organise mesons.

Physicists like to organise mesons in this way because certain symmetries appear if their strangeness is plotted against their electric charge.

Figure 1 shows a plot of the eightfold way. Six particles and their quark compositions are shown at the vertices of a hexagon. The centre of the hexagon represents particles called neutral pions.

Vertices are linked by horizontal axes, labelled **S**, and diagonal axes labelled **q**. Some additional information is missing from **Figure 1**.



Deduce the meaning of the labels **S** and **q** in **Figure 1** and explain your reasoning for both.

Answer

State the meaning of label **S**:

- **S** stands for strangeness; [1 mark]

Explain using the properties of the particles on the horizontal axes:

- Strange quarks have a strangeness of *S* = –1 and anti-strange quarks have a strangeness of *S* = 1
- Both particles on the top horizontal axes labelled **S** =1 have one anti-strange quark each

  OR

  Both particles on the bottom horizontal axes labelled **S**= –1 have one strange quark each; [1 mark]

State the meaning of the label **q**:

- **q** stands for charge; [1 mark]

Explain using the properties of the particles on the diagonal axes:

- Particles on the diagonal axes have charges corresponding to q = 1, q = 0 and q = –1; [1 mark]

[Total: 4 marks]

(4 marks)

(b) Use **Figure 1** to determine the quark composition of the K^+, π^+, $\overline{K}^0$ and K^-.

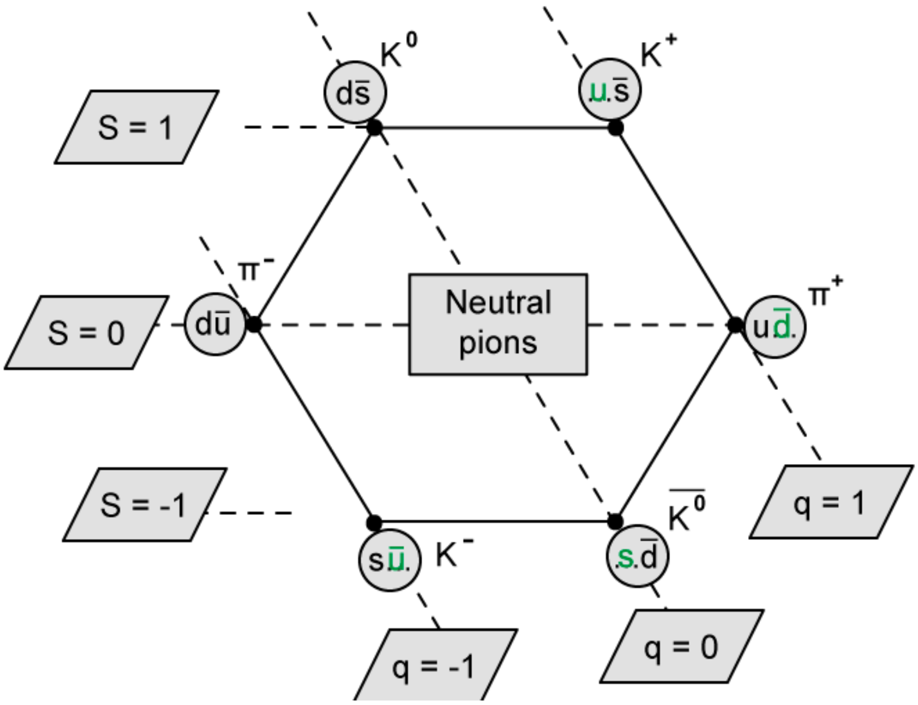
Answer

From the data sheet:

- Charge of an up quark, $u = +\frac{2}{3}e$; anti-up quark, $\overline{u} = -\frac{2}{3}e$
- Charge of a down quark, $d = -\frac{1}{3}e$; anti-down quark, $\overline{d} = +\frac{1}{3}e$
- Charge of a strange quark, $s = -e$; anti-strange quark, $\overline{s} = +\frac{1}{3}e$
- Strangeness of an up (u) or down quark (d) , S = 0
- Strangeness of a strange quark (s) , S = –1

- Strangeness of an anti-strange quark $(\bar{s})$, S = +1

One mark for each correct addition:



[Total: 4 marks]

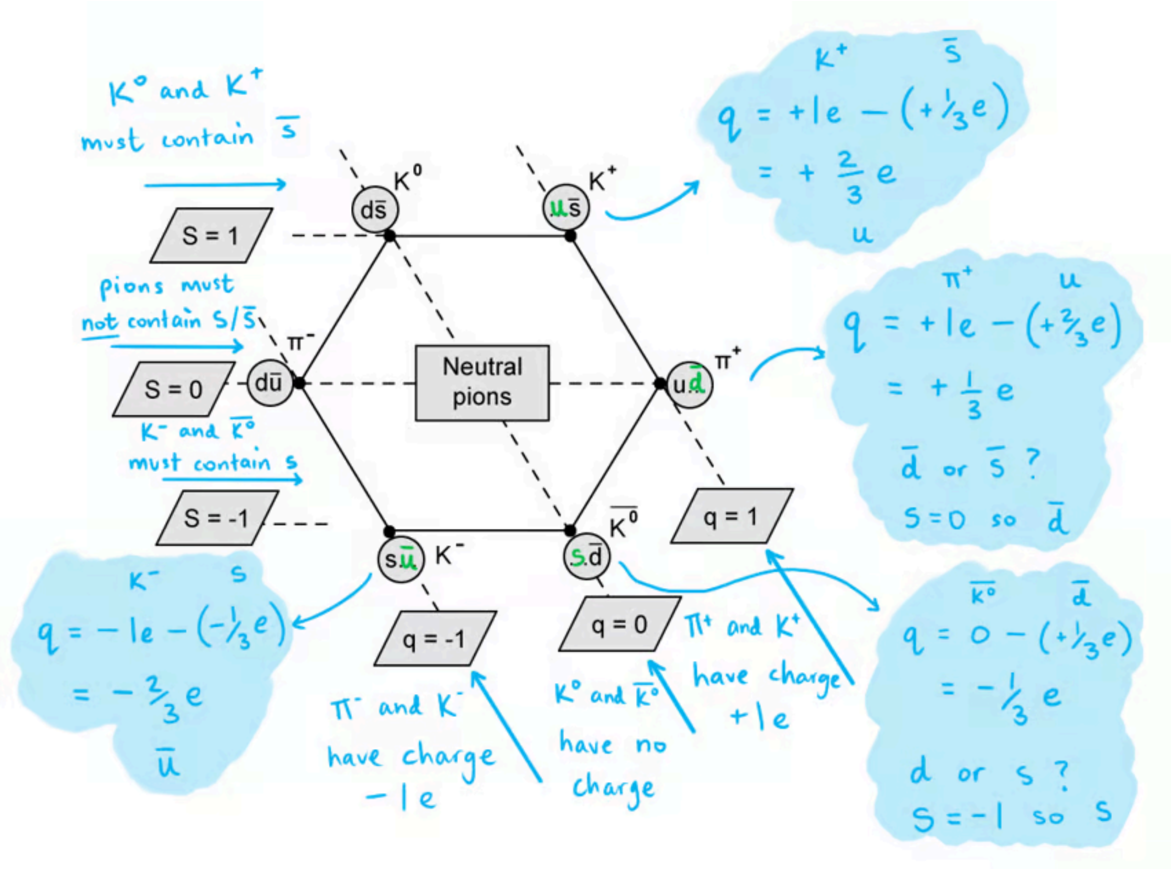
The 'Properties of quarks' table in the 'Particle Physics' section of your data sheet is really helpful in this topic. Usually, when considering quark composition, the first step is to think about the overall charge of the particle. It is really helpful to try and memorise the charge on each quark:

$$u = +\frac{2}{3}e$$

$$d = -\frac{1}{3}e$$

$$s = -\frac{1}{3}e$$

Then, anti-particles are simply the opposite charge. This will speed up your ability to identify potential candidates for quark compositions. It is also super important to remember that the strange quark has a strangeness S = –1 which is probably the opposite to what you'd expect (hence the anti-strange quark has a strangeness of $S = +1$).



(4 marks)

(c) Deduce a possible quark composition for a neutral pion and explain your reasoning.

Answer

State the quark composition for a neutral pion:

- $u\bar{u}$ **OR** $d\bar{d}$(allow $s\bar{s}$) [1 mark]

Explain the quark composition:

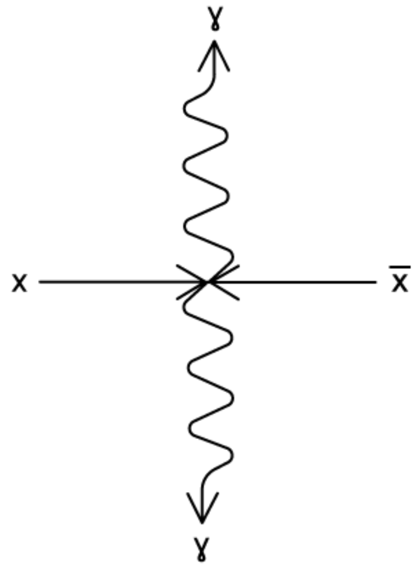
- This is the only combination of quarks which satisfies both **S** = 0 and **q** = 0; [1 mark]

[Total: 2 marks]

(2 marks)

4 (a) Annihilation is a type of particle interaction during which a colliding particle x and anti-particle $\bar{x}$ convert their masses into pure energy. This energy is radiated by at least two photons γ .

The Feynman diagram for such a process is shown in **Figure 1**



Explain why at least two photons result from annihilation.

Answer

Identify the appropriate conservation law:

- Momentum is conserved in all particle interactions; [1 mark]

Explain how momentum is conserved in this particle interaction:

- For two colliding particles, the total momentum is zero; [1 mark]

- At least two photons <u>travelling in opposite directions</u> is required to conserve momentum after annihilation; [1 mark]

[Total: 3 marks]

In this topic, while it's natural to focus on the newly learned conserved properties like lepton and baryon number, and charge, the specification reminds us that we should appreciate energy and momentum is **always** conserved in any particle interaction.

Examiners like to interrogate this on occasion, sometimes asking what 'other' quantities are conserved in some interaction. Always think, energy and momentum! Try to remember too that momentum is conserved provided **no external forces act on the system**: a cool fact is that the two photons radiate away at very *nearly* 180° to each other, but not exactly. This is because, in reality, a perfectly isolated system is hard to attain (i.e. a very faint external resultant force acts on the system).

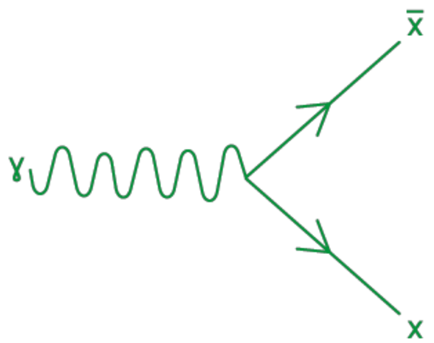
(3 marks)

(b) Pair production, the opposite process to annihilation, is the creation of a particle and anti-particle pair from the energy carried by a photon.

Sketch a Feynman diagram that represents pair production from a single photon.

Answer

A Feynman diagram that would score 3 marks is shown below:



- Correct symbol (γ) for photon shown; [1 mark]
- Particle and anti-particle bar correctly indicated; [1 mark]
- Correct directions of particle and anti-particle pair indicated (outgoing); [1 mark]

[Total: 3 marks]

For the second mark, you could label the correct symbols for quarks, or other particles, as long as the corresponding anti–particle is correctly labelled to gain the mark.

(3 marks)

(c) State and explain whether the photon in part (b) can be classified as a virtual photon.

Answer

Describe the nature of virtual photons:

- Virtual photons are not detectable; [1 mark]

Describe the nature of the photon in part (b):

- The photon in part (b) is not a virtual photon / is detectable; [1 mark]

- Pair creation is observed in the laboratory / real photons are responsible for pair production;

  OR

  Pair production is not an electromagnetic interaction; [1 mark]

[Total: 3 marks]

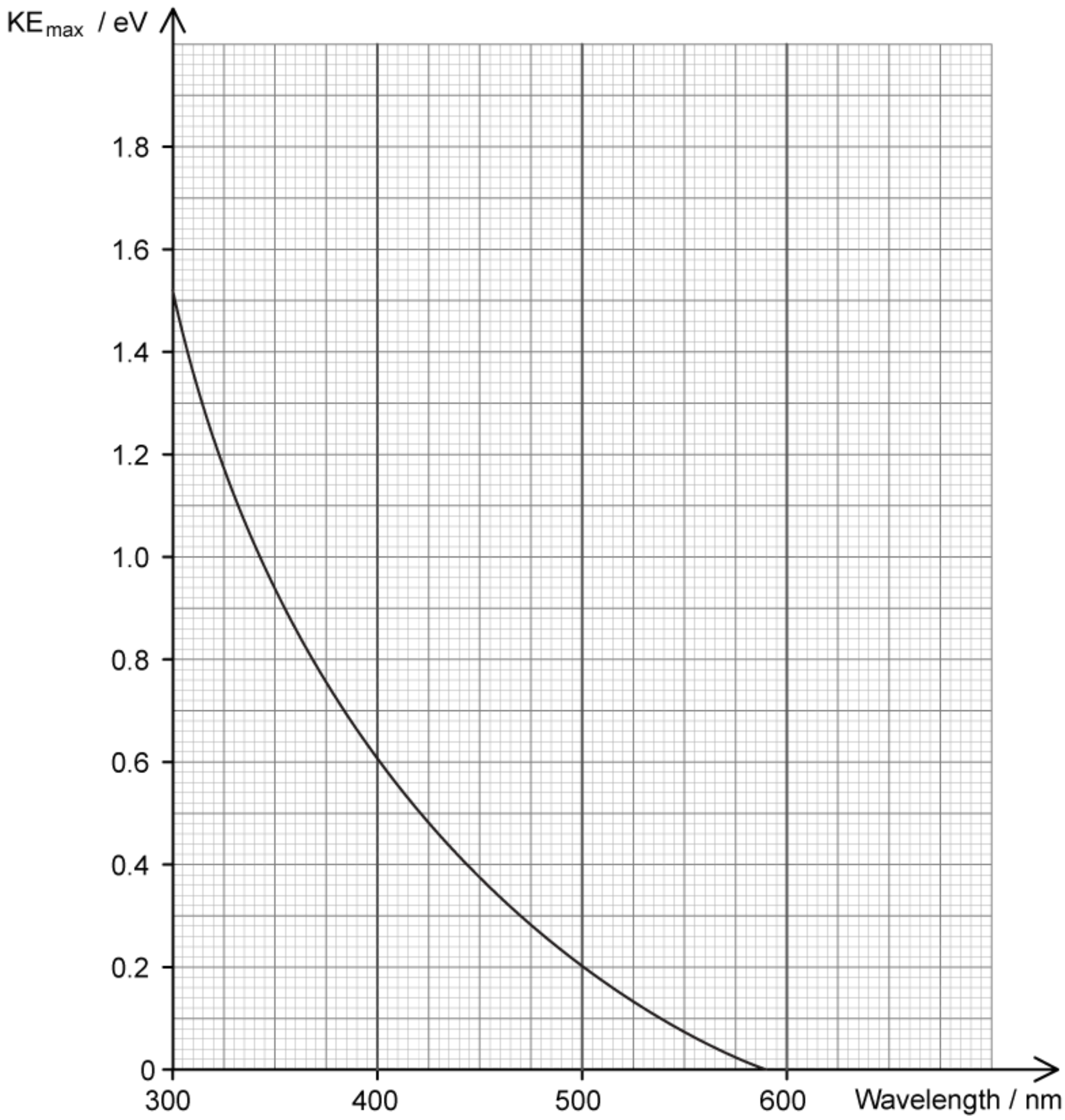
Pair production is a direct conversion of radiant energy into matter. It is Einstein's equation $E = mc^2$ in action. As such, the photon is a quantum of pure energy, which is measurable. Two neutrally charged particles can be created by pair production.

Virtual photons, on the other hand, are exchange particles which mediate the electromagnetic force. As such, virtual photons are exchanged when charged particles (and leptons) interact.

(3 marks)

5 (a) The maximum kinetic energy, KE_{max} of photoelectrons varies with the wavelength of incident light on a metal surface. This is shown in **Figure 1** below:

Figure 1



Use **Figure 1** to show that the work function of the metal is 2.1 eV.

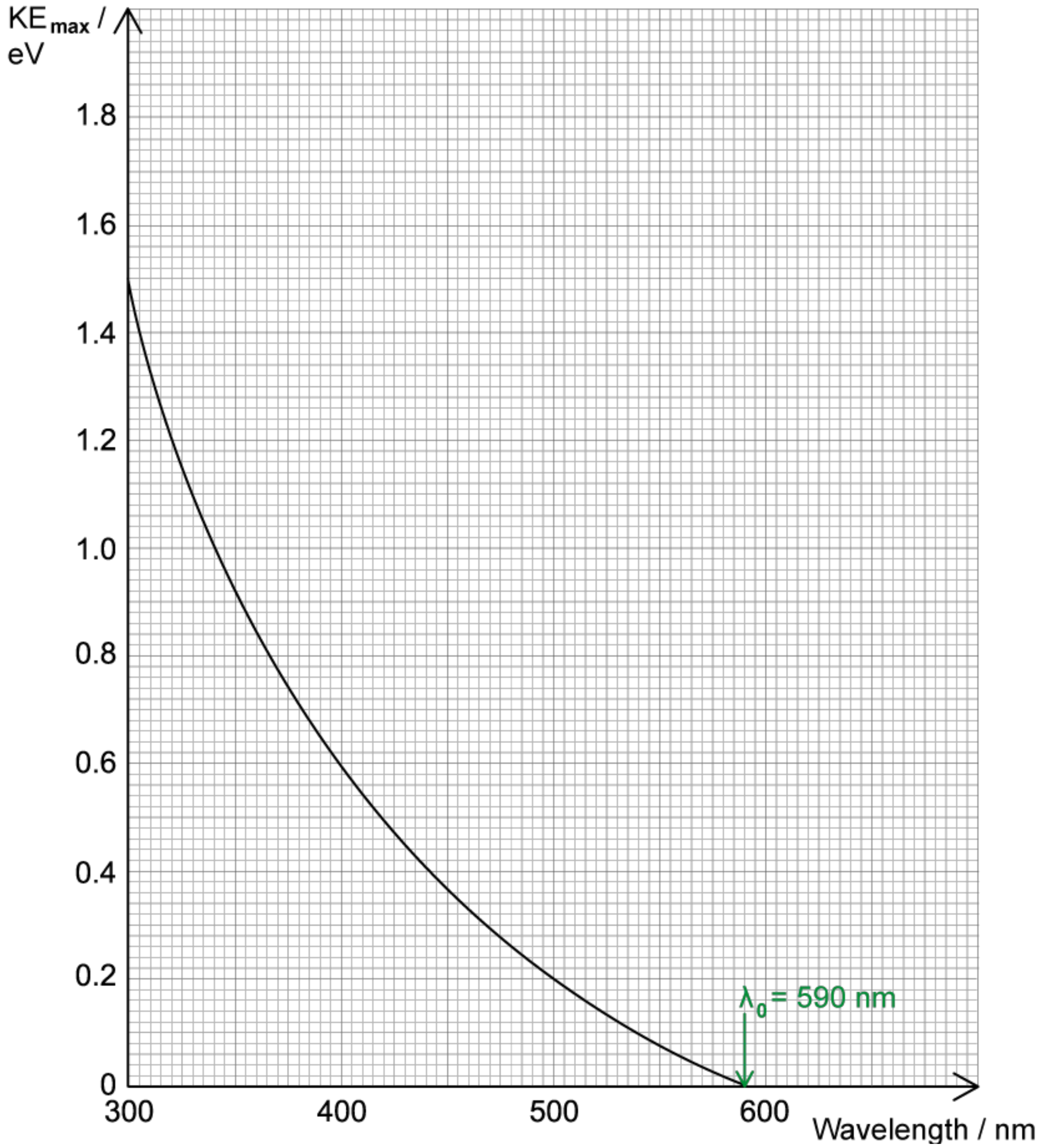
Answer

From data sheet:

- Speed of light in a vacuum, $c = 3 \times 10^8$ m s^{-1}
- Planck's constant, $h = 6.63 \times 10^{-34}$ J s

Use **Figure 1** to find the wavelength when KE_{max} = 0:

- Photoelectrons are only released to the surface if light is incident at the threshold frequency (or threshold wavelength,λ_0)
- λ_0= 590 nm when the KE_{max} = 0 from **Figure 1**; [1 mark]



Calculate the work function using the threshold wavelength, λ_0 :

- $hf = \phi + KE_{max}$

- When KE_{max} = 0, the work function $\phi = hf_0$
- $\phi = hf_0 = h \times \dfrac{c}{\lambda_0}$ = (6.63 × 10^{-34}) × $\left(\dfrac{3 \times 10^8}{590 \times 10^{-9}}\right)$ = 3.4 × 10^{-19} J [1 mark]

Calculate the work function in electronvolts, ϕ:

- 1 eV = 1.6 × 10^{-19} J
- ϕ =(3.4 × 10^{-19}) ÷ (1.6 × 10^{-19}) = 2.1 eV [1 mark]

[Total: 3 marks]

Remember that the wave function, ϕ is the energy needed for photoelectrons to reach the surface of a metal. This is unique to each metal. The extra kinetic energy is only for electrons which are further near the surface of the metal which have enough energy to reach the surface **and** leave the surface altogether. The extra energy left to leave the surface is the kinetic energy.

(3 marks)

(b) A student uses the shape of the curve in **Figure 1** to state that maximum kinetic energy is inversely proportional to the incident wavelength.

By referring to the photoelectricity equation, discuss the validity of this statement.

Answer

Rearrange the photoelectricity equation to make $E_{k\ max}$ the subject:

- The photoelectricity equation states $hf = \Phi + E_{k\ max}$
- Since **Figure 1** puts $E_{k\ max}$ on the y-axis, the photoelectricity equation should be rearranged to $E_{k\ max} = hf - \Phi$ [1 mark]

Write the photoelectricity equation in terms of wavelength:

- In terms of wavelength, this is $E_{k\ max} = \dfrac{hc}{\lambda} - \Phi$ [1 mark]

Discuss the validity of the student's statement by explaining the relationship between $E_{k\ max}$ and λ:

- The student's statement is invalid / not valid; [1 mark]
- Inverse proportionality means $E_{k\ max} \times \lambda$ = constant which is not the case here

  OR

  The work function ϕ is a term which cannot be ignored in the relationship;[1 mark]

[Total: 4 marks]

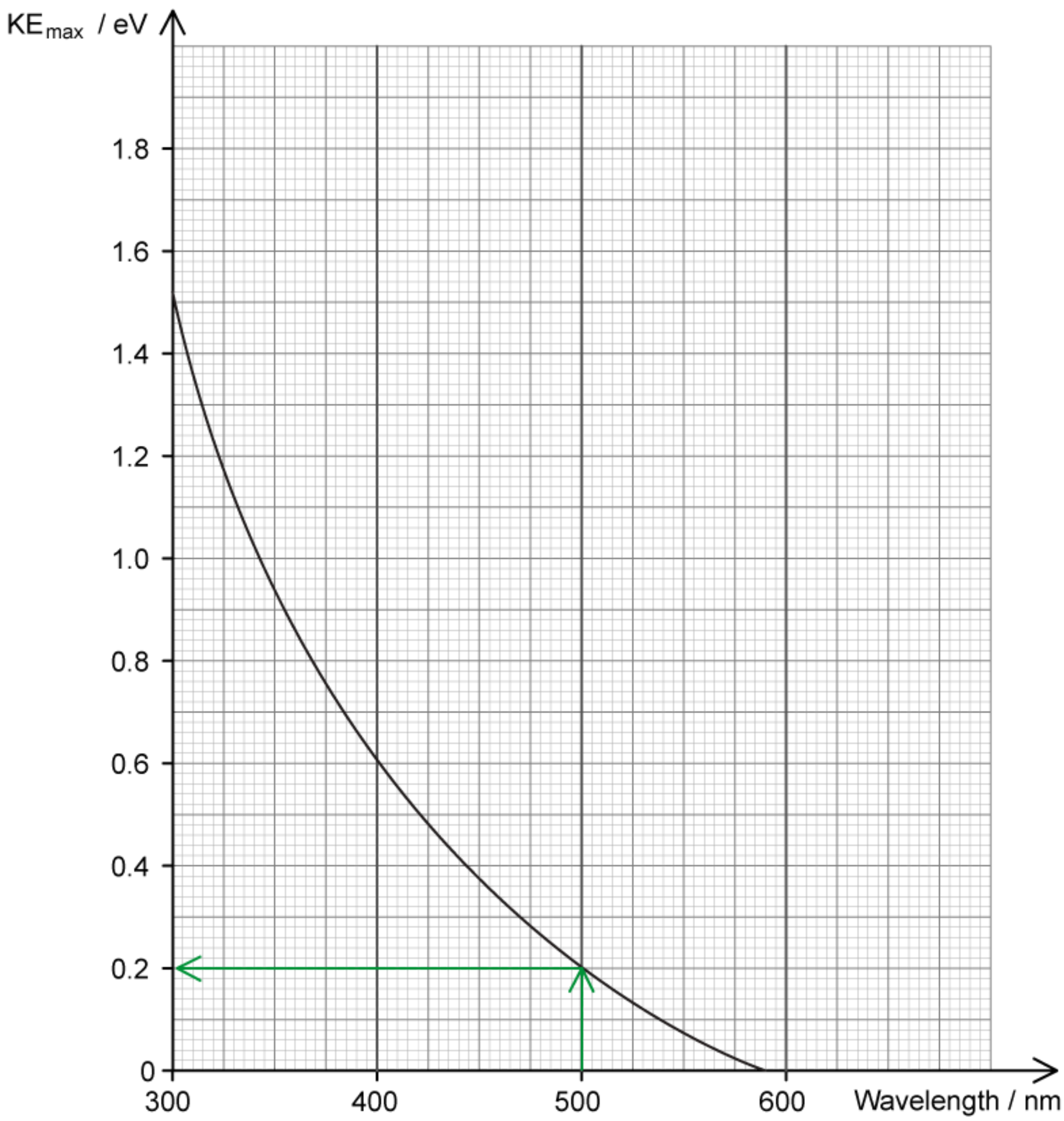
The equation of the incident photons is $E = hf = \frac{hc}{\lambda}$, which means photons with a shorter wavelength have more energy to give to the electrons in the metal. This is why the graph in **Figure 1** shows electrons have more kinetic energy when the incident light has a shorter wavelength.

(4 marks)

(c) Calculate the de Broglie wavelength of photoelectrons emitted from the metal surface for incident light of wavelength 500 nm.

Answer

Use **Figure 1** to determine $E_{k\ max}$ of photoelectrons emitted for the incident light:



- If the wavelength of incident light is 500 nm then the $E_{k\ max}$ of photoelectrons is 0.2 eV; [1 mark]

Convert the from eV to J:

- $E_{k\ max}$ = 0.2 eV = 0.2 × 1.6 × 10^{-19} = 3.2 × 10^{-20} J [1 mark]

Calculate the velocity of the emitted photoelectrons:

- $E_{k\ max} = \frac{1}{2} m \left(v_{max}\right)^2$

- $3.2 \times 10^{-20} = \frac{1}{2} \times (9.11 \times 10^{-31}) \times \left(v_{max}\right)^2$
- $6.4 \times 10^{-20} = (9.11 \times 10^{-31}) \times \left(v_{max}\right)^2$
- $\left(v_{max}\right)^2 = 7.03 \times 10^{10}$
- $v_{max} = \sqrt{7.03 \times 10^{10}} = 2.7 \times 10^5$ m s^{-1} [1 mark]

Calculate the de Broglie wavelength of the emitted photoelectrons:

- $\lambda = \frac{h}{mv}$
- $\lambda = \frac{6.63 \times 10^{-34}}{(9.11 \times 10^{-31}) \times (2.7 \times 10^5)} = 2.7 \times 10^{-9}$ m [1 mark]

[Total: 4 marks]

(4 marks)

(d) Discuss the experimental changes that would reduce the minimum de Broglie wavelength of the emitted photoelectrons.

Answer

State how to reduce the minimum de Broglie wavelength:

- The <u>maximum</u> velocity/momentum of photoelectrons must be <u>increased</u>; [1 mark]
- An increase in maximum velocity is achieved by an increase in $E_{k\ max}$; [1 mark]
  - This is shown by the equation $E_{k\ max} = \frac{1}{2} m \left(v_{max}\right)^2$

Suggest experimental changes to increase the $E_{k\ max}$ of emitted photoelectrons:

- $E_{k\ max}$ of emitted photoelectrons can be increased by using a metal with a smaller work function energy <u>for the same incident light source</u>; [1 mark]

- $E_{k\ max}$ of emitted photoelectrons can also be increased by increasing the frequency/decreasing the wavelength of the light source, for the same metal surface; [1 mark]

[Total: 4 marks]

Remember to be specific about what you need to change about the light source. Avoid saying 'increase the energy of the light source' because you cannot do that directly, this can only be achieved through changing the frequency or wavelength (since they are related by $E = hf = \frac{hc}{\lambda}$)

(4 marks)

6 (a) An experiment to investigate how the de Broglie wavelength λ of an electron varies with its velocity v is carried out. The results from this experiment are shown in **Table 1**.

Table 1

v / 10^4 km s^{-1}	λ / 10^{-11} m
1.4	4.9
2.5	2.8
3.5	2.1

Discuss whether the data in **Table 1** is consistent with the de Broglie equation:

$$\lambda = \frac{h}{mv}$$

Answer

State the proportionality between wavelength and velocity:

- The de Broglie equation suggests that the wavelength λ is inversely proportional to the electron velocity v; [1 mark]
- If the wavelength is inversely proportional to velocity, then λv = constant, or $\lambda_1 v_1 = \lambda_2 v_2$

Calculate the constant of proportionality:

- From the first row of **Table 1**, $\lambda v = (4.9 \times 10^{-11}) \times (1.4 \times 10^4 \times 10^3) = 6.9 \times 10^{-4}$
- From the second row of **Table 1**, $\lambda v = (2.8 \times 10^{-11}) \times (2.5 \times 10^4 \times 10^3) = 7.0 \times 10^{-4}$
- From the third row of **Table 1**, $\lambda v = (2.1 \times 10^{-11}) \times (3.5 \times 10^4 \times 10^3) = 7.4 \times 10^{-4}$
- All values correctly calculated; [1 mark]

Conclusion:

- The results support the de Broglie equation because the constant of proportionality between pairs of values for λ and v is approximately equal (to 1 significant figure); [1 mark]

[Total: 3 marks]

Demonstrating whether results from a table (or graph) are consistent with an equation are a common hard question in A-level Physics that is a similar method every time.

- If two variables are **proportional** to each other e.g. $a \propto b$, then the fraction $\frac{a}{b}$ = constant

- You must then find the value of $\frac{a}{b}$ for each value in the table, or graph, and see if the values are approximately equal (even just to 1 decimal place)

- If two variables are **inversely proportional** to each other e.g. $a \propto \frac{1}{b}$, then ab = constant

- You must then find the value of ab for each value in the table, or graph, and see if the values are approximately equal (even just to 1 decimal place)

In this particular question, because the powers are the same for all values of v and the same for values of λ, you can emit the powers from your calculation to save time.

(3 marks)

(b) Using the data in **Table 1** and the accepted value for Planck's constant, comment on the accuracy of the results.

Answer

From the data sheet:

- Planck's constant $h = 6.63 \times 10^{-34}$ J s

- Electron rest mass, $m = 9.11 \times 10^{-31}$ kg

Method 1: Calculate the mean constant of proportionality from part (a)

- From part (a), λv = constant
- The mean constant from the data is $\frac{(6.9+7.0+7.4)\times 10^{-4}}{3}$ = 7.1 × 10^{-4} [1 mark]

Use the mean constant λv to calculate a value for Planck's constant:

- Using de Broglie's equation, $h = m \times \lambda v$
- h = (9.11 × 10^{-31}) × (7.1 × 10^{-4}) = 6.47 × 10^{-34}J s (3 s .f) [1 mark]

Calculate the percentage difference to the accepted value for Planck's constant:

- The accepted value for Planck's constant is 6.63 × 10^{-34} J s
- The percentage difference Δ% = $\frac{(6.63-6.47)\times 10^{-34}}{6.63\times 10^{-34}}$ × 100 = 2% [1 mark]

Comment on the accuracy of the experiment:

- The percentage difference between the value of h suggested by the experiment and the accepted value is small **AND** so the experimental results are accurate [1 mark]

OR

Method 2: Calculate Planck's constant for each pair of values of λ and v

- Using de Broglie's equation, $h = m \times \lambda v$
- The constant from the pair of values in the first row of **Table 1**, λv = 6.9 × 10^{-4}
  - h = (9.11 × 10^{-31}) × (6.9 × 10^{-4}) = 6.29 × 10^{-34} (J s)
- The constant from the pair of values in the second row of **Table 1**, λv = 7.0 × 10^{-4}
  - h = (9.11 × 10^{-31}) × (7.0 × 10^{-4}) = 6.38 × 10^{-34} (J s)
- The constant from the pair of values in the third row of **Table 1**, λv = 7.4 × 10^{-4} so
  - h = (9.11 × 10^{-31}) × (7.4 × 10^{-4}) = 6.74 × 10^{-34} (J s)

- All values correctly calculated; [1 mark]

Calculate the mean value of Planck's constant from the data:

- The mean value of Planck's constant is $\frac{(6.29+6.38+6.74)\times10^{-34}}{3}$ = 6.47 × 10^{-34} (J s) [1 mark]

Calculate the percentage difference to the accepted value for Planck's constant:

- The accepted value for Planck's constant is 6.63 × 10^{-34} J s

- The percentage difference Δ% = $\frac{(6.63-6.47)\times10^{-34}}{6.63\times10^{-34}}$ × 100 = 2% [1 mark]

Comment on the accuracy of the experiment:

- The percentage difference between the value of *h* suggested by the experiment and the accepted value is small **AND** so the experimental results are accurate; [1 mark]

[Total: 4 marks]

Experimental accuracy is a measure of how **close** measurements are to the 'true' or 'accepted' value of a quantity. Anything less than 5% percentage difference between an accepted value and one deduced from experimental results – for A Level at least! – can be considered as reasonably accurate.

(4 marks)

(c) The experiment required electrons to travel at very high speeds. In order to achieve this, they were first accelerated through a very high voltage, *V*.

Show that the de Broglie wavelength *λ* of these electrons is related to the accelerating voltage *V* by the expression:

$$\lambda \propto \frac{1}{\sqrt{V}}$$

Answer

Consider energy transfers for the electron:

- The energy transferred (or work done) W on a particle with charge q as it moves across a potential difference V is given by $W = q\Delta V$

- For an electron, $q = e$ so the energy transferred by a potential difference is given by eV

- The energy is transferred is to the electron's kinetic energy, KE = $\frac{1}{2}mv^2$

- Therefore, $eV = \frac{1}{2}mv^2$ [1 mark]

Substitute the de Broglie wavelength:

- de Broglie wavelength $\lambda = \frac{h}{mv}$, therefore $v = \frac{h}{\lambda m}$

- Therefore, $eV = \frac{1}{2}m\left(\frac{h}{\lambda m}\right)^2 = \frac{1}{2}m\left(\frac{h^2}{\lambda^2 m^2}\right) = \frac{h^2}{2m\lambda^2}$ [1 mark]

Deduce the proportionality between λ and V:

- $eV = \frac{h^2}{2m\lambda^2}$

- $\lambda^2 = \frac{h^2}{2meV}$

- $\lambda = h\sqrt{\frac{1}{2meV}}$ therefore $\lambda \propto \frac{1}{\sqrt{V}}$ [1 mark]

[Total: 3 marks]

(3 marks)

7 (a) When electromagnetic waves are reflected from a shiny surface, such as a road sign, they often become polarised.

Suggest how to determine experimentally if visible light reflected from a road sign is polarised.

Answer

State the use of a polaroid filter:

- Hold a polaroid filter up to the reflected light; [1 mark]

Describe the effect of rotating the polaroid filter:

- Rotating the polaroid filter will determine if the light is polarised; [1 mark]
- If the light is polarised (along a certain plane) it will be blocked / the polaroid filter will not transmit it at some angle; [1 mark]

[Total: 3 marks]

Generally speaking, it's really good practice to specify angles of rotation in polaroid filter questions. For example, if the intensity of light transmitted behind two polaroid filters is 100% when the two filters are aligned at an angle θ, then the light intensity will fall to 0% when this angle is ($\theta + 90^\circ$) and then back up to 100% at ($\theta + 180^\circ$) and back to 0% at ($\theta + 270^\circ$) and then finally back to 100% at ($\theta + 360^\circ$) – yes really, this much detail! You will usually secure full credit for specific descriptions through a full rotation.

However, in this question, it is sufficient to refer to 'some angle θ' because you are only required to determine if the light is polarised or not. A full description of the effects on the intensity of transmitted light through a full rotation is not required.

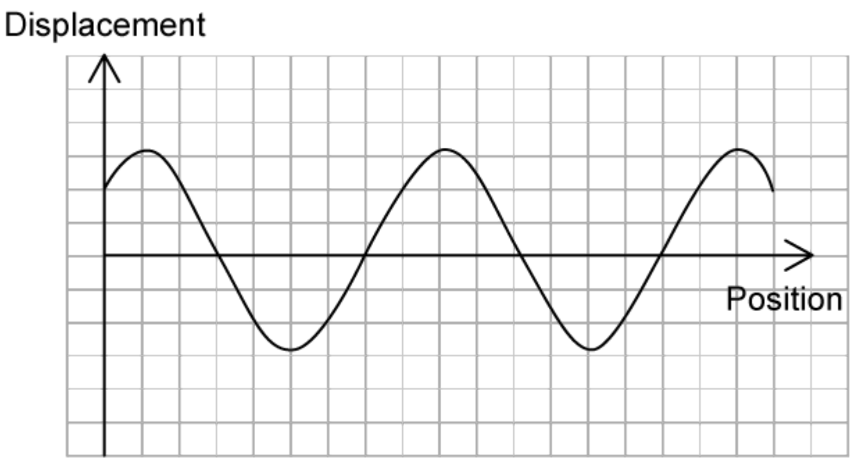
(3 marks)

(b) Changes in phase can also occur when electromagnetic waves are reflected from a surface.

If an electromagnetic wave is reflected at the boundary between a medium with a higher refractive index than the medium it is travelling in, the oscillating electric field undergoes a phase change of π radians.

Light is incident on an air-water boundary. A displacement-position sketch of the amplitude of the incident electric field is shown in **Figure 1**. The origin represents the boundary.

Figure 1

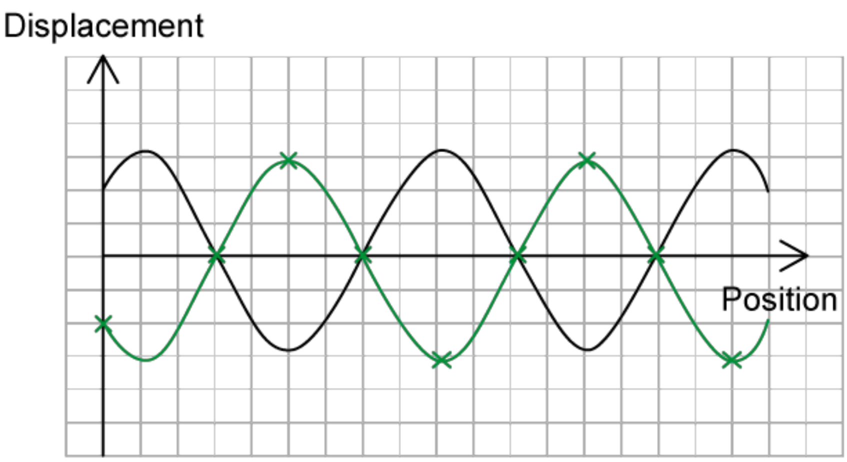


On **Figure 1**, sketch the amplitude of the reflected electric field.

Answer

Sketch the reflected electric field amplitude as a reflection in the position-axis:

- Since the reflection causes a phase change of π radians, the displacement of the reflected electric field will be exactly opposite to the displacement of the incident field



- Correct initial displacement of –2 units; [1 mark]

- Correct shape at all positions; [1 mark]

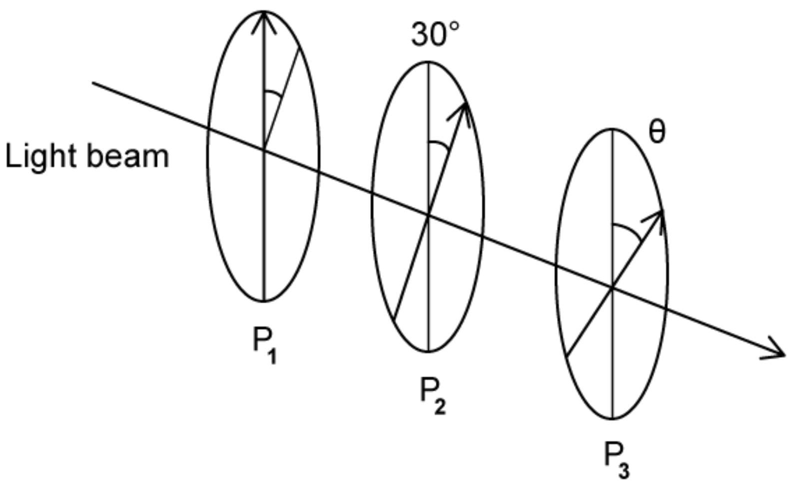
[Total: 2 marks]

(2 marks)

(c) Three polaroid filters P_1, P_2 and P_3 are aligned as shown in **Figure 2**.

Unpolarised light is incident on P_1 and subsequently passes through each of the three polaroid filters. P_1 and P_2 are fixed, but P_3 can be rotated to any angle θ to that of P_1.

Figure 2



Determine the angles of θ at which minima and maxima of emergent light intensity occur.

Answer

State the angles of θ with respect to the first polaroid filter P_1:

- Polarised light, parallel to the orientation of P_2, is incident on P_3
- This means that, when $\theta = 30^\circ$ emergent light is at maximum intensity (P_3 is parallel to the alignment of P_2); [1 mark]

Determine the angles of minimum and maximum intensity:

- $\theta = (30^\circ + 90^\circ) = 120^\circ$ is the first angle of minimum intensity; [1 mark]
- $\theta = (30^\circ + 180^\circ) = 210^\circ$ is the first angle of maximum intensity

AND

$\theta = (30^{\circ} + 270^{\circ}) = 300^{\circ}$ is the second angle of minimum intensity; [1 mark]

[Total: 3 marks]

(3 marks)

(d) Complete **Table 1**, by stating whether the waves listed are polarised or unpolarised, and giving a reason for your answer.

Table 1

Wave	**Polarised or Unpolarised**	**Reason**
Light from the sun		
Compression waves caused by an earthquake	Unpolarised	Longitudinal waves cannot be polarised
Electromagnetic waves emitted from a dipole aerial		
Ultra-sonic waves from an echo sounder		

Answer

Identify the properties of each wave and how it is generated:

Wave	Polarised or Unpolarised	Reason
Light from the sun	**Unpolarised**	**Light in the sun is generated randomly / oscillates in all planes**
Compression waves caused by an earthquake	Unpolarised	Longitudinal waves cannot be polarised
Electromagnetic waves emitted from a dipole aerial	**Polarised**	**Electromagnetic waves are generated by ordered oscillations of charged particles / oscillate parallel to the plane of the aerial**
Ultra-sonic waves from an echo sounder	**Unpolarised**	**Sound waves cannot be polarised**

- All waves correctly identified as polarised or unpolarised; [1 mark]
- At least one correct reason; [1 mark]
- All reasons correct; [1 mark]

[Total: 3 marks]

(3 marks)

8 (a) **Figure 1** shows an experiment designed by a student to investigate vibrations in a stretched nylon string of fixed length *l*.

The student measures how the frequency *f* of the first harmonic varies with the mass *m* suspended from the string.

Figure 1

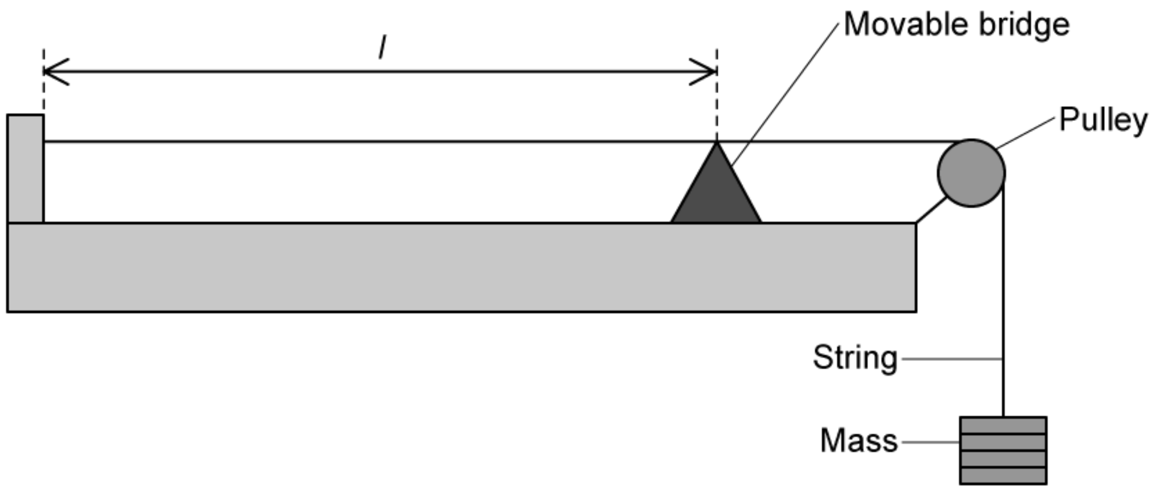


Table 1 shows the results they record from the experiment:

Table 1

m / g	*f* / Hz
500	110
800	140
1200	170

With reference to an appropriate proportionality, discuss the validity of the results in **Table 1**.

Answer

Refer to the relationship between the frequency of the first harmonic *f* and the tension *T*:

- The equation for the
- ovided by the suspended mass *m*

  AND

  Hence, $T = mg$ [1 mark]

Determine if the results are consistent with the equation for the first harmonic:

- If $f \propto \sqrt{T}$ this means $f \propto \sqrt{mg}$
- Therefore, (if the results are valid) $f = k \times \sqrt{m}$ [1 mark]
- Calculating the constant of proportionality $k = \frac{f}{\sqrt{m}}$ for each pair of readings gives

  $k_1 = \frac{110}{\sqrt{500}}$ = 4.9, $k_2 = \frac{140}{\sqrt{800}}$ = 4.9, and $k_3 = \frac{170}{\sqrt{1200}}$ = 4.9 [1 mark]

Comment on the validity of the results:

- The results are consistent with the equation for the frequency of the first harmonic / $f \propto \sqrt{T}$ <u>therefore the results are valid</u>; [1 mark]

[Total: 5 marks]

(5 marks)

(b) The student notes that the string has a uniform diameter of 4.0×10^{-4} m.

The fixed length *l* of the vibrating string was also measured to be 0.80 m.

Determine the density of the nylon string.

Answer

List known values:

- Mass of the load, m = 500 g = 0.5 kg

- Frequency at this mass, f = 110 Hz
- Length of string, l = 80 m
- Diameter of the string, $d = 4.0 \times 10^{-4}$ m

From the data sheet:

- Frequency of the first harmonic $f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

Use f and m to calculate the mass per unit length μ of the nylon string:

- Substitute a pair of values from **Table 1** into the equation for the first harmonic of the string
- $f = \frac{1}{2L}\sqrt{\frac{T}{\mu}} = \frac{1}{2L}\sqrt{\frac{mg}{\mu}}$
- $110 = \frac{1}{2 \times 0.80} \times \sqrt{\frac{0.5 \times 9.81}{\mu}}$ [1 mark]
- $176 = \sqrt{\frac{0.5 \times 9.81}{\mu}}$
- $30976 = \frac{0.5 \times 9.81}{30976}$
- $\mu = \frac{0.5 \times 9.81}{30976} = 1.6 \times 10^{-4}$ kg m^{-1} [1 mark]

Calculate the cross-sectional area A of the nylon string:

- $A = \pi \times \left(\frac{d}{2}\right)^2 = \pi \times \left(\frac{4.0 \times 10^{-4}}{2}\right)^2 = 1.3 \times 10^{-7}$ m^2 [1 mark]

Calculate the density of the nylon string:

- $\rho = \frac{\mu}{A} = \frac{1.6 \times 10^{-4}}{1.3 \times 10^{-7}}$ = 1300 kg m^{-3} (2 s .f.) [1 mark]

[Total: 4 marks]

(4 marks)

(c) The student realises that a graph of frequency *f* against $\sqrt{T}$ would be a straight line, where *T* is the tension in the string, if she used her readings in **Table 1**.

Discuss how the graph would look for tensions that are much larger than those used by the student in **Table 1**.

Answer

Comment on the diameter of the string:

- For tensions that are much larger than those used in **Table 1**, the diameter of the string would decrease; [1 mark]

Explain the effects on the mass per unit length for a reduced diameter:

- Since the diameter is smaller, the mass per unit of length is lower; [1 mark]

Explain the effects on the graph for a lower mass per unit length:

- Therefore, the frequencies for the first harmonic would be higher (than those predicted by the straight line); [1 mark]

- Hence the graph would curve upwards; [1 mark]

[Total: 4 marks]

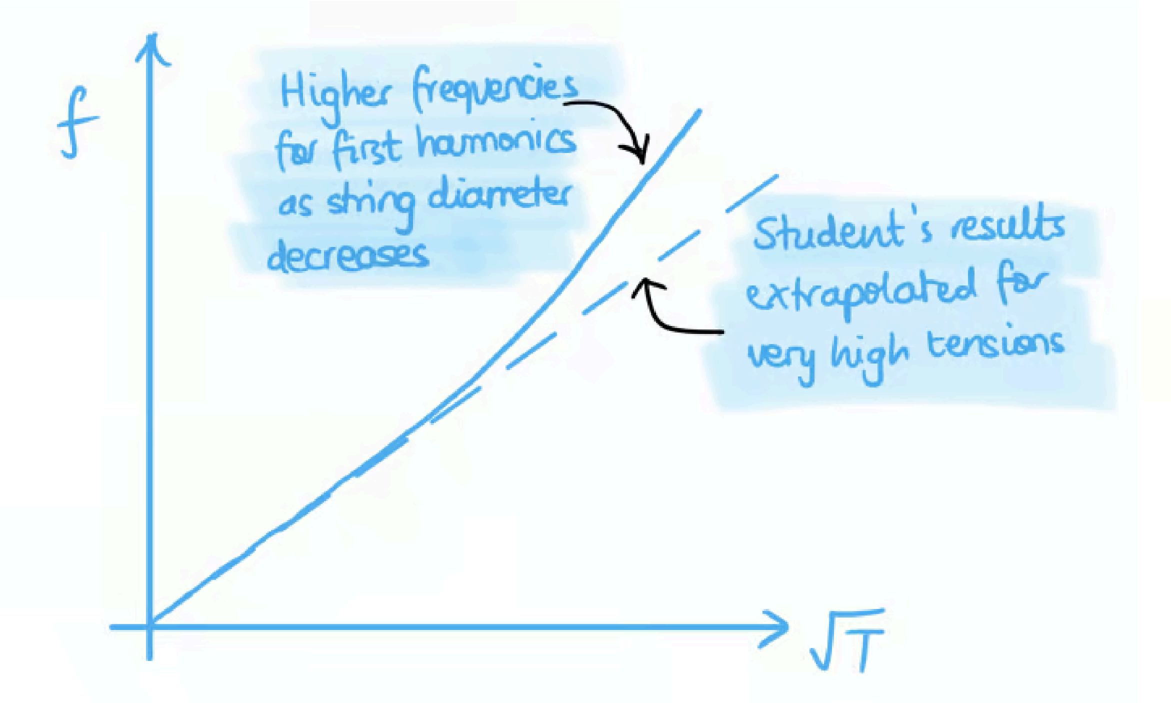
The proportionality between the frequencies of the first harmonics and the mass per unit of length is shown below:

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

$$f \propto \frac{1}{\sqrt{\mu}} \text{ where } \mu = \rho A$$

Since the diameter is proportional to the cross sectional area *A*, then a smaller *d* means a smaller *A*, and hence a smaller μ which in turn means a higher frequency *f*!

The student's results imply that *f* is proportional to $\sqrt{T}$ – which it is, *if* all things remain constant. But if the string's diameter decreases – which it will do for extremely large tensions, then the frequency for the first harmonic at a given tension will be much higher. This is shown below:



(4 marks)

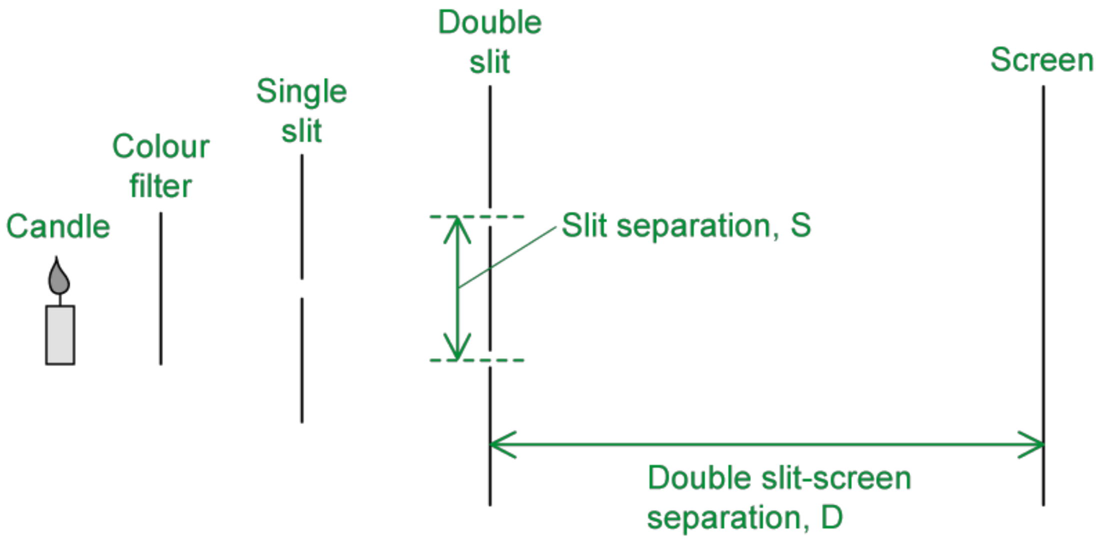
9 (a) A student designs an experiment to replicate Young's double slit demonstration.

The student opts to use a candle as a light source, with a piece of coloured filter paper to produce monochromatic light. They then consider additional apparatus required in order to observe an interference pattern.

Sketch a diagram, labelling all **apparatus** as well as any important **quantities**, to show the setup the student should use to produce and observe a diffraction pattern.

Answer

A possible sketch that would gain 3 marks is shown below:



- Candle, colour filter, single slit, double slit and screen; [1 mark]
- Slit separation *s* labelled as the distance between the centre of the slits; [1 mark]
- Distance between the double slit and the screen, *D*; [1 mark]

[Total: 3 marks]

In case you're wondering why the interference fringes are not included in this sketch, pay particular attention to the wording of the question. It asks you to sketch the *setup*, including important quantities, rather than the results or specifically the interference fringes that would be observed.

(3 marks)

(b) The student labels the two slits on the double-slit grating slit **X** and slit **Y**.

The student then paints over slit **X**, such that the intensity of light emerging from it is 50% of that emerging from slit **Y**.

Discuss the effects, if any, this tweak will have on the student's observations.

Answer

State the existence of an interference pattern:

- There will still be an interference pattern / bright and dark fringes observed (on the screen); [1 mark]

State the effect on the fringes:

- Dark fringes increase in intensity **OR** dark lines become even more dark / black; [1 mark]

- Bright fringes decrease in intensity **OR** bright lines become less bright; [1 mark]

Explain the observations:

- Even though overall intensity is reduced, light is still emerging from both slits

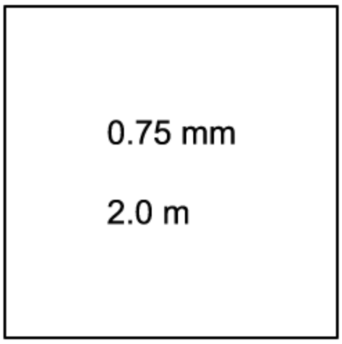
- Less intensity means less brightness / more darkness; [1 mark]

[Total: 4 marks]

(4 marks)

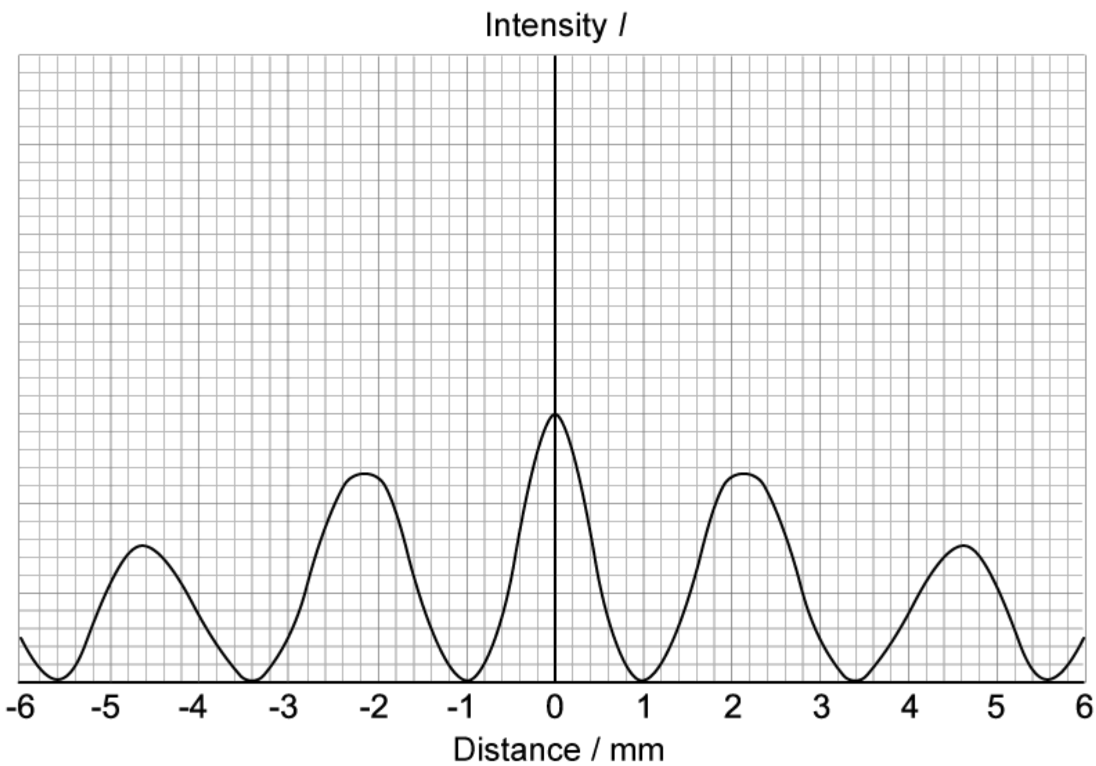
(c) The student finishes setting up their apparatus and makes a quick note of two separate measurements in a lab book.

Figure 1a



These are shown in **Figure 1a**. They then plot a graph of the intensity of light against the distance from the centre of the screen, represented by the origin. This is shown in **Figure 1b.**

Figure 1b



Using the information in **Figure 1a** and **Figure 1b**, determine which colour of filter paper the student most likely chose for this experiment.

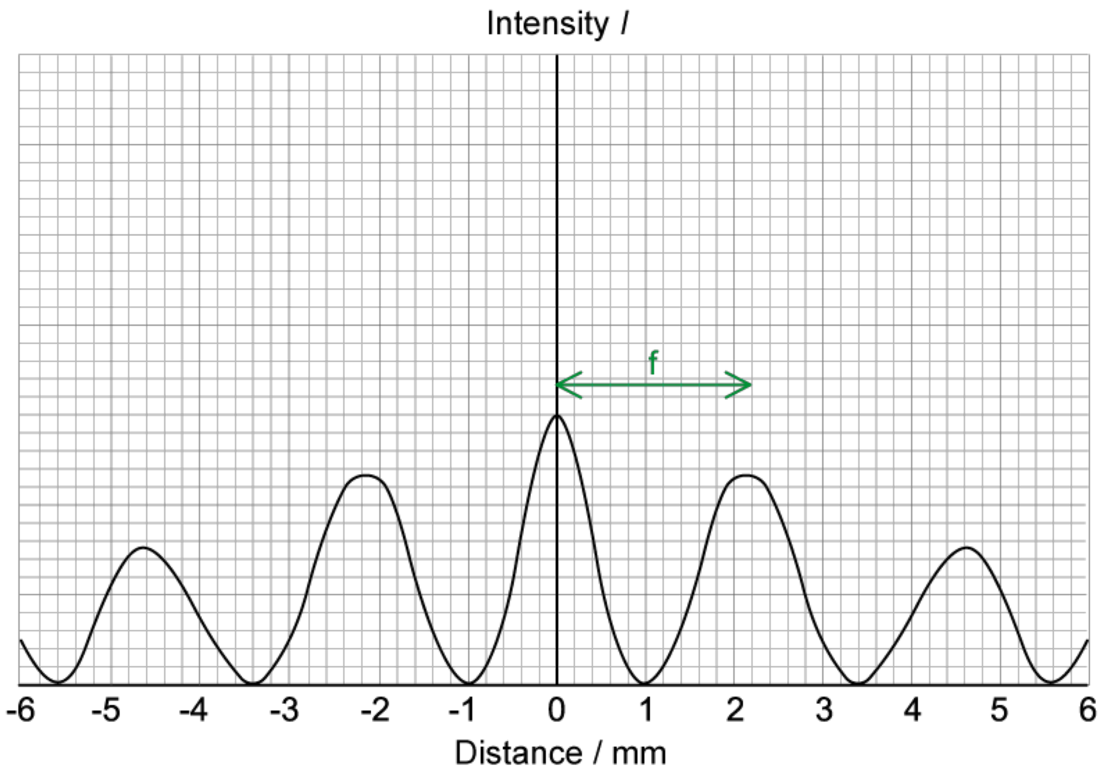
Answer

Interpret the information in **Figure 1a**:

- The two quantities in the equation for fringe spacing are the distance between the slits *s* and the distance between the double-slit and the screen, *D*
- *s* is a much smaller length than *D*, *s* << D
- Therefore *s* = 0.75 mm **AND** *D* = 2.0 m [1 mark]

Calculate the fringe spacing *f* using **Figure 1b**:

- The fringe spacing is determined as the distance between the centre of each maxima



- Hence, the fringe spacing f = 2.2 mm = 2.2 × 10^{-3} m [1 mark]
  - *The acceptable range is 2 to 2.2 mm*

Whenever you have multiple fringes, always **measure across as many as possible** and **divide by the number of fringes** to get the most accurate value

Calculate the wavelength λ of light incident on the screen:

- $W = \frac{\lambda D}{s}$
- $2.2 \times 10^{-3} = \frac{\lambda \times 2.0}{0.75 \times 10^{-3}}$
- $\lambda = \frac{(2.2 \times 10^{-3}) \times (0.75 \times 10^{-3})}{2.0}$ = 8.3×10^{-7} m [1 mark]

Determine the most likely colour corresponding to this wavelength:

- Visible light lies in the range 400 nm – 900 nm
- Red light is nearer the longer wavelength end, around 900 nm
- Since the wavelength of light incident on the screen is about 830 nm, the coloured filter paper was likely red; [1 mark]

[Total: 4 marks]

Full marks will be awarded for an answer calculated correctly using an fringe spacing within the acceptable range.

(4 marks)

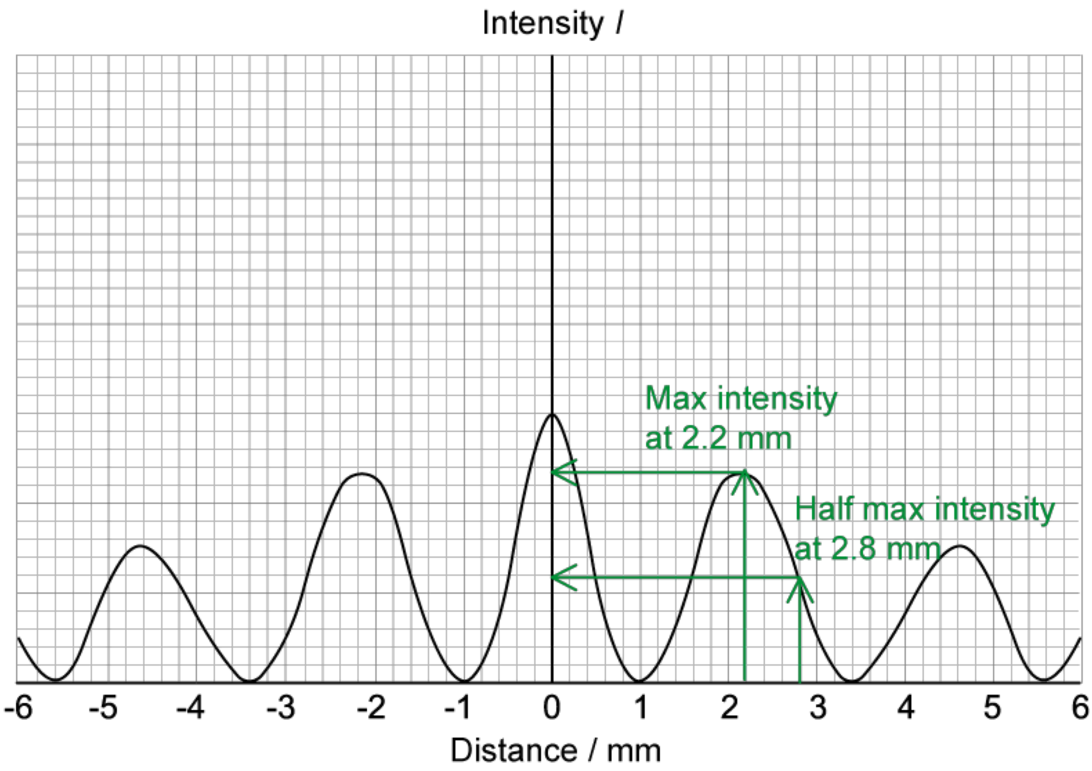
(d) Determine the phase difference between the waves meeting at the point that is 2.8 mm from the centre of the screen.

Show how you arrive at your answer.

Answer

Use **Figure 1b** to determine the relative intensity of light at a point 2.8 mm from the centre:

- At 2.8 mm away from the screen, the intensity falls to 50% of the centre of the bright fringe



- Lines drawn at 2.2 and 2.8 mm to show 50% drop in intensity; [1 mark]

Determine the phase difference between waves at this point:

- Since the intensity is halfway between maximum and minimum, the phase difference must be halfway between 0° and 180° / 0 or 2π radians and π radians; [1 mark]

- Therefore the phase difference between the waves arriving at this point is 90° or $\frac{1}{2}$ π radians; [1 mark]

[Total: 3 marks]

(3 marks)

10 (a) A narrow beam of coherent light of wavelength 460 nm is incident upon a diffraction grating that has 2.80×10^2 lines per mm. The diffraction pattern is seen on a screen.

Calculate the total number of minima seen on the screen.

Answer

List the known quantities:

- Wavelength, λ = 460 nm = 460×10^{-9} m
- $N = 2.80 \times 10^2$ lines per mm = 2.80×10^5 lines per m

From the data sheet:

- Diffraction grating, $d \sin \theta = n\lambda$

Calculate adjacent spacing between slits, *d*:

- $d = \frac{1}{N}$
- $d = \frac{1}{2.80 \times 10^5}$ = 3.57×10^{-6} m [1 mark]

Calculate the highest order possible:

- The highest order would be at θ = 90° **OR** $\sin \theta = 1$
- $d \sin 90° = n\lambda \quad \Rightarrow \quad d = n\lambda$
- $n = \frac{d}{\lambda} = \frac{3.57 \times 10^{-6}}{460 \times 10^{-9}} = 7.76$
- Therefore, the highest visible order is the seventh (n = 7) [1 mark]

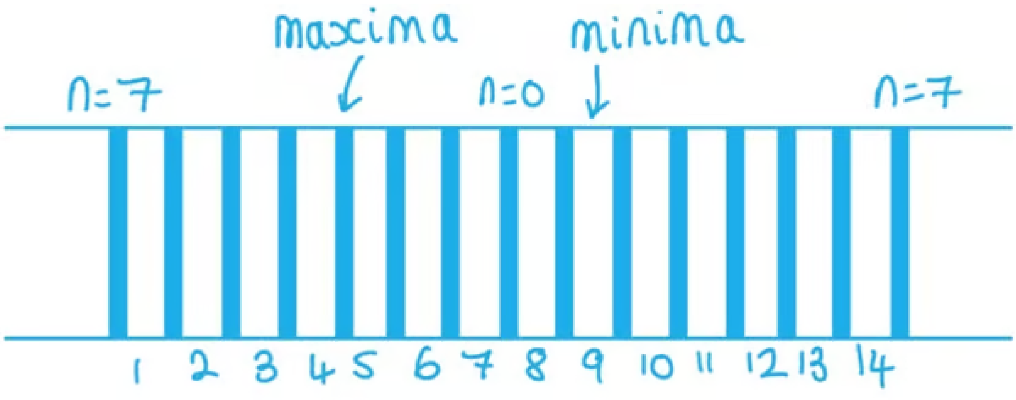
Determine the total number of minima:

- Total minima = (2 × 7) = 14 [1 mark]

Allow 16 only if correct statement explaining the minima either side of the seventh orders have been counted

[Total: 3 marks]

Remember *n* is the order of the **maxima (bright fringes)**, and the minima are the dark fringes in between these.



(3 marks)

(b) A different light source is incident upon the same diffraction grating. A student suspects there are two wavelengths of light in this incident beam, one of wavelength 570 nm and another of wavelength 570.4 nm.

Determine the order of diffracted light where the two wavelengths are most distinguishable.

Answer

List the known quantities:

- $d = 3.57 \times 10^{-6}$ m (from part a)
- Wavelength, λ = 570 nm = 570×10^{-9} m

Explain where the different wavelengths of light are most distinguishable:

- They are most likely to be distinguishable at the highest order; [1 mark]

Calculate highest visible order:

- The highest order would be at $\theta = 90°$ **OR** $\sin\theta = 1$

- $n = \frac{d}{\lambda} = \frac{3.57 \times 10^{-6}}{570 \times 10^{-9}} = 6.26$ [1 mark]

- The highest visible order is the sixth (n = 6)

Conclusion:

- The two wavelengths are most distinguishable at n = 6 (the sixth order); [1 mark]

[Total: 3 marks]

Remember to always round **down** for the value of your highest order, since n indicates the number of visible maxima. For example, n = 4.8 means there will be at most 9 visible maxima (one central maxima, and four maxima on either side of this). Note, we cannot round up to 5, because there are not 5 maxima visibly created.

(3 marks)

(c) The minimum angular separation for which the two wavelengths may be differentiated is 0.12°.

Deduce whether the student observed the two wavelengths as distinct images at this order. Support your answer with calculations.

Answer

List the known quantities:

- Wavelength, λ_1 = 570 nm = 570 × 10^{-9} m

- Wavelength, λ_2 = 570.4 nm = 570.4 × 10^{-9} m

- Slit spacing, d = 3.57 × 10^{-6} m (from part a)

- Highest order visible n = 6 (from part b)

Calculate angle for the highest order visible for the first wavelength, θ_1:

- $d \sin \theta = n\lambda$

- $\theta = \sin^{-1}\left(\frac{n\lambda}{d}\right)$

- $\theta_1 = \sin^{-1}\left(\frac{6 \times (570 \times 10^{-9})}{3.57 \times 10^{-6}}\right)$ = 73.332°

Calculate the angle the highest order visible for the second wavelength, θ_2 :

- $\theta_2 = \sin^{-1}\left(\frac{6 \times (570.4 \times 10^{-9})}{3.57 \times 10^{-6}}\right)$ = 73.467°

Calculate the difference in angles:

- $\Delta\theta = \theta_2 - \theta_1$ = 73.467° – 73.332° = 0.135° [1 mark]

Conclusion

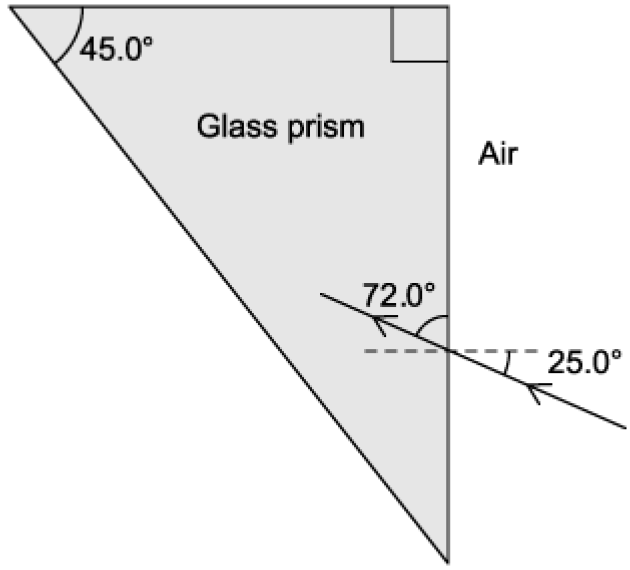
- Since 0.135° > 0.12° the student can observe the two wavelengths as separate, distinct images; [1 mark]

[Total: 2 marks]

(2 marks)

11 (a) A ray of light passes from air into a glass prism as shown in **Figure 1**.

Figure 1



As the light ray passes through the prism, it emerges back into the air.

Calculate the critical angle from the glass to the air.

Answer

List the known quantities:

- Angle of incidence, θ_1 = 25.0°
- Angle of refraction, θ_2 = 90° – 72.0° = 18.0°
- Refractive index of air, n_1 = 1

Calculate the refractive index of the glass, n_2:

- Snell's Law: $n_1 sin(\theta_1) = n_2 sin(\theta_2)$
- $n_2 = \dfrac{n_1 \times sin(\theta_1)}{sin(\theta_2)} = \dfrac{1 \times sin(25.0)}{sin(18.0)}$ [1 mark]

- n_2 = 1.37 [1 mark]

Calculate the critical angle, θ_c:

- $sin(\theta_c) = \frac{n_2}{n_1} = \frac{1}{1.37}$
- $\theta_c = \sin^{-1}\left(\frac{1}{1.37}\right)$ = 46.9° (3 s. f.) [1 mark]

[Total: 3 marks]

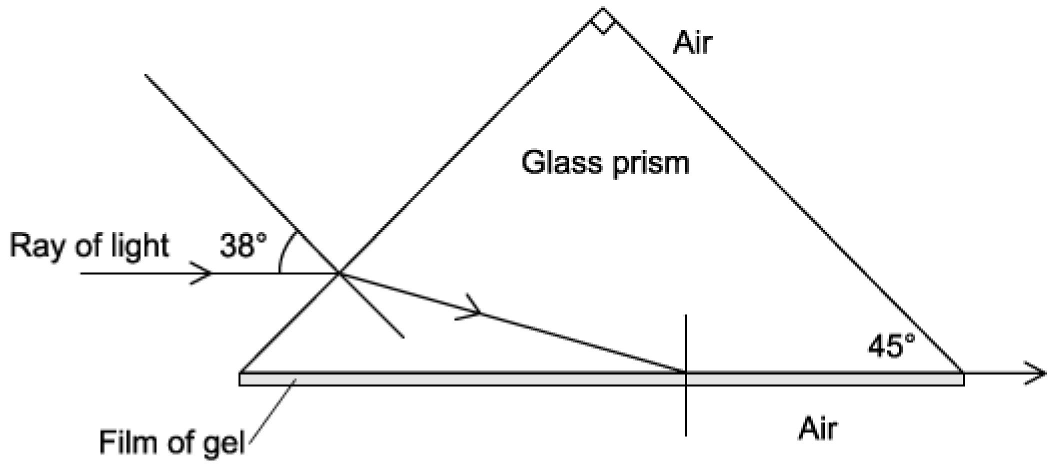
Remember that the angle of incidence and angle of refraction is always the angle from the **normal** to the light ray.

Using the relation $sin\theta_c = \frac{sin\theta_1}{sin\theta_2}$ is a quicker method than the one indicated above but it requires some thought regarding getting the angles of incidence and refraction the correct way around for the glass–air boundary (they will be reversed compared to the air–glass boundary).

(3 marks)

(b) **Figure 2** now shows the prism rotated and one side is coated with a film of transparent gel. A ray of light strikes the prism, at an angle of incidence of 38°, and continues through the glass to strike the glass–gel boundary at the critical angle.

Figure 2



Calculate the refractive index of the gel.

Answer

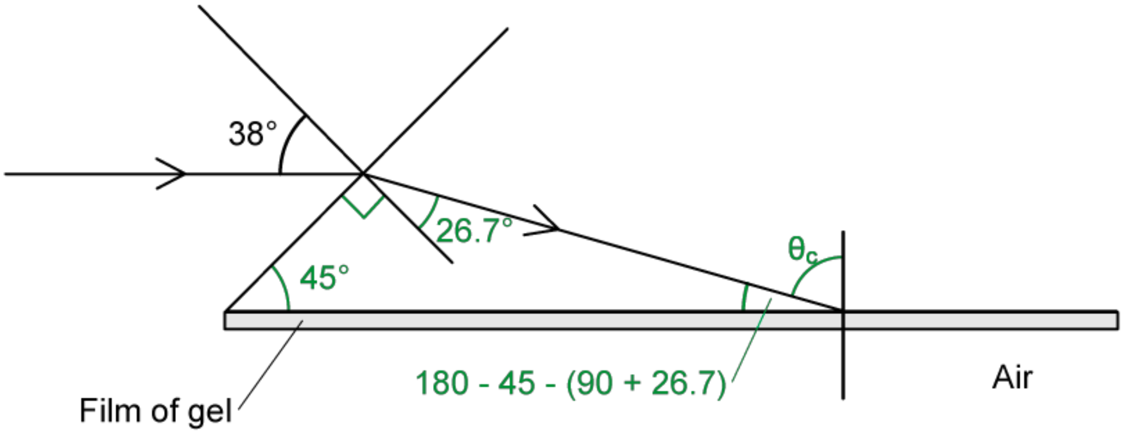
List the known quantities:

- Angle of incidence at air–glass boundary, θ_1 = 38°
- Refractive index of the glass, n_{glass} = 1.37 (from part a)
- Refractive index of air, n_{air} = 1

Calculate the angle of refraction at the air–glass boundary, θ_2:

- Snell's Law: $n_1 sin(\theta_1) = n_2 sin(\theta_2)$
- $sin(\theta_2) = \frac{n_{air} \times sin(\theta_1)}{n_{glass}}$
- $\theta_2 = sin^{-1}\left(\frac{1 \times sin(38)}{1.37}\right)$ = 26.7° [1 mark]

Calculate the critical angle of the gel–glass boundary:



- 180 – 45 – (90 + 26.7) = 18.3°
- θ_c = 90 – 18.3 = 71.7° [1 mark]

Calculate the refractive index of the gel, n_{gel}:

- $sin(\theta_c) = \frac{n_2}{n_1} = \frac{n_{gel}}{n_{glass}}$
- $n_{gel} = sin(\theta_c) \times n_{glass} = sin(71.7) \times 1.37$ [1 mark]
- n_{gel} = 1.30 [1 mark]

[Total: 4 marks]

Always do a sanity check whenever you're asked to calculate the refractive index since they should always be greater than or equal to 1.

Remember to use the diagram given by labelling any missing angles.

(4 marks)

(c) A ray of light now strikes the prism at an angle of incidence which means that it now refracts straight through the gel at the glass–gel boundary.

Without calculation, explain how the critical angle for the glass–gel boundary differs from the critical angle for the gel–air boundary.

Answer

EITHER

- The critical angle for the glass–gel boundary is larger; [1 mark]
- There is a smaller difference between the refractive index of the glass and gel than there is between the gel and air; [1 mark]

Allow $\frac{1.30}{1.37} > \frac{1}{1.37}$ *or vice versa*

OR

- The critical angle for the gel–air boundary is smaller; [1 mark]
- There is a larger difference between the refractive index of the gel and air than there is between the glass and gel; [1 mark]

Allow $\frac{1}{1.37} < \frac{1.30}{1.37}$ *or vice versa*

[Total: 2 marks]

(2 marks)

12 (a) A plane flying across the Lake District sets off from base camp to lake Windermere, 28 km away, in a direction of 20.0° north of east.

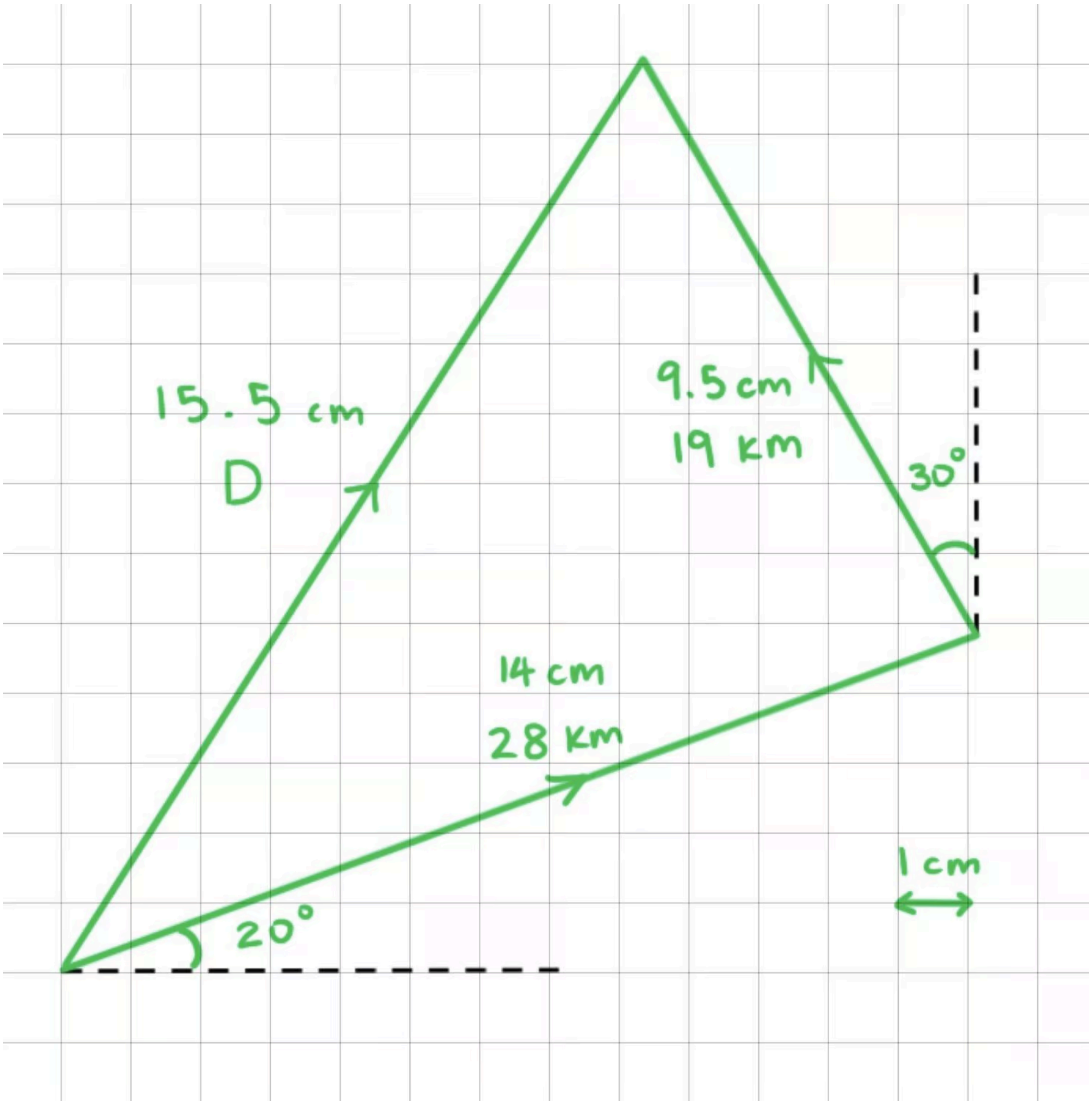
After dropping off supplies it flies to lake Coniston, which is 19 km at 30.0° west of north from lake Windermere.

Determine the distance from lake Coniston to base camp.

Answer

Method 1: Scale Drawing

An example of a scale diagram that would earn 2 marks:



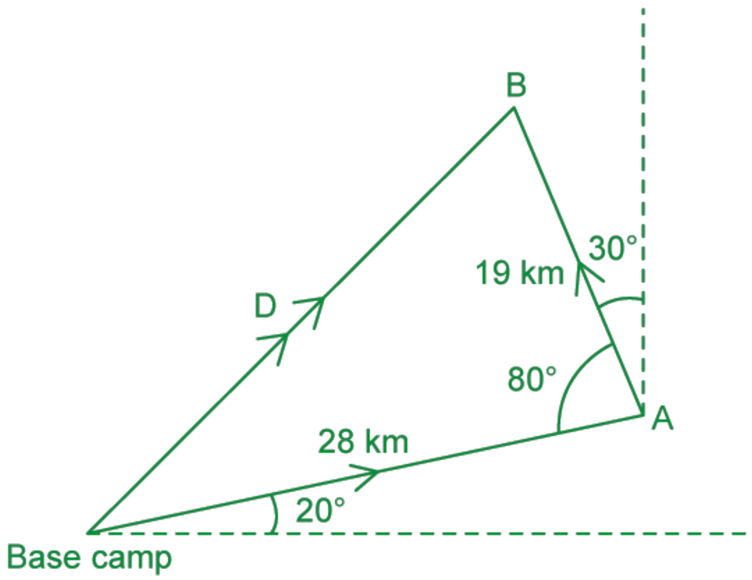
Scale 1 cm: 2km or otherwise suitable scale; [1 mark]

- Correct resultant direction with clear arrow; [1 mark]

Measurement of displacement, D:

- For a scale of 1 cm: 2km
- Acceptable range for measurement: 15.3 – 15.7 cm
- Acceptable range for displacement, D: 30.6 – 31.4 km [1 mark]

Method 2: Using the cosine rule



List the known quantities:

- a = D
- b = 28 km
- c = 19 km
- A = 60° + 20° = 80° [1 mark]

Use the cosine rule to calculate displacement, D:

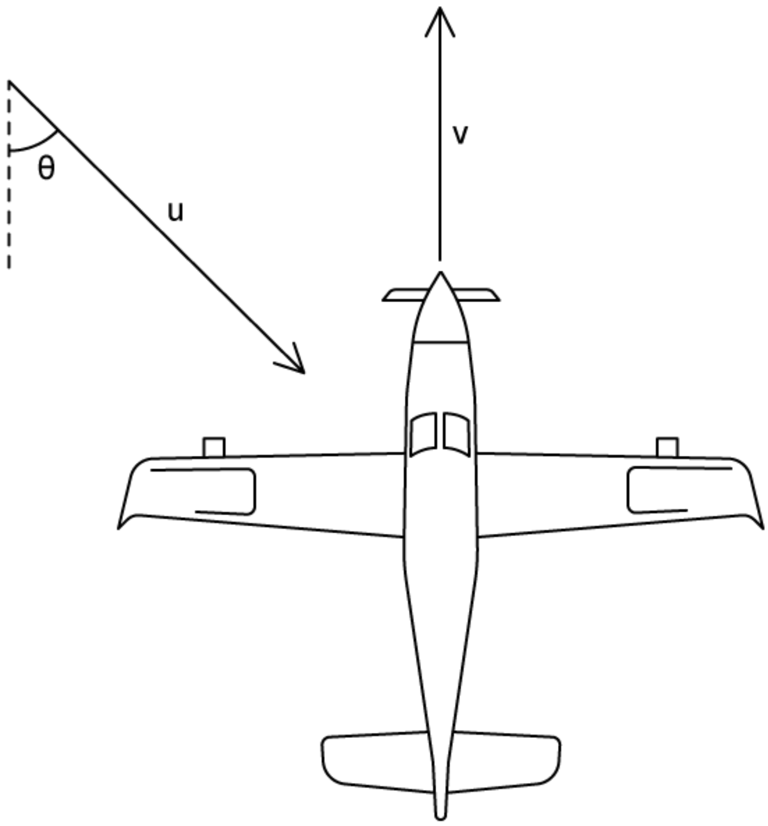
- $D^2 = 28^2 + 19^2 - 2(28)(19)\cos 80°$ [1 mark]
- $D = \sqrt{960.238} = 30.988 = 31$ km [1 mark]

[Total: 3 marks]

(3 marks)

(b) **Figure 1** shows the plane flying due north through still air with a speed *v*.

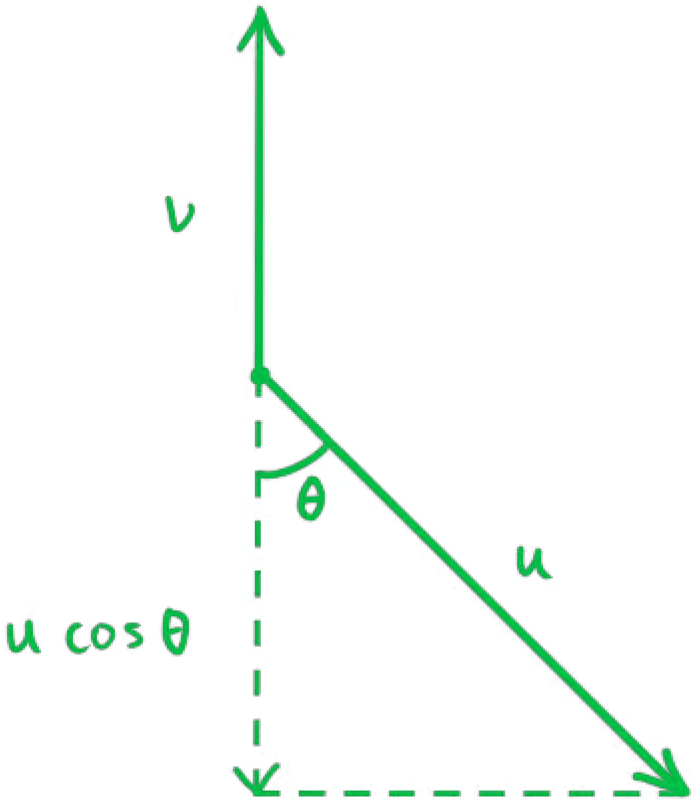
Figure 1



The plane enters a region where the wind is blowing with a speed u from a direction which makes an angle of θ with due south.

Write an expression for the time taken for the plane to fly a distance D due north of its current position in this windy region.

Answer



Resolve the velocity vertically:

- Resultant velocity $= v - u \cos\theta$ [1 mark]

Use the equation relating speed, distance and time to write an expression

- Speed $= \dfrac{distance}{time}$
- Time $= \dfrac{distance}{speed}$
- Time $= \dfrac{D}{v - u \cos\theta}$ [1 mark]

[Total: 2 marks]

Even though not expressly mentioned in the question to do so, drawing a vector triangle around the wind's velocity is a helpful way to identify the correct component in this case

(2 marks)

(c) When there is no wind, the aircraft travels 180 km every 30 minutes. When there is a wind blowing 53° south, the aircraft takes an extra 4 minutes to travel the same distance.

Assuming the orientation of the plane does not change, calculate the speed of the wind in km h^{-1}.

Answer

List the known quantities:

- $t = \dfrac{D}{v - u\cos\theta}$ (from part b)

Calculate the speed, v, when there is no wind:

- Aircraft speed $v = 2 \times 180 = 360$ km h^{-1} [1 mark]

Calculate the speed of the wind, u:

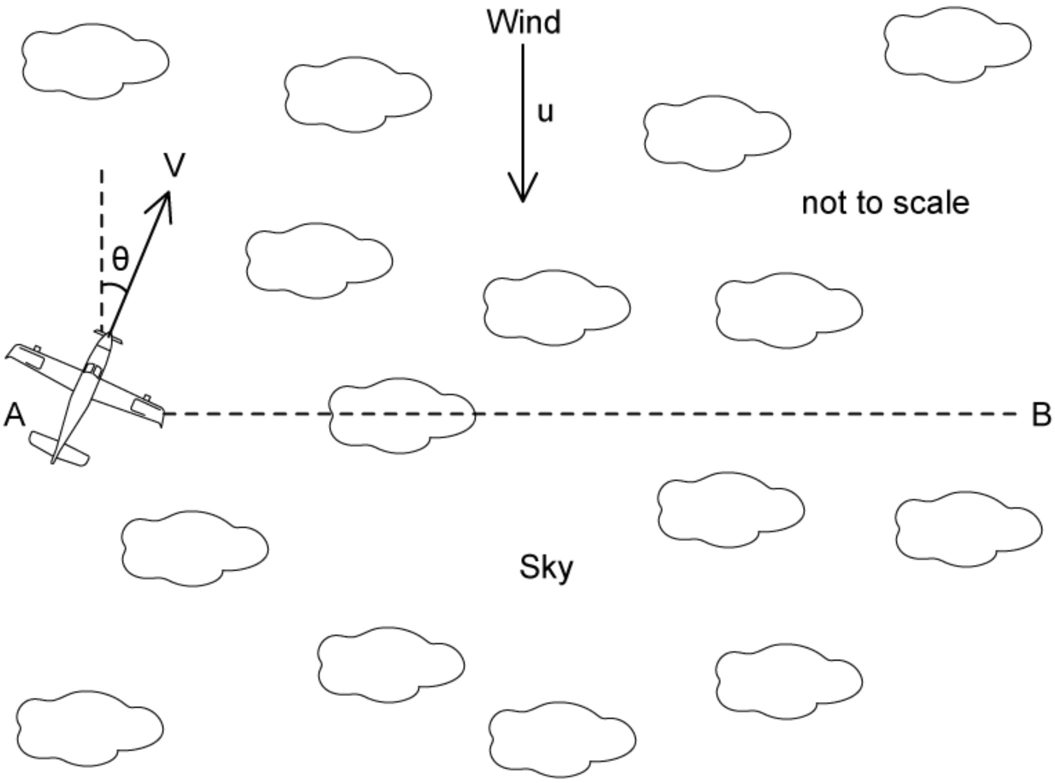
- Aircraft travels 180 km in 34 minutes
- $\dfrac{34}{60} = \dfrac{180}{360 - u\cos 53°}$ [1 mark]
- $360 - u\cos 53° = \dfrac{180 \times 60}{34}$
- $u = \dfrac{1}{\cos 53°}\left(360 - \dfrac{180 \times 60}{34}\right)$
- $u = 70.4 = 70$ km h^{-1} [1 mark]

[Total: 3 marks]

(3 marks)

(d) The pilot wishes to cross the sky in a straight line between two points labelled **A** and **B** as shown in **Figure 2**.

Figure 2



The wind is now blowing due south with the same speed as in part (c). The plane continues to travel at the same speed in the wind as in part (c).

To cross from **A** to **B** the pilot has to turn the plane at an angle θ to the direction of the wind, as shown in **Figure 2**.

Determine the size of the angle θ using a scale drawing.

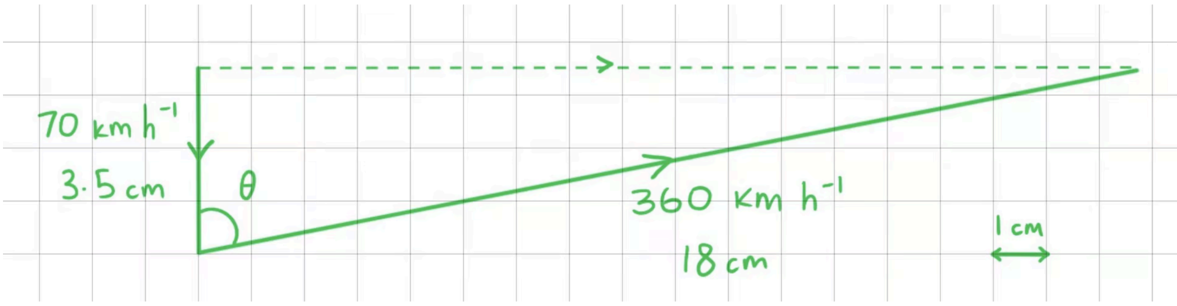
Answer

List the known quantities:

- Plane speed, v = 360 km h^{-1}
- Wind speed, u = 70 km h^{-1}

Method 1: Scale drawing

An example of a scale diagram that would earn 2 marks:

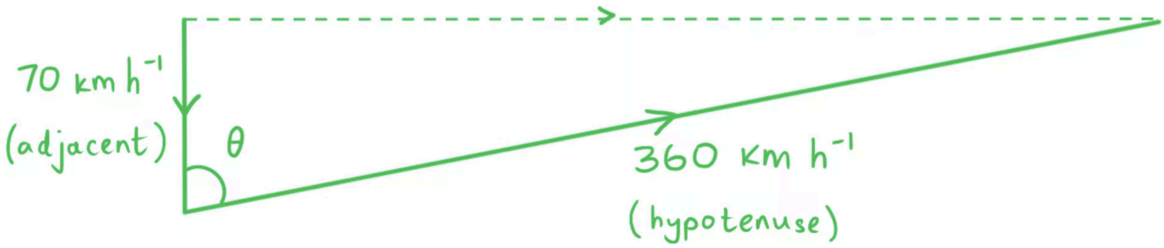


- Scale 1 cm: 20km h^{-1} or otherwise suitable scale; [1 mark]
- Correct labels with clear arrows; [1 mark]

Measurement of angle, θ:

- For a scale of 1 cm: 20 km h^{-1}
- Acceptable range for angle, θ: 78 – 80° [1 mark]

Method 2: Calculation (max 1 mark)



Determine θ using Pythagoras theorem:

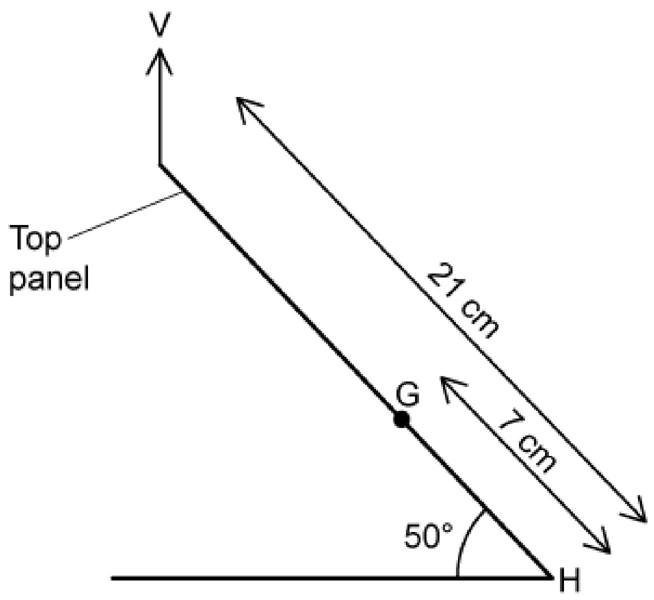
- $\cos\theta = \frac{adjacent}{hypotenuse} = \frac{70}{360}$
- $\theta = \cos^{-1}\left(\frac{70}{360}\right) = 78.788 = 79°$ [1 mark]

[Total: 3 marks]

(3 marks)

13 (a) **Figure 1** shows the opening of the top panel of a laptop. The panel is held open at an angle of 50º to the horizontal by a vertical force *V* applied at one end of the panel. The panel is 21 cm long, has a mass of 0.5 kg and its centre of gravity, **G** is 7 cm from the hinge at **H**.

Figure 1



Calculate the magnitude of the force acting at the hinge **H**.

Answer

Calculate the magnitude of the force acting at the hinge **H**:

List the known quantities:

- Angle, θ = 50º
- Length of panel = 21 cm = 0.21 m
- Centre of gravity distance from **H** = 7 cm = 0.07 m
- Mass of panel, m = 0.5 kg

Calculate the weight, W:

- $W = mg = 0.5 \times 9.81 = 4.905$ N

Calculate the vertical force *V*:

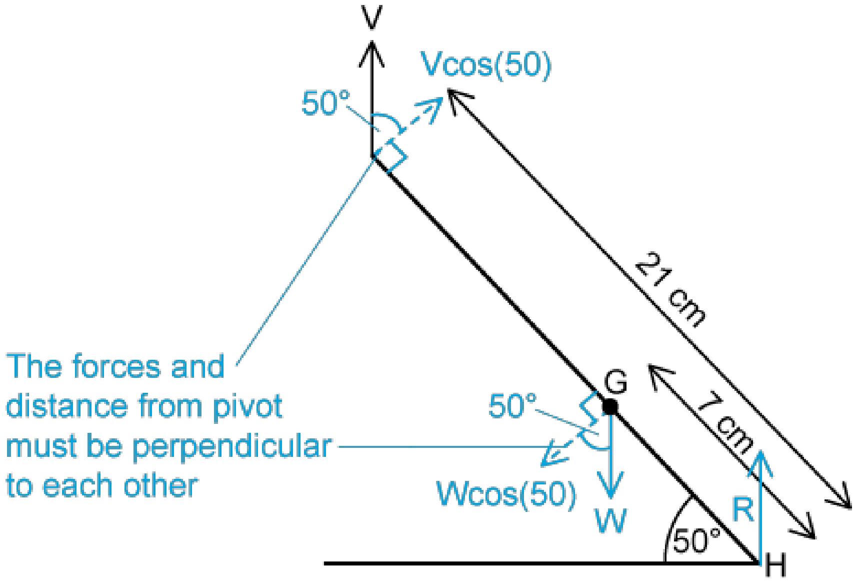
- Principle of moments: Sum of anti–clockwise moments = sum of clockwise moments about a pivot
- Clockwise moment about **H** = $V \cos(50) \times 0.21$
- Anti–clockwise moments about **H** = $W \cos(50) \times 0.07$
- $V \cos(50) \times 0.21 = 4.905 \cos(50) \times 0.07$ [1 mark]
- $V = \frac{4.905 \cos(50) \times 0.07}{0.21 \cos(50)} = 1.635$ N [1 mark]

Calculate the force acting at the hinge **H**, *R*:

- $V + R = W$
- $R = W - V = 4.905 - 1.635$
- $R = 3.3$ N (2 s .f) [1 mark]

[Total: 3 marks]

When calculating moments, the distance between the **line of action** of the force and the pivot must be **perpendicular**. Identifying this perpendicular distance may involve taking components of vectors. In this case, since the forces are vertical, we need the components of these forces that are perpendicular to the panel. Alternatively, you can calculate the perpendicular distance (instead of $V \cos(50) \times 0.21$ this would be $V \times 0.21\cos(50)$). This would give the same answer.



We do not take moments at **H** because **H** is the pivot therefore the moment would be $R \times 0 = 0$. However, the reaction force is required for the forces on the laptop to be in equilibrium. Make sure to look out for this reaction force for anything on a surface or against a wall!

Since cos(50) cancels out in the fraction when calculating V, you will get the same answer whether you take components or not, and be awarded a mark for your final answer of V. However, you will not be awarded a mark for the working out if this does not include the cos(50).

(3 marks)

(b) At an angle of 65°, in order to open the top panel of the laptop further, V must now act perpendicular to the top of the panel.

Calculate the magnitude of the new force acting at the hinge **H**.

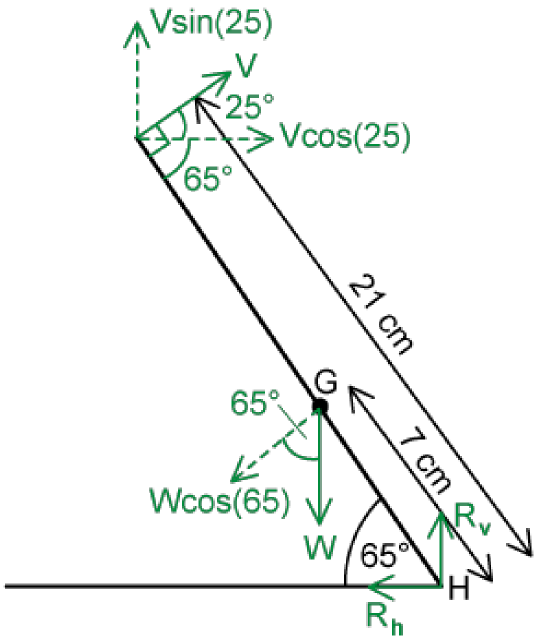
Answer

Calculate the magnitude of the new force acting at the hinge **H**:

List the known quantities:

- Angle, θ = 65°

- Length of panel = 21 cm = 0.21 m
- Centre of gravity distance from **H** = 7 cm = 0.07 m
- Weight, *W* = 4.905 N (from part a)



Calculate the perpendicular force, *V*:

- Principle of moments: Sum of anti–clockwise moments = sum of clockwise moments about a pivot
- Clockwise moment about **H** = *V* × 0.21
- Anti–clockwise moment about **H**: *W* cos (65) × 0.07
- *V* × 0.21 = 4.905 cos (65)× 0.07 [1 mark]
- $V = \frac{4.905\cos(65)\times 0.07}{0.21}$ = 0.691 N [1 mark]

Calculate vertical force acting at the hinge **H**, R_v:

- *V* sin (25) + R_v = *W*
- R_v = *W*–*V* sin(25) = 4.905 – 0.691 sin(25)

- R_v = 4.613 N

Calculate horizontal force acting at the hinge **H**, R_h:

- R_h = V cos(25) = 0.691 cos(25)
- R_h=0.626

Calculate the resultant force at the hinge **H**, *R*:

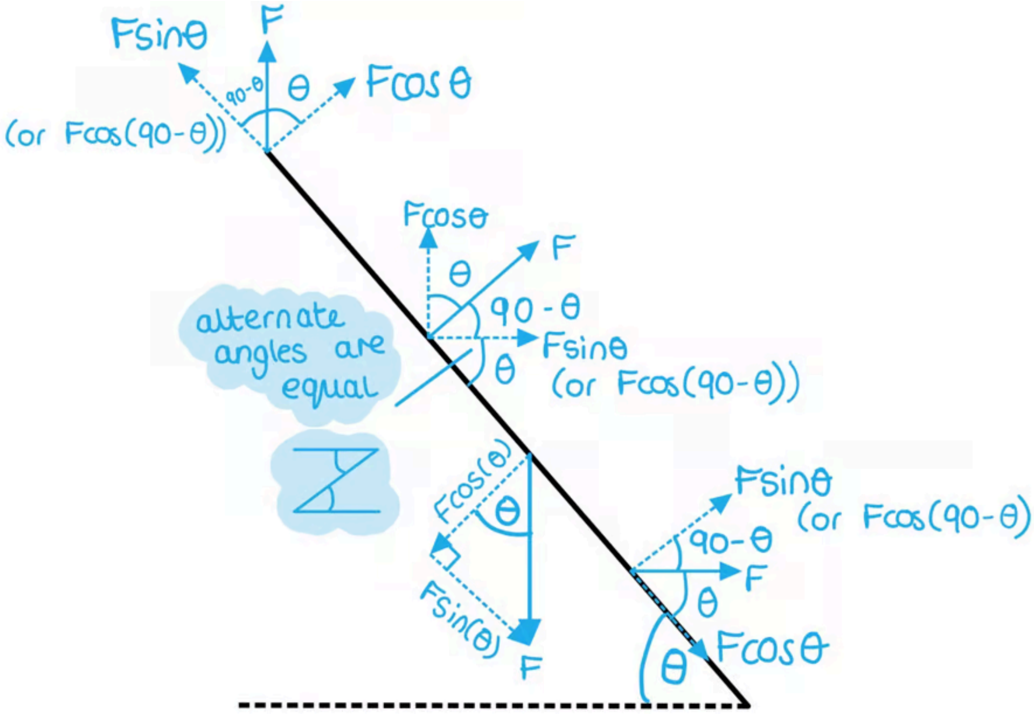
- $R = \sqrt{(R_v)^2 + (R_h)^2} = \sqrt{(4.613)^2 + (0.626)^2}$ [1 mark]
- *R* = 4.7 N (2 s. f) [1 mark]

[Total: 4 marks]

In part (a), the reaction force at the hinge only had a vertical component. This was because *V* was only in the vertical direction. However, since *V* is now perpendicular to the top panel of the laptop, it now has a **horizontal** component, meaning there must be a horizontal reaction force as well to balance it (for all forces to be in equilibrium).

Labelling all the forces on the diagram (or drawing your own diagram) before starting a moments question is important. This is to make sure you're resolving the correct components and calculating the correct perpendicular distances between the forces and the chosen pivot.

NB: cos(25) = sin(65) and sin(25) = cos(65); either is valid in the calculations when resolving the components of *V*. If you have trouble visualising the angles, the sketch below shows a few examples of some forces *F* acting in various directions on the top panel:



(4 marks)

(c) The largest angle the top panel of this laptop can open to is 90º.

Determine whether it would be easier for someone to continuously apply the force on the top panel, V vertically upwards on the panel, like in part (a), or continuously perpendicular to the panel, like in part (b).

Answer

Determine whether it would be easier for someone to continuously apply the force on the top panel, V vertically upwards on the panel, like in part (a), or continuously perpendicular to the panel, like in part (b):

- It would be easier to apply V perpendicular to the panel (like in part (b)); [1 mark]
- Since the moment for V perpendicular (from part b) will always be larger than the moment for V vertical (from part a); [1 mark]
- At any angle θ , the moment for V vertical = $V\cos(\theta) \times 0.21$ and the moment for V perpendicular = $V \times 0.21$ **AND** since $-1 \leq \cos(\theta) \leq 1$ **OR** $\cos(\theta)$ is always < 1 (V vertical will always be less than V perpendicular); [1 mark]

[Total: 3 marks]

(3 marks)

14 (a) A bus driver drives with a constant acceleration of 6.0 m s^{-2}. After 3.0 seconds, they slow down to stop at a set of traffic lights at 5.0 m s^{-2}.

Given that the bus initially travelled at 4.0 m s^{-1} and that the bus fully stops at the traffic lights, calculate the distance between the traffic lights and the point where the bus began to accelerate.

In this question, assume that air resistance is negligible.

Answer

Calculate the distance between the traffic lights and the point where the bus began to accelerate:

List the known quantities:

- Initial acceleration, a_1 = 6.0 m s^{-2}
- Time for acceleration, t = 3.0 s
- Deceleration, a_2 = – 5.0 m s^{-2}
- Initial velocity, u = 4.0 m s^{-1}

Calculate distance travelled by bus whilst accelerating, S_1:

- $s = S_1$, u = 4.0 m s^{-1}, v = not given, a = 6.0 m s^{-2}, t = 3.0 s
- SUVAT equation: $s = ut + \frac{1}{2}at^2$
- $s = (4.0 \times 0) + (\frac{1}{2} \times 6.0 \times (3.0)^2) = 39$ m [1 mark]

Calculate final velocity after acceleration, v:

- s = – , u = 4.0 m s^{-1}, v = ?, a = 6.0 m s^{-2}, t = 3.0 s
- SUVAT equation: $v = u + at$
- $v = 4.0 + (6.0 \times 3.0) = 22$ m s^{-1} [1 mark]

Calculate distance travelled by bus whilst decelerating, s_2:

- The bus reaches a maximum velocity of 22 m s^{-1} when accelerating. This is the new initial velocity for when it starts to decelerate
- $s = s_2$, u = 22 m s^{-1}, v = 0, a = –5.0 m s^{-2}, t = not given
- SUVAT equation: $v^2 = u^2 + 2as$
- $s = \dfrac{v^2 - u^2}{2a}$
- $s = \dfrac{0 - 22^2}{2 \times -5.0}$ = 48.4 m [1 mark]

Calculate total distance travelled before stopping at the traffic lights, s:

- $s = s_1 + s_2$ = 39 + 48.4 = 87.4 = 90 m (2 s. f) [1 mark]

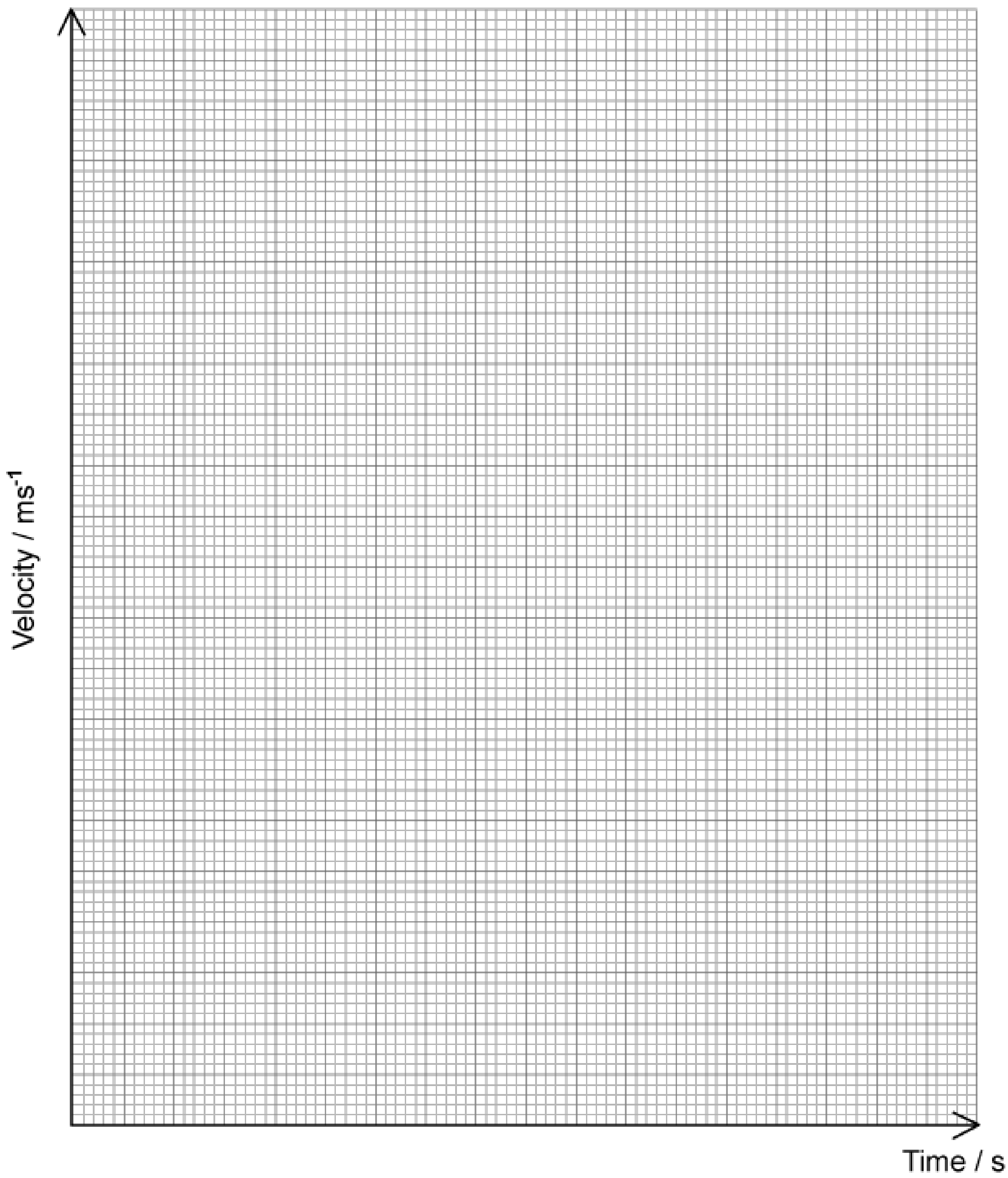
[Total: 4 marks]

This question is split into two parts – when the bus is accelerating, and when it is decelerating. The velocity of the bus is increasing whilst it accelerates, therefore when we use the SUVAT equation for deceleration, we need to make sure to have the correct **initial** velocity for this. This is equal to the **final** velocity **after** the **acceleration**. Remember that a deceleration is just a negative acceleration, so make sure a deceleration has a minus sign (e.g –5 m s^{-2}) before substituting into the SUVAT equations!

Using multiple SUVAT equations is a common expectation in harder questions, so make sure to think through the modelling of the scenario before substituting in any numbers!

(4 marks)

(b) **Figure 1**



Draw the velocity–time graph for the motion of the bus on **Figure 1**.

Answer

Draw the velocity–time graph for the motion of the bus on **Figure 1**:

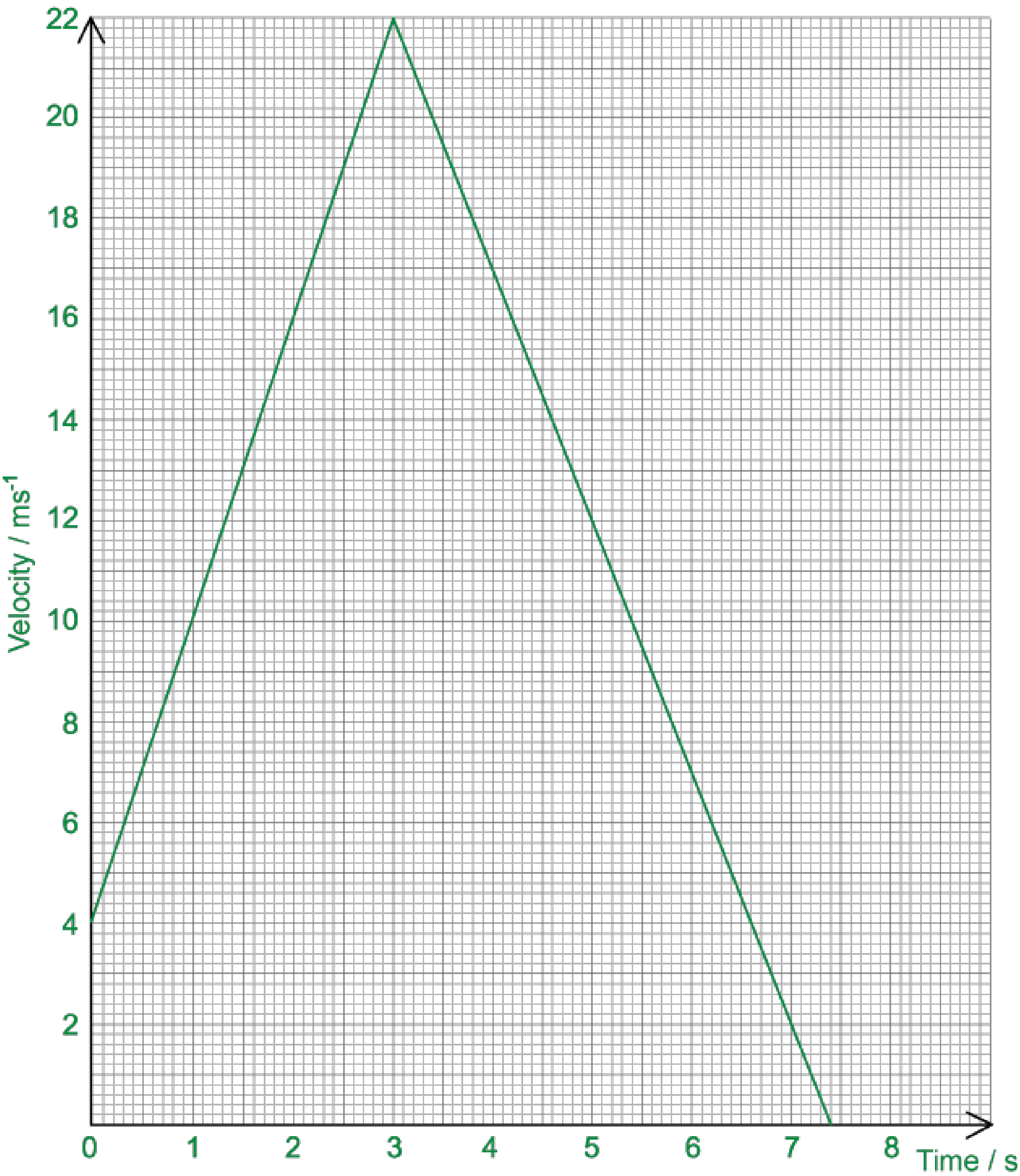
List the known quantities:

- Initial velocity = 4.0 m s^{-1}
- Final velocity after acceleration = 22 m s^{-1} (from part a)
- Time for acceleration = 3 s
- Final velocity after deceleration = 0 m s^{-1} (at traffic lights)

Calculate time taken to decelerate:

- s = not given, u = 22 m s^{-1}, v = 0, a = –5.0 m s^{-2}, t = ?
- SUVAT equation: $v = u + at$
- 0 = 22 – 5.0t
- $t = \frac{22}{5.0}$ = 4.4 s [1 mark]

A graph that would score 3 marks should look like:



- Appropriate scale on time and velocity axis; [1 mark]
- Straight diagonal line upwards from 4 m s^{-1} to 22 m s^{-1} in 3 s; [1 mark]
- Straight diagonal line downwards from 22 m s^{-1} to 0 m s^{-1} at 7.4 s (3 + 4.4); [1 mark]

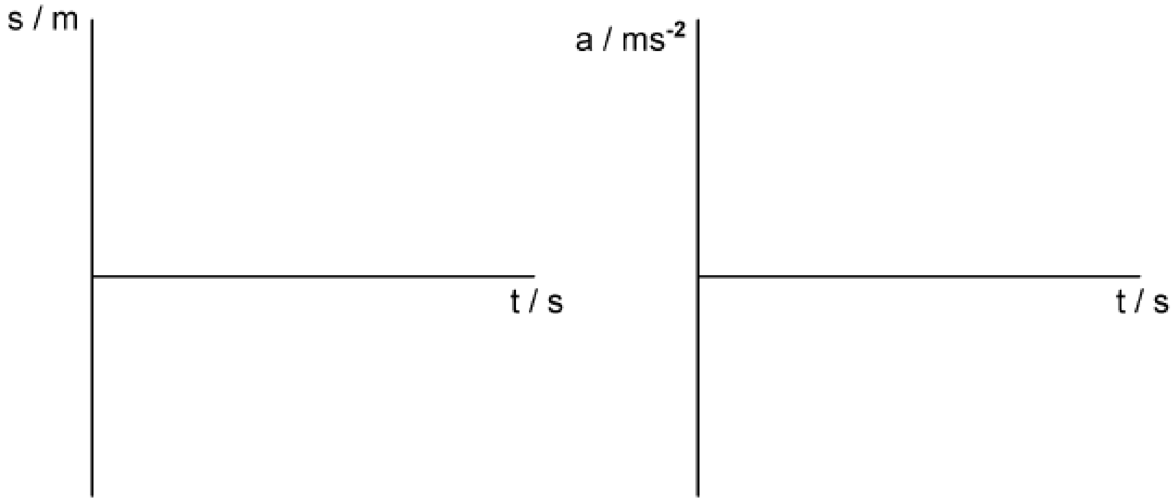
[Total: 4 marks]

When not given the axes on a table, think about the minimum and maximum values you need to plot (in this case 0 – 22 m s^{-1} and 0 – 7.4 s) and then pick an appropriate scale. This should be in either multiples of 1, 2, 5 or 10 for each large square and your graph must take up at least half the scales on both axes for full marks.

Error carried forward marks will apply here from part (a), although you will not get full marks in part (a). Therefore, make sure the shape of your graph is accurate and axes are all correctly labelled as per your value, and don't worry too much about getting the correct values if you are running out of time to get as many marks as possible.

(4 marks)

(c) **Figure 2**



Sketch the displacement–time and acceleration–time graphs in **Figure 2** for the bus. Label the axes appropriately and the graphs do not have to be to scale.

Answer

Sketch the displacement–time and acceleration–time graphs in **Figure 2** for the bus:

List the known quantities:

- Time for acceleration = 3 s
- Time for deceleration = 4.4 s (from part b)

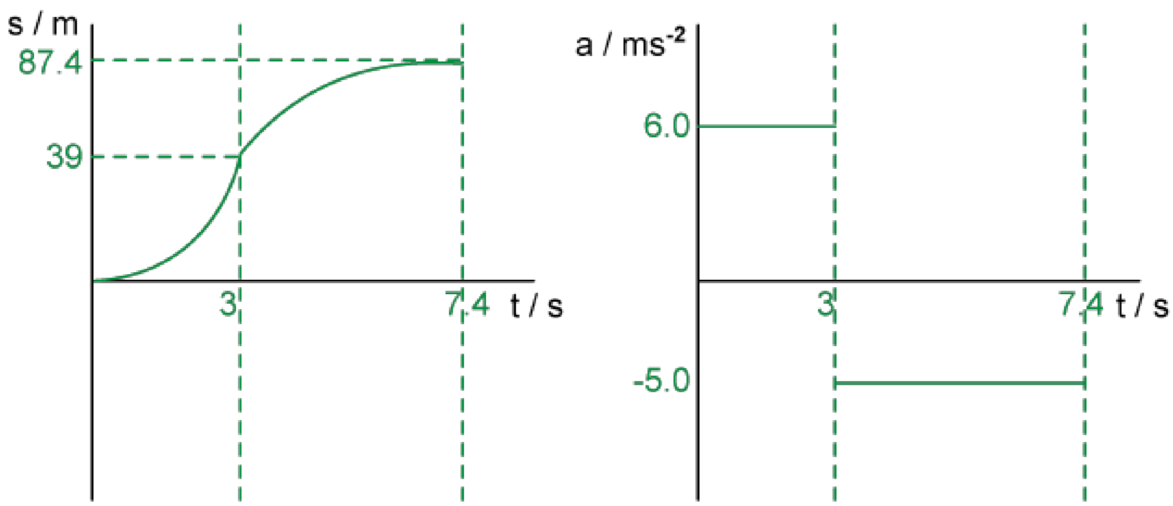
- Displacement whilst accelerating = 39 m

- Displacement while decelerating = 48.4 m

- Initial acceleration = 6.0 m s^{-2}

- Final deceleration = –5.0 m s^{-2}

Graphs that would score 2 marks each should look like:



Displacement–time graph:

- Graph curving upwards (acceleration) to 39 m in 3 s correctly labelled; [1 mark]

- Graph curving downwards (deceleration) to 87.4 m in 7.4 s correctly labelled; [1 mark]

Acceleration–time graph:

- Straight horizontal line at 6.0 m s^{-2}; [1 mark]

- Straight horizontal line at –5.0 m s^{-2}; [1 mark]

[Total: 4 marks]

Errors are carried forward for these graphs as well, as long as the shape of the graphs and the times and distances you have calculated are clearly annotated on the sketches.

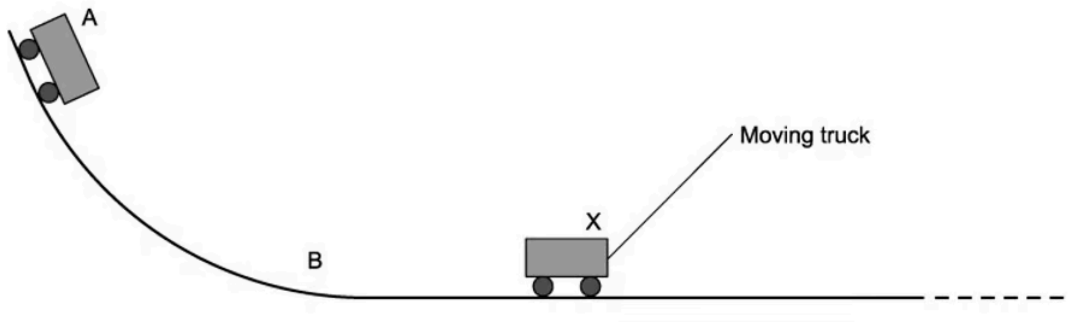
The bus will be accelerating then decelerating, meaning the displacement-time graph will have **curved** lines instead of straight. This is because the gradient of the *s*–*t* graph is velocity, and acceleration is the **rate of change** in velocity. Therefore, an increasing gradient means an increasing velocity (acceleration) and vice versa.

There is constant acceleration and deceleration for the bus. However, you will be expected to remember that deceleration is **negative** acceleration, and hence between 3 – 7.4 s this horizontal line should be in the **negative** section of the graph.

(4 marks)

15 (a) **Figure 1** shows the motion of a truck as it moves on a curved track.

Figure 1

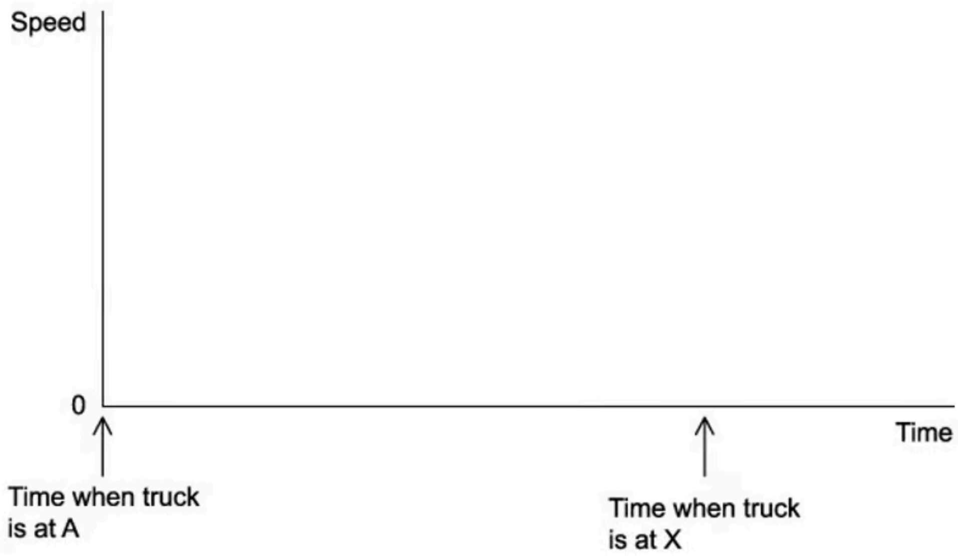


The slope beyond position **B** is horizontal.

On the axes below, sketch a speed – time graph for the truck from its release at position **A** until it reaches position **X,** as shown in **Figure 1**.

Indicate on your graph the time when the truck is at position **B**.

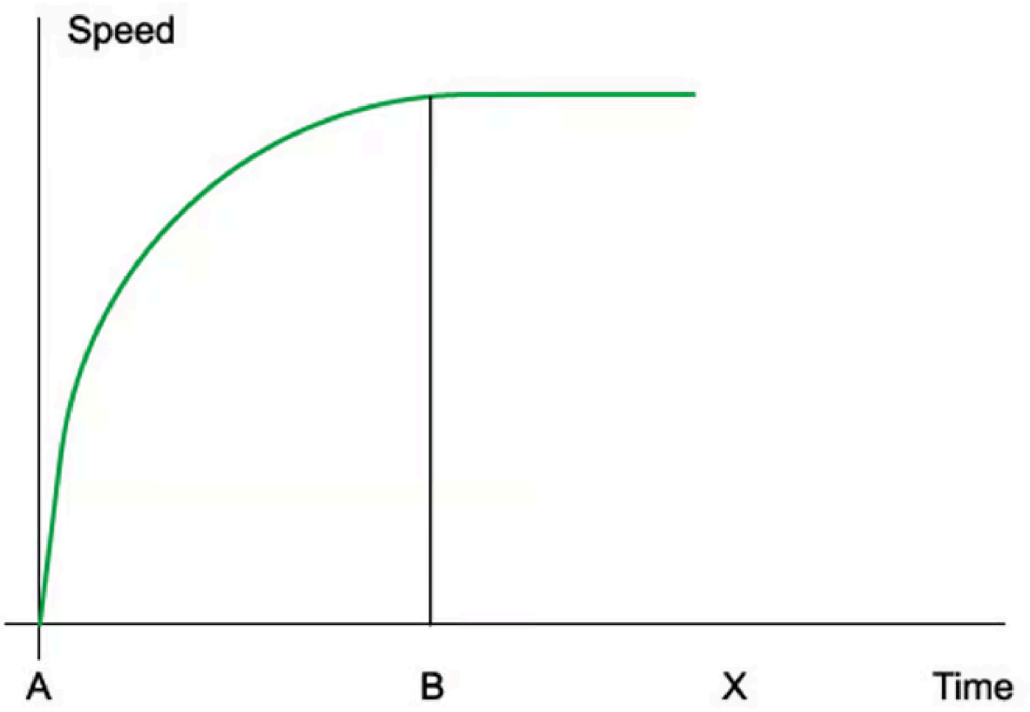
You can assume that frictional forces are negligible.



Answer

Sketch on your graph the time when the truck is at position **B**:

An example of a graph that would earn 3 marks:



- Initial curve with decreasing gradient **AND** reaching constant maximum speed before **X** and maintaining constant speed up to **X**; [1 mark]

- **B** labelled in correct place; [1 mark]

- After **B**, line is horizontal to show constant speed is maintained for remainder of time; [1 mark]

[Total: 3 marks]

(3 marks)

(b) Explain how Newton's first law of motion is illustrated by the motion of the truck between **B** and **X**.

Answer

State Newton's first law and how it relates to the motion of the truck between positions **B** and **X**:

- An object remains in the same state of motion unless acted upon by a resultant force

  OR

  There is no **unbalanced / resultant** force acting on the truck; [1 mark]

Explain how the motion of the truck demonstrates Newton's first law:

Any **one** from:

- Truck travels in a straight line at a constant speed / constant velocity; [1 mark]
- Truck maintains uniform motion / experiences no change in motion; [1 mark]
- Truck has zero acceleration; [1 mark]

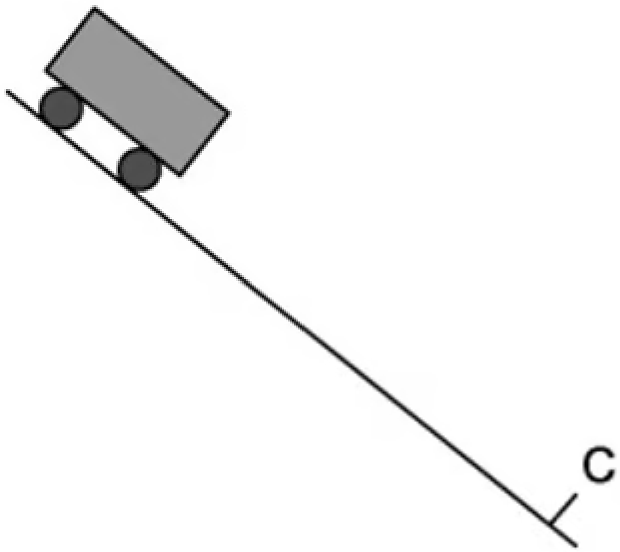
[Total: 2 marks]

If you state that the speed is constant, you must also acknowledge the truck moves in a straight line. Be careful not to say there are 'no forces' acting on the truck; instead, say the forces are **balanced** or there is no/zero **resultant force**.

(2 marks)

(c) **Figure 2** shows another truck moving freely down a ramp inclined at an angle to the horizontal.

Figure 2



The truck starts from rest at the top of the ramp and reaches point **C**. Friction and air resistance are negligible.

Discuss how the acceleration of the truck in **Figure 2** differs from the acceleration of the truck in **Figure 1**.

Answer

Discuss how the acceleration of the truck in **Figure 2** differs from the acceleration of the truck in **Figure 1**:

Describe the acceleration of the truck in **Figure 2**:

- Acceleration of truck in **Figure 2** is constant; [1 mark]

Compare the accelerations of the two trucks:

Any **two** from:

- Acceleration is greater at start / top of the ramp in **Figure 1** than in **Figure 2**; [1 mark]

- Acceleration decreases (for the truck) in **Figure 1**, whereas it is constant in **Figure 2**; [1 mark]

- Zero acceleration / uniform velocity between **B** and **X** in **Figure 1**, but acceleration is constant in **Figure 2**; [1 mark]

- The component of weight / acceleration parallel to the slope changes in **Figure 1**, but remains constant in **Figure 2**; [1 mark]

[Total: 3 marks]

The question asks you to discuss the acceleration rather than the speed, so make sure to do this.

Also, you would lose marks for only discussing **Figure 1** or **Figure 2**, so ensure to make statements about both diagrams using comparative language, such as:

- greater than / less than

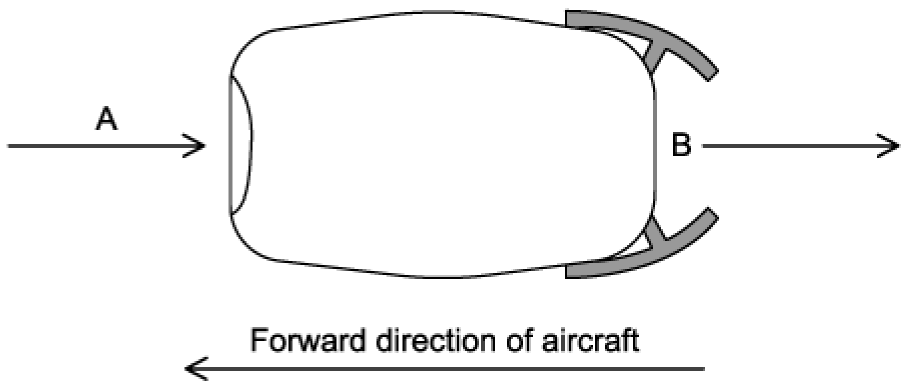
- whereas

- but / however

(3 marks)

16 (a) **Figure 1** shows a jet engine.

Figure 1



Air enters the engine at **A** and is heated before leaving **B** at a much higher speed.

By referring to the momentum of the air as it passes through the engine, explain using appropriate laws of motion, why the air exerts a force on the engine in the forward direction.

Answer

Explain what happens to the momentum of the air:

- (The momentum of air) <u>increases</u>; [1 mark]

Explain why the momentum increases:

Newton's second law:

- Force = rate of change of momentum
- (Momentum increases so) <u>force</u> acts <u>on</u> the <u>air</u>; [1 mark]

Newton's third law:

- <u>Air</u> exerts a <u>force</u> (on the engine) of the <u>same / equal magnitude</u> **AND** But acts in <u>opposite</u> in direction; [1 mark]

[Total: 3 marks]

(3 marks)

(b) In one second, a mass of 350 kg of air enters at **A**. The speed of this mass of air increases by 590 m s^{-1} as it passes through the engine.

Calculate the force that the air exerts on the engine.

Answer

List the known quantities:

- Speed, v = 590 m s^{-1}
- Rate at which the mass of air passes through the engine (mass per second) = $\frac{\Delta m}{\Delta t}$ = 350 kg s^{-1}

From the data sheet:

- Force $F = \frac{\Delta(mv)}{\Delta t}$

Calculate the force exerted by the air:

- Force = rate of change of momentum
- $F = \frac{\Delta mv}{\Delta t} = \frac{\Delta m}{\Delta t}v = 350 \times 590$ [1 mark]
- F = 206 500 N = 210 000 N (2 s.f.) [1 mark]

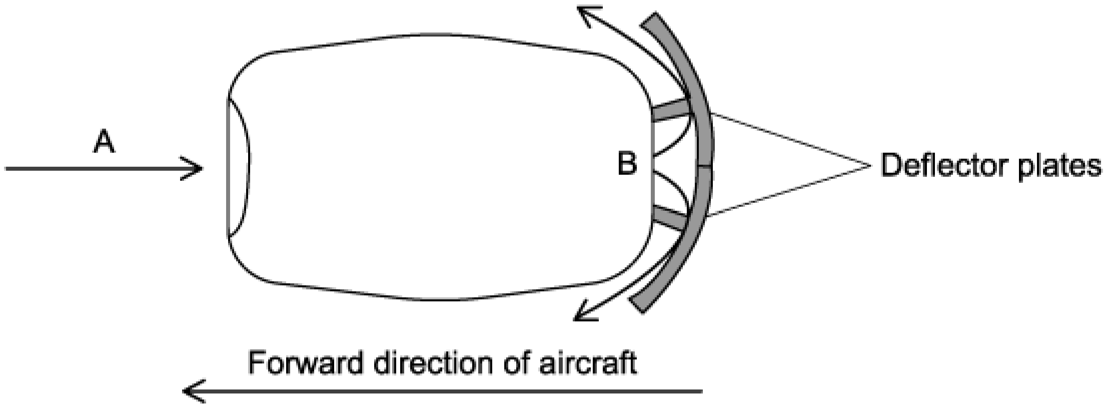
[Total: 2 marks]

(2 marks)

(c) When an aircraft lands, its jet engines exert a decelerating force on the aircraft by making use of deflector plates.

These cause the air leaving the engines to be deflected at an angle to the direction the aircraft is travelling as shown in **Figure 2**.

Figure 2



The speed of the air leaving **B** is the same as the speed of the deflected air.

Explain why the momentum of the air changes, and suggest why in practice the decelerating force provided by the deflector plates may not remain constant.

Answer

Explain why the momentum of the air changes:

- Momentum / velocity is a vector / has direction; [1 mark]
- There is a change (in the air's) direction; [1 mark]

Suggest why in practice the decelerating force may not remain constant:

- Rate of intake of air decreases (as aircraft slows)

  OR

  Volume / mass / amount of air (passing through engine) per second decreases; [1 mark]

- (Because) there is a smaller rate of change of momentum / momentum change (as the aircraft slows);[1 mark]

[Total: 4 marks]

(4 marks)

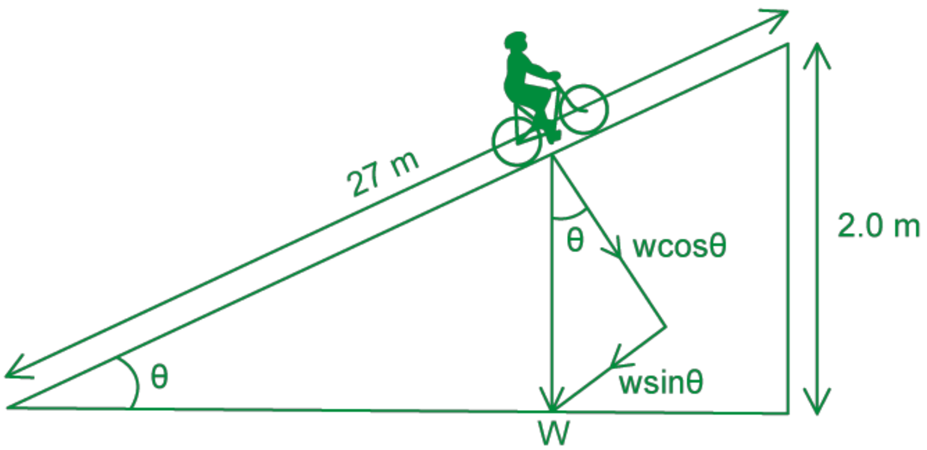
17 (a) A cyclist rides along a road up an incline at a steady speed of 5.0 m s^{-1}. The mass of the rider and bicycle is 70 kg and the bicycle travels 27 m along the road for every 2.0 m gained in height. Neglect energy loss due to frictional forces.

Calculate the power required for the cyclist to ride up the slope.

Answer

List the known quantities:

- Speed, v = 5.0 m s^{-1}
- Mass, m = 70 kg
- Distance, s = 27 m for every 2.0 m gained in height



Determine an equation for the force on the cyclist, F:

- The only eternal force on the cyclist is their weight pulling them down the slope
- $F = W \sin\theta = mg \sin\theta$
  - W is the weight of the rider and bicycle

Calculate sinθ:

- $\sin\theta = \dfrac{opposite}{hypotenuse} = \dfrac{2.0}{27}$ [1 mark]

Substitute in values to calculate the force, F:

- $F = 70 \times 9.81 \times \frac{2.0}{27} = 50.87$ N [1 mark]

Calculate the power, P:

- $P = Fv = 50.87 \times 5.0 = 254.35 = 250$ W (2 s. f.) [1 mark]

[Total: 3 marks]

The cyclist must do work **against** the force of gravity (weight) to travel up the incline. The force is the **component** of the weight that acts **along** the incline, since $P = Fv$ is derived from $W = Fs$ where W is the work done, F is the force and s is the distance moved in the direction of the force.

$P = Fv$ is very commonly misused. It can only be used if the object is moving at **constant** velocity (which is the case in this question) and the force must be in the direction **parallel** to the distance moved. Therefore, the most common way students lose marks in this question is by not resolving components of the weight.

(3 marks)

(b) The cyclist now travels along a steeper incline at an angle of 6.5° to the horizontal.

Calculate the power output required from the cyclist on this new plane and discuss how this value differs from that in part (a).

Answer

List the known quantities:

- Speed, $v = 5.0$ m s^{-1}
- Mass, $m = 70$ kg
- Angle of incline, $\theta = 6.5°$

Write an equation with power P and work done W:

- The work done by cyclist equals their gravitational potential energy gained
- GPE = mgh

- Power, $P = \dfrac{W}{t} = \dfrac{mgh}{t}$

Calculate height, *h*:

- $\text{Sin}\theta = \dfrac{opposite}{hypotenuse} = \dfrac{h}{5.0}$

- h = 5.0 sinθ = 5.0 sin(6.5) [1 mark]

Substitute in the values:

- Since power is the change in energy per unit **time**, we can substitute time as 1 second

- $P = \dfrac{70 \times 9.81 \times 5.0 \times \sin(6.5)}{1}$ = 390 W (2 s.f.) [1 mark]

Discussion:

- The power output on the steeper incline is higher; [1 mark]

- This is because the cyclist has to do more work against gravity; [1 mark]

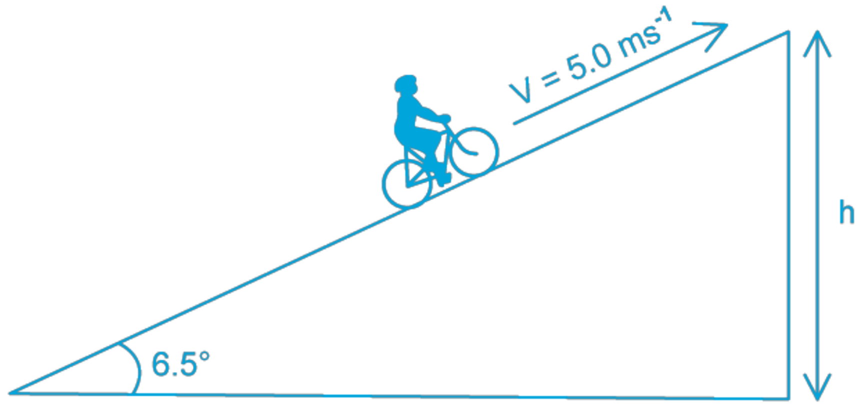
[Total: 4 marks]

This question might look a little confusing as to why we would use the distance of the hypotenuse in m s^{-1} (the velocity). This is because we want the unit of power to be in J s^{-1} (W) or N m s^{-1}, which means we don't just want the distance but the distance **each second**. This might make more sense when looking at the units:

$$P = \frac{mgh}{t} = \frac{[kg][N\,kg^{-1}][m]}{s} = \frac{[N][m]}{[s]}$$

Therefore, $\dfrac{m}{s}$ or m s^{-1} must come from the velocity (which has units m s^{-1})

Since the angle of incline has now changed for this question, make sure **not** to use the value of *h* given in part (a), since this has now changed!



(4 marks)

(c) The cyclist stops pedalling and the bicycle freewheels up the incline for a short time.

Considering the energy transfers, calculate the distance travelled along the slope from when the cyclist stops pedalling to where the bicycle comes to rest.

Answer

List the known quantities:

- Speed, v = 5.0 m s^{-1}
- Mass, m = 70 kg
- Angle of incline, θ = 6.5°

Determine the energy transfers:

- The initial kinetic energy of the cyclist is transferred to the gain in gravitational potential energy

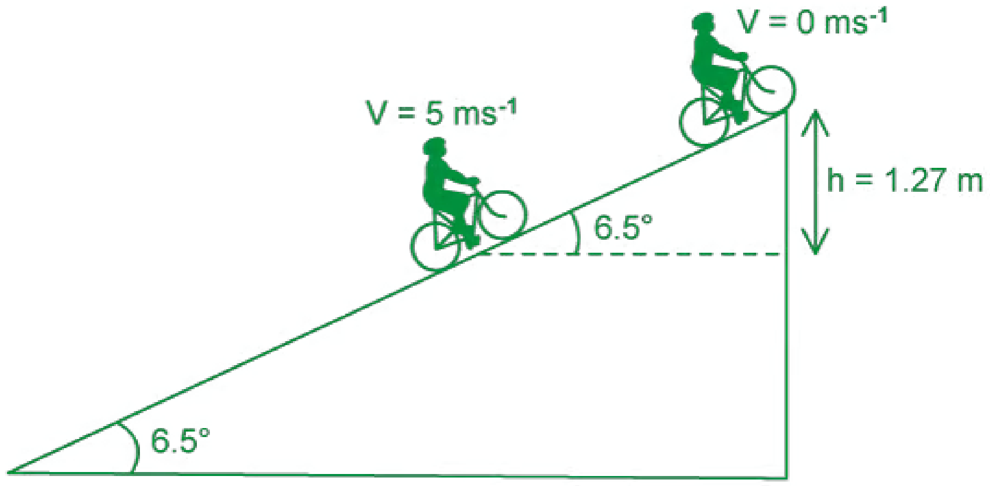
Calculate the initial kinetic energy, E_k:

- $E_k = \frac{1}{2}mv^2 = \frac{1}{2} \times 70 \times (5.0)^2 = 875$ J
- Therefore, the gain in GPE, E_p is also 875 J; [1 mark]

Calculate the height the cyclist has travelled, *h*:

- $E_p = mgh$
- $h = \frac{E_p}{mg} = \frac{875}{70 \times 9.81}$ = 1.27 m [1 mark]

Calculate distance travelled, *s*:



- $\sin\theta = \frac{opposite}{hypotenuse} = \frac{h}{s}$
- $s = \frac{h}{\sin\,\theta} = \frac{1.27}{\sin(6.5)}$ = 11 m (2 s.f.) [1 mark]

[Total: 3 marks]

(3 marks)

18 (a) A steel ball of density 8.05 g cm^{-3} has a diameter of 6.50 cm.

Calculate the weight of this ball.

Answer

List the known quantities:

- Density, ρ = 8.05 g cm^{-3}
- Diameter, d = 6.50 cm

Calculate the volume of ball, V:

- Volume of a sphere, $V = 4/3\ \pi\ r^3$
- $V = \frac{4}{3} \times \pi \times \left(\frac{6.50}{2}\right)^3$ = 143.79 cm^3 [1 mark]

Calculate the mass, m:

- Density, $\rho = \frac{m}{V}$
- $m = \rho V$ = 8.05 × 143.79 = 1157.5095 g
- m = 1.158 kg [1 mark]

Calculate the weight, W:

- $W = mg$ = 1.158 × 9.81 = 11.4 N [1 mark]

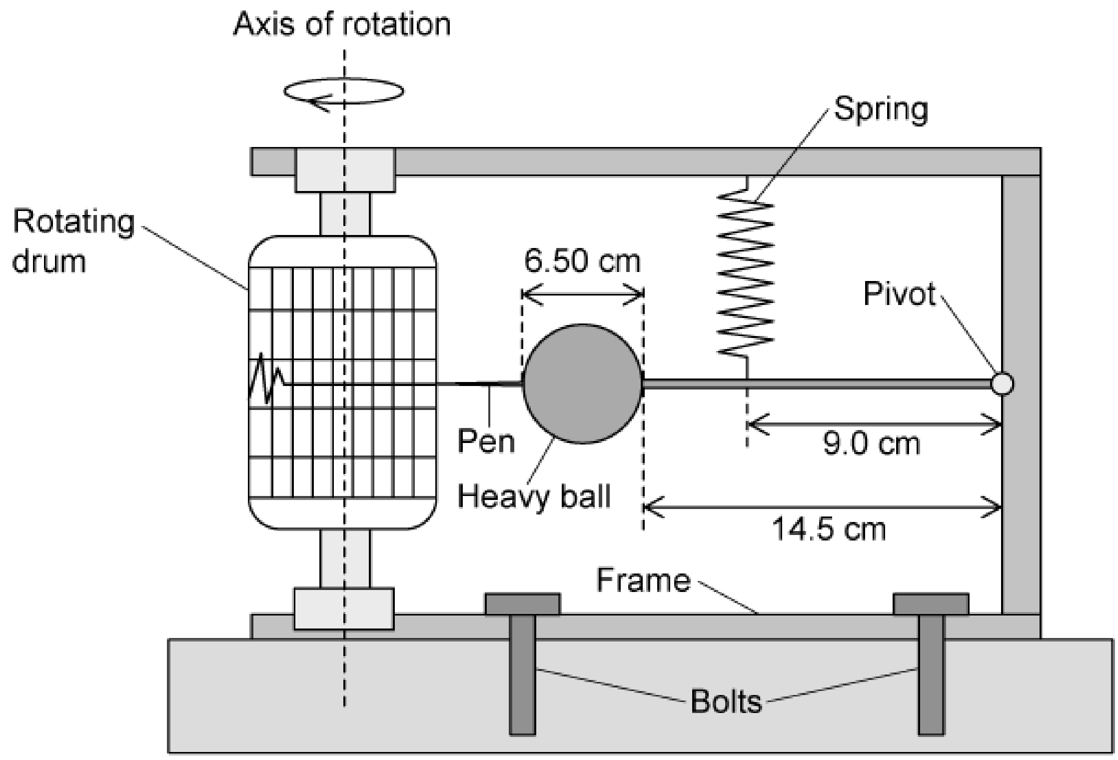
[Total: 3 marks]

This question can also be done by converting g cm^{-3} into kg m^{-3} first and changing the diameter into m to find the volume in m^3. Both will give the same answer. However, as long as the volume and density are in the same length units (cm or m etc.) then it doesn't matter (since cm^{-3} and cm will cancel out, as will m^{-3} and m)

(3 marks)

(b) A seismometer is a device that is used to record the movement of the ground during an earthquake. A simple seismometer is shown in **Figure 1**.

Figure 1



The steel ball from part (a) is attached to a pivot by a rod so that the rod and ball can move in a vertical plane. The rod is suspended by a spring so that, in equilibrium, the spring is vertical and the rod is horizontal. A pen is attached to the ball. The pen draws a line on graph paper attached to a drum rotating about a vertical axis. Bolts secure the seismometer to the ground so that the frame of the seismometer moves during the earthquake.

When the rod is horizontal and the spring is vertical, the spring extends by 8.3 cm.

Show that the spring constant of the spring is around 240 N m^{-1}.

Answer

List the known quantities:

- Extension, ΔL = 8.3 cm = 0.083 m
- Distance from pivot to spring = 9.0 cm = 0.09 m
- Distance of surface of the ball to pivot = 14.5 cm = 0.145 m

- Weight of the ball, W = 11.4 N (from part a)
- Diameter of ball = 6.5 cm = 0.065 m

Take moments about the pivot:

- When the rod is horizontal and spring is vertical, the system is in equilibrium
- Principle of moments:
  - Clockwise moment from spring = anticlockwise moment from ball
- Moment = force × perpendicular from the pivot

Calculate anticlockwise moment from ball:

- Anticlockwise moment from ball = $W \times (0.145 + \frac{0.065}{2})$
- $11.4 \times (0.145 + \frac{0.065}{2})$ = 2 N m [1 mark]

Calculate force from spring, F:

- Moment from spring (clockwise) = $F \times 0.09$ = 2 (anticlockwise)
- $F = \frac{2}{0.09}$ = 22.22 N [1 mark]

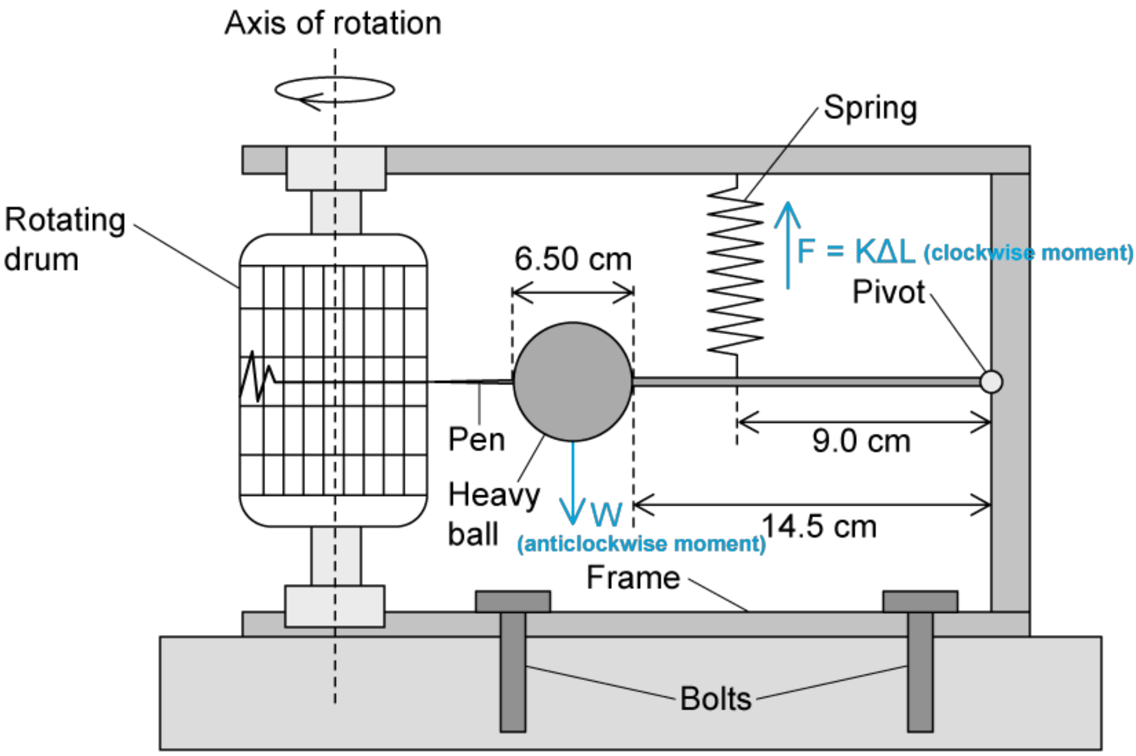
Calculate spring constant of spring, k:

- Hooke's law: $F = k\,\Delta L$
- $k = \frac{F}{\Delta L} = \frac{22.22}{0.083}$ = 267.71 = 270 N m^{-1} [1 mark]

[Total: 3 marks]

A common mistake in this question is not to take the perpendicular distance from the pivot to the **centre of mass** of the ball. Hence, the distance from the pivot to the centre of mass of the ball is 0.145 cm to the surface then the extra ($\frac{0.065}{2}$) 0.0325 m.

Remember that when the spring is extended, its force is always towards the equilibrium position. In this case, because it is a vertical spring its **upwards** and hence has a clockwise moment.



(3 marks)

(c) Discuss why it is important that the spring constant of the spring is not too low or high.

Answer

If the spring constant is too low:

- The spring will be less stiff / extend more **AND** the rod will shake / vibrate / move the ball more during an earthquake; [1 mark]
- The vibrations would cause the pen to go off / would not be marked accurately on the graph paper (on the rotating drum) **OR** the vibration readings / graph would be too large; [1 mark]

If the spring constant is too high:

- The spring will be too stiff **AND** the rod will not shake / vibrate / move the ball much during an earthquake; [1 mark]
- The vibrations would not be marked much / at all on the graph paper by the pen **OR** the vibration readings / graph would be too small; [1 mark]

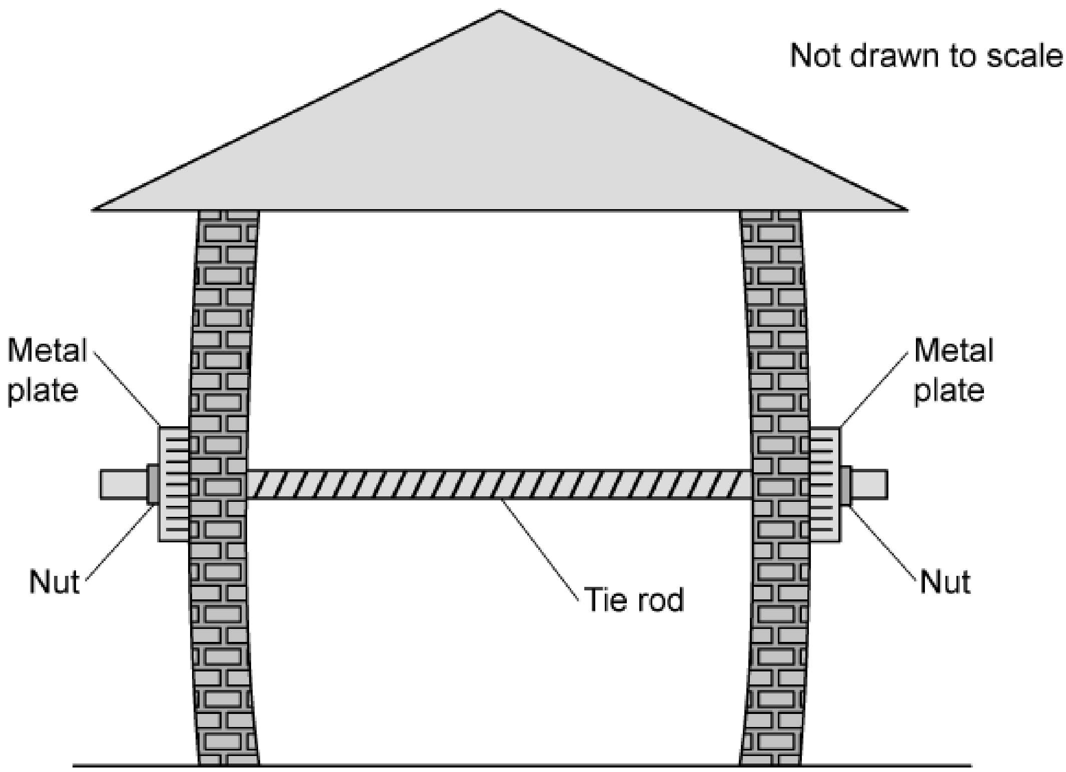
[Total: 4 marks]

Remember in question with the 'discuss' command word to always relate back to the scenario at hand where possible. The solution for this question requires information about **why** the whole experiment wouldn't work if the spring constant isn't chosen carefully. Have a look back at the original diagram to help with the wording and make sure to mention objects in the diagram in your answer.

(4 marks)

19 (a) In order to prevent the collapse of walls of old buildings a metal rod is often used to tie opposite walls together, as shown in **Figure 1** below.

Figure 1



In one case a steel tie rod of diameter 24 mm is used. When the nuts are tightened, the rod extends by 7 %. The Young Modulus of steel is 2.1×10^{11} Pa.

Calculate the force exerted on the walls by the rod.

Answer

List the known quantities:

- Diameter, d = 24 mm = 24×10^{-3} m
- Young Modulus of steel = 2.1×10^{11}Pa
- Extension of rod = 7 %

Calculate the strain:

- Strain = $\frac{\Delta L}{L}$ = 7 % = 0.07 [1 mark]

Calculate cross–sectional area, A:

- $A = \pi r^2 = \frac{\pi d^2}{4}$

- $A = \frac{\pi \times (24 \times 10^{-3})^2}{4}$ = 4.52 × 10^{-4} m^2 [1 mark]

Calculate the stress:

- Young Modulus = $\frac{stress}{strain}$ therefore, Stress = Strain × Young Modulus

- Stress = 0.07 × (2.1 × 10^{11}) = 1.47 × 10^{10} Pa [1 mark]

Calculate force, F:

- Stress = $\frac{F}{A}$ therefore, F = Stress × A

- F = (1.47 × 10^{10}) × (4.52 × 10^{-4})

- F = 6.64 × 10^6 N = 6.6 MN [1 mark]

[Total: 4 marks]

If the rod extends by 7 % that means the **ratio** of it extension to the original length is equal to 7 % or 0.07. You can check this by putting in some numbers:

For example:

- If L = 5 m, therefore $L + \Delta L$ = 5 × 1.07 = 5.35 (increased by 7 %)

- Strain = $\frac{\Delta L}{L} = \frac{5.35 - 5}{5}$ = 0.07

(4 marks)

(b) Another part of the old building requires 6 steel bolts, each of diameter 8.5 cm. When an earthquake occurs, this produces a strain on each bolt of 5.3 × 10^{-4}.

Calculate the maximum force exerted on the bolts during this earthquake.

Answer

List the known quantities:

- Young Modulus of steel = 2.1×10^{11} Pa
- Strain = 5.3×10^{-4}
- Diameter of each bolt, d = 8.5 cm = 8.5×10^{-2} m

Calculate the stress on each bolt:

- Young Modulus = $\frac{stress}{strain}$ therefore, Stress = Young Modulus × Strain
- Stress = $(2.1 \times 10^{11}) \times (5.3 \times 10^{-4}) = 111.3 \times 10^{6}$ Pa [1 mark]

Calculate the cross–sectional area of each bolt, A:

- $A = \pi r^2 = \frac{\pi d^2}{4}$
- $A = \frac{\pi \times (8.5 \times 10^{-2})^2}{4} = 5.6745 \times 10^{-3}$ m^2 [1 mark]

Calculate the force on each bolt, F:

- Stress = $\frac{F}{A}$ therefore F = Stress × A
- $F = (111.3 \times 10^{6}) \times (5.6745 \times 10^{-3}) = 631.571 \times 10^{3}$ N [1 mark]

Calculate the force on all the bolts:

- $F \times 6$ bolts = $(631.571 \times 10^{3}) \times 6 = 3.789 \times 10^{6}$ = 3.8 MN [1 mark]

[Total: 4 marks]

A common reason why students don't end up with full marks on calculation questions is from rounding answers too early. In this question, there are values that are substituted into other calculations multiple times. To avoid rounding errors, store the full values from a calculation in your calculator. Or write down your answer to at least 3–4 decimal places until the final answer (which should be to the lowest number of significant figures in the question), as done in this mark scheme.

(4 marks)

(c) The ultimate tensile stress of steel is 5.0×10^8 Pa.

Determine the minimum number of bolts that is required to make sure the building stays intact during this earthquake.

Answer

List the known quantities:

- Maximum force, $F = 3.789 \times 10^6$ N (from part b)
- Ultimate tensile stress of steel, UTS = 5.0×10^8Pa
- Young Modulus of steel = 2.1×10^{11}Pa
- Area of one bolt, $A = 5.6745 \times 10^{-3}$ m^2 (from part b)

Calculate the total area of steel bolts needed to hold the building, A_T:

- UTS = $\frac{F}{A_T}$ therefore, $A_T = \frac{F}{UTS}$
- $A_T = \frac{3.789 \times 10^6}{5.0 \times 10^8}$ = 7.578×10^{-3} m^2 [1 mark]

Calculate number of bolt:

- Number of bolts = $\frac{A_T}{Area\ of\ one\ bolt}$ = $\frac{7.578 \times 10^{-3}}{5.6745 \times 10^{-3}}$ = 1.34 [1 mark]
- Number of bolts = 2 bolts [1 mark]

[Total: 3 marks]

The number of bolts must be a whole number, so remember to round up to the next whole number for your final answer.

(3 marks)

20 (a) An electron in a cathode ray tube is accelerated from rest through a potential difference of 150 kV and aimed towards a screen.

Calculate the kinetic energy of the electrons when they collide with the screen.

Answer

List the known quantities:

- Potential difference which accelerates the electron, V = 150 kV

From data and formula sheet:

- Magnitude of the charge of an electron, e = 1.60 × 10^{-19} C
- pd, $V = \frac{W}{Q}$

Calculate the kinetic energy of the electrons after being accelerated by potential difference of 150 kV, E_k:

- $E_k = eV$ [1 mark]
- E_k = (1.60 × 10^{-19})(150 × 10^3) = 2.40 × 10^{-14} J [1 mark]

[Total: 2 marks]

The potential difference across the electrodes of the cathode ray tube will cause the electrons to accelerate. The electrical energy which the potential difference creates will equal the gain in kinetic energy of electrons.

(2 marks)

(b) If the current in the tube is 32 mA, calculate the rate at which the heat must be dissipated from the screen when it reaches its working temperature.

Answer

List the known quantities:

- Current in the tube, I = 32 mA
- Kinetic energy of the electrons, E_k = 2.40 × 10^{-14} J (from part a)

From data and formula sheet:

- Magnitude of the charge of an electron, e = 1.60 × 10^{-19} C
- Current, $I = \frac{\Delta Q}{\Delta t}$

Calculate the total charge which strikes the screen each second, ΔQ:

- $\Delta Q = I\Delta t$ = (32 × 10^{-3})(1) = 32 × 10^{-3} C [1 mark]

Calculate the number of electrons which strike the screen each second, N_e:

- $N_e = \frac{Q}{e} = \frac{32 \times 10^{-3}}{1.60 \times 10^{-19}}$ = 2 × 10^{17} [1 mark]

Calculate rate at which heat be dissipated from the screen:

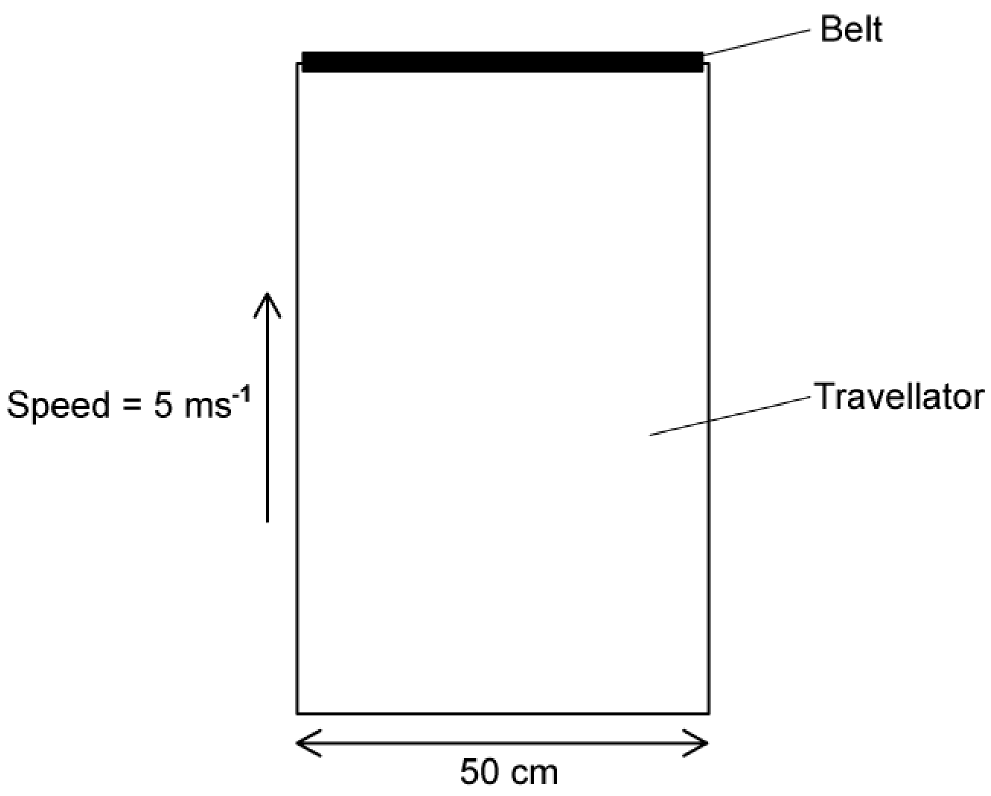
- Heat dissipated from screen = $E_k \times N_e$ = (2.40 × 10^{-14})(2 × 10^{17}) = 4800 J [1 mark]

[Total: 3 marks]

(3 marks)

(c) A travellator of width 50 cm has a surface containing a layer of electrons. It moves through a brush, which collects the electrons and transports them away as electric current, at a speed of 5 m s^{-1}. **Figure 1** shows the arrangement of the conveyor belt and the brush.

Figure 1



Given that there are 10 000 electrons per mm^2 of the travellator, calculate the magnitude of the electric current through the wire brush.

Answer

List the known quantities:

- Width of the travellator, W = 50 cm
- Speed of the travellator, v = 5 m s^{-1}
- Number of electrons in 1 mm^2 = 10 000 electrons

From data and formula sheet:

- Magnitude of the charge of an electron, e = 1.60 × 10^{-19} C
- Current, $I = \frac{\Delta Q}{\Delta t}$

Calculate the area of the travellator which passes the brush in 1 second, A:

- $A = W \times v$ = 500 × (5 × 10^3) = 2.5 × 10^6 mm^2 [1 mark]

Calculate the number of electrons which are collected by the brush in 1 second, N_e:

- N_e = 10 000× (2.5 × 10^6) = 2.5 × 10^{10} electrons [1 mark]

Calculate the magnitude of the electric current through the wire brush, I:

- $I = \frac{N_e e}{\Delta t} = \frac{(2.5 \times 10^{10})(1.60 \times 10^{-19})}{1}$ = 4 × 10^{-9} A [1 mark]

[Total: 3 marks]

We are told in the question that the travellator is moving at a speed of 5 m s^{-1}. This means that a length of 5 m will pass through the travellator each second.

(3 marks)

21 (a) At room temperature a metal has a resistivity of 3.0×10^{-7} Ω m.

A 2.5 m length of wire made from this metal has a cross-sectional area of 0.50 mm^2.

Calculate the power dissipated in this length of wire when it carries a current of 10 mA.

Answer

List the known quantities:

- Resistivity of wire (at room temperature), $\rho = 3.0 \times 10^{-7}$ Ω m
- Cross-sectional area, $A = 0.50\ mm^2 = 0.50 \times 10^{-6}\ m^2$ [1 mark]
- Length of the wire, $L = 2.5$ m
- Current, $I = 10$ mA $= 10 \times 10^{-3}$ A

From the data sheet:

- Resistivity $\rho = \frac{RA}{L}$
- Power, $P = VI$

Calculate the resistance of the wire at room temperature:

- $\rho = \frac{RA}{L}$ therefore $R = \frac{\rho L}{A}$
- $R = \frac{(3.0 \times 10^{-7}) \times 2.5}{0.50 \times 10^{-6}} = 1.5$ Ω [1 mark]

Calculate the power dissipated:

- $P = I^2R = (10 \times 10^{-3})^2 \times 1.5$
- $P = 1.5 \times 10^{-4}$ W [1 mark]

[Total: 3 marks]

(3 marks)

(b) Explain the assumption that is necessary for calculating the answer to part (a).

Answer

State the assumption:

- In order to calculate the power dissipated it must be assumed that the resistance of the wire is constant; [1 mark]

Explain why the resistance must be assumed constant:

- If the resistance changes (e.g. because the temperature of the wire increases) then the current (hence power) will change; [1 mark]

[Total: 2 marks]

(2 marks)

(c) The wire is cooled from room temperature θ_r and becomes superconducting.

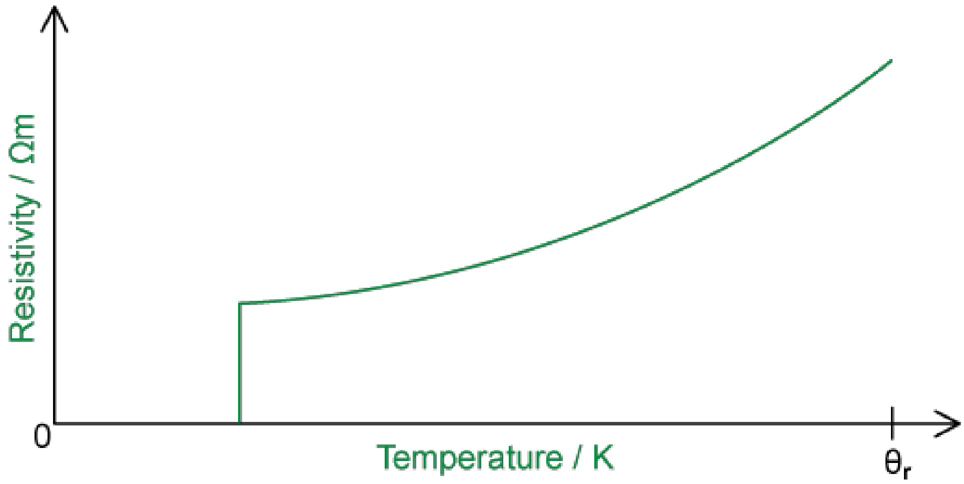
Sketch a graph on the axes provided in **Figure 1** below to show how the wire's resistivity would vary with temperature as it is cooled from room temperature θ_r.

Include any appropriate labels on your sketch.

Figure 1

Answer

A graph which would score full credit is shown below:



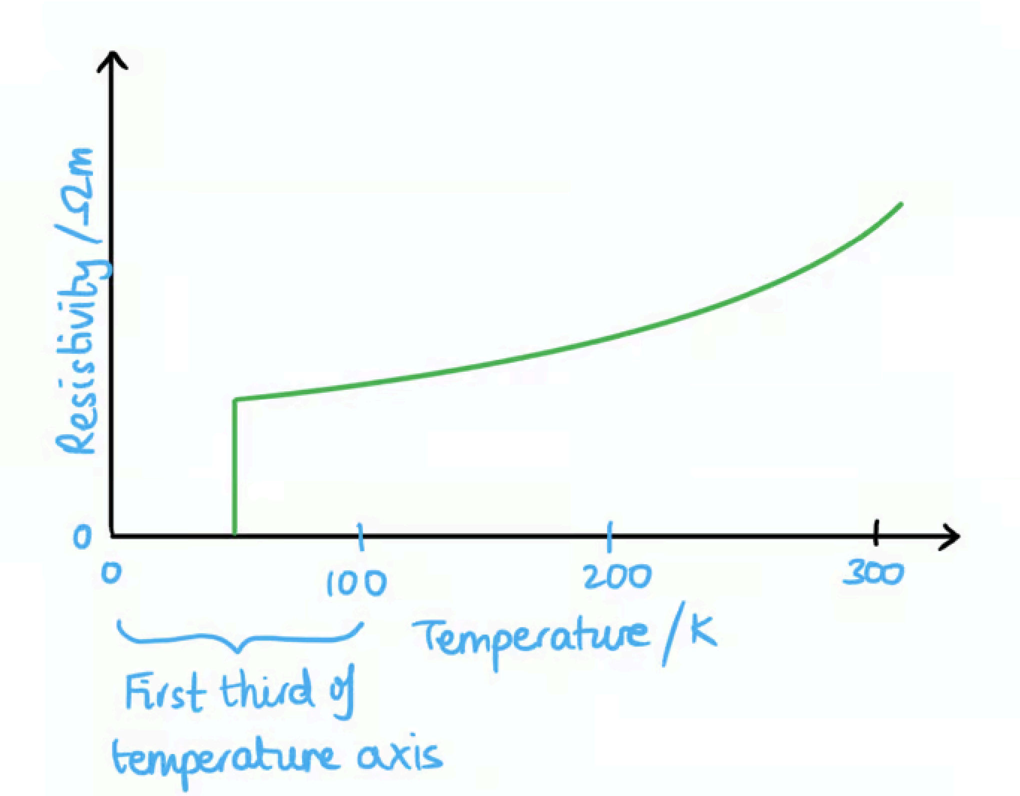
- Axes labelled with correct units; [1 mark]
    - Resistivity / Ω m AND Temperature / K
- Curve upwards / line with a positive gradient; [1 mark]
- Vertical line to some non-zero temperature within the first 1/3 of the temperature axis; [1 mark]

[Total: 3 marks]

This question gets you to think carefully about sensible units and ranges of temperature.

You should remember that the resistivity (and the resistance) of superconducting metals falls abruptly to zero at very cold temperatures, i.e., temperatures close to absolute zero. The clue that the axis in this question is appropriately labelled with Kelvin for units of temperature is that there is no space for negative temperature.

Most superconducting metals have transition temperatures between 0 K and 10 K. Compared to room temperature (20 $^{\circ}$C is roughly 300 K rounded to the nearest 100) this is clearly *a lot* colder! So, the mark scheme specifying the transition temperature must be drawn within the first third of the temperature axis (shown below) is very lenient indeed!



(3 marks)

(d) Explain why the efficiency of electrical power transmission is improved when conventional copper wires are replaced with metal wires that are superconducting.

Answer

State the value of resistance for all superconducting metals:

- When a metal is superconducting the (electrical) resistance is zero; [1 mark]

Explain the effect of zero resistance on power transmission:

- If the resistance is zero, then there is no power loss / the waste energy transferred as heat is zero; [1 mark]

[Total: 2 marks]

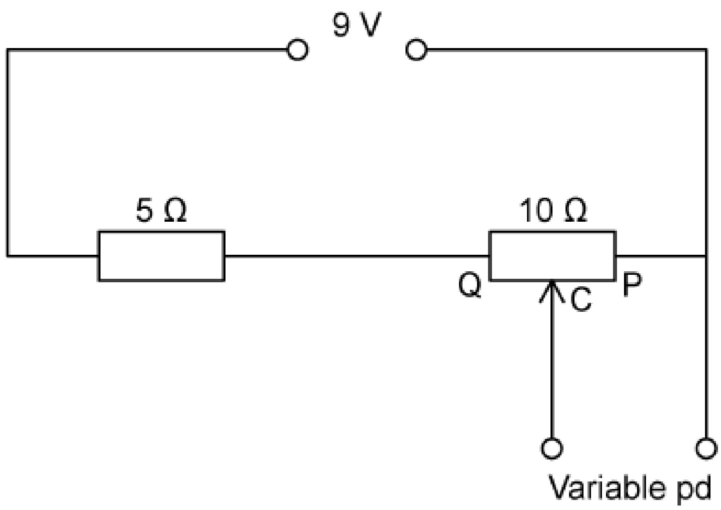
(2 marks)

22 (a) Varying the potential difference across components can be achieved in several different ways in electric circuits.

Potentiometers are devices that can vary an output potential difference (pd). Potentiometers consist of a sliding contact which can move along a wire. If connections are made across one end of the wire and the sliding contact, the output pd can be varied.

Figure 1a shows a potentiometer in action. The wire in the potentiometer is represented by a fixed resistor of resistance 10 Ω and the sliding contact **C** can move from one end of the wire (at position **P**) to the other end of the wire (position **Q**). The potentiometer is in series with another fixed resistor of 5 Ω.

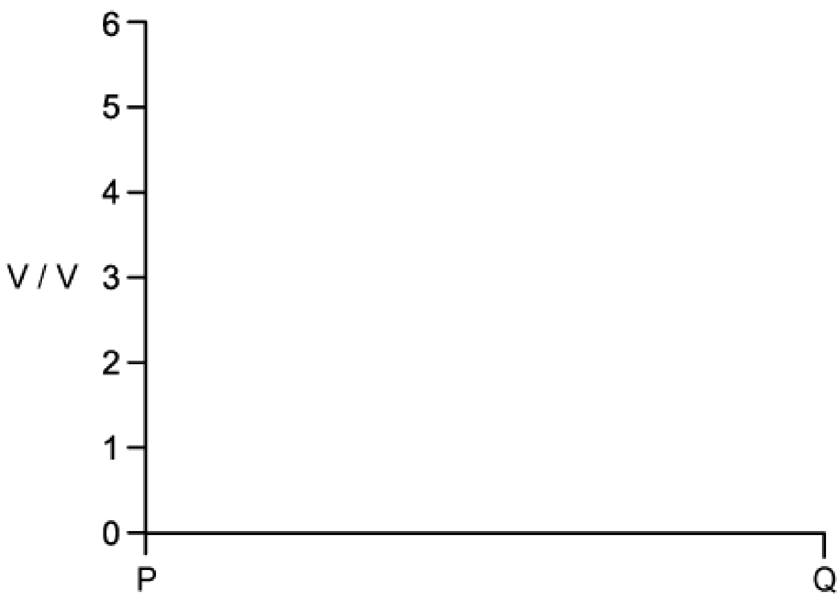
Figure 1a



This circuit is powered by a 9 V power supply and is used to supply a variable potential difference (pd) to another circuit.

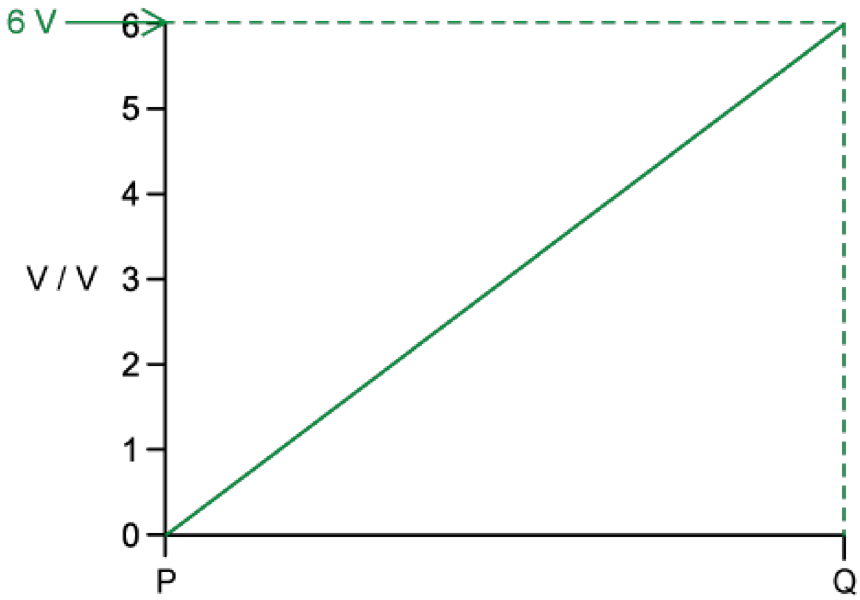
Sketch a graph on the axes provided in **Figure 1b** to show how the supplied pd *V* varies as the moving contact **C** is moved from position **P** to position **Q**.

Figure 1b



Answer

- A graph which would score 2 marks should look like:

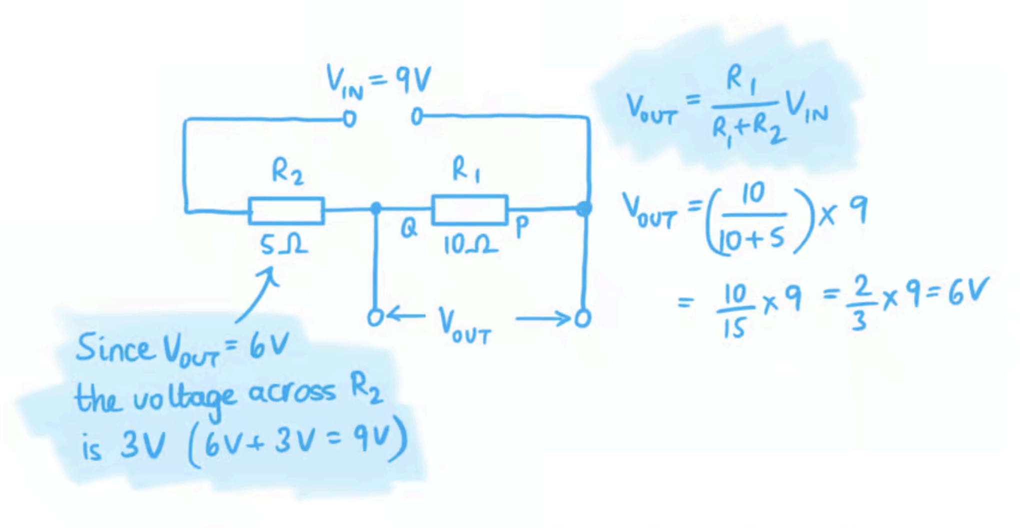


- Straight line with a positive gradient; [1 mark]
- At **P**, *V* = 0 V **AND** at **Q**, *V* = 6 V [1 mark]

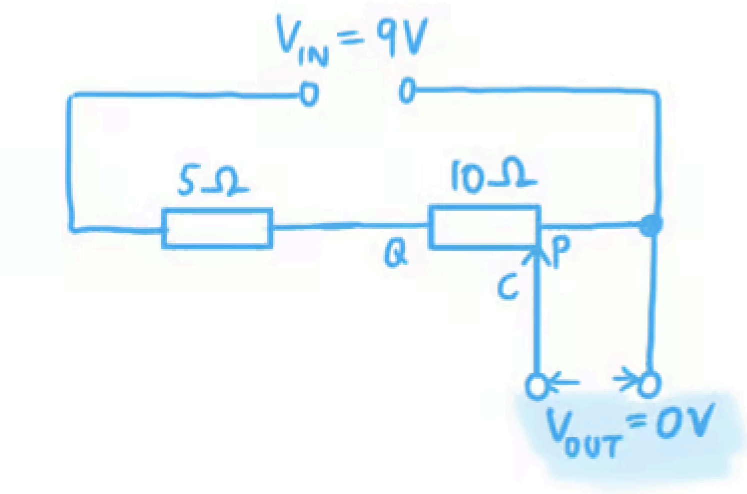
[Total: 2 marks]

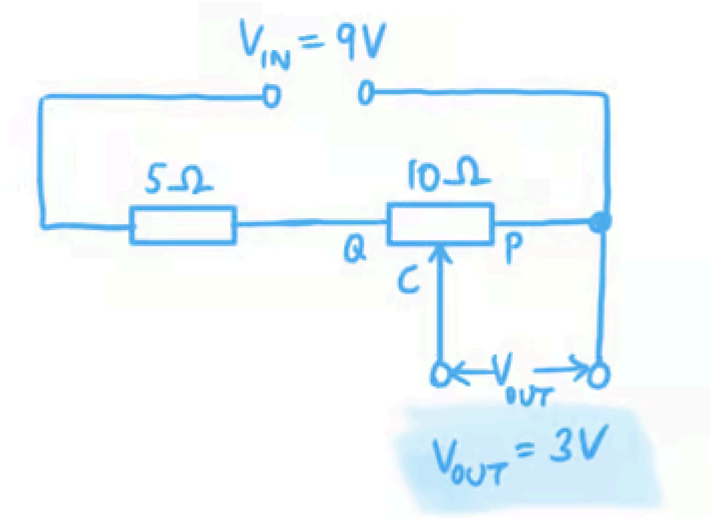
This circuit is a potential divider, because there are two resistors in series sharing the same EMF. As the slider moves, the potential is 'divided' between them.

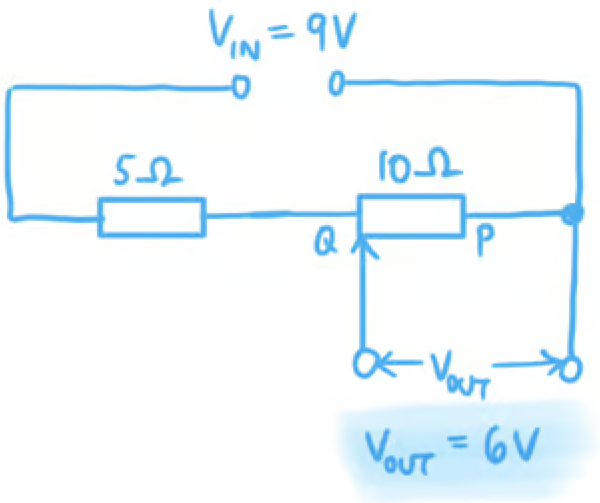
By inspection, the resistances are in the ratio 1:2, so the potential is also split in the ratio 1:2 (with the bigger share of voltage across the larger resistance). Hence, the 9 V EMF is divided as 3 V and 6 V. This can be seen more explicitly using the potential divider equation, shown below:



Note that if the sliding contact C in **Figure 1a** is at position P, then there is no potential difference across the variable pd (the voltage supplied is 0 V). The output pd (in parallel with the resistor) increases as more of the resistor is included between one end (position P) and the sliding C. This is illustrated below:







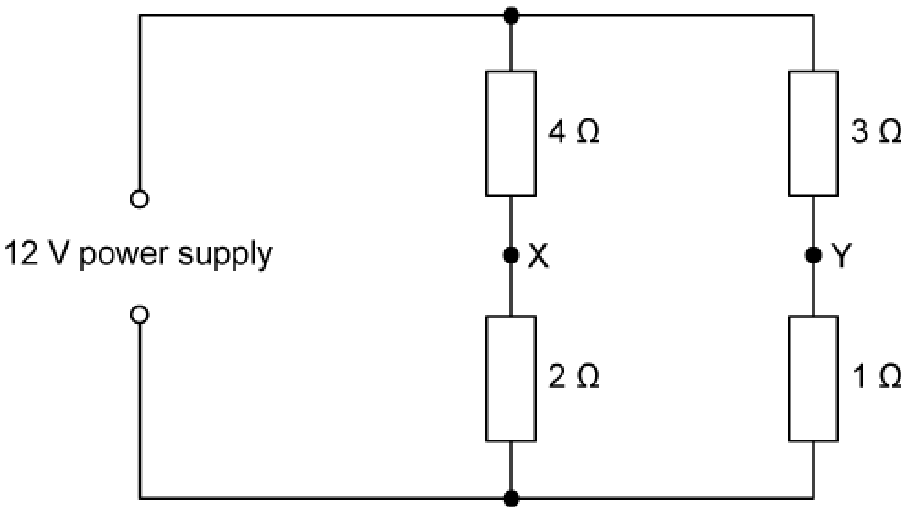
Therefore, the potentiometer can provide a variable resistance between 0 V and 6 V.

(2 marks)

(b) Potential divider circuits generally comprise of two or more resistors in series with each other.

Some potential divider circuits have multiple branches connected in parallel, such as that shown in **Figure 2**:

Figure 2



Show that the potential difference between **X** and **Y** is 1 V.

Answer

List the known quantities:

- V_{IN} = 12 V
- Resistor R_1 = 4 Ω
- Resistor R_2 = 2 Ω
- Resistor R_3 = 3 Ω
- Resistor R_4 = 1 Ω

Method 1

Determine the potential difference across R_1 :

- The potential divider equation for resistor R_1 (which is in a series loop with resistor R_2) is $V_{OUT} = \frac{R_1}{R_1 + R_2} V_{IN}$
- Therefore $V_{OUT} = \frac{4}{4+2} \times 12 = \frac{4}{6} \times 12 = 8$ V [1 mark]

Determine the potential difference across R_3 :

- The potential divider equation for resistor R_3 (which is in a series loop with resistor R_4) is $V_{OUT} = \frac{R_3}{R_3 + R_4} V_{IN}$
- Therefore $V_{OUT} = \frac{3}{3+1} \times 12 = \frac{3}{4} \times 12 = 9$ V [1 mark]

Determine the potential at **X** and **Y**:

- Charge transfers 8V across resistors R_1
- The potential at **X** is therefore 12 V – 8 V = 4 V

AND

- Similarly, there is a 9 V drop in potential across resistor R_3

- The potential at **X** is therefore 12 V – 9 V = 3 V [1 mark]

Show the potential difference between **X** and **Y** is 1 V:

- The potential difference between **X** and **Y** is therefore 4 V – 3 V = 1 V [1 mark]

Method 2

Determine the potential difference across R_2 :

- The potential divider equation for resistor R_2 (which is in a series loop with resistor R_1) is $V_{OUT} = \frac{R_2}{R_2 + R_1} V_{IN}$

- Therefore V_{OUT} = $\frac{2}{2+4}$ × 12 = $\frac{2}{6}$ × 12 = 4 V [1 mark]

Determine the potential difference across R_4 :

- The potential divider equation for resistor R_4 (which is in a series loop with resistor R_3) is $V_{OUT} = \frac{R_4}{R_4 + R_3} V_{IN}$

- Therefore V_{OUT} = $\frac{1}{1+3}$ × 12 = $\frac{1}{4}$ × 12 = 3 V [1 mark]

Determine the potential at **X** and **Y**:

- Charge transfers 4 V across resistor R_2

- The potential at **X** is therefore 12 V – 4 V = 8 V

  AND

- Charge transfers 3 V drop across resistor R_4

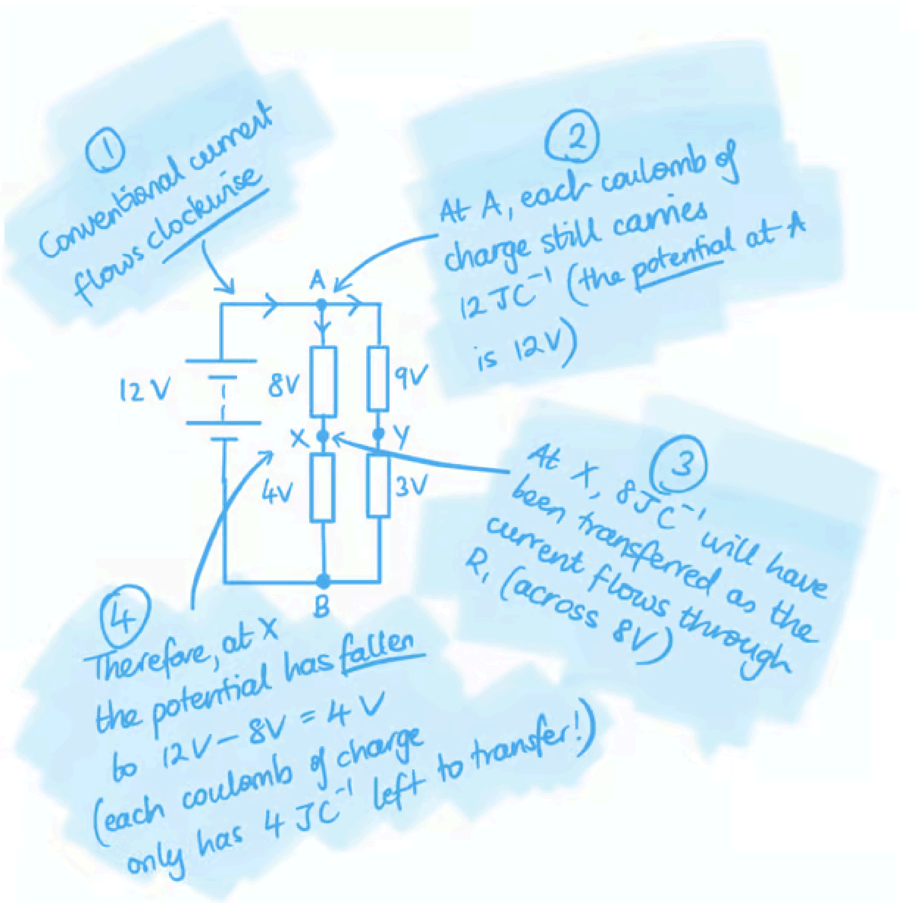
- The potential at **X** is therefore 12 V – 3 V = 9 V [1 mark]

Show the potential difference between **X** and **Y** is 1 V:

- The potential difference between **X** and **Y** is therefore 9 V – 8 V = 1 V [1 mark]

[Total: 4 marks]

Method 1 assumes that the current flows in a clockwise direction through the circuit: in other words, that the terminals of the power supply are oriented as shown in the sketch below. You should work your way through the four steps, to convince yourself that the potential at point X is 4V **if the current flows clockwise** through the circuit.



Following this reasoning, you should be able to check and see that the potential at Y is 12 V – 9 V = 3 V, so the potential **difference** between X and Y is 1 V.

Method 2 assumes the current flows in an anti-clockwise direction through the circuit; in that case, simply reverse the direction of the current in the illustration above and you should see that the potential at X is 12 V – 4 V = 8 V. Similarly, the potential at Y is 12 V – 3 V = 9 V, giving a potential difference between X and Y as 1 V again.

Clearly, either assumption is valid and will give the correct **potential difference** between **X** and **Y**. This question introduces you to a fact you will explore more in A Level physics: 'potential' as a quantity is relative (i.e., it depends on how you define directions), but potential *difference* is always absolute – so long as your working is consistent!

(4 marks)

(c) A wire joins **X** and **Y** in the potential divider circuit in **Figure 2**.

Based on **your reasoning** in part (b), deduce and explain whether a conventional current flows from **X** to **Y** or from **Y** to **X**.

Answer

EITHER

If **Method 1** is used in part (b):

- The potential at **X** is 4 V **AND** the potential at **Y** is 3 V if a conventional current flows clockwise from the power supply; [1 mark]
- Therefore, conventional current flows from **X** to **Y**; [1 mark]
- This is because (conventional) current flows from high to low potential; [1 mark]

OR

If **Method 2** is used in part (b):

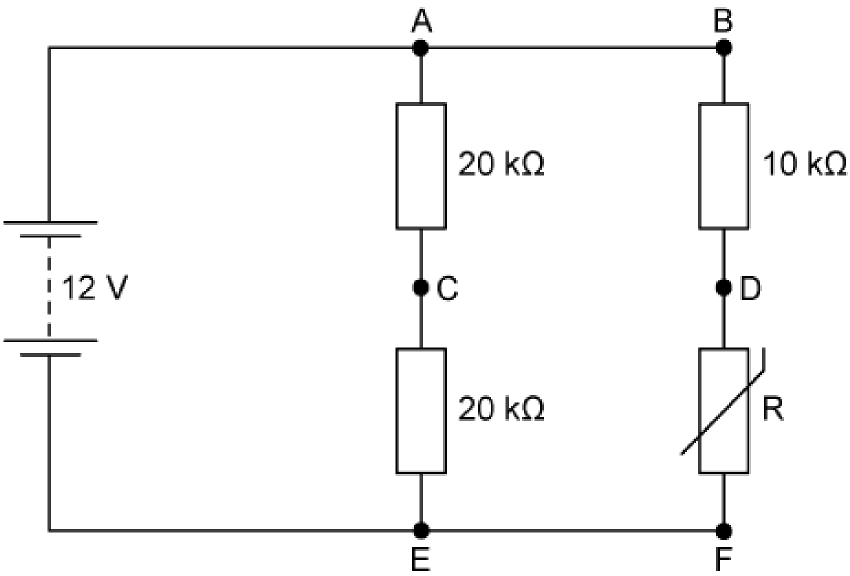
- The potential at **X** is 8 V **AND t**he potential at **Y** is 9 V if a conventional current flows anti-clockwise from the power supply; [1 mark]
- Therefore, conventional current flows from **Y** to **X**; [1 mark]
- This is because (conventional) current flows from high to low potential; [1 mark]

[Total: 3 marks]

(3 marks)

(d) **Figure 3** shows another potential divider circuit which includes a thermistor with resistance R.

Figure 3



The battery has an EMF of 12 V, with negligible internal resistance. At room temperature, the resistance of the thermistor is 5 kΩ.

Calculate the current in the battery at room temperature.

Answer

List the known quantities:

- Resistance of the thermistor (at room temperature), R = 5 kΩ = 5000 Ω
- Resistance between **AC** = 20 kΩ = 20 000 Ω
- Resistance between **CE** = 20 kΩ = 20 000 Ω
- Resistance between **BD** = 10 kΩ = 10 000 Ω
- EMF of the battery = 12 V

Determine the series resistances:

- Resistors in series $R_T = R_1 + R_2 + R_3 + \cdots$
- There are two resistors in series between **A** and **E**, so $R_T = R_1 + R_2$

- Therefore, the total resistance between **A** and **E** = resistance between **AC** + resistance between **CE** = 20 000 + 20 000 = 40 000 Ω

AND

- There are two resistors in series between **B** and **F**, so $R_T = R_1 + R_2$
- Therefore, the total resistance between **B** and **F** = resistance between **BD** + resistance of the thermistor (at room temperature) = 10 000 + 5000 = 15 000 Ω [1 mark]

Determine the total resistance of the circuit:

- Resistors in parallel $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$
- The resistance between **A** and **E** is in parallel to the resistance between **B** and **F**
- Therefore $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{40\ 000} + \frac{1}{15\ 000} = \frac{3}{120\ 000} + \frac{8}{120\ 000} = \frac{11}{120\ 000}$
- Therefore $R_T = \frac{120\ 000}{11}$ = 10 909 Ω [1 mark]

Determine the current through the battery:

- Using Ohm's law across the battery, $V = IR$
- V is the EMF ϵ of the cell
- Therefore 12 = I × 10 909
- $I = \frac{12}{10\ 909}$ = 1.1 × 10^{-3} A [1 mark]

[Total: 3 marks]

(3 marks)

23 (a) The Maximum Power Transfer theorem says the maximum amount of electrical power is dissipated in a load resistance R_L when it is exactly equal to the internal resistance of the power source *r*.

Figure 1a shows a circuit used to investigate maximum power transfer. A variable resistor, which acts as the load resistance R_L, is connected to a power source of e.m.f. ε and internal resistance *r*, along with a switch S and an ammeter and voltmeter:

Figure 1a

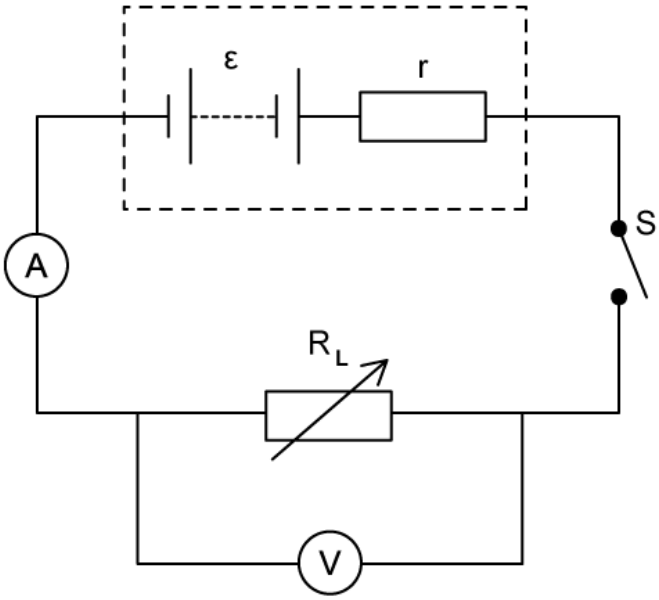
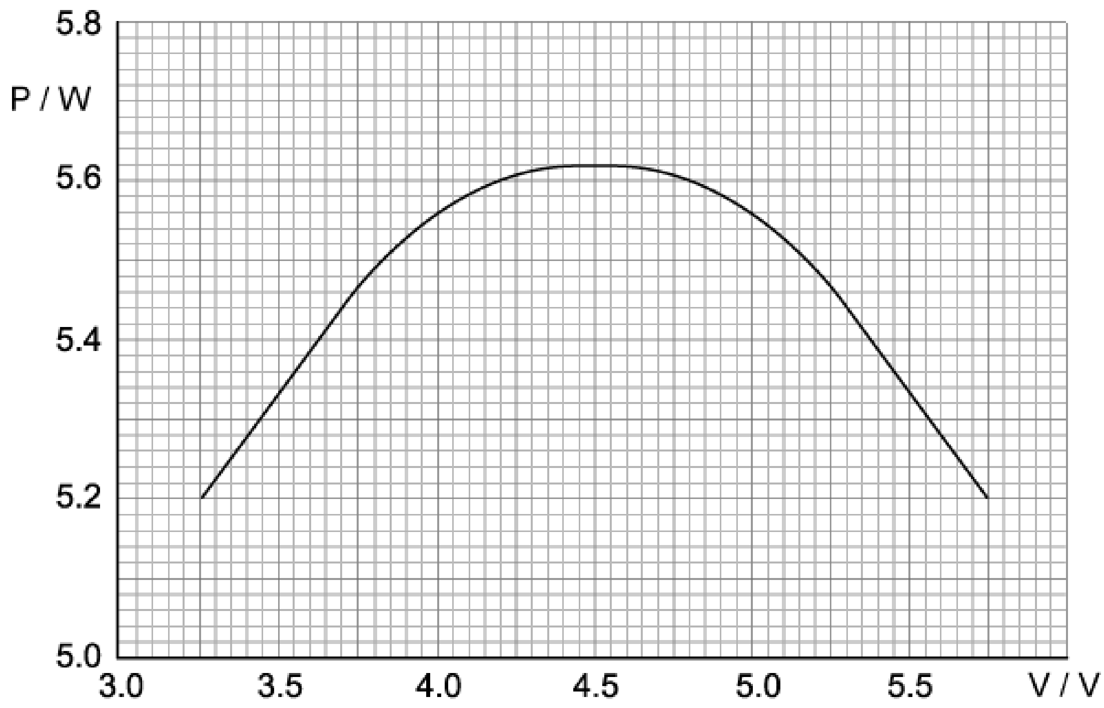


Figure 1b shows the graph obtained for the power *P* dissipated in R_L as the potential difference *V* across R_L is varied:

Figure 1b

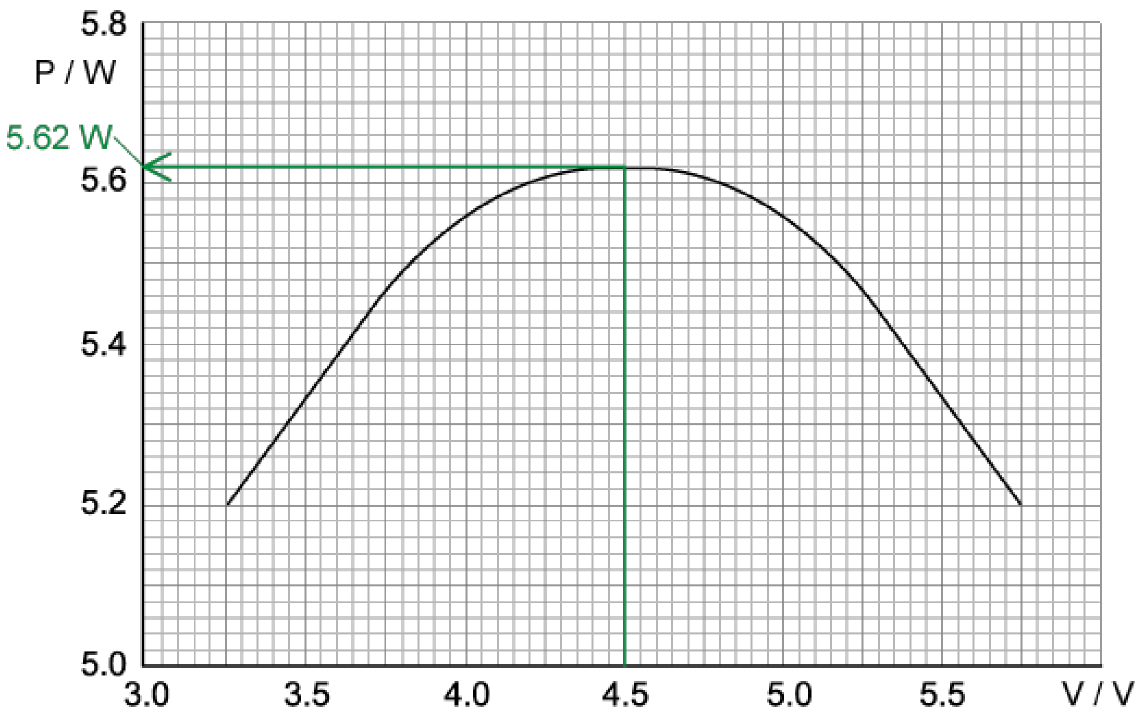


Assuming the Maximum Power Theorem is valid, use **Figure 1b** to determine the internal resistance of the power source.

Answer

Determine the potential difference *V* at maximum power *P*:

- Using **Figure 1b**, the potential difference *V* across the load resistance R_L is 4.5 V **AND** at maximum power *P* = 5.62 W [1 mark]
- This is shown below:



Calculate the load resistance R_L:

- Power, $P = \frac{V^2}{R}$
- Therefore, 5.62 = $\frac{(4.5)^2}{R}$
- $R = \frac{(4.5)^2}{5.62}$ = 3.60 Ω [1 mark]

Identify the value of the internal resistance, r:

- If the Maximum Power Theorem is valid, when the maximum power is dissipated in the load resistance R_L, it must be equal to the internal resistance r
- Therefore, $r = R_L$ = 3.60 Ω [1 mark]

[Total: 3 marks]

Though you do not have to memorise the Maximum Power Theorem for your examination, you can use the results from it in this question to infer that $R = r$ when power transfer is maximised

In fact, this is a pretty cool result to actually derive, using the equations for Power $P = I^2R$ and e.m.f. $\varepsilon = I\,(r + R)$. Again – you will not be required to do this for the physics examination – but a fun activity if you can use some differentiation from A Level mathematics and a bit of algebra.

(3 marks)

(b) Hence, show that the e.m.f. of the power supply is 9 V.

Answer

List the known quantities:

- Maximum power, P = 5.62 W (from part (a))
- Potential difference, V = 4.5 V (from part (a))
- Load resistance, R_L = 3.60 Ω (from part (a))

Calculate the current I in the circuit at maximum power:

- $P = VI$ so $I = \frac{P}{V}$
- $I = \frac{5.62}{4.5}$ = 1.25 A [1 mark]

Apply the result from the Maximum Power Theorem:

- According to the Maximum Power Theorem, when maximum power is dissipated in the load resistance R_L, it must be equal to the internal resistance r
- Therefore, e.m.f. $\varepsilon = I\,(R + r) = I\,(2R)$ **OR** $= I\,(2r)$ [1 mark]

Calculate the e.m.f. of the power supply:

- $\varepsilon = I\,(2R)$ = 1.25 × (2 × 3.60) = 9 V [1 mark]

[Total: 3 marks]

The final mark can also be awarded if r = 3.60 Ω is used from part (a) and substituted into ε = I(2r) for the same final value of e.m.f.

(3 marks)

(c) Describe what happens to each of the following quantities as the value of the load resistance R_L becomes infinitely large:

i) Current

ii) Potential difference across R_L

iii) Power dissipated in R_L

Answer

The impact on current if load resistance gets infinitely large:

- If the load resistance *R* gets infinitely large, the current in the circuit gets infinitely small / tends towards zero; [1 mark]

The impact on the potential difference across R_L as the load resistance gets infinitely large:

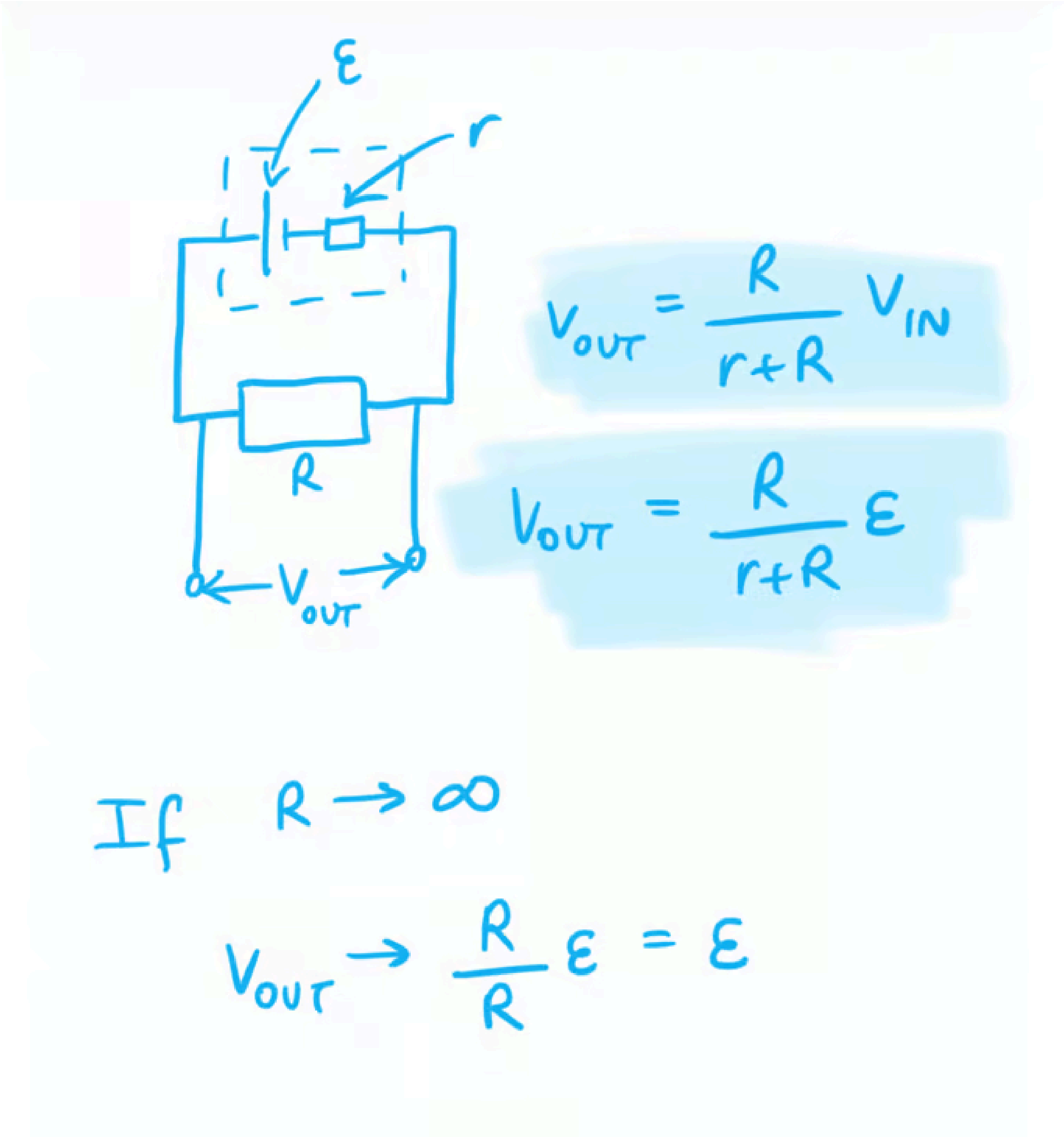
- As the load resistance gets infinitely large, the potential difference across it will tend toward the e.m.f. of the power supply; [1 mark]

The impact on the power dissipated in R_L:

- Since $P = I^2R$, as the current becomes infinitely small the power dissipated in the load resistance becomes infinitely small / tends to zero; [1 mark]

[Total: 3 marks]

This question gives you a feel for how the power dissipated changes with varying load resistance. You should be comfortable with larger resistance resulting in smaller current (which is the first awardable mark of this question). To see why the potential difference across R_L tends towards the e.m.f. of the power supply, you should treat the circuit as a simple **potential divider**. This is fine to do, because in reality, the potential of the e.m.f. is indeed divided between two series resistors – the internal resistance *r* and the load resistance . This is shown explicitly below:



Notice that if *R* ? infinity, we can effectively 'ignore' the tiny internal resistance *r* in the denominator, so the fraction becomes $\frac{R}{R}$ which is equal to 1. Overall, the potential difference across the load resistance *R* becomes equal to the e.m.f. of the cell.

This should make sense physically too: for an infinitely large external resistance, most of the electric energy per unit of charge will be used up across it (and very little in comparison used up across the internal resistance).

(3 marks)

(d) It can be shown that the power P dissipated in the load resistance R_L is zero when the load resistance is zero.

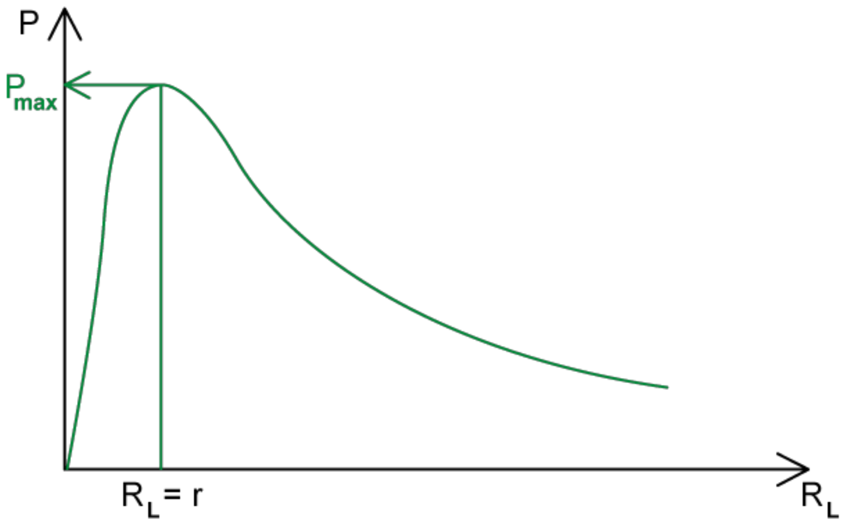
Using this, together with the result of the Maximum Power Theorem given in part (a) and your answer to part (c)(iii), sketch a graph on the axes provided in **Figure 2** to show how the power dissipated P varies with load resistance R_L.

Label the **position** of the internal resistance, r.

Figure 2

Answer

- A sketch that would gain full marks is shown below:



- Sketch begins at the origin; [1 mark]
- Line reaches a maximum power **AND** labels at $R_L = r$; [1 mark]
- Line decreases toward $P = 0$ for increasing R_L; [1 mark]

[Total: 3 marks]

This sketch brings together all of your learning from the question: namely, the power is zero when the load resistance is zero (given in the question), it decreases to zero again for very large load resistance (from part c) and that there is a maximum dissipation when $R = r$, explored in part (a).

Remember that even though you do not need to remember the Maximum Power Theorem for your examination, manipulating these equations in different contexts, and visualising them as graphs, are super useful techniques to practise.

(3 marks)

24 (a) The gravitational field strength on the surface of the moon is 1.63 N kg^{-1}.

Assuming that the moon is a uniform sphere of radius 1.74×10^6 m, calculate the mass of the moon.

Answer

List the known quantities:

- Gravitational field strength on moon, g_m= 1.63 N kg^{-1}
- Radius of the moon, r_m = 1.74×10^6 m

From data and formula sheet:

- Gravitational constant, G = 6.67×10^{-11} N m^2 kg^{-2}
- Magnitude of gravitational field strength in a radial field: $g = \frac{GM}{r^2}$

Calculate the mass of the moon:

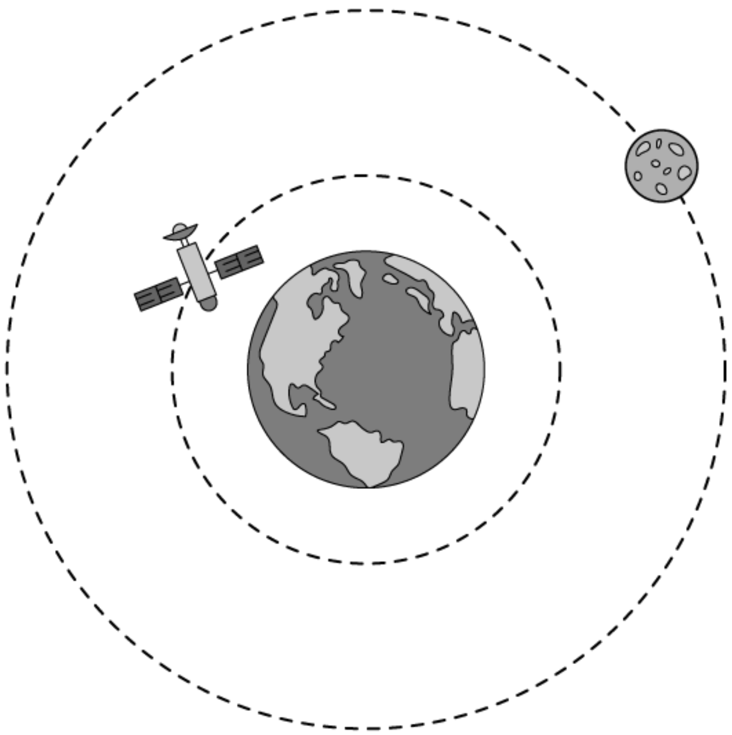
- $g_m = \frac{GM_m}{r^2}$ Therefore,
- $M_m = \frac{gr^2}{G}$
- $M_m = \frac{1.63 \times (1.74 \times 10^6)^2}{6.67 \times 10^{-11}}$ [1 mark]
- $M_m = 7.40 \times 10^{22}$ kg [1 mark]

[Total: 2 marks]

(2 marks)

(b) **Figure 1** shows the orbital paths of the International Space Station (ISS) and the moon around the Earth.

Figure 1



The ISS orbits the Earth at an average distance of 408 km from the surface of the Earth. The average distance between the centre of the Earth and the centre of the Moon is 3.80 × 10^8 m.

Using your answer from (a), calculate the maximum gravitational field strength which could be experienced by the ISS.

You can assume that both the Moon and the ISS can be positioned at any point on their orbital path.

Answer

List the known quantities:

- Separation distance between Earth and Moon, $r_{E \to M}$ = 3.80 × 10^8 m
- Mass of the Moon, m_m = 7.40 × 10^{22} kg (from part a)
- Orbital height of ISS, r_{orbit} = 408 km

From data sheet:

- Gravitational constant, $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}
- Mass of the Earth, $M_E = 5.97 \times 10^{24}$ kg
- Radius of the Earth, $r_E = 6.37 \times 10^6$ m
- Magnitude of gravitational field strength in a radial field, $g = \frac{GM}{r^2}$

Determine the separation between the ISS and centre of Earth:

- $r_{ISS \to E} = (408 \times 10^3) + (6.37 \times 10^6)$
- $r_{ISS \to E} = 6.78 \times 10^6$ m [1 mark]

Determine the separation between the ISS and the centre of the moon:

- $r_{ISS \to m} = r_{ISS \to m} + r_{E \to M}$
- $r_{ISS \to m} = 6.78 \times 10^6 + 3.80 \times 10^8$
- $r_{ISS \to m} = 3.87 \times 10^8$ m [1 mark]

Determine the equation for maximum gravitational field strength on the ISS, g_{ISS}:

- $g_{ISS} = \frac{GM_E}{(r_{ISS \to E})^2} + \frac{GM_m}{(r_{ISS \to m})^2}$ [1 mark]

Calculate the maximum gravitational field strength on the ISS, g_{ISS} :

- $g_{ISS} = \frac{(6.67 \times 10^{-11}) \times (5.97 \times 10^{24})}{(6.78 \times 10^6)^2} + \frac{(6.67 \times 10^{-11}) \times (7.40 \times 10^{22})}{(3.87 \times 10^8)^2}$
- $g_{ISS} =$ 8.66 N kg^{-1} [1 mark]

[Total: 4 marks]

Remember that the separation distance, r, between objects is taken to be the distance between the centre of mass of the objects. This means the radius of the object needs to be considered.

You need to demonstrate that you understand the maximum gravitational field strength will occur when the Earth and Moon are positioned on the same straight line as the ISS and that they will be aligned on the same side of the ISS, as shown below. When the Earth and the moon are on the same side of the ISS then the gravitational field strength each exerts on the ISS will be in the same direction, hence the equation used must show the gravitational field strengths being added together.

(4 marks)

(c) Show that for any planet or radius, r, the relationship between the gravitational field strength and the planets density can be expressed as:

$$g = \frac{4}{3}G\pi r\rho \quad \textbf{[Equation 1]}$$

where ρ is the density of the planet and G is the Gravitational constant.

Answer

From data and formula sheet:

- Magnitude of gravitational field strength in a radial field, $g = \frac{GM}{r^2}$
- Rearranging for mass, $M = \frac{gr^2}{G}$

Determine volume of the planet, V:

- $V = \frac{4}{3}\pi r^3$
- Density of planet, $\rho = \frac{M}{V}$

Substitute expressions for mass and volume:

- $\rho = \dfrac{gr^2/G}{\frac{4}{3}\pi r^3}$

- $\rho = \dfrac{3gr^2}{4G\pi r^3}$

- $\rho = \dfrac{3g}{4G\pi r}$ [1 mark]

Rearrange the equation into the required form:

- $g = \dfrac{4}{3}G\pi r\rho$ 1 mark]

[Total: 2 marks]

Remember that if a question asks you to show that an equation is correct then you should use the information you have to arrive at the equation you are being asked to show.

(2 marks)

(d) Two planets, **X** and **Y** have the same mass. Planet **X** has a radius R and the gravitational field strength on its surface is g.

The radius of Planet **Y** is twice that of planet **X** and the gravitational field strength at the surface of Planet **Y** is a quarter the value of the gravitational field strength on Planet **X**.

Use **Equation 1** to determine the volume of Planet **Y**, V_Y, in terms of the volume of Planet **X**, V_X.

Answer

List the known quantities:

- Radius of **X** = R

- Radius of **Y** = $2R$

- Gravitational field strength of **X** = g

- Gravitational field strength of **Y** = ¼ g
- Mass of **X** = m
- Mass of **Y** = m

Substitute $\rho = \frac{m}{V}$ into Equation 1and rearrange to get equation for volume:

- $g = \frac{4G\pi rm}{3V}$
- $V = \frac{4G\pi rm}{3g}$

Substitute values to get volume of Planet **X**, V_X:

- $V_X = \frac{4G\pi Rm}{3g}$ [1 mark]

Substitute values to get volume of Planet **Y**, V_Y:

- $V_Y = \frac{4G\pi(2R)m}{3\left(\frac{1}{4}g\right)}$
- $V_Y = \frac{32G\pi Rm}{3g}$ [1 mark]

Rewrite expression in terms of V_X

- $V_Y = 8 \times \frac{4G\pi Rm}{3g}$
- $V_Y = 8V_X$ [1 mark]

[Total: 3 marks]

(3 marks)

25 (a) The International Space Station (ISS) orbits the Earth at a height of 400 km above the Earth's surface. The Soyuz spacecraft is used to transport astronauts to the ISS. At lift-off, the spacecraft, with all its contents and fuel, has a mass of 308 000 kg .

Calculate the work done in taking the Soyuz spacecraft from the Earth's surface to the ISS.

Assume that the mass of the Soyuz spacecraft remains constant.

Answer

List the known quantities:

- Height of ISS above the Earth's surface, h = 400 km
- Mass of Soyuz spacecraft, m_s = 308 000 kg

From data booklet:

- Gravitational constant, $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}
- Mass of the Earth, $M_E = 5.97 \times 10^{24}$ kg
- Mean radius of the Earth, $r_E = 6.37 \times 10^6$ m
- Gravitational potential, $V = -\frac{GM}{r}$
- Work done, $W = m\Delta V$

Calculate the gravitational potential difference, ΔV, between the surface of the Earth and the ISS:

- $\Delta V = V_{ISS} - V_E$
- $\Delta V = \left(-\frac{GM_E}{h + r_E}\right) - \left(-\frac{GM_E}{r_E}\right)$ [1 mark]
- $\Delta V = -\frac{(6.67 \times 10^{-11})(5.97 \times 10^{24})}{(400 \times 10^3 + 6.37 \times 10^6)} + \frac{(6.67 \times 10^{-11})(5.97 \times 10^{24})}{(6.37 \times 10^6)}$

- $\Delta V = 3.69 \times 10^6$ J kg^{-1} [1 mark]

Calculate work done in taking the Soyuz spacecraft from the Earth's surface to the ISS:

- $W = m_s \Delta V$
- $W = (308\ 000) \times (3.69 \times 10^6)$
- $W = 1.14 \times 10^{12}$ J [1 mark]

[Total: 3 marks]

Remember that for questions which involve sending an object into space we cannot use the equation for gravitational potential energy, E = $mg\Delta h$ to calculate the work required because the gravitational field strength, g, changes appreciably between the two points.

(3 marks)

(b) In reality it takes less work than your answer to part (a) to get the Soyuz rocket from the surface of the Earth to the ISS.

Discuss a possible reason for the difference in values of work done.

Answer

State the reason for difference in work done values

- When the Soyuz travels from the surface of the Earth to the ISS it will burn fuel
- This will cause the mass of the Soyuz to decrease during its flight; [1 mark]

Explain the reason for the difference in work done values:

- From the equation $W = m_s \Delta V$, the mass of the Soyuz is **directly proportional** to work done in getting it from the surface of the Earth to the ISS
- Therefore, if the mass decreases, then the work done will decrease; [1 mark]

[Total: 2 marks]

(2 marks)

(c) Calculate the work done in taking the Soyuz spacecraft from the Earth's surface to a point where the Earth's gravitational effect is negligible.

Answer

List the known quantities:

- Mass of Soyuz spacecraft, m_s = 308 000 kg
- Gravitational potential at infinity, V_∞ = 0

From data and formula sheet:

- Gravitational constant, $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}
- Mass of the Earth, $M_E = 5.97 \times 10^{24}$ kg
- Mean radius of the Earth, $r_E = 6.37 \times 10^6$ m
- Gravitational potential, $V = -\frac{GM}{r}$
- Work done, $\Delta W = m\Delta V$

Calculate the gravitational potential difference between the surface of the Earth and ISS:

- $\Delta W = m_s \Delta V = m_s\left(V_\infty - V_E\right)$
- $\Delta W = 308\,000 \times \left(0 - \left(-\frac{GM_E}{r_E}\right)\right)$
- $\Delta W = 308\,000 \times \left(0 - \frac{(6.67 \times 10^{-11})(5.97 \times 10^{24})}{(6.34 \times 10^6)}\right)$ [1 mark]
- $\Delta W = 308\,000 \times \left(6.25 \times 10^7\right)$
- $\Delta W = 1.93 \times 10^{13}$ J [1 mark]

[Total: 2 marks]

Gravitational potential is the work done per unit mass in bringing an object from infinity to that point. When an object is at infinity then the gravitational potential is zero, it is no longer affected by the gravitational field produced by other objects.

(2 marks)

(d) Explain why there is no work done by the ISS when it maintains a constant orbit around the Earth.

Answer

From data and formula sheet:

- Gravitational potential, $V = -\frac{GM}{r}$
- Work done, $\Delta W = m\Delta V$

Explanation:

- From the equation $V = -\frac{GM}{r}$, the gravitational constant G, and the mass of the Earth M, are constants
- So if the ISS maintains a constant orbital radius, r, around the Earth, then the gravitational potential, V, at every point in the orbit will be equal; [1 mark]
- As there is no change in gravitational potential in the orbit, $\Delta V = 0$
- Using the work done equation, $\Delta W = m\Delta V$, where $\Delta V = 0$, then $\Delta W = 0$; [1 mark]
- Therefore, there is no work done by the ISS when it maintains a constant orbit around the Earth

[Total: 2 marks]

(2 marks)

26 (a) The orbits of the Earth and Jupiter are very nearly circular, with radii of 150 × 10^9 m and 778 × 10^9 m respectively. It takes Jupiter 11.8 years to complete a full orbit of the Sun.

Show that the values in this question are consistent with Kepler's third law.

Answer

List the known quantities:

- Orbital radius of the Earth, r_E = 150 × 10^9 m
- Orbital radius of Jupiter, r_J = 778 × 10^9 m
- Periodic Time for Jupiter to complete an orbit, T_J = 11.8 years
- Periodic Time for the Earth to complete an orbit, T_E = 1 year

To verify values in the question are consistent with Kepler's third law:

- $T^2 \propto r^3$
- $\frac{T^2}{r^3}$ = constant

Calculate the constant using values for the Earth:

- $\frac{1^2}{(150 \times 10^9)^3}$ = 2.96 × 10^{-34}

Calculate the constant using values for Jupiter:

- $\frac{11.8^2}{(778 \times 10^9)^3}$ = 2.96 × 10^{-34} [1 mark]

Must show calculation for both Earth ***and*** *Jupiter*

Conclusion:

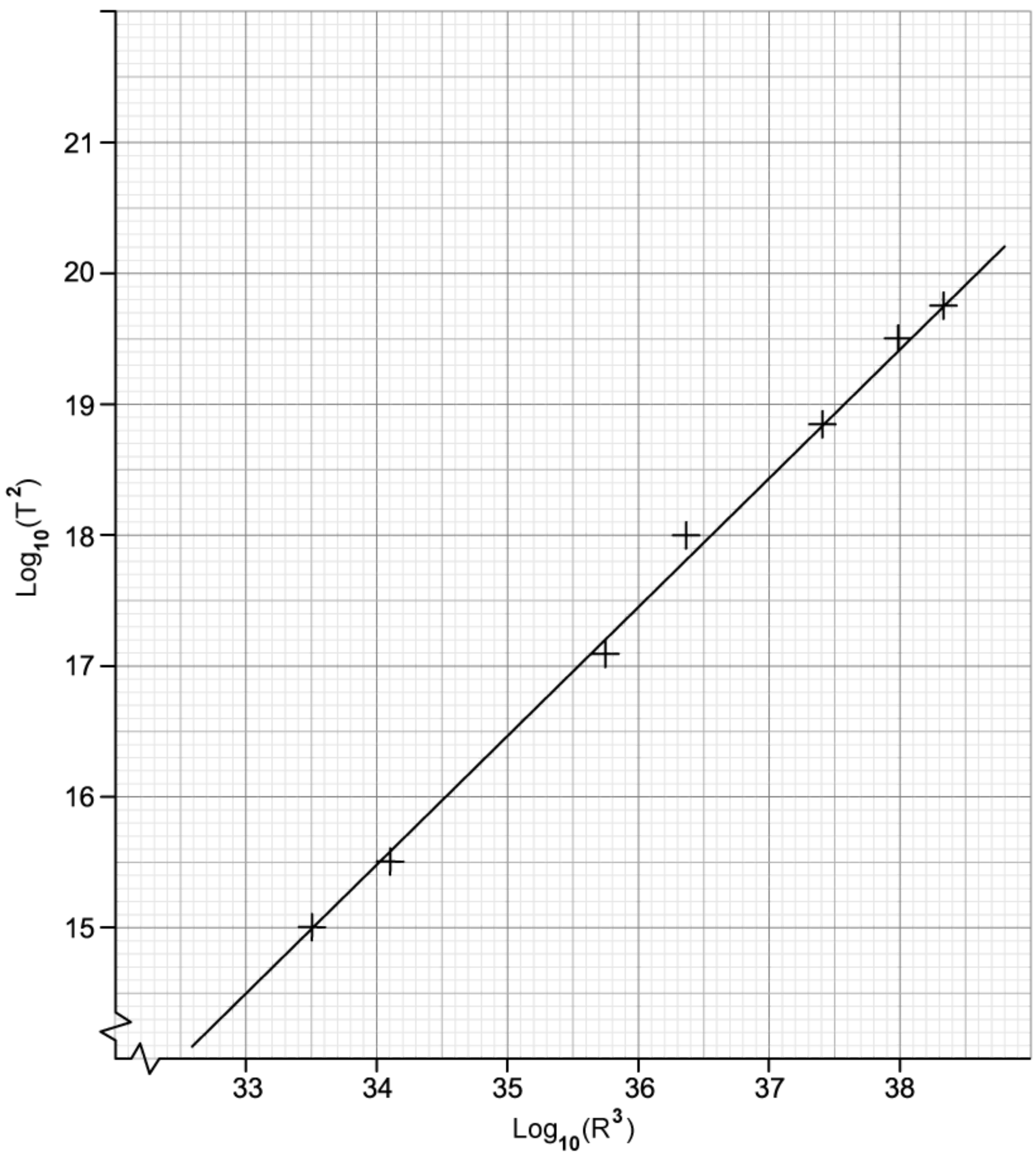
- The constants calculated from the values for the Earth and Jupiter are the same
- This shows that they are consistent with Kepler's third law; [1 mark]

[Total: 2 marks]

(2 marks)

(b) Data from the orbits of different planets around our Sun is plotted in a graph of log (T^2) against log (R^3) as shown in **Figure 1**, where *T* is the orbital period and *R* is the radius of the planets orbit.

Figure 1



Calculate the percentage error for the mass of the Sun obtained from the graph in **Figure 1**.

Answer

From data sheet:

- Newton's gravitational constant, G = 6.67 × 10^{-11} N m^2 kg^{-2}
- Mass of the Sun, M_s = 1.99 × 10^{30} kg

State Kepler's third law equation:

- $T^2 = \frac{4\pi^2 r^3}{GM_S}$

Relate Kepler's equation to the graph in **Figure 1**:

- $\log\left(T^2\right) = \log\left(\frac{4\pi^2 r^3}{GM_S}\right)$
- From log (AB) = log(A) + log(B), $\log\left(T^2\right) = \log\left(\frac{4\pi^2}{GM_S}\right) + \log\left(r^3\right)$
- $\log\left(T^2\right) = \log\left(r^3\right) + \log\left(\frac{4\pi^2}{GM_S}\right)$ [1 mark]

By relating to the straight-line equation y = mx + c:

- y-axis = $\log\left(T^2\right)$
- x-axis = $\log\left(r^3\right)$
- y-intercept value = $\log\left(\frac{4\pi^2}{GM_S}\right)$

Choose a co-ordinate of a point on the line in **Figure 1**:

- Co-ordinates of a point on the line: (33.5, 15)

Substitute this into the equation above:

- $15 = 33.5 + \log\left(\frac{4\pi^2}{(6.67\times10^{-11})M_S}\right)$ [1 mark]

- $-18.5 = \log\left(\frac{4\pi^2}{(6.67\times10^{-11})M_S}\right)$

- $3.16 \times 10^{-19} = \frac{4\pi^2}{(6.67\times10^{-11})M_S}$

- $M_S = \frac{4\pi^2}{(6.67\times10^{-11})(3.16\times10^{-19})}$

- $M_S = 1.87 \times 10^{30}$ kg [1 mark]

Calculate the percentage error for the mass of the Sun obtained from the graph in **Figure 1**:

- % Error = $\frac{Actual\ value - Calculated\ value}{Actual\ value} = \frac{(1.99\times10^{30})-(1.87\times10^{30})}{(1.99\times10^{30})}$ [1 mark]

- % Error = 6.03% [1 mark]

[Total: 5 marks]

This question requires you to know that log (AB) = log (A) + log (B).

(5 marks)

(c) The Sun is about 8 kpc (kilo-parsecs) from the centre of the Milky Way Galaxy and moves at about 230 km/s. Calculate the orbital period of the Sun around the centre of the Milky Way Galaxy.

1 kpc = 3260 light years

Answer

List the known quantities:

- Distance between the Sun and the centre of the Milky Way galaxy, r_S = 8 kpc

- Speed of the Sun in orbit, v_S = 230 km s^{-1} = 230 × 10^3 m s^{-1}
- 1 kpc = 3 260 light years

Convert 8 kpc to m:

- 8 kpc = 8 × 3 260 = 26 080 light years
- 26 080 light years = 26 080 × (3 × 10^8) × (365.25 × 24 × 60 × 60) = 2.47 × 10^{20} m
- 8 kpc = 2.47 × 10^{20} m [1 mark]

Calculate the period of the orbit:

- T = $\frac{2\pi r}{v}$ [1 mark]
- T = $\frac{2\pi \times (2.47 \times 10^{20})}{230 \times 10^3}$ = 6.74 × 10^{15} s = 213 million years [1 mark]

[Total: 3 marks]

Remember that one light year is the **distance** light will travel in one year. To calculate that distance in metres you need to multiply the speed of light, 3 × 10^8 m s^{-1}, by the number of seconds in one year.

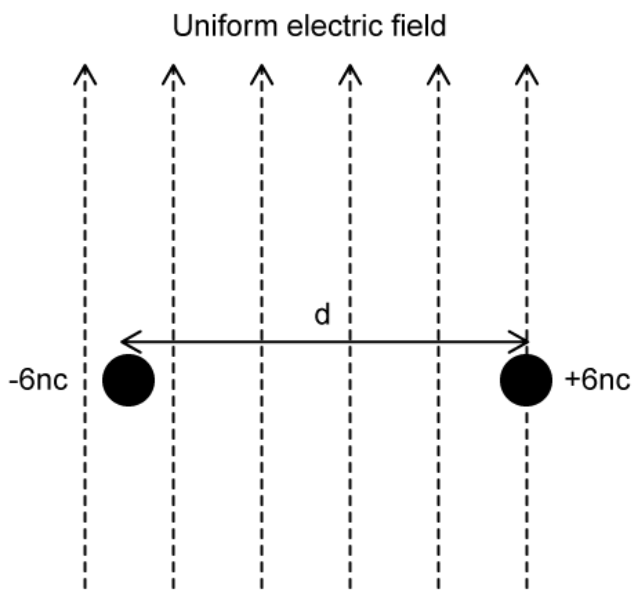
As we know the orbital speed and the radius of the orbit we can calculate the period of the orbit using *time = distance × speed*. We are given the speed in the question and the distance for one complete orbit can be found by calculating the circumference of the orbital path using 2πr.

(3 marks)

27 (a) An electric dipole shown in **Figure 1** consists of two separated point charges of the opposite charge.

A pair of charges +6 nC and –6 nC are separated by a distance *d* of 3.15×10^{-14} m to make an electric dipole and is placed in a uniform electric field.

Figure 1



Describe what happens to the dipole in the field.

Answer

The dipole:

- Rotates clockwise **OR** the –6 nC charge travels upwards as the +6 nC travels downwards (at the same time); [1 mark]
- Creates a couple; [1 mark]

[Total: 2 marks]

Remember to specify the direction of the rotation for full marks.

(2 marks)

(b) The electric field strength of the uniform electric field is 5.1 N.

Calculate the moment of the electric dipole.

Answer

List the known quantities:

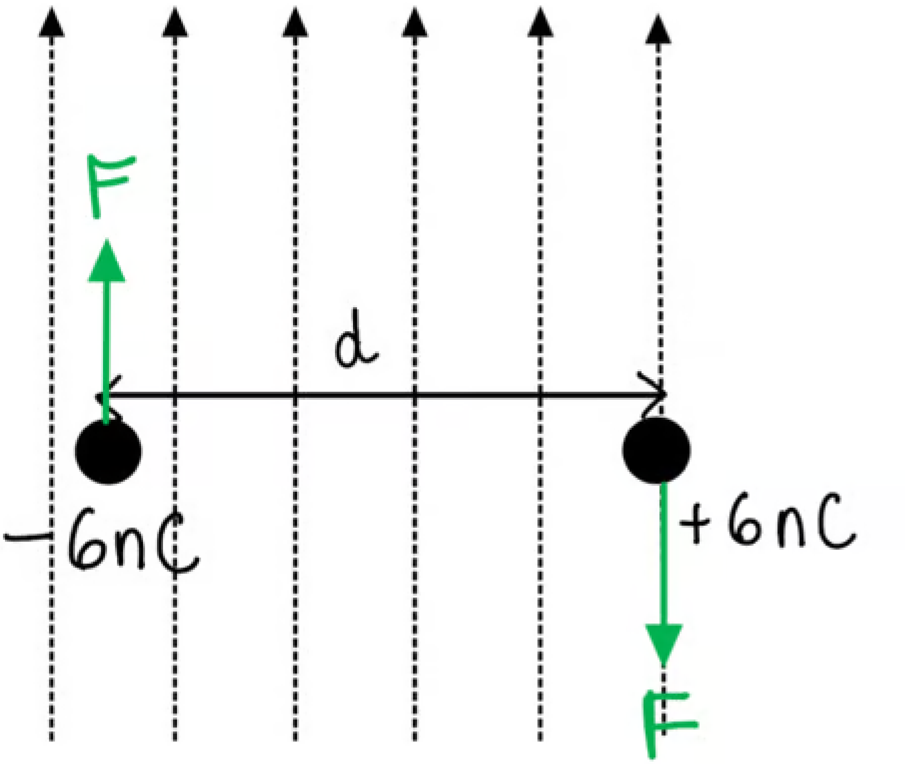
- Electric field strength, E = 5.1 N
- Distance between charge, $d = 3.15 \times 10^{-14}$ m
- Magnitude of each charge, Q = 6 nC = 6×10^{-9} C

From data sheet:

- Force on a charge, $F = EQ$
- Moment = Fd

Draw a diagram of the situation:

- Since the force on each charge is equal and opposite, this forms a couple
- The moment of a couple (or the torque) = force F × perpendicular distance, d



Calculate electric force on each charge, *F*:

- $F = 5.1 \times (6 \times 10^{-9}\ \text{C}) = 3.06 \times 10^{-8}$ N [1 mark]

Calculate moment of the electric dipole:

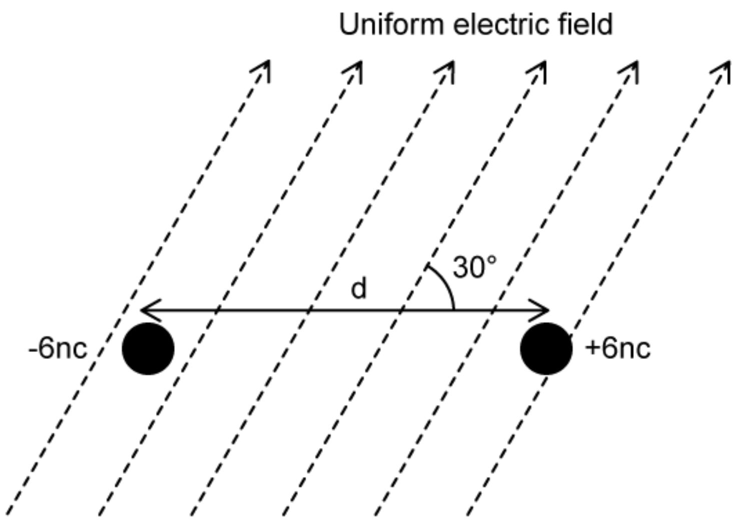
- Moment of a couple = $(3.06 \times 10^{-8}) \times (3.15 \times 10^{-14})$
- Moment = 9.64×10^{-22} N m [1 mark]

[Total: 2 marks]

(2 marks)

(c) The dipole is reset to its original horizontal position, and the same electric field is now placed at an angle of 30º to the horizontal plane of the dipole as shown in **Figure 2**.

Figure 2



Show that the moment of the electric dipole is now halved.

Answer

List the known quantities:

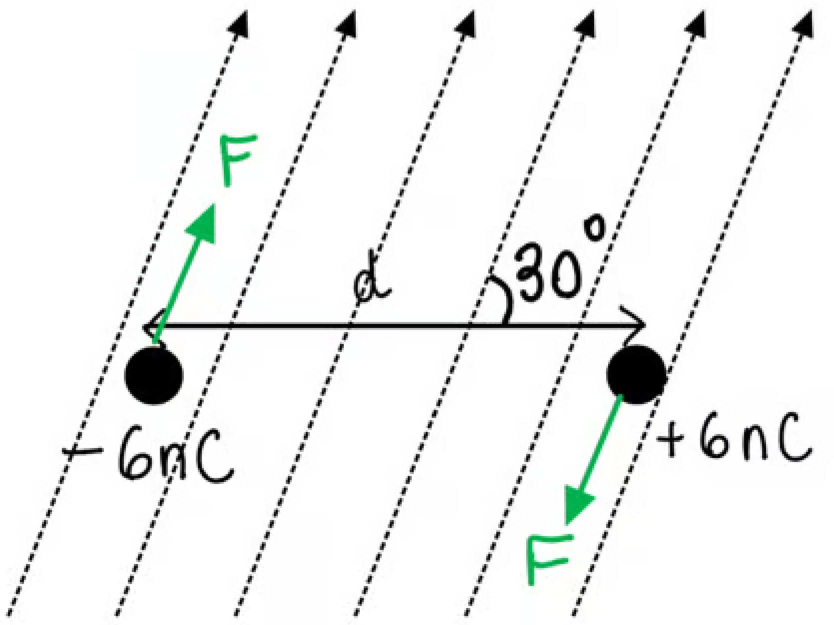
- Electric force on each charge, $F = 3.06 \times 10^{-8}$ N (from part b)
- Angle = 30°

From data sheet:

- Moment = Fd

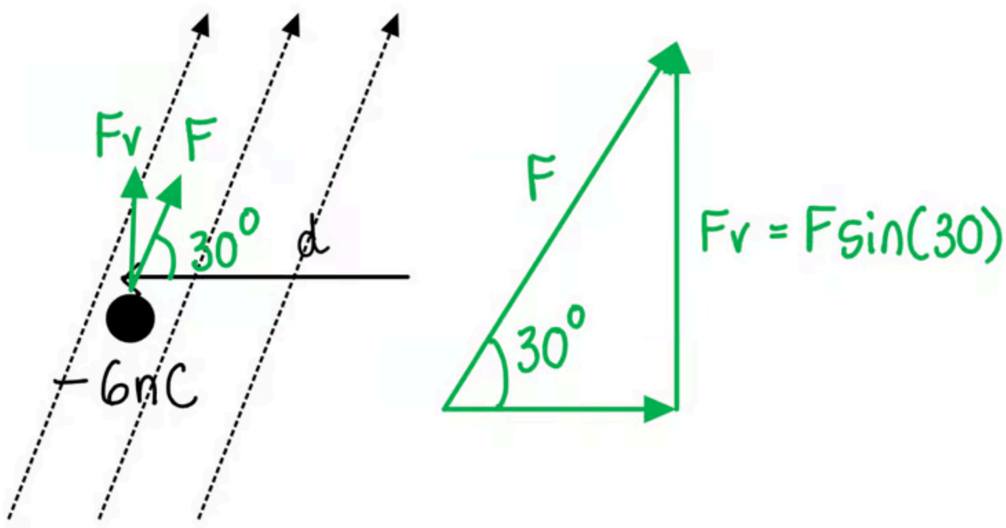
Draw a diagram of the situation:

- The electric force on each charge is still equal and opposite, and parallel to the electric field lines



- The moment of a couple (or the torque) = force *F* × perpendicular distance *d*
- Therefore, the force needed is the vertical component of the electric force since this is perpendicular to the distance between the charges

Calculate vertical component of the force, F_v:



- F_v = *F* sin(30) [1 mark]

- $\sin(30) = \frac{1}{2}$ therefore the $F_v = \frac{F}{2}$ [1 mark]
- E and Q are constant

Determine the moment of the electric dipole:

- Since moment of the couple (dipole) = $F_v d = \frac{Fd}{2}$, the moment of the electric dipole has halved; [1 mark]
- d is constant

[Total: 3 marks]

(3 marks)

28 (a) Calculate the kinetic energy of an α particle travelling at a speed of 3.50×10^{7} m s^{-1} when relativistic effects are ignored.

Answer

List the known quantities:

- Speed, $v = 3.50 \times 10^{7}$ m s^{-1}
- Permittivity of free space, $\varepsilon_0 = 8.85 \times 10^{-12}$ F m^{-1}
- Mass of proton / mass of neutron = 1.67×10^{-27} kg

Calculate the mass of an alpha particle, m:

- An alpha particle is a helium nucleus with 2 protons and 2 neutrons
- Total mass = $4 \times (1.67 \times 10^{-27}) = 6.68 \times 10^{-27}$ kg [1 mark]

Calculate kinetic energy, KE:

- $KE = \frac{1}{2}mv^2 = \frac{1}{2} \times (6.68 \times 10^{-27}) \times (3.50 \times 10^{7})^2$ [1 mark]
- $KE = 4.09 \times 10^{-12}$ J [1 mark]

[Total: 3 marks]

(3 marks)

(b) Calculate the closest distance of approach for a head-on collision between an α particle travelling at a speed of 3.50×10^{7} m s^{-1} and a gold nucleus. The proton number of gold is 79.

Assume that the gold nucleus remains stationary during the collision.

Answer

List the known quantities:

- Kinetic energy of particle, $KE = 4.09 \times 10^{-12}$ J (from part a)
- Permittivity of free space, $\varepsilon_0 = 8.85 \times 10^{-12}$ F m^{-1}

- Charge of an electron, e = 1.60 × 10^{-19} C

Calculate the charge of alpha particle and gold nucleus:

- An alpha particle is a helium nucleus with 2 protons and 2 neutrons
- Charge of alpha particle, Q_α = +2e **AND** charge of gold nucleus, Q_{gold} = +79e [1 mark]

Calculate the closest distance of approach:

- The kinetic energy lost by the alpha particle = gain in electric potential energy as it approaches the gold nucleus
- Gain in potential energy = $\frac{Q_\alpha Q_{gold}}{4\pi\varepsilon_0 r}$ = loss in KE [1 mark]
- $KE = \frac{Q_\alpha Q_{gold}}{4\pi\varepsilon_0 r}$ therefore, $r = \frac{Q_\alpha Q_{gold}}{4\pi\varepsilon_0 KE}$
- $r = \frac{2\times 79\times(1.60\times 10^{-19})^2}{4\pi\times(8.85\times 10^{-12})\times(4.09\times 10^{-12})}$ [1 mark]
- r = 8.89 × 10^{-15} m = 8.89 fm [1 mark]

[Total: 4 marks]

(4 marks)

(c) The alpha particle is fired towards a polonium nucleus, which contains more protons than the gold nucleus. The α particle is given the same initial kinetic energy as in part (a).

State and explain, without further calculation, any changes that occur to the closest distance of approach.

You can ignore any recoil effects.

Answer

The distance of closest approach will:

- Increase; [1 mark]

Explanation:

- An increased number of protons in polonium will result in a **greater** (electrostatic) **repulsive force** between the polonium nucleus and the alpha particle; [1 mark]

[Total: 2 marks]

(2 marks)

29 (a) An air-filled parallel-plate capacitor is charged from a source of e.m.f. The electric field has a strength E between the plates. The capacitor is disconnected from the source of e.m.f. and the separation between the isolated plates is doubled.

State the final value of electric field strength between the plates and explain your answer.

Answer

Final value of electric field:

- Unchanged / E; [1 mark]

Explanation:

Any **two** from:

- When disconnected the charge on the plates remains constant; [1 mark]
- The p.d. between the plates increases as the plates are separated; [1 mark]
- The electric field strength remains constant when work is done on the plates to separate them; [1 mark]
- Charge remains constant so electric field strength remains constant since $E \propto Q$ / $E = \frac{V}{d} = \frac{Q}{\varepsilon_0 A}$ [1 mark]
- The energy of the capacitor increases due to the work done in moving the plates apart; [1 mark]

[Total: 3 marks]

E being unchanged when the distance between the plates, d increases may seem counter-intuitive. The extra energy required to keep E the same comes from the work put in to separate the plates. Doubling d also doubles V.

(3 marks)

(b) The capacitor consists of two parallel square plates of side l separated by distance d. The capacitor contains air as a dielectric. The capacitance of the arrangement is C.

Determine the capacitance of a second capacitor with square plates of side $2l$ separated by air at a distance of $\frac{d}{4}$.

Answer

List the known quantities:

- Area of second capacitor, $A = (2l)^2$
- Separation of second capacitor, $d = \frac{d}{2}$
- Relative permittivity of air, $\varepsilon_r = 1.0$

From data sheet:

- Capacitance, $C = \frac{A\varepsilon_0\varepsilon_r}{d}$

Calculate the new capacitance, C_2:

- $C_2 = \frac{(2l)^2\varepsilon_0}{\frac{d}{4}} = \frac{4 \times 4l^2\varepsilon_0}{d} = \frac{16l^2\varepsilon_0}{d}$ [1 mark]
- Capacitance of second capacitor, $C_2 = 16C$ [1 mark]

[Total: 2 marks]

(2 marks)

(c) The first capacitor is then filled with a dielectric which increases its capacitance to 2*C*.

The second capacitor still has air as its dielectric.

Determine the relative permittivity of the dielectric in the first capacitor.

Answer

List the known quantities:

- Capacitance of first capacitor, $C_1 = 2C = \frac{l^2 \varepsilon_0 \varepsilon_r}{d}$

- Capacitance of second capacitor, $C_2 = 16C = \frac{16 l^2 \varepsilon_0}{d}$ (from part b)

Calculate the ratio, $\frac{C_1}{C_2}$:

- $\frac{C_1}{C_2} = \frac{2C}{16C} = \frac{2}{16}$

- $\frac{C_1}{C_2} = \frac{l^2 \varepsilon_0 \varepsilon_r}{d} \div \frac{16 l^2 \varepsilon_0}{d} = \frac{\varepsilon_r}{16}$ [1 mark]

- $\frac{\varepsilon_r}{16} = \frac{2}{16}$

Calculate relative permittivity, ε_r

- $\varepsilon_r = 2 \text{ F m}^{-1}$ [1 mark]

[Total: 2 marks]

(2 marks)

(d) One of the capacitors is a variable capacitor which can be varied from 8.0 to 4.0 × 10^{-12} F.

The capacitor is set to its maximum capacitance and fully charged using a 36 V supply. The capacitor is then disconnected from the supply and isolated. Finally, the capacitance is reduced to its minimum value without any charge being lost by the capacitor.

Determine the potential difference (p.d.) across the capacitor and the charge it stores after the capacitance has been reduced.

Answer

List the known quantities:

- Maximum capacitance, $C = 8.0 \times 10^{-12}$F
- Minimum capacitance, $C = 4.0 \times 10^{-12}$F
- m.f. = 36 V

From data sheet:

- Capacitance, $C = \frac{Q}{V}$

Calculate the change in p.d, ΔV:

- $C \propto \frac{1}{V}$
- Change in capacitance = $\frac{4.0 \times 10^{-12}}{8.0 \times 10^{-12}} = \frac{1}{2}$
- Therefore, $\Delta V = 2 \times 36 = 72$ V [1 mark]

Calculate the charge stored:

- Charge, $Q = CV = (4.0 \times 10^{-12}) \times 72$ [1 mark]
- $Q = 2.88 \times 10^{-10} = 2.9 \times 10^{-10}$C [1 mark]

[Total: 3 marks]

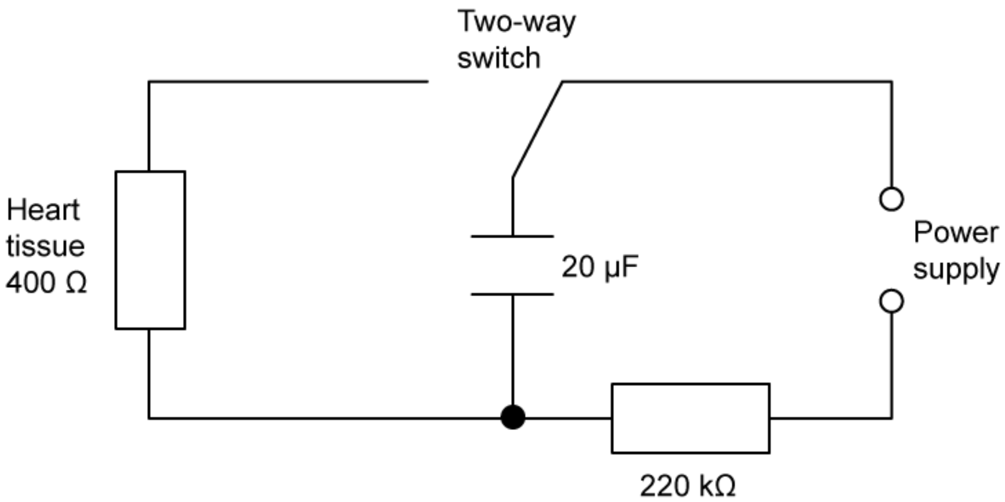
When the capacitor is discharged, its variable capacitance is reduced. Therefore the potential difference across it must increase so that the charge remains unchanged.

(3 marks)

30 (a) Pacemakers are devices which are used to deliver short pulses of charge to a patient who is suffering from a cardiac arrest.

Figure 1 shows a circuit that is used in such a pacemaker.

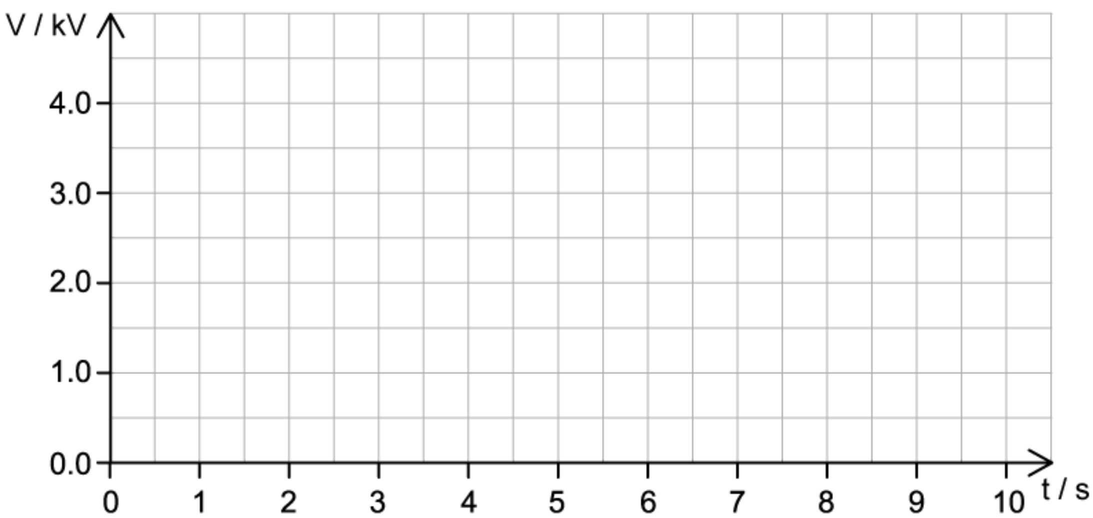
Figure 1



The capacitor is initially uncharged when at t = 0 s, then the two-way switch connects it to a power supply of 4.0 kV.

Sketch the graph on the axes provided in **Figure 2** that shows how the voltage across the capacitor varies over 10 seconds.

Figure 2



Answer

List the known values:

- Resistance R in the charging circuit = 220 kΩ
- Capacitance C = 20 μF
- Heart tissue resistance R_H = 400 Ω
- Power supply voltage = 4 kV

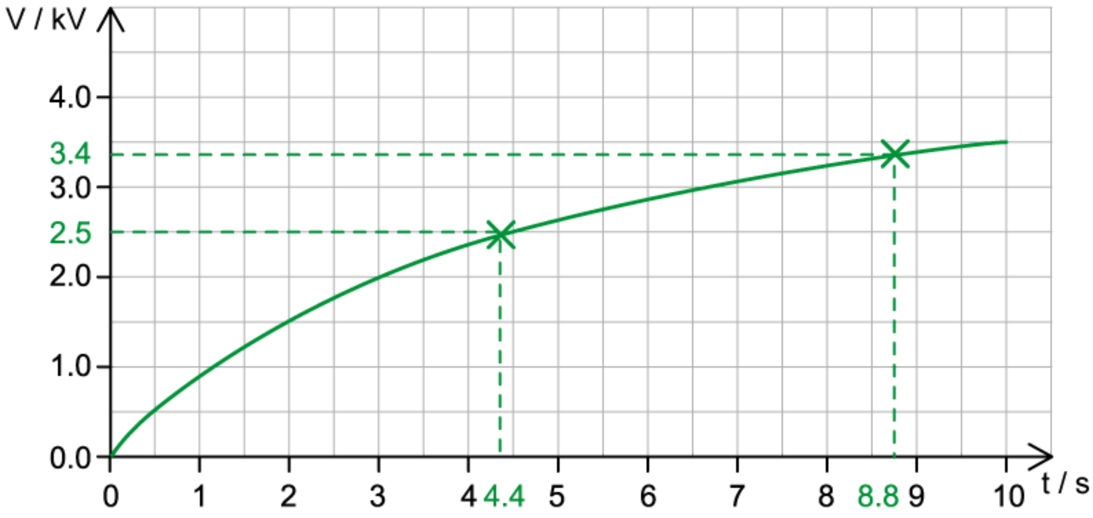
Calculate the time constant of the charging circuit:

- $\tau = RC$
- $\tau = (220 \times 10^3) \times (20 \times 10^{-6}) = 4.4$ s [1 mark]

Calculate the voltage across the capacitor after two time-constants:

- In the first 10 seconds of charging, two time-constants will pass
- After one time constant, the voltage across the capacitor will be 63% of the supplied voltage, which is 4000V
- 0.63 × 4000 = 2520 V ≈ 2.5 kV
- After another time constant (i.e., at 8.8 s) the voltage across the capacitor will be 86% of the supplied voltage
- 0.86 × 4000 = 3440 V ≈ 3.4 kV

Sketch the graph:



- Correct charging curve with decreasing gradient; [1 mark]
- Passing through (4.4, 2.5) **AND** (8.8, 3.4); [1 mark]

[Total: 3 marks]

It's useful to know how to calculate the voltage across the charging capacitor in terms of the number of time constants that pass.

In order to do this, you just substitute a multiple of the time constant into the charging equation, e.g., $t = 2RC$ represents a time of two time-constants. If you do this, you can show that the voltage across the capacitor V is 86% of the maximum voltage V_0. This is illustrated below:

$$V = V_0\left(1 - e^{-t/RC}\right)$$ charging equation

When $t = 2RC$ (i.e. two time constants)

$$V = V_0\left(1 - e^{-(2RC)/RC}\right)$$

$$= V_0\left(1 - e^{-2}\right)$$ $e^{-2} = \frac{1}{e^2} = 0.135...$

$$= V_0\left(1 - 0.135...\right)$$

$$V = 0.86\,V_0$$ so after $t = 2RC$, the voltage is 86% of the maximum voltage!

(3 marks)

(b) Show that the time taken for the charge stored in the capacitor to reach 5.0 mC is approximately 0.3 s.

Answer

List the known quantities:

- Charge, Q = 5.0 mC = 5.0 × 10^{-3} C

- Time, t = 0.3 s
- Maximum voltage, V_0= 4 kV = 4000 V
- Capacitance C = 20 μF = 20 × 10^{-6} F
- Time constant, τ = 4.4 s (from part a)

Calculate the maximum charge:

- The maximum charge $Q_0 = CV_0$
- Q_0= (20 × 10^{-6}) × 4000 = 0.08 C [1 mark]

Rearrange charge equation for time, t:

- The charge stored on a charging capacitor, Q, is given by $Q = Q_0(1 - e^{-\frac{t}{\tau}})$
- $\frac{Q}{Q_0} = 1 - e^{-\frac{t}{\tau}}$
- $e^{-\frac{t}{\tau}} = 1 - \frac{Q}{Q_0}$
- $\ln(e^{-\frac{t}{\tau}}) = \ln(1 - \frac{Q}{Q_0})$
- $-\frac{t}{\tau} = \ln(1 - \frac{Q}{Q_0})$
- $t = -\tau \ln(1 - \frac{Q}{Q_0})$

Substitute values into equation time t equation:

- $t = -(4.4) \times \ln(1 - \frac{5.0 \times 10^{-3}}{0.08})$ [1 mark] = 0.28 s [1 mark]

[Total: 3 marks]

(3 marks)

(c) Pacemaker manufacturers need efficient ways of measuring how quickly the charge and energy are released by capacitors.

One approach uses the 'exponential factor' **σ**, which is a ratio of the energy remaining in a capacitor E to its initial energy stored E_0 . For a given time interval, a smaller ratio indicates a quicker greater discharge of energy.

Show that the exponential factor **σ** is given by:

$$\sigma = e^{-\frac{2t}{RC}}$$

Answer

State equation for the voltage V of a discharging capacitor:

- $V = V_0 e^{-\frac{t}{RC}}$

Write equations for the initial energy E_0 and final energy E stored in a discharging capacitor:

- Initial energy stored $E_0 = \frac{1}{2} C(V_0)^2$

  AND

  Final energy stored $E = \frac{1}{2}C(V)^2 = \frac{1}{2}C(V_0 e^{-\frac{t}{RC}})^2$ [1 mark]

Take the ratio of the two equations and simplify fully:

- $\frac{E}{E_0} = \frac{\frac{1}{2}C\left(V_0 e^{-\frac{t}{RC}}\right)}{\frac{1}{2}C(V_0)^2}$

Simplify equations fully:

- $\dfrac{E}{E_0} = \dfrac{\frac{1}{2}C(V_0)^2 e^{-\frac{2t}{RC}}}{\frac{1}{2}C(V_0)^2} = e^{-\frac{2t}{RC}}$ [1 mark]

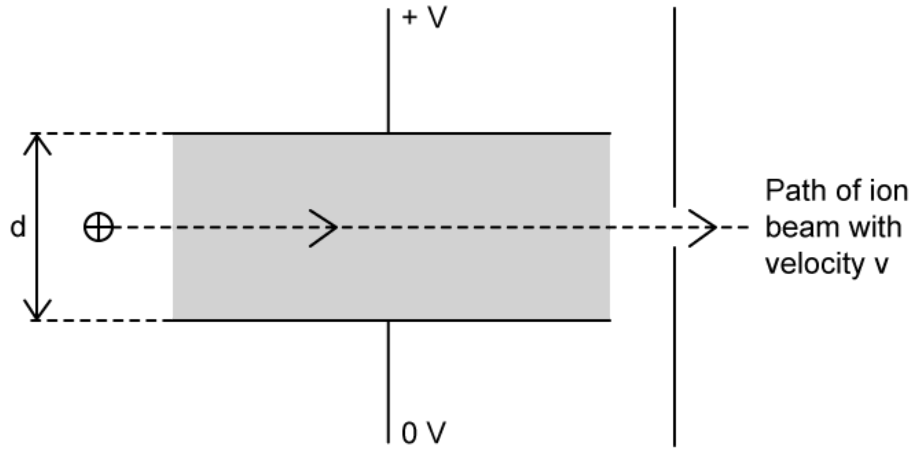
[Total: 2 marks]

Remember when raising a power in an exponential expression, the new power is found by **multiplying** the two powers together. E.g $(x^a)^b = x^{ab}$ therefore for exponentials, $(e^x)^2 = e^{2x}$.

(2 marks)

31 (a) A mass spectrometer is an instrument used to analyse the atoms present in a sample of gas. The atoms are ionised in order to produce positive ions and then passed through a velocity selector, as shown in **Figure 1**.

Figure 1



The velocity selector consists of electric and magnetic fields acting at right angles to each other.

The magnetic field is directed into the page. The electric field is produced by a pair of parallel plates of separation *d* with a high potential difference, *V*, across them.

Discuss how the velocity of an ion affects the path it takes through the velocity selector.

Answer

The effect of the electric and magnetic fields on the ion:

- The electric field produces a constant downward force on the ion; [1 mark]
- The magnetic field produces an upward force on the moving ion; [1 mark]

The effect of velocity on the path of the ions:

- Ions entering the velocity selector with greater velocities will emerge higher than those with lower velocities

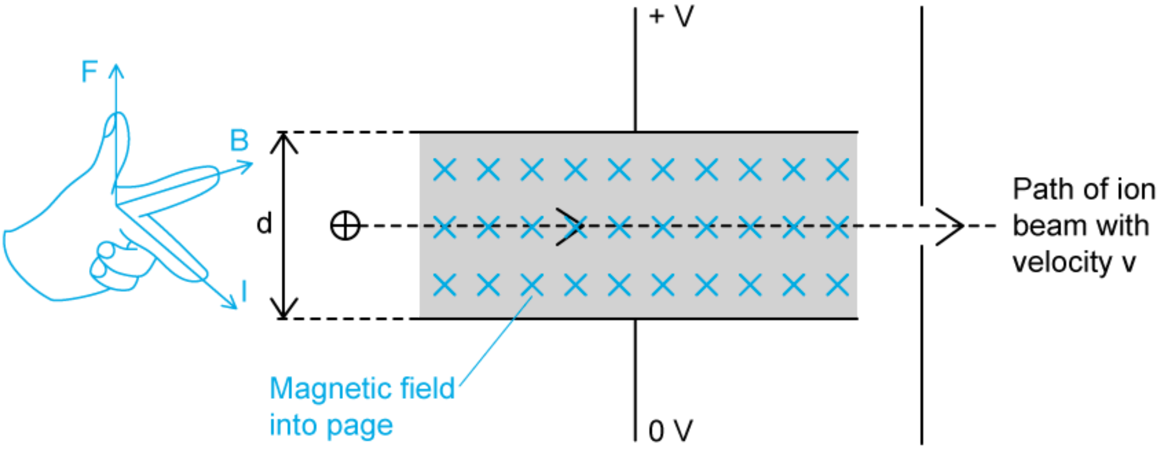
OR

- Ions entering the velocity selector with lower velocities will emerge lower than those with higher velocities; [1 mark]

[Total: 3 marks]

The magnetic force acting on the ion can be calculated using $F = BQv$. B, the magnetic field and Q, the charge of the ion, are both **constant**, therefore the force acting on the ion is directly proportional to its velocity.

The direction of the magnetic force can be determined using Fleming's left hand rule, where the first finger (direction of magnetic field) points into the page, the second finger (direction of current) points to the right and the thumb then points upwards to give the direction of the magnetic force.



(3 marks)

(b) The magnetic field strength is 260 mT, the potential difference across the plates is 1200 V and the plate separation is 2 cm.

Calculate the velocity of each ion passing straight through the selector.

Answer

List the known quantities:

- Magnetic field strength, B = 260 mT
- Potential difference across the plates, V = 1200 V
- Plate separation, d = 2 cm = 0.02 m

Equate force due to magnetic field and force due to electric field:

- Force on a moving charge, $F = BQv$
- Force on a charge due to electric field, $F = EQ$
- Field strength for a uniform field, $E = \frac{V}{d}$
- $BQv = \frac{VQ}{d}$ [1 mark]

Rearrange equation to find the velocity of the ions, v, and substitute in values:

- $v = \frac{V}{Bd}$
- $v = \frac{V}{Bd} = \frac{1200}{(260 \times 10^{-3}) \times 0.02} = 2.31 \times 10^5$ m s^{-1} [1 mark]

[Total: 2 marks]

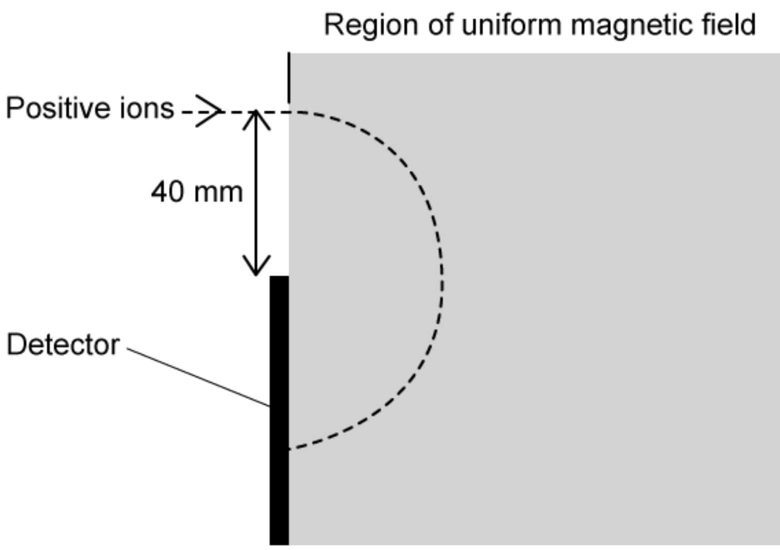
We are told in the question that the ion passes straight through the velocity selector, therefore the **upwards** force on the ion due to the magnetic field must be equal to the **downwards** force on the ion due to the electric field.

(2 marks)

(c) A mixture of singly-ionised lithium isotopes of mass 6u and 7u leave the velocity selector with a velocity of 4.8×10^4 m s^{-1}.

The ions enter a uniform magnetic field which exerts a force on them causing them to strike the detector. The upper edge of the detector is 40 mm from the point at which the charged ions enter the field, as shown in **Figure 2**.

Figure 2



Calculate the maximum magnetic field strength that can be applied in order for both ions to be detected.

Answer

List the known quantities:

- Velocity of lithium ions, $v = 4.8 \times 10^4$ m s^{-1}
- Charge of each singly-ionised ion, $Q = +e$
- Distance between the top of detector and entry point of ions, d = 40 mm

From data sheet:

- Atomic mass unit, $u = 1.661 \times 10^{-27}$ kg
- Elementary charge, e = 1.60×10^{-19} C

Determine an equation for magnetic field strength, B:

- The centripetal force is the magnetic force on the ions
- Force on a moving charge, $F = Bev$

- Centripetal force, $F = \frac{mv^2}{r}$
- $Bev = \frac{mv^2}{r}$
- $B = \frac{mv}{er}$ [1 mark]

Determine the **minimum** radius of the path of the ions which will be detected, *r*:

- $r = \frac{d}{2} = \frac{40 \times 10^{-3}}{2}$ = 0.02 m [1 mark]

Determine which ion will emerge at the top of the detector if *B* is constant:

- Since $m \propto r$, the lighter ions (smaller *m*) will travel in a circular path with a smaller radius (smaller *r*)
- Hence the 6u ion will emerge at the top of the detector; [1 mark]

Calculate the maximum magnetic field strength that can be applied in order for both ions to be detected:

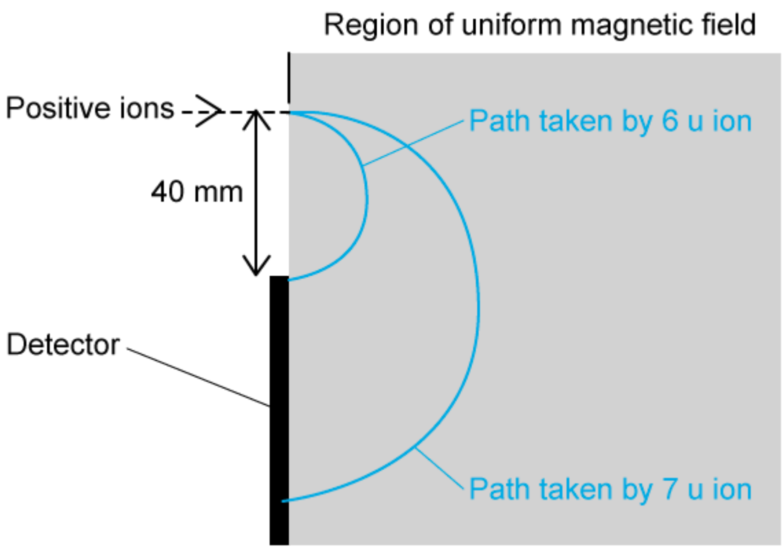
- $B = \frac{(6 \times 1.661 \times 10^{-27}) \times (4.8 \times 10^4)}{(1.6 \times 10^{-19}) \times (0.02)}$ = 0.149 T [1 mark]

[Total: 4 marks]

By equating the magnetic force and the centripetal force we get the equation, $B = \frac{mv}{er}$. The mass, velocity and charge of each ion are all constants, therefore $B \propto \frac{1}{r}$. Hence the largest value of *B* will be achieved when *r* has the smallest possible value (which is for the 6 u ion mass). The detector is 40 mm below the point of entry of the ions, therefore the smallest radius which the path of the ion can have is 20 mm.

To work out which ion will have the smallest radius we need to rearrange the centripetal force equation to give: $r = \frac{mv^2}{F}$. The velocity, *v*, and the force, *F*, are both constants, therefore $r \propto m$. As we have previously determined that the smallest radius will require the largest magnetic field, we need to use the ion with the smallest mass, hence we use the 6u ion in the calculation.

The paths which the 6u ion and the 7u ion would take in the magnetic field are shown below:



(4 marks)