



This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

- + *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.
- + *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.
- + *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.
- + *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at <http://books.google.com/>

NYPL RESEARCH LIBRARIES



3 3433 06909996 2









TEXT-BOOK OF PHYSICS

BY

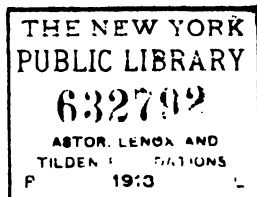
C. E. LINEBARGER

LAKE VIEW HIGH SCHOOL, CHICAGO

NEW YORK
CHICAGO
BOSTON

D. C. HEATH & CO., PUBLISHERS

BOSTON NEW YORK CHICAGO



COPYRIGHT, 1910,
BY D. C. HEATH & Co.

I E 2

NEW YORK
1910
1911

PREFACE

IN preparing this book, the author's aim has been to dovetail the fund of information about things physical, already possessed by the student, with results obtained in the laboratory. Common knowledge of everyday phenomena is constantly referred to and used as a basis for studying the investigations of physicists. The mode of treatment of most topics is as follows: (1) appeal to the student's ready-at-hand knowledge of the topic; (2) mention, when helpful, of pertinent historical items of interest; (3) clear-cut enunciation of definitions; (4) careful description of apparatus well illustrated; (5) logical derivation of laws and principles so worded as not to inflict too great a burden on the memory; (6) numerous applications of principles; (7) exercises and problems (not too hard) requiring for solution some constructive thought.

The material selected is that considered to be of most value to the student. Some traditional topics may have been slighted in the exercise of this choice, but in each case the advantage to the student has been carefully considered. Recent views and theories, especially those referring to the constitution of matter and of electricity, have not been neglected; but the author frankly acknowledges that he considers anything more than the briefest mention of them quite out of place in an elementary text written for the average student.

In some schools it may be desirable to shorten or modify the course. To indicate where omissions may be made without interfering with continuity, two sizes of type have been used; the large type alone gives a coherent briefer course, to which may be added such portions of the smaller type as may seem best suited to individual needs.

Certain fundamental topics of physics present great difficulties to high-school students, and yet they are essential to a clear understanding of the science as a whole. These have not been slurred over and left to the teacher to explain, but have been accorded so thorough and elementary a discussion as to render them readily intelligible. While perhaps for this reason the number of topics covered may be less than that commonly found in high-school texts, it is believed that the thorough presentation of a smaller number will yield better results than the scant presentation of a larger number. The treatment has been made intensive rather than extensive.

Special stress is laid upon the practical applications of physical principles; these are numerous and varied. Yet undue prominence is not given to appliances (however interesting and important in themselves) that cannot be presented without going into details of construction and of operation difficult to grasp at a first reading. Many carefully prepared diagrams aid in making clear the working of the appliances.

The illustrations are not pictures or "catalogue cuts" of more or less obsolete forms of apparatus, but are in most cases working diagrams that present the salient features of apparatus of the latest and most approved type. These diagrams are designed to show the principle of the thing rather than the thing itself, and therefore fit well into the equipment of apparatus to be found in most

schools. In numerous instances explanatory legends accompany the diagrams, their purpose being to place special insistence upon the important points to be noted, as well as to supplement the text.

Of course, in these days, it is hardly necessary to mention that a laboratory course should be carried on along with the text-book course, supplementing and sometimes forestalling the text-book presentation. Because of this collateral laboratory course, some things may be omitted from the text that should otherwise be presented, and the treatment of certain topics may be curtailed, since they will be amply treated in the laboratory. While any laboratory manual can be used in connection with this text, the author has prepared a laboratory guide that is closely correlated with it so that the two constitute a coherent first course in physics.

For valuable criticism and suggestions on the manuscript, the author wishes to acknowledge his indebtedness to Mr. C. H. Smith of the Hyde Park (Chicago) High School, Mr. W. E. Tower of the Englewood (Chicago) High School, Mr. C. M. Turton of the South Chicago High School, and to Mr. J. E. Kimmons of the Austin (Ill.) High School. He has been ably assisted in reading the proofs by Mr. A. W. Augur of the Lake (Chicago) High School, Mr. O. C. Kenyon of the Central High School, Syracuse, N.Y., Mr. E. J. Rendtorff of the Lake Forest (Ill.) Academy, Professor F. W. Stevens of the Lake Forest College, and Professor E. C. Woodruff of the James Milliken University.

CONTENTS

CHAPTER I. INTRODUCTORY	1
CHAPTER II. MATTER AND FORCE	2
Units of Measurement	2
Measurement of Force	2
CHAPTER III. MECHANICS OF LIQUIDS	3
Pressure in Liquids	3
The Principle of Archimedes	3
Specific Gravity	3
CHAPTER IV. MECHANICS OF GASES	4
Pressure in Gases	4
Pneumatic Appliances	4
CHAPTER V. FORCE AND MOTION	5
Uniformly Accelerated Motion	5
Laws of Motion	5
CHAPTER VI. COMPOSITION AND RESOLUTION OF FORCES	6
Center of Gravity and Equilibrium	6
CHAPTER VII. MOLECULAR MOTIONS AND FORCES	7
CHAPTER VIII. GRAVITATION	8
Curvilinear Motion	8
CHAPTER IX. WORK, ENERGY, AND MACHINES	9
Work and Activity	9
Energy	9
Friction	9
Machines	9
CHAPTER X. NATURE, PRODUCTION, AND TRANSMISSION OF HEAT	10

CONTENTS

vii

	PAGE
CHAPTER XI. TEMPERATURE AND CHANGE OF SIZE	163
Thermometry	163
Expansion of Solids and Liquids	168
Expansion of Gases	173
CHAPTER XII. QUANTITY OF HEAT AND CHANGE OF STATE	178
Specific Heat	178
Change of State	183
CHAPTER XIII. HEAT AND WORK	199
Mechanical Equivalent of Heat	199
Heat Engines	202
CHAPTER XIV. WAVE MOTION	211
CHAPTER XV. PRODUCTION AND PROPAGATION OF SOUND	227
Reflection of Sound	234
CHAPTER XVI. LOUDNESS AND PITCH OF SOUNDS	238
Loudness of Sound	238
Musical Scales	243
Resonance and Interference	245
CHAPTER XVII. QUALITY OF SOUND	252
Vibrations of Strings	252
Vibration of Air in Pipes	253
Fundamentals and Overtones	257
Harmony and Discord	262
Table of Overtones	265
CHAPTER XVIII. RADIANT ENERGY	270
CHAPTER XIX. REFLECTION AND REFRACTION OF LIGHT	283
Reflection at Plane Surfaces	283
Refraction at Plane Surfaces	289
Reflection at Curved Surfaces	299
Refraction at Curved Surfaces	305
Optical Instruments	309

	PAGE
CHAPTER XX. COLOR	320
Color and Wave Length	320
Spectra	325
Color by Transmitted and Reflected Light	329
CHAPTER XXI. STATIC ELECTRICITY	334
Electrical Distribution	340
Potential and Electromotive Force	342
Electrostatic Induction	347
Electrical Condensers	353
Atmospheric Electricity	357
CHAPTER XXII. MAGNETISM	359
Magnetization	359
Terrestrial Magnetism	367
Electromagnetism	370
CHAPTER XXIII. CURRENT ELECTRICITY	374
The Voltaic Cell	374
Magnetic Effects	380
Heating Effects	383
Chemical Effects	389
Primary Cells	392
Some Important Cells	395
Connection of Cells and Circuits	402
Electrical Measurements	407
CHAPTER XXIV. INDUCED CURRENTS	414
Electromagnetic Induction	414
Transformers and Induction Coils	418
The Dynamo and the Motor	423
X-Rays and Radioactivity	430
CHAPTER XXV. COMMERCIAL APPLICATIONS	436
Transmission of Messages	436
Production of Heat and Light	445
Generation of Power	452
INDEX	465

TEXT-BOOK OF PHYSICS

CHAPTER I

INTRODUCTORY

1. **Subject-matter.** — Much of the subject-matter of physics is familiar to every one; but this popular knowledge lacks the orderly arrangement characteristic of science. The facts of everyday experience are as a rule isolated and independent; their interrelationships and mutual bearings are unknown. Further, this knowledge is gained by superficial observation merely, and not by a thoroughgoing investigation of all the facts pertaining to a *phenomenon* (happening) of nature. Who does not know, for instance, that a stone, if unsupported, falls to the ground? But who knows from everyday experience alone how fast it falls, and can predict how long it will take to fall through a given distance? Who does not know that electricity can produce motion and light and heat? But who knows without thorough study just how these effects are brought about? Who does not know that a rainbow may appear when sunshine strikes a cloud? But who knows without painstaking observation just how the white sunlight gives rise to the colored bow? The main purpose of the study of physics is to put law and order into this mass of knowledge, and to give training in good methods of acquiring and interpreting facts.

2. Physics Defined. — Physics may be defined as the branch of science that treats of matter and energy; but this definition, while enjoying the merit of brevity, suffers from vagueness, because other sciences, such as chemistry and astronomy, for example, also treat of matter and energy. The limits between the different branches of science are not easily fixed; indeed, we have border sciences, such as physical chemistry and astrophysics. But as we proceed in our study we shall see in a measure what limitations are to be applied to the above definition of physics, and we shall learn wherein physics differs from the allied branches of science.

3. Matter. — We learn through experience that different portions of space around us, as well as the same portion of space at different times, affect our senses differently; and this fact leads us to believe that there must be something in such portions of space which gives rise to these sensations and impressions. This something which occupies space and affects our senses is called *matter*.

4. Constitution of Matter. — Matter may be regarded as having a granular structure, that is, as being made up of parts more or less separated in space; or it may be considered as continuous in structure, without breaks or gaps. Physicists are agreed that ordinary matter possesses a granular structure, and that the extraordinary kind of matter known as the *ether* has a continuous structure. This ether makes itself manifest mainly in electrical and optical phenomena; the discussion of it will therefore be deferred until those topics are treated.

5. Divisibility of Ordinary Matter. — A body of ordinary matter may be divided without undergoing any change in substance and the division may be made very minute.

A kernel of wheat, for example, may be ground into flour so fine that the unaided eye cannot distinguish the separate fragments. Let a piece of some substance, such as glass, be broken into smaller pieces. Each piece will still be glass. Let any of these pieces be broken into smaller and smaller fragments, and let the division be extended in imagination further and further. We should finally obtain particles so small that another division would result in breaking them up into fragments that are not glass. The particles of a substance which are so small that they cannot be divided further without destroying the identity of the substance are called *molecules*. These molecules are in turn made up of *atoms*, which, when the word was coined, were supposed themselves to be indivisible; but they are now known to be composed of particles thousands of times smaller yet, called *corpuscles* or *electrons*.

6. Different Kinds of Matter. — The number of different kinds of matter is limitless. It has been found, however, that all these different substances are composed of certain simple substances, the *elements*, of which about eighty are at present known. As the elements may combine with one another in countless ways, myriads of compound substances may be formed.

7. Energy. — It is a common experience that matter is continually changing its position, condition, temperature, form, volume, composition, and so on. These changes are accompanied by changes in energy. Energy is apparently inseparable from matter in its manifestations. What energy is, will be stated in Chapter VIII.

8. Conservation of Matter and Energy. — It was shown toward the end of the seventeenth century that matter is indestructible; and about the middle of the nineteenth

century, it was shown that the same is true of energy. Present-day physics is based upon the generalization: *Neither matter nor energy can be created or destroyed.*

9. The Methods of Physics. — I. *Experimental.* It may be observed that a certain phenomenon stands in a causal relationship to some other phenomenon. A change in the one brings about a change in the other. If the phenomena involved attract the attention of the physicist, he seeks to study the relationship more thoroughly. To this end he endeavors to isolate these particular phenomena from all others, so that the relationship may be as little obscured as possible. He then devises an apparatus by means of which he may learn how a variation in the one phenomenon may affect the other, everything except the cause and the effect being kept in so far as possible invariable and constant. If practicable, he measures the variations; that is, he makes the experiment *quantitative*. Otherwise, he observes the variations without measuring them; in other words, he performs a *qualitative* experiment. If a relationship is found from the results of his series of experiments, it is formulated as a law.*

10. Illustration. — Let us apply the method just outlined to a simple case. Every one knows that a rod of wood or metal may be bent, and that the greater the force applied, the greater will be the amount of bending. But let us as physicists study the relationship between the amount of bending and the bending force, and let us make the experiment quantitative. We clamp one end of a rod firmly to the table top and hang on it at the same distance from the clamped end various weights, noting by means of a scale the amounts of bending or deflec-

* "A law of Nature is a statement of our belief concerning certain phenomena; it is suggested by a number of observations and measurements, and is, in fact, a generalization of these. It is shown to be in accord with all observations, to within the range of error inherent in the experimental instruments used, but can never be perfectly *verified*." — ROWLAND AND AMES.

tions (Fig. 1). Suppose that the results of a series of such measurements are as follows :

<i>Weights</i>	100	200	300	400	500	600	700	800
<i>Deflections</i>	13	26	39	52	65	78	91	104

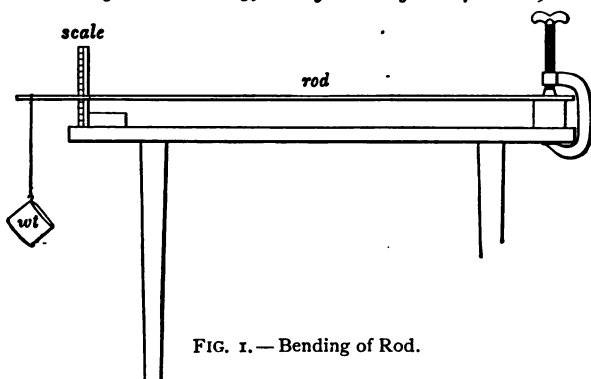


FIG. 1.—Bending of Rod.

We see that the deflections increase in the same ratio as do the weights. Indeed, if we multiply the weights by .13, we get the corresponding deflections. We have a right to assume that, as the above results may be considered typical, this same relationship will obtain for other weights, at least, within certain limits.

11. The Methods of Physics.—II. Theoretical. A law found by experimenting upon a certain class of phenomena may be thought applicable to other phenomena also. The law may act as prophet in foretelling what will occur when new experiments in perhaps quite different fields are tried. Again, facts and laws may be given an interpretation that frequently proves to be of great value, not only in understanding the laws themselves, but also in applying them to other cases. These interpretations are based upon *hypotheses*, and give rise to *theories*.* In physics the

* "From the point of view of the physicist, a theory is a policy rather than a creed ; its object is to connect or coördinate apparently diverse phenomena, and above all to suggest, stimulate, and direct experiment. It ought to furnish a compass which, if followed, will lead the observer further and further into previously unexplored regions."—J. J. THOMSON.

deductions from theories are often made by mathematical operations, and may not perhaps accord with the facts themselves as obtained by experiment. The theories must therefore be modified to conform with the facts; for theories are to be considered acceptable only when they are useful in leading to discoveries of new facts, and must be rejected whenever facts are found that contradict them.

12. Illustration. — We have seen that the deflection is proportional to the bending force in the case of a particular rod. Trials with other rods lead to the same conclusion. May we then assume that there is proportionality between the amount of force of any kind which changes a body in any way and the amount of change in the body? We shall find that we do have such a right, for in every trial that has been properly made the proportionality has been found to exist. We have then to do with a law of nature. This law can be applied to cases of quite a different kind and will frequently be found to suggest ways of explaining such cases, as well as modes of attack that help in the finding out of new laws.

13. Variation and Proportion. — Natural laws are frequently discovered by observing in what way and by how much, variations in one set of phenomena cause variations in another set. We have learned that the deflection of a rod is proportional to the force that bends it; and we may express this relationship or law by saying that the deflection *varies directly* as the force applied. In the language of algebra, if d denote the deflection, f , the force, and if $\propto$ be the symbol of variation,

$$d \propto f.$$

In the particular case cited above, the deflections were found to be numerically equal to .13 of the weights, so that

$$d = .13f.$$

This constant number will have other values if rods of different materials, or of the same material but of different

dimensions, are experimented upon; its value will also vary with the units of measurement employed. If the conditions are not altered, however, the value of this "constant" will remain the same. Denoting it by k , we have

$$d = kf, \text{ or } \frac{d}{f} = k;$$

and in case we employ such units of measurement that $k = 1$,

$$d = f.$$

Let d' and f' represent any other values of the deflections and forces for which k has the same value as for d and f . Then

$$\frac{d'}{f'} = k = \frac{d}{f},$$

or

$$\frac{f}{f'} = \frac{d}{d'},$$

which may also be written

$$f : f' :: d : d'.$$

If we call these terms of a proportion *variables*, it is apparent that *one variable varies directly as another when their quotient is a constant*. To test for direct variation or proportionality, proceed as follows. *Divide each of the measured quantities of one kind (deflection) by the corresponding quantity (force) of the other kind; a constant quotient indicates direct variation.*

14. Inverse Proportion. — Let us now investigate the relationship between the deflections and the widths of the rods. We employ the same apparatus (Fig. 1) and use rods of the same material, length, and thickness, but of different widths (horizontal). We clamp these rods in succession to the table and hang from them at the same distance from the clamp always the same weight. The

data for a set of determinations made under these conditions are as follows :

<i>Widths</i>	12	8	6	4	3	2	1
<i>Deflections</i>	1	1.5	2	3	4	6	12

As the widths decrease, the deflections increase; the deflection *varies inversely* as the width. The product of each width w by the corresponding deflection d is a constant k (12 for the case in hand). Hence, generally,

$$dw = k.$$

When the product of two variables is a constant, the variables vary inversely. Denoting by d' and w' any other corresponding set of values, we have

$$d'w' = k = dw,$$

or
$$d'w' = dw;$$

whence
$$\frac{d}{d'} = \frac{w'}{w},$$

or
$$d : d' :: w' : w.$$

The deflection of a rod varies then directly as the force and inversely as the width, so that

$$d = k \frac{f}{w}.$$

Investigations of the relationships between the deflection and (1) the length l , and (2) the thickness (vertical) t , have shown that the deflection varies directly as the cube of the length and inversely as the cube of the thickness. Combining all these relationships, we have

$$d = k \frac{f l^3}{w t^3}.$$

The value of k depends upon the nature of the material of which the rod is made, as well as upon the units of

measurement; but these once chosen, it is constant except for the unavoidable errors of experimentation.

EXERCISES

1. Weights were hung in succession from one end of a long wire and the corresponding elongations measured. From the following values find out in what proportion the elongations and the weights stand.

<i>Weights</i>	100	200	300	400	500	600	700	800
<i>Elongations</i>	2	4	6	8	10	12	14	16

2. The volumes and corresponding pressures of a certain quantity of air were as follows:

<i>Volumes</i>	6	9	10	12	15	18
<i>Pressures</i>	60	40	36	30	24	20

In what relationship do these volumes and pressures stand?

CHAPTER II

MATTER AND FORCE

15. Properties of Matter. — All bodies have certain *general properties*, such as extension, impenetrability, inertia, and elasticity. Every substance has certain *special properties*, such as color, hardness, and density. It is by the study of these properties of matter that we attain to a more intimate knowledge of matter itself.

16. Extension. — All bodies occupy space ; they all have length, breadth, and thickness ; and when any one of these three is lacking, there can be no *extension*. A point, line, or surface includes no matter.

UNITS OF MEASUREMENT

17. Measuring and Units. — Measuring consists in comparing some definite quantity which is called a *unit* with the quantity to be measured. The number of times the unit is contained in the quantity is its *measure*. The unit of measurement must be a definite quantity of the same kind as the thing measured. Thus, the unit for measuring length must be a certain length ; the unit for measuring volume must be a certain volume ; the unit for measuring mass must be a certain mass.

Certain units have been adopted as standards by different nations, and it is one of the duties of a government to see that the units employed by the people are in strict agreement with the standards. Many different units have

been legalized at various times, but the units of the *French* or *metric system* are used almost exclusively in scientific work.

18. Metric Units of Space. — All the units of the metric system are based upon a fundamental unit of length named the *meter* (m.), which is defined to be the distance at the temperature of melting ice between two cross lines that are engraved upon the surface of a bar made of a very durable metal. This bar, made in 1796 and kept at Paris, is known as the International Prototype Meter. Accurate copies of it, called National Prototype Meters, are preserved in most civilized countries. The unit of length most in use among physicists is the *centimeter* (cm.), the one hundredth part of a meter. The *millimeter* (mm.), the one thousandth part of a meter, is also frequently used, when the distances to be measured are small.

The units of surface and of volume are, respectively, square surfaces and cubical volumes, having as their edges various units of length; such as the square centimeter (cm.²) or the cubic millimeter (mm.³). The volume of 1000 cm.³, when used to measure the capacity of a vessel, has been given the special name of *liter*.

19. Metric Units of Mass. — Mass is the amount of matter in a body. The physicist's unit of mass is the *gram* (g.), which is the one thousandth part of the kilogram (kg.). The International Prototype Kilogram is the mass of a certain cylinder made of a very durable metal preserved at Paris; and every civilized government has an accurate copy of it, called the National Prototype Kilogram.

20. Metric Units and their English Equivalents. — The meter, the liter, and the gram have decimal multiples and submultiples. The names of the multiple units are formed

by adding to the name of the fundamental unit the Greek prefixes, *deca* (ten), *hecto* (hundred), and *kilo* (thousand) while the names of the submultiples are obtained by adding the Latin prefixes, *deci* (tenth), *centi* (hundredth), and *milli* (thousandth).

The relations between the units, multiples, and submultiples of the metric system are shown in the

TABLE OF THE METRIC SYSTEM

LENGTH	MASS	VOLUME	NOTATION
Kilometer (km.) .	Kilogram (kg.) .	Kiloliter . . .	1000.
Hectometer . . .	Hectogram . . .	Hectoliter . . .	100.
Decameter . . .	Decagram . . .	Decaliter . . .	10.
METER (m.) .	GRAM (g.) .	LITER (l.) . .	1.
Decimeter . . .	Decigram . . .	Deciliter . . .	.1
Centimeter (cm.)	Centigram (cg.)	Centiliter . . .	.01
Millimeter (mm.)	Milligram (mg.)	Milliliter . . .	.001

In the following table is given the relation between the commonest metric and English units.

1 cm. = .3937 inches	1 inch (in.) = 2.54 cm.
1 m. = 39.37 inches	1 foot (ft.) = 30.48 cm.
1 km. = .6214 miles	1 mile (mi.) = 1.609 km.
1 cm. ² = .155 square inch (in. ²)	1 in. ² = 6.45 cm. ²
1 cm. ³ = .061 cubic inch (in. ³)	1 in. ³ = 16.39 cm. ³
1 l. = 1.057 quarts	1 quart (qt.) = .946 l.
1 g. = 15.44 grains	1 grain (gr.) = 64.8 mg.
1 g. = .0353 oz.	1 ounce = 28.35 g.
1 kg. = 2.204 lbs.	1 lb. = .4536 kg.

21. Density.—Equal volumes of different substances rarely contain equal quantities of matter. The greater the mass of a substance for a given volume, the denser or

heavier it is. By *density* is meant the *mass of a substance per unit volume*. Thus one cubic foot of water has a mass of 62.42 lb.; and therefore when the units of measurement are the pound and the cubic foot, the density of water is expressed by the number 62.42. With the same units the density of kerosene is 49.94, for one cubic foot of this liquid has a mass of 49.94 lb. In like fashion, the densities of all other substances may be expressed. But with different units other numbers are obtained to express densities. In the metric system, for instance, one cubic centimeter of water has a mass of one gram, and its density is therefore unity. As .8 g. of kerosene occupy a volume of one cubic centimeter, its density is .8 g. per cubic centimeter, or $.8 \frac{\text{g.}}{\text{cm.}^3}$. In the metric system the number standing for density is the same as the number expressing the mass of a substance in grams occupying one cubic centimeter. One of the great advantages of the metric system is this adoption of a standard of mass standing in such a simple relationship to the standard of volume.

EXERCISES

1. What is the mass of one liter of kerosene?
2. 42 g. of aluminum occupy a volume of 15 cm.³. What is the density of aluminum?
3. The density of sulphuric acid is $1.84 \frac{\text{g.}}{\text{cm.}^3}$. What is the mass of 50 cm.³ of the acid?
4. What is the height of a column of mercury having a cross-sectional area of 1 cm.², if the mass of the mercury is 1033.3 g. and its density is 13.596?

MEASUREMENT OF FORCE

22. Forces, Balanced and Unbalanced. — *Force is whatever produces or tends to produce motion.* The purpose of our muscles is to produce motions of various sorts; and it is

from this muscular effort that we derive our primitive notion of force. The very words *push*, *pull*, *strike*, *thrust*, *bite*, and *kick*, indicate how closely and variously our muscular actions are associated with the notion of force. Many other words also express the *force* idea, as *attraction*, *repulsion*, *pressure*, and *tension*. Now all these terms imply motion, attempted or produced. Force and motion are inseparably connected in thought. If we know that force is being applied to a body without motion resulting, we at once suspect that another force or other forces are also acting upon the body and counteracting the first force. In such cases the forces balance one another, and the body is said to be in equilibrium. A single force, not balanced by other forces, necessarily produces motion. *Unbalanced forces produce motion; balanced forces cannot produce motion.*

23. Weight. — Weight is the force that makes bodies when unsupported (*i.e.* not acted upon by sufficient force in the opposite direction) fall toward the earth. The amount of this force of attraction at a given place depends only upon the mass of the body under consideration; the greater its mass, the greater is its weight. The pull of the earth on a body may then be taken as a measure of the amount of matter in the body.

24. Weight Units of Force. — In engineering operations and in the affairs of everyday life, the amounts of matter in bodies are measured in terms of the earth-pull on given pieces of matter. Thus a gram weight is the attraction of the earth for a piece of matter possessing a mass of one gram. Similarly, we use pound weights and ton weights, to indicate the earth-pulls on pound masses and ton masses.

As the attraction between a body and the earth varies

slightly with the locality, it is necessary in very accurate work to specify both the distance from the equator and the distance above or below the level of the sea. It should be borne in mind that the mass of a body is always the same, wherever it may be, but that its weight may vary according to its position with respect to other bodies.

25. Stress and Strain.—Force may be applied to a body and yet not change the motion of the body as a whole. Its effect may be to cause relative motions of the molecules of the body so that its shape or size is changed. Common instances are the stretching of a rubber band and the bending of a piece of paper or wood. Change of size or shape of a body due to the action of force is called *strain*. *Stress* is the name given to a pair of forces producing a strain; two opposite unbalanced forces act upon a body until the change is complete, and after this the forces are balanced. If the forces act away from each other, the stress is called a *pull* or *tensile stress*; if toward each other, the stress is known as a *push*, *pressure*, or *compressive stress*. Thus, in stretching a rubber band, the two hands act in opposite directions and tend to pull the band apart, thereby putting it under tension; and in pinching a rubber eraser, the thumb and finger act toward each other and tend to crush the rubber, the resulting strain being a compression.

26. Elasticity and Elastic Force.—Force has to be exerted to stretch a rubber band or to bend a strip of wood or metal, and the stretched band or bent strip exerts force to regain its original condition. *Elasticity is the property of a substance which imparts to it the tendency to recover from a strain, and elastic force is the stress exerted by the strained body.* Substances such as a stretched rub-

ber band or a bent steel strip, which regain their original form when the force that has distorted them ceases to act, are said to possess *elasticity of form*. Substances such as air and water, which, when compressed, tend to regain their original volume but not their form, are said to possess *elasticity of volume*. Solids such as butter and clay, which have elasticity of volume but not of form, are said to be *plastic*.

When the force distorting a solid body is great enough or acts long enough, the solid usually undergoes a permanent change (*set*), and does not regain its original shape when the distorting force is removed. When such is the case, the body is said to have passed its *elastic limit*.

27. Hooke's Law. — In 1676, Hooke published at the end of one of his books the anagram *veiiostttinu*, the key to which he gave two years later. This is "*Ut tensio ut vis*" which, translated from the Latin, reads: *Stress is proportional to strain*.*

28. Measure of Stresses. — Inasmuch as stress is proportional to strain, the strains of bodies may be taken as measures of the stresses producing them. In instruments based upon this relationship, the body to be strained is usually given the form of a spiral of wire of some elastic metal, such as steel or brass. By choosing fine wire and winding it in a long and wide spiral, exceedingly slight forces will lengthen it enough to be measured with accuracy. By using coarse wire wound in a short and narrow spiral, large forces may be measured. The use of spiral springs to relieve jars in vehicles and for other purposes

* Hooke himself translated the law thus: "The Power of any Spring is in the same proportion with the tension thereof: That is, if one Power stretch or bend it one space, two will bend it two, and three will bend it three, and so forward."

is common; and their application to force measuring in the spring scales is familiar in everyday life.

29. Tensile or Longitudinal Stress.—When a force acts upon a body so as to tend to increase one of its dimensions, as its length, the body is said to be subjected to a *tensile* or *longitudinal stress*. Thus the pull on the coupling that connects two cars, on a rope in a “tug of war,” and of a hanging lamp on its chain are examples of tensile stress.

When the body is in the form of a wire or a rod, it has been established by experiment that the *longitudinal strain depends upon the material, and varies* (1) *directly with the stress*, (2) *directly as the length*, and (3) *inversely as the cross-sectional area*.

30. Compressive Stress.—If the forces acting upon a body tend to diminish one of its dimensions, as its length, the body is said to be under *compressive stress* or *pressure*. The strain is known in this case as *compression*. The facts that have been ascertained in regard to the relationship between longitudinal stress and strain also apply, with appropriate changes, to compression and pressure.

31. Bending Stress.—When the ends of a rod are fixed, and force is applied at some place between its ends, the rod is said to be under *bending stress*. The strain is the deflection, and it is measured by the distance through which the rod is bent. Experiment shows that the *deflection depends upon the material, and varies* (1) *directly as the load*, (2) *inversely as the breadth*, (3) *directly as the cube of the length*, and (4) *inversely as the cube of the depth*. The case discussed on pp. 5-8 is also one of bending stress.

32. States or Conditions of Matter.—Most substances can be made by suitable change of temperature to assume the solid, liquid, or gaseous state. Thus water (a liquid) may be converted into ice (a solid) or into steam (a gas or vapor). The following are the characteristic differences between these states.

A *gas* (not in motion as a whole) has no shape or size of its own, but assumes that of the containing vessel. It spreads out and fills any space offered it, and presses upon the walls of the containing vessel uniformly. It

offers no resistance, or very little, to a stress tending to change its shape; and, compared with solids and liquids, but feeble resistance to a stress tending to change its volume. A gas possesses perfect elasticity of volume but not of form.

A *liquid* (at rest) has a level free surface and assumes the shape of the vessel in which it is. A liquid, like a gas, possesses perfect elasticity of volume, not of form. While it offers but slight resistance to forces tending to change its shape, its resistance to stresses tending to change its volume is enormous.

Gases and liquids are collectively called *fluids* because of their comparative ease of flow.

A *solid* has a definite shape and volume of its own, which, within its elastic limits, it assumes when released from a distorting stress.

In brief, solids tend to have permanency of form and of volume; liquids, permanency of volume only; and gases, permanency of neither form nor volume.

EXERCISES

1. If a body is acted upon by a force and yet does not move, what is to be inferred? Give some examples.
2. State the laws showing the relationships between compression, compressive stress, and the dimensions of a body.
3. If a spiral spring is lengthened 23.4 cm. by a weight of 15.0 g., what elongation will 31.7 g. give it?
4. If 15.6 g. stretch a spiral spring 24.8 cm., what weight will be required to stretch it one millimeter?
5. How much stiffer is a beam made by doubling (1) its width, (2) its depth?
6. How does the bending of two similar boards in a walk compare, if the cross supports are one meter apart under one board and 1.5 m. apart under the other?

7. (a) A plank is placed over a brook so that it is 20 cm. from the surface of the water. If 4 kg. at its center make it sag 1 cm., what will be the weight of a man who, in passing over it, causes it barely to touch the water?
(b) If the plank were twice as wide, how much would it bend under 4 kg.?
(c) If only half as wide?

8. An iron rod of square cross section is 200 cm. long and 10 cm. thick. If the density of iron is $7.8 \frac{\text{g.}}{\text{cm.}^3}$, what is the mass of the rod?

9. A glass ball has a diameter of 7.71 cm. and a mass of 600 g. Find the density of glass. (The volume of a sphere is equal to the cube of its diameter multiplied by .5236.)

10. If the average density of the earth is $5.527 \frac{\text{g.}}{\text{cm.}^3}$, and its average diameter is 12,740 km., what is its mass in metric tons? (A metric ton is 1000 kg.)

SUGGESTIONS AS TO SOLVING PROBLEMS

The difficulty of solving problems is lessened by adopting the mode of attack suggested below.

I. Read the problem through carefully, and in case you do not understand certain words or terms, look them up in the preceding part of the text (refer to the Index, if necessary), or in a dictionary.

II. Search out the principles or laws involved, and if your knowledge of them is vague, review them thoroughly.

III. Change, if necessary, the units so as to have them of the same kind. Thus, if lengths are expressed in both inches and feet, reduce either the inches to feet or the feet to inches.

IV. Many of the laws are given as formulas, *i.e.* in algebraic language. Whenever possible, use the formulas and solve the problem by algebraic methods, as these are usually simpler than the corresponding methods of arithmetic.

CHAPTER III

MECHANICS OF LIQUIDS

33. Mechanics is the division of physics that treats of forces and their effects upon matter; it is subdivided into dynamics and statics. *Dynamics* deals with *unbalanced forces* which make a body move as a whole; *statics* has to do with *balanced forces* which cause no motion of the body as a whole but maintain some strain in it. Prefixes are employed to distinguish the dynamics and statics of liquids and gases from those of solids. Thus *hydrodynamics* and *hydrostatics* denote the dynamics and statics of liquids; and *aërodynamics* and *aërostatics*, the dynamics and statics of gases.

34. Pressure on Fluids.—When a single force acts upon a solid body, it will tend to move in the direction in which the force is acting; while in the case of a fluid, the force will not tend to make the body move in any particular direction, but will be transmitted as a pressure in all directions. Liquids subjected to compressive stress change but slightly in volume; a cubic centimeter of water requires the enormous pressure of 3000 kg. to reduce its volume a tenth. Liquids are practically incompressible. Gases, on the other hand, are readily compressible. The pressure in a fluid is due either to its weight or to the action of external force. For the sake of clearness we shall assume only one of these causes of pressure to act at a time; that is, in considering pressure externally applied to a fluid, we

shall neglect its weight, and in considering its weight we shall neglect any external forces acting upon it.

PRESSURE IN LIQUIDS

35. Pascal's Law. — Blaise Pascal was the first to state, in 1653, the general facts in regard to the transmission of

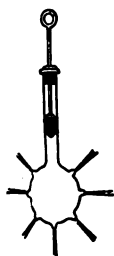


FIG. 2.—When the piston is pressed in, the water squirts out of all the holes; pressure is transmitted equally in all directions.

pressure in fluids. He found that *when pressure is applied to any part of a confined fluid, it is transmitted undiminished in all directions; and that the total pressures upon any surfaces of a fluid at rest are di-*

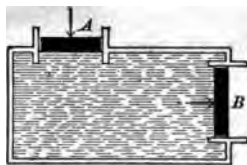


FIG. 3.—When the weight of a liquid is neglected, its pressure on equal areas, wherever placed, is the same.

rectly proportional to the areas of those surfaces. This law applies to gases as well as to liquids. Three points in regard to the pressure of liquids should be emphasized: *Liquid pressure (1) acts at right angles (normal) to the surface; (2) has the same value in all directions at any single point in the liquid; (3) if due to an external force, is the same, at any given time, at all points throughout the liquid* (Figs. 2 and 3).

36. Measurement of Fluid Pressure. — The amount of pressure exerted by a fluid is expressed as a force per unit of area, as 10 lb. per square inch ($10 \frac{\text{lb.}}{\text{in.}^2}$) or 50 g. per square centimeter ($50 \frac{\text{g.}}{\text{cm.}^2}$). A pressure measurement involves the measurement of the total force and of the area acted upon, and the *pressure per unit area* is obtained by dividing the total force by the total area.

37. Popular Notions of Pressure.—The notion of pressure is so interwoven with everyday affairs that certain terms, and elliptical expressions with reference to it are in common use. Thus "head of water" means the pressure that water exerts, a "head of 50 ft." being the pressure (due to its weight) of a column of water 50 ft. high; and the phrase, "a boiler carries 60 lb. of steam," means that every square inch of the boiler's walls is withstanding a pressure equal to the weight of 60 lb.

In the strength of materials, it is the pressure per unit area rather than the total pressure which is the important thing. Thus, if the area be made very small, as in the case of the sharpened edge of a cutting tool, a comparatively slight total pressure can overcome the strength of a material; and, on the other hand, if the total pressure is distributed over a large area, as in the case of snowshoes, the pressure per unit area is so diminished that the material (crust of snow) can withstand it.

38. Multiplication of Pressure.—Suppose that we push the piston (Fig. 4) into the tube which is filled with a fluid (either liquid or gas). The plug will be forced against the spiral spring, and if we neglect friction and assume

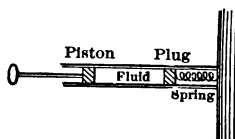


FIG. 4.—The Pressure is Unchanged.

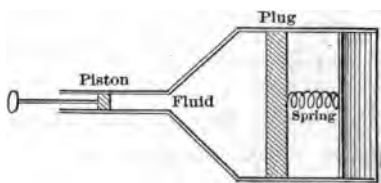


FIG. 5.—The Pressure is Multiplied.

the bore of the tube to be uniform, the pressure against the spring will be equal to that applied on the piston. If the tube flares out (Fig. 5), the pressure on the spring is as many times greater than that applied on the piston as the area of the plug is times greater than that of the piston. If, for example, the area of the plug is 100 cm.^2 , and that of the piston is 2 cm.^2 , a total pressure of 10 g. on the piston will balance a total pressure of 500 g. on the plug.

The pressure per unit area is the same in both cases, for in the case of the piston, $\frac{1}{2}^0 = 5$, and in the case of the plug, $\frac{500}{100} = 5$.

If two pistons are pressing upon the same liquid (Fig. 6), and such weights are placed upon them as will keep them from moving, it is found that the weights are proportional to the areas of the pistons. If, then, pressure be applied to a liquid by means of a small piston, and there be a large piston also in contact with the liquid, the pressure applied will be multiplied, and the amount of the multiplication will be equal to the ratio of the area of the large piston to that of the smaller.*

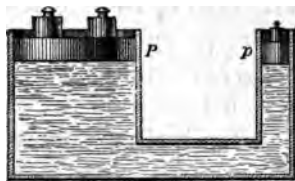


FIG. 6.—The area of the small piston is $\frac{1}{16}$ that of the large piston; an ounce weight can therefore balance a pound.

39. The Hydraulic Press. — The principle of the multiplication of pressure is applied in a class of machines

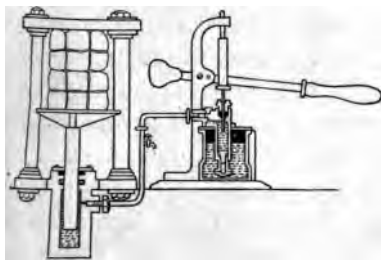


FIG. 7.—Hydraulic Press.

known as *hydraulic presses* or *jacks*. Their action is essentially as follows. Two cylinders fitted with pistons are connected by a pipe (Fig. 7). The small cylinder is provided with valves like those of a

* Pascal's words in this connection are: "If a vessel full of water, closed in all parts, has two openings of which the one is a hundred times the other, placing in each a piston which fits it, the man pushing the small piston will equal the force of a hundred men who push that which is a hundred times as large, and will surpass that of ninety-nine. Whatever proportion these openings have, and whatever direction the pistons have, if the forces that apply on the pistons are as the openings, they will be in equilibrium."

force pump, and its lower end is placed under water in a reservoir. By working the handle attached to the small piston, water is forced through the pipe into the large cylinder. The large piston supports a platform sliding in a framework on top of which is a fixed platform. The article to be compressed is placed upon the platform, and water pumped into the large cylinder. The large piston slowly rises and presses upon the object with a force which is as many times greater than that applied to the small piston as the area of the large piston exceeds that of the small. The hydraulic press may be used in operations requiring enormous pressures slowly applied. Thus hay or cotton is baled, lead pipe is made, car wheels are forced upon their axles, and heavy weights are lifted by means of hydraulic presses or jacks.

40. Pressure, Weight, and Depth of a Liquid.—Push a can with a small hole in its bottom down into water (Fig.

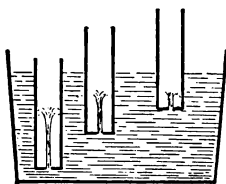


FIG. 8.—Pressure and Depth.

8), and note that the deeper you push it, the harder you have to press and the higher the water spouts up through the hole. This indicates that *pressure increases with depth*.

Conceive of a liquid as made up of a large number of thin horizontal layers. Each of these layers has to support the weight of those above it. The lower the layer, the greater is the weight it has to support, and hence the greater the pressure exerted upon it. *The downward pressure of a liquid is proportional to the depth.*

41. Pressure and Density.—The greater the density of a liquid, the heavier are the horizontal layers into which it

may be considered to be divided. Hence *the downward pressure of a liquid is proportional to its density.*

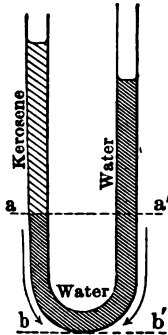


FIG. 9. — Balancing Columns of Non-miscible Liquids.

42. Balanced Columns. — The U-tube (Fig. 9) is partially filled with water, and kerosene is then poured into the left-hand branch. The kerosene pushes down the water on the left side and makes it rise on the right side. The downward pressure of the left-hand water column balances the oppositely directed downward pressure of the right-hand water column from a' to b' . The pressures (weights) of the kerosene and water columns above aa' (their common level) are equal; but since the density of kerosene is .8 and that of water is one, it takes a longer column of the lighter liquid to balance the heavier liquid. How many times longer above aa' should the kerosene column be than the water column?

43. Pressure at Different Points in the Same Level of a Liquid. — If the pressure at any one part of the same horizontal layer of a liquid were greater than at another, there would be a motion of the liquid from the place of greater pressure to that of lower. But as the fluid is at rest there can be no such motion. Hence *the pressure at all points equally distant from the free surface of a liquid at rest is the same.*

44. Total Pressure on any Horizontal Surface. — As liquid pressure is transmitted equally in all directions and depends only upon the depth and density of the liquid, *the downward and upward*

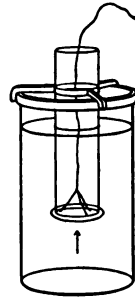


FIG. 10. — Downward and Upward Pressure Equal. The disk closing the lower end of the tube is held in place by the pressure of the water up against it, and drops only when enough water is poured into the tube to make the downward pressure due to the weight of the water added to that of the disk become equal to the upward pressure.

pressures at any point must be the same, and the total pressure upward or downward of a liquid on any horizontal surface is equal to the weight of a column of the liquid, the base and length of which are, respectively, the area pressed upon and the depth of this area below the free surface of the liquid (Fig. 10).

45. Total Pressure on any Vertical Surface; Lateral Pressure. — The pressure against the vertical side of the vessel (Fig. 11) at the surface *a* of the liquid is zero, for

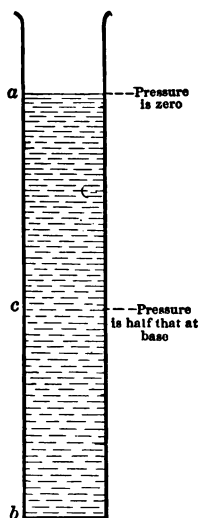


FIG. 11.—Lateral Pressure.

there the liquid has no depth. The pressure increases with the depth until at the lowest point in the side *b* the pressure against the side is equal to the pressure against the bottom. As the pressure increases uniformly with the depth, the average pressure per unit of surface against the side is equal to the pressure just at the middle of the side *c*, for the pressures from the middle to the top decrease at the same rate as the pressures from the middle to the bottom increase. The lateral pressure is therefore the average of the pressures at the surface of the liquid and at the bottom of the vessel, and this average pressure must be equal to half the pressure per unit of surface against the bottom.

Hence the *total pressure of a liquid upon any portion of a vertical side of a vessel is equal to the weight of a column of the liquid, the base and length of which are, respectively, the area of that portion of the side and its average depth.*

46. Pressure Independent of Shape of Vessel.—The lower openings of the three glass vessels (Fig. 12) have the same size and can be tightly closed by a ground-glass disk hung by a cord from one arm of a balance. The cylindrical vessel is supported by a ring stand and enough

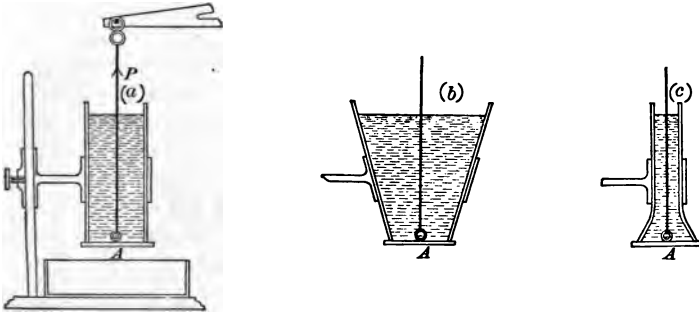


FIG. 12.—It is the area of the base alone, if depth and density are the same, that determines the total pressure upon that area.

weights are placed in the balance pan to keep the disk from letting water leak out when the vessel is nearly full. The downward pressure on the disk is then equal to the weight of water contained in the vessel. When the other vessels are in turn substituted for the cylindrical one, it is found that, with the same weights in the pan as before, water does not begin to leak out until the vessels are filled to the same depth as the first one. This shows that the same pressure in all three cases is exerted upon the disk. That equal pressures may be produced by unequal weights of a liquid seems contrary to what one might commonly expect; such behavior seems paradoxical. But this "hydrostatic paradox" may be accounted for if we take into consideration the pressures exerted upon the liquid by the sides of the vessels. While the weight of the water in vessel (b) is greater than that in vessel (a), the excess of

weight is supported by the upward pressure of the outward sloping sides; and in vessel (*c*), which contains less water than vessel (*a*), the deficit in weight is made up by the downward pressure of the inward sloping sides.

47. Surface of a Liquid at Rest is Horizontal. — By jolting a vessel the surface of a liquid in it may be made to assume some such form as is shown in Figure 13. As we know, it retains such a form only for an instant. Why is this? Consider two surfaces *a* and *b* at the same level within the liquid. The total downward pressures upon *a* and *b* are proportional to their respective depths, but the depth of *b* is greater than that of *a*. Hence the unbalanced forces tend to lower the surface above *b* and to raise that above *a*. Motion ensues, and the liquid comes to rest only when the surfaces *a* and *b* are at the same depth.

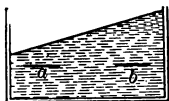


FIG. 13.—The Surface tends to become Horizontal.

48. Liquids seek their own Level. — When the same body of liquid is contained in any number of communicating vessels, the free surfaces in all must be at the same level; for if the level be higher in some than in others, the pressures of the raised amounts of liquid will force up the levels of the other surfaces (Fig. 14).

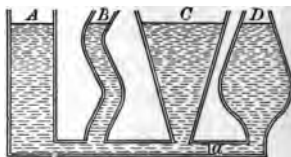


FIG. 14.—Communicating Vessels.

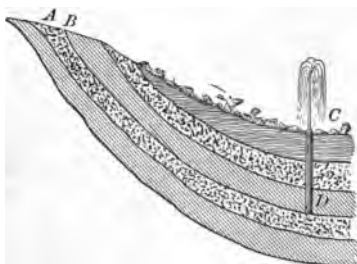


FIG. 15.—Artesian Well.

49. Water Works. — The water supply of cities depends upon the property that liquids have of seeking their own level. If the source is at a higher level than the city, the weight of the water alone will distribute it through the pipes. Otherwise the requisite pressure has to be exerted through the agency of pumps.

50. Artesian Wells are such as spout water above the level of

the ground. They may occur wherever porous strata of gravel or sand lie between impervious strata of rock or clay and the strata dip from a higher to a lower part of the country (Fig. 15). When a well is sunk in the lower part of the country to the porous strata, the water is forced to flow out because of the pressure of the water in the higher strata.

EXERCISES

1. Pascal placed a cube of wood on the bottom of a vessel, and, holding it there in close contact with the bottom, poured in enough mercury to cover the cube, no mercury being allowed to come below the wood (Fig. 16). He found that the wood, although much lighter than mercury, refused to rise. But on lifting it up a little, so as to let mercury get underneath the wood, he saw that it promptly rose to the surface. Account for this phenomenon.

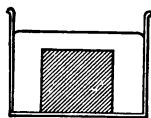


FIG. 16.

2. Calculate the pressure in grams per square centimeter at a depth of 10 m. in a fresh-water lake.

3. If the pressure at the bottom of a fresh-water lake is four times the pressure at a depth of 2 m., how deep is the lake there?

4. If a hole 10 cm.² in area is bored in a ship's bottom 500 cm. below the water line, what pressure (kilograms) must be applied to a board placed over the hole in order to keep the water from coming in?

5. Suppose that the head of water in a system of waterworks is 50 m. above the distributing pipes. What is the pressure ($\frac{\text{kg.}}{\text{cm.}^2}$) in the pipes?

6. What head of water in meters must there be in a system of waterworks so that the pressure in the distributing pipes may be $2.4 \frac{\text{kg.}}{\text{cm.}^2}$?

7. What is the pressure on a gate 2 m. long and 3 m. high, in the side of a dam, when the water stands at the height of 12 m. above the top of the gate?

8. The water in a square swimming-tank is 2.6 m. deep; the sides are vertical and 20 m. long. Calculate the pressure in kilograms (*a*) on the bottom; (*b*) on one side.

9. An iron bar 25 cm. long, 12 cm. wide, and 6 cm. high, is suspended horizontally with its lower largest side 44 cm. below the surface of water. What is the amount and direction of the pressure on each face?

10. A cubical box, 10 cm. on each edge, is filled half with water and half with mercury. What is the total pressure (*a*) on the bottom; (*b*) on the sides? (Density of mercury is 13.6.)

11. The diameter of the small plunger of a hydraulic press is 10 cm. that of the large plunger is 1 m. If a pressure of 200 kg. is applied to the small plunger, what load can be sustained by the large plunger?

12. A certain hydraulic elevator is designed to carry 10,000 kg. If the

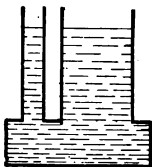


FIG. 17.

pressure of the water supplied is $50 \frac{\text{kg.}}{\text{cm.}^2}$, what should be the diameter of the large plunger?

13. The cross-sectional areas of the pipes in Figure 17 are in the ratio of 10 : 1. How do the weights of the liquid in each pipe compare? How do the (1) total pressures, (2) pressures per unit of area at the bottoms of the pipes compare?

THE PRINCIPLE OF ARCHIMEDES

51. **Buoyant Force of Liquids.** — Consider the cube *abcd* (Fig. 18) immersed in a liquid so that two of its faces are horizontal. The pressures on the vertical sides are equal, since they have equal areas and are at the same depth. Since the directions of these equal pressures on the opposite sides are towards each other, they offset each other, and their only effect is to produce a strain in the material of the cube. The downward pressure at *d* is less than the upward pressure at *c*, because the depth is less. There is then an unbalanced force acting upon the cube tending to make it rise; this is called the buoyant force of the liquid. Because of this buoyant force, a body apparently weighs less in a liquid than in the air, and less in the air than in a vacuum.

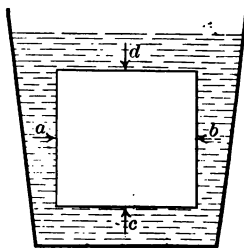


FIG. 18.—Buoyant Force.

52. Archimedes's Principle. — Archimedes (287–212 B.C.) made the observation that when a body is sunk in a liquid, an amount of liquid equal in volume to that of the body is displaced upward, that is, is raised; and he drew therefrom the conclusion that *a submerged body is buoyed up by a force (apparently losing a part of its weight) equal to the weight of the liquid displaced.* Of the various experimental proofs of this principle, that shown by the “bucket and plug” experiment (Fig. 19) is perhaps most readily grasped.



FIG. 19.—Bucket and Plug.

The cylindrical plug fits accurately into the bucket; the volumes of the bucket and of the plug are therefore equal. The bucket is hung from the short pan of a hydrostatic balance, and the plug hooked beneath the bucket. The balance is brought to equilibrium by placing weights upon the other pan. A glass of water is then brought up under the plug until it is just submerged when the beam is held horizontal. As soon as the beam is released, the plug is pushed upward by the buoyant force of the water. But when the bucket is filled just full with water, the scales again balance.

53. Floating Bodies. — A body placed in a liquid will completely sink if its weight is greater than that of an equal volume of the liquid. But if its weight is less than that of an equal volume of the liquid, it will only partially sink until the weight of the liquid it displaces is just equal to its own weight. *A floating body displaces a volume of liquid the weight of which is equal to its own weight.*

54. Displacement. — It is customary in comparing the sizes of ships to speak of them as being of so many tons burden or displacement. Thus a ship of 10,000 tons displacement sinks until the weight of water displaced equals 10,000 tons. The downward force of the ship equals the upward buoyant force of the water; hence the weight of the ship when loaded is also 10,000 tons.

EXERCISES

1. Steel is nearly eight times as heavy as water, and yet a steel ship floats. How may this be accounted for?
2. Why can stones in water be moved so much more easily than on land?
3. A floating cork displaced 20 cm.³ of water. What was the weight of the cork?
4. A barge with vertical sides is 30 ft. long and 10 ft. wide. When an elephant is driven on board, the barge sinks 4 in. Find the weight of the elephant.

SPECIFIC GRAVITY

55. Specific Gravity. — *The specific gravity of a substance is the number of times greater that the weight of a given volume of the substance is than the weight of an equal volume of some substance taken as a standard; it is the ratio of the weights of equal volumes.* For solids and liquids, water at a temperature of 39.2° Fahrenheit or 4° Centigrade has been chosen as the standard. Hence for all solids and liquids:

$$\text{Specific gravity} = \frac{\text{Weight of the substance}}{\text{Weight of an equal volume of water}}.$$

56. Density and Specific Gravity. — Care must be taken not to confuse density with specific gravity. Density, being the mass of unit volume, varies with the units employed, while *specific gravity*, being a *ratio of weights* of equal volumes, is not affected by a change of units. If the centimeter and the gram are chosen as units, then, since one gram of water at 4° C. occupies one cubic centimeter, the number expressing the weight of the water is the same as the number that denotes the volume of the water. With these units the density and the specific gravity of a substance are numerically equal.

EXAMPLE.—One cubic centimeter of iron has a mass of 7.8 g. Its density is therefore $7.8 \frac{\text{g}}{\text{cm.}^3}$. Since one cubic centimeter of water weighs one gram, the specific gravity of iron is also 7.8.

One cubic foot of iron has a mass of 486.72 lb. Its density in the English system of units is therefore $486.72 \frac{\text{lb.}}{\text{ft.}^3}$. As, however, one cubic foot of water weighs 62.4 lb., the specific gravity of iron is found to be the same as before, for $\frac{486.72}{62.4} = 7.8$.

57. Application of Archimedes's Principle to Specific Gravity Determinations.—When a body sinks below the surface of a liquid, it is manifest that the volume of the liquid displaced equals that of the body; *equality of volumes* is thereby assured. The weight apparently lost by the immersed body is, by Archimedes's principle, equal to the weight of the liquid displaced. The difference between the weights (the *loss in weight*) of a body outside of and within a liquid gives then the weight of a volume of the liquid equal to the volume of the body.

$$\text{Specific gravity} = \frac{\text{Weight in air.}}{\text{Loss of weight}}$$

58. Methods of determining Specific Gravities.—(1) *Solid heavier than water and insoluble in it.* Suppose a piece of iron to weigh 78 g. in air and 68 g. in water. The loss in weight is 10 g., which, by Archimedes's principle, is the weight of an equal volume of water. Hence the specific gravity of iron is $\frac{78}{78 - 68} = 7.8$.

(2) *Solid heavier than water and soluble in it.* The loss in weight has to be determined in some liquid that does not dissolve the solid. Suppose a piece of rock candy to weigh 40 g. in air and 20 g. in kerosene. The specific gravity of the candy referred to kerosene is therefore $\frac{40}{40 - 20} = 2$. The specific gravity of kerosene referred to water has been found to be .8. Hence the specific gravity of rock candy referred to water is $2 \times .8 = 1.6$.

(3) *Solid lighter than water.* Suppose that a piece of wood weighs 80 g. in air, that a piece of lead weighs 100 g. in water, and that the two when tied together weigh 60 g. in water. The buoyant force of the wood not only supports it, but also takes $100 - 60 = 40$ g. away from the weight of the lead when submerged in water. Hence the buoyant force upon the wood is $80 + 40 = 120$ g., and this is the weight, by Archimedes's principle, of a volume of water equal to that of the wood. The specific gravity of the wood is therefore $\frac{80}{120} = .67$.

(4) *Liquid by Specific Gravity Bottle.* Weigh a bottle, first empty, then filled with water, and finally filled with the liquid. Suppose the weights to be 20 g., 60 g., and 80 g., respectively. Then the weights of equal volumes of the liquid and of the water are 60 g. and 40 g., respectively. Therefore the specific gravity of the liquid is $\frac{60}{40} = 1.5$.

(5) *Liquid by Sinker Method.* Suppose that a piece of glass weighs 33 g. in air, 23 g. in water, and 15 g. in sulphuric acid, the glass sinker losing 10 g. in water and 18 g. in the acid. These losses in weight are, by Archimedes's principle, the weights of equal volumes of water and acid. Therefore the specific gravity of the acid is $\frac{18}{10} = 1.8$.

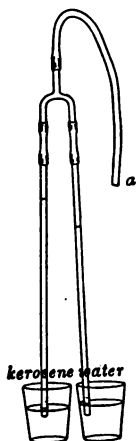


FIG. 20.—Specific Gravity by Balancing Columns Method.

(6) *Liquid by Balancing Columns.* The lower ends of two tubes dip into two liquids, say kerosene and water (Fig. 20), while their upper ends are joined to a T-tube connected by its third opening to a rubber tube. Suction is applied at *a* until the liquids rise nearly to the T-tube, and then the rubber tube is pinched together. The weights of the liquid columns are equal (why?) and their volumes proportional to their lengths. Suppose the length of the kerosene column to be 100 cm. and that of the water 80 cm. Then the specific gravity of the kerosene is $\frac{100}{80} = 1.25$.

When the liquids do not mix, a U-tube (Fig. 9) may also be used. The principle of the method is the same in both cases.

59. Hydrometers.—When a body is floated in water and then in another liquid, the weights of the displaced liquids are equal, but their volumes may be different. If these volumes be measured, the specific gravity of the

liquid may be found by dividing the volume of the water by that of the liquid. Instruments designed to determine specific gravities by means of volume measurements are called *constant-weight hydrometers*.

When a floating body is made to sink to the same depth in various liquids, the volumes of the liquids displaced will be equal, but their weights may be unequal. Instruments designed to find specific gravities by determining the weights needed to make them displace equal volumes of liquids are known as *constant-volume hydrometers*.

60. Constant-weight Hydrometers.—As commonly made, a hydrometer of constant weight consists of a glass tube near one end of which are two glass bulbs (Fig. 21). The lower bulb is loaded with mercury or shot, and the upper bulb as well as the rest of the tube contains air. The weight of the hydrometer is so adjusted that when placed in pure water, it will sink to a certain point which is usually marked 1.000 on the scale attached to the instrument. The scale is so graduated that when the hydrometer is floated in a liquid, the scale reading at the liquid level gives the specific gravity directly. The specific gravity of liquid mixtures of certain kinds gives information as to their composition, and special forms and graduations of hydrometers have been devised to give indications as to the strength and purity of certain liquids. Thus a *lactometer* is a hydrometer for finding the specific gravity of milk. The names *acidometer* and *alcoholometer* suggest the uses of these instruments.

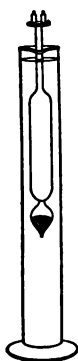


FIG. 22.—Fahrenheit's Hydrometer.

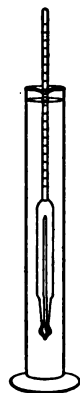


FIG. 21.—Constant-weight Hydrometer.

61. Constant-volume Hydrometers.—*Fahrenheit's hydrometer* (Fig. 22) is used to determine the specific gravity of liquids. It resembles a constant-weight hydrometer, but has no scale.

and is provided with a little pan at the upper end of the stem. The hydrometer is weighed, floated in water, and weights are placed in the pan until the level of the water is at a certain mark on the stem. Suppose the weight of the instrument to be 30 g., and 10 g. to be placed upon the pan. Then, by Archimedes's principle, the weight of the water displaced is 40 g. Suppose that when the instrument is

placed in another liquid, say kerosene, 2.0 g. have to be placed in the pan to sink the hydrometer to the same depth as before. The weight of kerosene displaced is accordingly 32 g., and its specific gravity is $\frac{32}{40} = .8$.

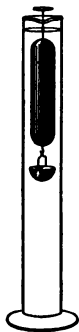


FIG. 23.—Nicholson's Hydrometer.

(2) *Nicholson's hydrometer* (Fig. 23), which is used to determine the specific gravity of solids, is a modification of Fahrenheit's instrument in that it has an additional pan beneath the water. Suppose that 24 g. must be placed in the upper pan in order to sink the instrument to a certain mark on the stem, and that, when some sand is in the upper pan, only 16 g. are required to bring the mark level with the liquid surface. Manifestly, the weight of the sand in the air is $24 - 16 = 8$ g. Suppose further that when the same sand is placed under water on the lower pan,

19.2 g. are needed in the upper pan to sink the instrument to the mark. The sand loses weight in the water to an amount equal to $19.2 - 16 = 3.2$ g., and this loss in weight is, by Archimedes's principle, equal to the weight of the water displaced by the sand. Hence the specific gravity of the sand is $\frac{8}{3.2} = 2.5$.

EXERCISES

1. Why does a vessel draw more water (*i.e.* sink deeper) in a river than in the ocean?
2. What is the specific gravity of a substance having a density of 312 lb. per cubic foot?
3. A body weighs 604 g. in air and 453 g. in water. What is its specific gravity?
4. If a piece of zinc weighs 175 g. in air and 150 g. in water, what is its specific gravity?
5. What is the volume in cubic centimeters of 49 g. of zinc?

6. What loss in weight will 125.3 g. of zinc undergo when immersed in water?

7. How much will 350 g. of zinc weigh in water? In kerosene?

8. If a certain body weighs 437.5 g. in air and 202.8 g. in a liquid, the specific gravity of which is 1.432, what is the specific gravity of the body?

9. What is the pressure ($\frac{\text{kg.}}{\text{cm.}^2}$) at a depth of 70 m. in the ocean, the density of sea water being 1.026?

10. A piece of metal of density 8 having a volume of 5 cm.³, and a piece of wood of density .6 having a volume of 150 cm.³, are fastened together and placed in water. How many cubic centimeters of the combination will remain above the surface?

11. If a force of 50 $\frac{\text{kg.}}{\text{cm.}^2}$ crushes a certain kind of brick which is 2.5 times as heavy as water, how high can a wall of this brick be carried without danger of its collapsing?

12. A glass stopper weighs in air 40 g., in water, 24 g., and in milk, 234 g. What is the specific gravity (a) of the glass? (b) of the milk?

13. If the specific gravity of copper is 8.92 and a certain piece of it weighs 52.45 g. in water, how much will it weigh in air?

14. If a silver dollar weighing in air 26.73 g. is suspended in water, what will be the tension in the supporting string, the specific gravity of silver being 10.57?

15. A lump of metal weighing in air 70.4 g. when placed in a vessel brimming full of water, caused 8.30 g. of the liquid to overflow. Calculate the specific gravity of the metal.

16. If a man weighs 75 kg., and his volume exclusive of his head is 72 l., find how many cubic decimeters of cork (sp. gr. = .25) are needed to keep him floating with his head out of water.

17. A copper sulphate crystal weighs 3.03 g. in air and 1.86 g. in turpentine (sp. gr. = .88). Find the specific gravity of the crystal.

18. If a cake of soap (8 cm. $\times$ 5 cm. $\times$ 3 cm. and sp. gr. = .96) is put into a washbasin brimming full of water, what volume of water overflows?

19. If a cylinder floats vertically in water with 4.5 cm. of its length immersed, to what depth will it sink in a liquid with a specific gravity of .92?

20. A piece of gold weighs 9.73 g., and a flask brimming full of water weighs 95 g. The gold is dropped into the flask, displacing some of the water, so that the flask and its contents now weigh 104.23 g. Calculate the specific gravity of the gold.

CHAPTER IV

MECHANICS OF GASES

PRESSURE IN GASES

62. Air and Water Typical Substances.—The statements made in Chapter III with respect to pressure in liquids apply also to pressure in gases, except such as refer to the free surface and the depth of liquids. Just as in studying the behavior of water under stress, we assumed that all liquids act in similar fashion so that the laws established in such a study could be applied to all of them, so now we shall take air as a type for gases, and shall consider the physical laws ascertained with regard to this typical gas, as true of all gases.

63. Weight of Air.—Air is so light that it is hard to realize that it, like any other kind of matter, is attracted by the earth,—that it has weight. Yet it can be readily shown that if air be compressed into a vessel, the weight of the vessel and contents will increase.

The density of gases is so small (the mass of a cubic foot of air is but an ounce), that large amounts of a gas are required to render its pressure due to gravity appreciable. Yet the weight of the air in a room, $15 \times 30 \times 60$ ft., exceeds a ton. The pressure of a gas due to its weight varies with its depth and density, as is the case with a liquid. Since, however, a gas is easily compressed, the density of its upper layers is less than that of its lower layers, which is not the case to an appreciable extent with

liquids. Gas pressure and depth are therefore not directly proportional, but vary in a quite complex fashion.

64. Torricelli's Experiment.—Torricelli, in 1643, in order to obtain a means of measuring the heaviness of air, devised the following experiment. A glass tube nearly a meter long and sealed at one end is completely filled with mercury (Fig. 24). The open end is stopped with the finger, the tube inverted, the temporarily closed end placed under mercury in a dish, and the finger removed. Some of the mercury runs out of the tube, but enough is left to form a column about 76 cm. (30 in.) high. Since the tube was completely filled with mercury, the space left vacant

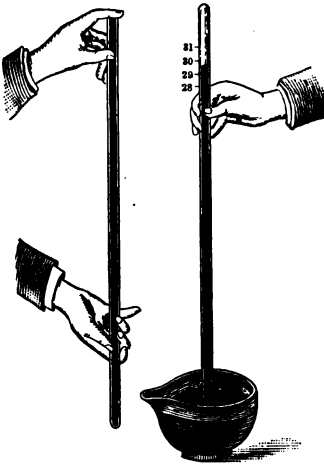


FIG. 24.—A Torricellian Tube.

cannot contain any air; it is empty of everything but a little mercury vapor. But the mercury in the dish is in contact with the atmosphere. These being the facts, Torricelli divined that it was the pressure due to the weight of the atmosphere which supported the column. The weight of the column of mercury of a given cross-sectional area should then be equal to the weight of the column of air of equal cross section which balances it.

If this be true, the column of mercury should be shorter where the density of the atmosphere is less. The higher the altitude of a locality, the less is the amount of the air above that place. Hence, at the top of a mountain, the

mercury column should be shorter than at its base. This conclusion was verified in 1648 by Pascal, who found that the mercury did not stand as high at the top of a tall tower as it did at its base. Later, some friends of Pascal, at his suggestion, found the same to be true on ascending a mountain.

Another conclusion from Torricelli's experiment is that the lighter the liquid is, the higher it should rise in the tube. Thus the density of mercury being 13.6 and that of water unity, the weight of the atmosphere should support a column of water 13.6 times as high as one of mercury.

65. Measurement of Atmospheric Pressure.—Since the pressure of a liquid depends upon its depth and density, the pressure of the atmosphere can be found by measuring the height and determining the density of the liquid column supported by it. The prevailing height of a column of mercury at the level of the sea is 76 cm. As pressure is given in units of weight (grams or pounds) per unit of area (square centimeter or square inch), if the area of the cross section of the mercury column is one square centimeter, the column occupies 76 cm.³, and since one cubic centimeter of mercury weighs 13.596 g., the weight of the column is $76 \times 13.596 = 1033.3$ g. At sea level, then, the atmosphere exerts an average pressure of $1033.3 \frac{\text{g.}}{\text{cm.}^2}$. This is equivalent to $14.7 \frac{\text{lb.}}{\text{in.}^2}$. Either is known as the pressure of one atmosphere.

To sum up, we have the choice of the following equivalent methods of expressing the amount of atmospheric pressure: One atmosphere = 760 mm. = 30 in. of mercury = 10.333 m. of water = $1033.3 \frac{\text{g.}}{\text{cm.}^2} = 14.7 \frac{\text{lb.}}{\text{in.}^2}$.

66. The Barometer.—The barometer is an instrument for measuring the pressure of the atmosphere. It is made in two different forms—the mercury, and the aneroid.



FIG. 25.—Mercury Barometer.

The *mercury barometer* is a Torricellian tube fastened by the side of a scale for measuring the height of the column of mercury (Fig. 25).

The *aneroid barometer* consists of a round, flat box from which part of the air has been



FIG. 26.—Aneroid Barometer.

taken (Fig. 26). Its top is made of thin metal which is corrugated in such a way that it bends under very slight forces. Feeble variations of atmospheric pressure suffice to make it move in or out. This motion is multiplied by means of a mechanism that causes a pointer to move round a scale, readings on which give the barometric pressure. While aneroids have the advantages of small size and portability, they must be frequently compared with mercury barometers, as they are subject to changes that impair the reliability of their indications.

67. Boyle's Law. — Every one knows that the volume of a gas depends upon its pressure. The very act of breathing proclaims this; we expand the chest to fill the lungs

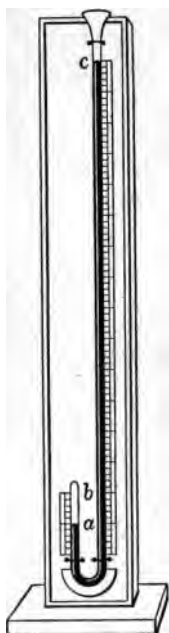


FIG. 27. — Boyle's Law Apparatus. The pressure upon the gas is equal to that of the atmosphere plus that of the mercury column *ac*.

with air, and then by contracting the chest we diminish the volume of the air taken in so that its pressure is greater than that of the atmosphere, when it is expelled. Other familiar instances present themselves in these days of automobile tires and other pneumatic appliances. But the first to show in what way the volume of a gas depends upon its pressure by making a series of measurements of gas volumes and their pressures was Robert Boyle. In 1662 he took (Fig. 27) "a long glass tube, which, by a dexterous hand and the help of a lamp, was in such a manner crooked at the bottom, that the part turned up was almost parallel to the rest of the tube, and the orifice of this shorter leg of the siphon (if I may so call the whole instrument) being hermetically sealed, the length of it was divided into inches (each of which was subdivided into eight parts) by a straight list of paper, which containing those divisions, was carefully pasted all along it."*

He introduced enough mercury to fill the crook in the tube, noted the volume, and then poured in more mercury, the weight

* The quotation is from Boyle's essay on "A Defense of the Doctrine Touching the Spring and Weight of the Air."

sponding volumes and pressures was made, from which he deduced the general conclusion. *The volume of a gas varies inversely as its pressure, the temperature remaining the same.*

If v and v' denote the volumes which a given mass of gas occupy at the pressures p and p' , respectively, then, by Boyle's law,

$$\frac{v}{v'} = \frac{p'}{p};$$

whence

$$pv = p'v'.$$

Boyle's law may therefore also be stated thus: *The products of the pressures by the corresponding volumes of a given mass of gas are always the same, the temperature remaining constant.*

Boyle tested the relationship for pressures less than one atmosphere, as well as for pressures greater than one atmosphere (Fig. 28). Subsequent and more accurate work has proved that the law is not strictly true, its inaccuracy becoming greater the nearer a gas approaches a temperature and a pressure at which it condenses into a liquid.

68. Density, Volume, and Pressure of a Gas. — When the pressure of a given mass of gas is increased, its volume is diminished; the same amount of matter is made to occupy less room. Now density is the amount of matter occupied by the unit volume; a decrease of volume, therefore, corresponds to an increase of density. *The density of a gas is accordingly directly proportional to its pressure.*

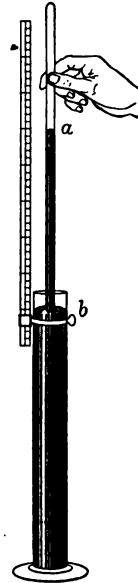


FIG. 28.—Boyle's Law Apparatus. The pressure upon the gas is equal to that of the atmosphere minus that of the mercury column ab .

69. Height of the Atmosphere.—At an altitude of about 3.5 miles, the mercury in a barometer stands only half as high as it does on an average at the earth's surface. This indicates that half of the mass of the atmosphere is within 3.5 miles of the surface. If the air were of uniform density, the height of the atmosphere could be found as follows.

Since the average pressure of the atmosphere is $1033.3 \frac{\text{g}}{\text{cm}^2}$, and since one cubic centimeter (this is the same as a column one centimeter high and one square centimeter in cross-sectional area) of air weighs .00129 g., a column of air of one square centimeter cross-sectional area would have to be high enough to weigh 1033.3 g.; that is, its height would be $\frac{1033.3}{.00129} = 8 \times 10^5 \text{ cm}$. But because of the compressibility of air,

the density of the atmosphere decreases with the altitude, so that such a result is hardly even a gross approximation, the actual altitude being much greater. In consequence of the decrease of pressure with the altitude, the barometer may be used to estimate the heights of mountains. Up to elevations of several thousand feet, it has been found that a fall of one inch in the barometric height corresponds to an elevation of about 900 ft.

70. Buoyant Force of Gases.—Archimedes's principle

applies to gases as well as to liquids. *Any body immersed in a gas is buoyed up by a force equal to the weight of the gas displaced.* Any body the weight of which is less than that of the air displaced by it will tend to move upward.

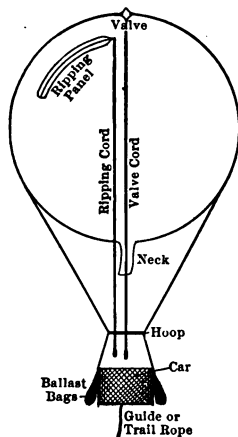


FIG. 29.—Balloon.

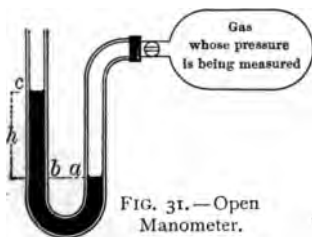
71. Balloons.—A balloon (Fig. 29) is a large bag made of cotton or silk, treated with a special varnish to make it as nearly gas-tight as possible. A network of cordage envelops the bag, the lower parts being joined to ropes which pass through the hoop and support the car. The neck is open so as to permit the free expansion of the gas, and a valve is

at the top by means of which the balloonist may slowly empty
g of gas. To enable him to let the
it rapidly (as is frequently necessary at
re of landing), a section or panel is so
in that a vigorous tug on a cord will
away. Around the car are hung bags
id, which can be emptied so as to
the balloon and make it rise.

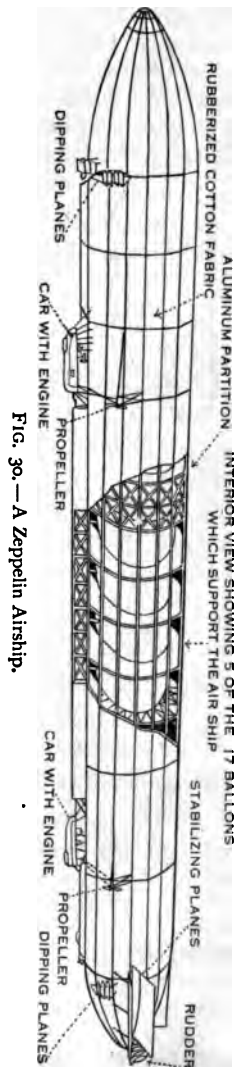
loons are filled with hydrogen or the
r illuminating gas, or even with hot air.
one liter of hydrogen weighs .09 g.
e liter of air weighs 1.29 g., the lifting
s equal to $1.29 - .09 = 1.20$ g. per liter.
one liter of illuminating gas weighs
how does the lifting force of a balloon
with hydrogen compare with its lifting
when filled with illuminating gas?

Dirigible Balloons. — By providing
ns with propellers driven by gas en-
and with rudders, they can be made in-
sure independent of the wind and can
even in any desired direction. A type
uccessful dirigible balloon is shown in
e 30.

Manometers. — A manometer
instrument for measuring the



sure of a gas. Two forms are
e, the *open* and the *closed*.



The *Open Manometer* is used to measure small pressures. It consists of a U-shaped tube (Fig. 31) attached to a support having a graduated scale between the arms of the tube. The tube is filled with a liquid (usually mercury or water) nearly up to the part of one arm where there is a horizontal branch at which connection is made with the vessel containing a gas. The gas presses against the liquid and forces it down in one arm and up in the other. The difference in the levels of the two columns is a measure of the amount of pressure. By multiplying this by the density of the liquid used, the pressure in units of weight per unit area is obtained.



FIG. 32.—Closed Manometer.

The *Closed Manometer* (Fig. 32) is similar to the open manometer, but is used to measure greater pressures, its action involving the application of Boyle's law. Before the pressure is applied, the liquid levels in the two arms are equalized. The confined gas is then under a pressure equal to that of the atmosphere. When the pressure is applied, the gas volume decreases and the mercury rises in the closed branch.

If the tube is assumed to have the same bore throughout its length, the change in the length of the gas column may be taken as a measure of its change in volume. Two volumes and one pressure are then known, from which, by Boyle's law, the unknown pressure may be calculated.

EXERCISES

1. Why do bubbles of air grow larger as they ascend in water?
2. A tumbler brimming full of water and covered with a piece of paper may be inverted without the water running out. How is this?
3. When will a balloon remain at a constant height above the earth?
4. The tube *abcd* (Fig. 33) is twice as long as a barometric tube. It is inverted and completely filled with mercury when the opening *b* is closed. The end *a* is closed with the finger and immersed in a dish of mercury. On taking the finger away from *a*, the mercury in *dc* falls down and fills the enlarged portion *R*; and the mercury in *ba* falls until it forms a column equal in height to the prevailing barometric column. How is this accounted for? If now *b* is opened, the mercury all runs out of *ba*, but rises in *dc* to a height equal to that of the barometer. Account for this behavior.



FIG. 33.

5. If the air is partially removed from the two tubes (Fig. 34), how will the heights to which the liquid rises in them compare, a being twice as wide as b ?

6. A bubble of air has a volume of 2.72 cm.^3 at a depth of 634 cm. below the surface of water. What will be its volume at the depth of 317 cm., the pressure of the atmosphere being neglected?

7. What will be the volume of a given mass of gas under a pressure of $10 \frac{\text{lb.}}{\text{in.}^2}$, if under a pressure of $100 \frac{\text{lb.}}{\text{in.}^2}$ it occupies 4 ft.^3 ?

8. When acetylene gas burns it generates double its volume of carbonic acid gas. How many cubic centimeters of carbonic acid gas under a pressure of 75 cm. of mercury are produced by the combustion of 100 cm.^3 of acetylene under a pressure of 125 cm. of mercury?

9. The manometer (Fig. 31) is filled with water and connected with an illuminating gas cock, the pressure of the gas making bc equal to 6.4 cm. What would bc equal if (1) mercury, (2) kerosene, were substituted for the water?

10. The length of bc (Fig. 32) is 10 cm. and the length of the air column above b is 30 cm. when the atmospheric pressure is 76 cm. of mercury. When pressure is applied, the length of the air column becomes 10 cm. What is the value of the pressure applied?

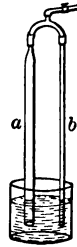


FIG. 34.

PNEUMATIC APPLIANCES

74. **Siphons.** — The siphon (Fig. 35) consists essentially

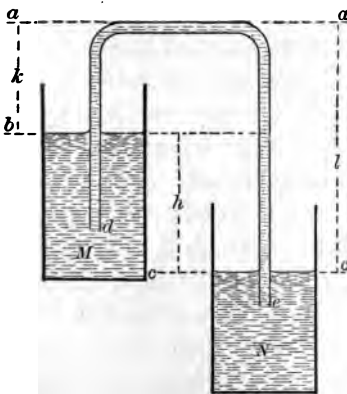


FIG. 35.—The Siphon.

of a bent tube with branches of unequal length; it serves to transfer a liquid from a higher to a lower place over some intervening obstacle, as the side of a vessel. It is set in operation by filling it with the liquid and introducing the shorter (inlet) branch into the liquid that is at the higher level. Two forces act in the working of a siphon — the pressure

of the atmosphere and the weight of the liquid within the tube. Atmospheric pressure serves merely to keep the siphon filled with the liquid. If the bend of the siphon is so high above the surface of the liquid to be transferred that the pressure of the atmosphere cannot push the liquid up to it, the siphon refuses to work, for the liquid does not reach around over the bend. A siphon cannot act within a vacuum, for there is no pressure against the liquid's surface to force it up the tube. Again, if the end of the outlet branch is at the same level as the surface of the liquid in the higher vessel, the liquid does not flow; for in this case the pressure of the atmosphere acts equally upon the liquid in both branches and merely keeps the siphon filled. When, however, the outlet is lower than the surface of the upper liquid, and the tube is full, the flow takes place.

Suppose the siphon to be in action, water being transferred from M to N . At the level of the water surfaces b and c , the pressure of the air is not appreciably different and tends to force the water up in both branches. The bend a should not be higher above b than about 1033 cm., as the atmospheric pressure cannot support a column of water higher than that. The downward pressure at c is equal to the weight of the column of water ac or l , while that at b is equal to the weight of the column ab or k . The difference between the two columns is h , and the downward pressure in the longer arm therefore exceeds the downward pressure in the shorter arm by the weight of the column of water h , which, not being balanced by any upward force, causes the liquid to move downward. But as fast as water leaves the outlet, fresh portions are forced up through ba by atmospheric pressure, so that a continuous flow ensues. As the cause of the flow is the excess of

the weight of the liquid in the longer branch over that in the shorter branch, the greater this excess, that is, the longer h is, the more rapid will be the flow.

75. Lift Pumps. — The *lift* or *suction pump* is a contrivance that applies the pressure of the atmosphere in raising water by means of a cylinder and perforated piston fitted with valves which allow the water to pass upward

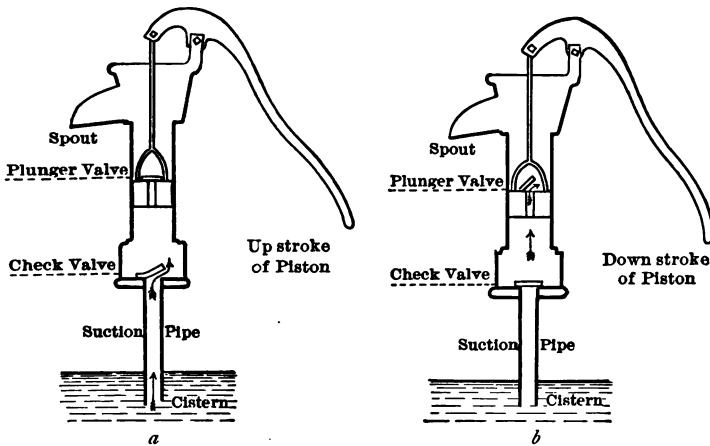


FIG. 36.— Lift Pump.

but not downward. When the piston (Fig. 36 *a*) is lifted, the valve in it (the plunger valve) remains closed, while the valve in the cylinder (the check valve) opens because the atmospheric pressure upon the water in the cistern makes the water rise and fill the partial vacuum formed. With a downward motion of the piston (Fig. 36 *b*), the pressure of the air or water trapped in the cylinder closes the check valve and opens the plunger valve. At the end of the downward stroke the air or water that was in the

cylinder has passed through the plunger valve and is above the piston. The second upstroke lifts this air or water up, while more water rises into the cylinder. Repetitions of the strokes result in the water filling the pipe and flowing from the spout. As it is atmospheric pressure that causes the pump to act, it is evident that the check valve must be near enough to the surface of the water for the atmospheric pressure to succeed in raising water up to it. Experience has shown that it is not practicable to have the valve more than about 28 feet above the water (Fig. 37).

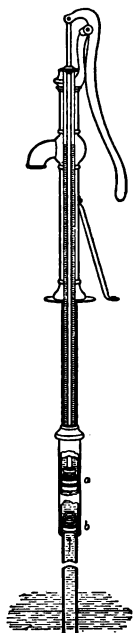


FIG. 37.—Deep Well Pump.
 δ , Check Valve.

76. The Force Pump.—Each stroke of a lift pump discharges a limited amount of water, its action being intermittent. Furthermore, a lift pump can raise water only as high as its spout. The force pump, on the other hand, not only gives a nearly constant stream of water, but also forces it above the level of the pump. This is accomplished by substituting (Fig. 38) a solid piston of the lift pump and replacing th

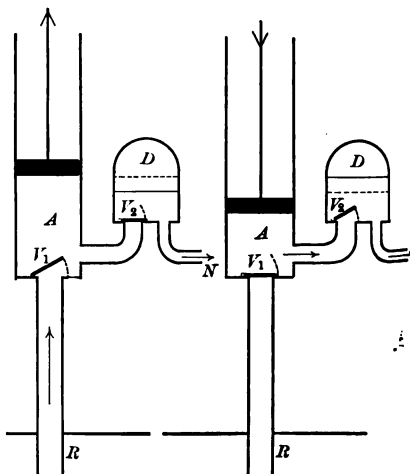


FIG. 38.—Force Pump for Liquids.

for the hollow piston, of the lift pump and replacing th

plunger valve by a valve V_2 that closes a pipe leading to an air dome or chamber D . During an upward stroke of the piston, the pressure of the air in A is reduced and the pressure of the atmosphere forces V_1 open, while the pressure of the air in the dome keeps valve V_2 closed. At the same time the pressure of the atmosphere upon the water in the cistern R forces water up the pipe. During the downward stroke of the piston valve V_1 is closed and valve V_2 opened. Water then enters the air dome and compresses the air therein. As the pipe N is smaller than the supply pipe, water can be forced during the down-stroke into the air dome faster than it can escape. During the upstroke, when no water is entering the dome, the compressed air in it expands and forces the water out; the outflow is thereby made nearly continuous. By connecting a vertical pipe at N , the water may be forced to a considerable elevation.

77. The Air Pump. — Air pumps serve either to take air out of a vessel (receiver) or to force air into it. The name is commonly applied to pumps that exhaust the air within a receiver. Those which compress air are called *compressors, condensing pumps, blowing engines, or blowers.*

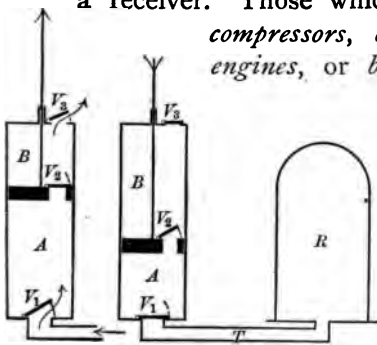


FIG. 39. — Exhausting Air Pump.

The construction of the ordinary mechanical exhausting air pump resembles that of the lift pump. The bell jar or receiver R has its rim ground smooth and flat so as to fit tightly upon a metal plate in the center of which is

a tube communicating with the cylinder (Fig. 39). During the upward stroke of the piston, the air below it is rarefied so that the greater pressure of the air within the receiver forces valve V_1 open. The air already in the cylinder is thereby compressed and keeps valve V_2 closed, but opens valve V_3 . During the downward stroke, the air which has passed from the receiver into the cylinder is compressed, thereby closing valve V_1 but forcing valve V_2 open. As the pressure of the air that now passes through the cylinder is less than that of the atmosphere, valve V_3 is shut by the pressure of the outside air. With each up and down stroke, then, a volume of air equal to the capacity of the cylinder is drawn from the receiver. But even with an unlimited number of strokes a perfect vacuum cannot be secured; for only a part of the air is removed by each stroke, the rest remaining in the cylinder.

78. Degree of Exhaustion Possible. — Suppose the capacity of the cylinder to be one liter and that of the receiver 9 liters, while the barometric height is 76 cm. When the piston is raised, the 9 liters of air in the receiver expand to 10 liters, while its pressure, by Boyle's law, is reduced to $76 \times \frac{9}{10} = 68.4$ cm. By lowering the piston, one liter of the air under this diminished pressure is expelled. During a second up-stroke, the remaining air expands so that its pressure is further reduced in the same proportion, becoming $76 \times \frac{9}{10} \times \frac{9}{10} = 61.56$ cm. In like fashion after n strokes the pressure will be $(\frac{9}{10})^n \times 76$ cm. The density of the air or the amount of it left in the receiver will diminish at the same rate.

79. Air-compressing Pumps. — If in the exhausting air pump (Fig. 39), valve V_3 be done away with and the action of the other valves reversed, a compressing air pump results (Fig. 40). When the piston moves to the right, valve V_2 is closed and valve V_1 opened so that the air that was in the cylinder under atmospheric pressure is forced

into the receiver R . When the piston moves to the left, valve V_1 is closed and V_2 opened, and the cylinder is again filled with air under atmospheric pressure. Each stroke of the pump crowds a cylinderful of air into the pump. Familiar air compressors

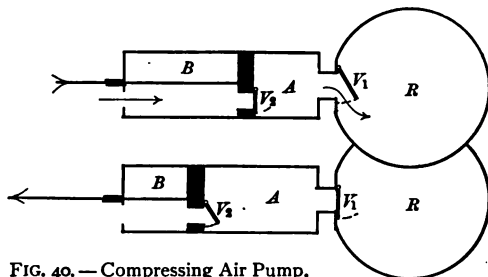
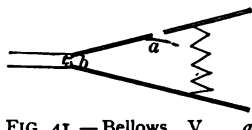


FIG. 40. — Compressing Air Pump.

are the bellows (Fig. 41) and the bicycle or automobile tire pump. Compression pumps driven by steam or electrical power are extensively used to furnish compressed air to operate pneumatic tools of various kinds as well as diving bells and caissons.

FIG. 41. — Bellows. V opens in, and valve b , out.

80. Mechanical Valve Air Pumps. — The degree of exhaustion of such an air pump as has been described is limited by the pressure it takes to lift the valves, which is rarely less than $\frac{1}{20}$ of an atmosphere. If, however, the valves are directly operated by the piston and if oil is used as a packing for the piston, much greater degrees of exhaustion are obtainable (Fig. 42). The *Hemenway Oil Seal Air Pump* (Fig. 43) has two cylinders CC' in which moves a double-ended piston PP' that acts as a sliding valve for the intakes EE' . The outer ends of the cylinder are covered by the check valves VV' held by coil

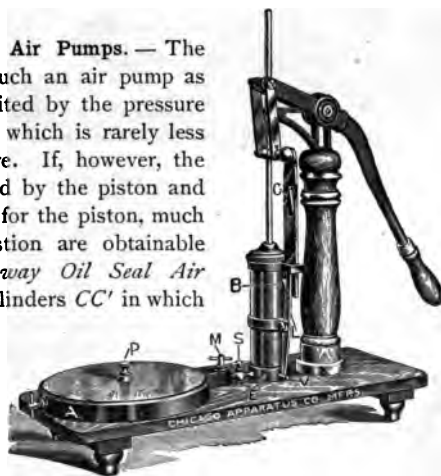


FIG. 42. — Air Pump with Mechanical Valves.

springs against the openings OO' that lead to the hollow balls RR' . The valves are submerged in oil so that leakage cannot occur.

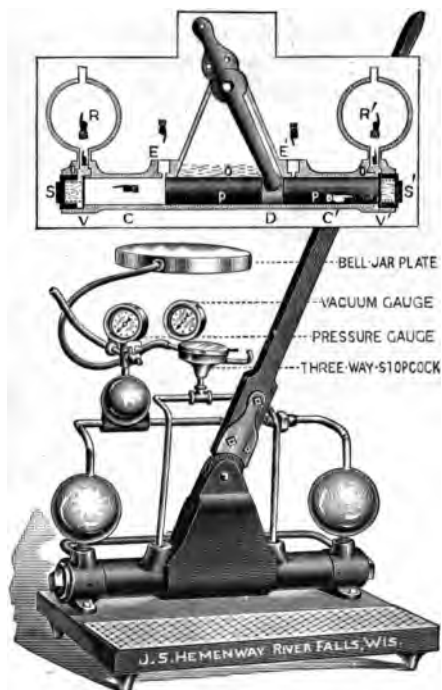


FIG. 43. — Hemenway Oil Seal Air Pump.

as an exhausting or as a compressing pump, and reduces the pressure in a vessel to nearly $\frac{1}{30}$ of an atmosphere, or increases its pressure to about seven atmospheres.

81. Mercury Exhausting Pump. — Mercury is allowed to drop through a narrow tube to which is attached a branch leading to the vessel from which the air is to be exhausted, such as an incandescent lamp bulb (Fig. 44). As the drops of mercury pass by the side opening, the air which is caught between them is dragged down and forced out at the bottom of the tube.

With the piston in the position shown, the intake E is open, and air can pass from the bell jar into the cylinder C . By moving the piston to the left, the intake E is closed and the air is compressed in the cylinder C . The valve V is forced open either by the pressure of the air or by the piston itself, and the air passes through the oil at O into R , where it may be compressed or allowed to escape. During this time the reverse order of events has been taking place in the other cylinder, one cylinder being emptied and the other filled at each stroke of the pump. The Hemenway pump may be used either



FIG. 44. — Mercury Exhausting Pump.

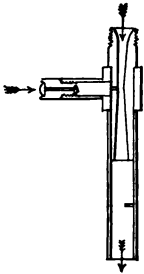


FIG. 45. — Water Exhaust.

82. Aspirators.— An aspirator or water exhaust is similar to a mercury pump, water, however, instead of mercury being used (Fig. 45). The aspirator is connected with the water supply, and a rapid current passed through it. Air is caught at the side opening which is connected with the receiver.

83. Ejectors.— When a strong current of air is blown at a certain angle across the narrowed opening of a tube, the lower end of which is immersed in a liquid to which the atmosphere

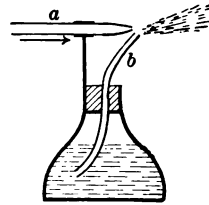


FIG. 46. — Atomizer.

has free access, the pressure within the tube is lessened so that the atmospheric pressure forces

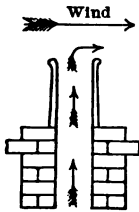


FIG. 47. — A Chimney acts as an Ejector.

the liquid up through the orifice. The current of air separates the particles of the liquid and scatters them in a fine spray. Such an *ejector* is the familiar atomizer (Fig. 46). A chimney may be considered to act as an ejector when the wind is blowing. When the wind is in the right direction, a rapid current of air

passes up through the chimney, even if the fire is out (Fig. 47).

84. The Diving Bell.— By pressing the rim of an empty and inverted tumbler down upon the surface of water, some air is entrapped which cannot escape as long as the rim is horizontal. The deeper the tumbler is immersed, the more the air is compressed, the water rising proportionally higher within it. A diving bell resembles a huge, heavy tumbler

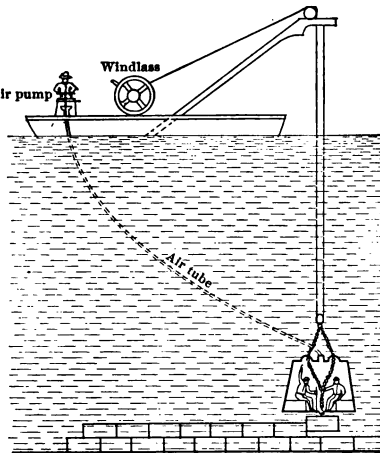


FIG. 48. — Diving Bell.

in which men can work under water at laying foundations for bridges and lighthouses. Air is pumped into diving bells (Fig. 48) by means of a compressor for the double purpose of furnishing fresh air to the workmen and of keeping the water level within the bell near its bottom, the excess of air bubbling out from under the bell's rim. The pressure of the air within the bell is that of a column of water, the length of which is equal to the depth of the rim. If the bell, for example, is 34 ft. below the water's surface, the pressure is twice that of the atmosphere. *Caissons* are immense stationary diving bells.



FIG. 49. — Diving Suit.

85. The Diving Suit. — The diving suit may be regarded as a one-man diving bell (Fig. 49). Air is forced in through a pipe or tube leading to a pump on board a boat, and the used air escapes through a non-return valve in the suit. Or the diver may carry on his back a tank of compressed air from which to draw his supply, such an arrangement rendering him more independent of the surface. The soles of the shoes are weighted so as to enable him to maintain his equilibrium under water.

EXERCISES

1. John Canton in 1761 filled a thermometer tube with water, boiled the water in it so as to expel the air, and sealed the tip air-tight. On breaking off the tip so as to admit the air after the thermometer came to the temperature of the room, the level of the water at once sank a trifle. To what is this change in volume due?

2. What is the purpose of the large copper bulb or dome so conspicuous on a fire engine?

3. In a deep dish are placed two similar small dishes in which is mercury, the dish *d* (Fig. 50) containing more than the dish *e*. In the two dishes are placed two ends of a forked tube, the longer ("handle") end of which is open to the air. When water is poured into the large dish, the mercury rises in the tubes, and at last flows over from *d* to *e*. Give an adequate explanation for this behavior.

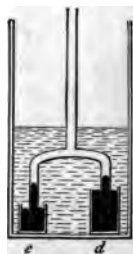


FIG. 50.

4. To reduce the pressure of the air within a receiver from 746 mm. to 37.3 mm. of mercury, what per cent of the air must be removed?

5. If air is removed from the receiver of an air pump until its pressure supports a column of mercury only 10 mm. high, what is the pressure of the air in the receiver in grams per square centimeter?

6. How high can a liquid with a specific gravity of 1.8 be raised by a perfect lift pump, the barometer standing at 720 mm.?

7. Over how high a hill can water be transferred by means of a siphon when the mercury stands at 74 cm.?

8. If the cylinder of a bicycle pump has a capacity of 200 cm.³, and an automobile tire has a capacity of 4000 cm.³, how many strokes of the pump are required to fill the vacuous tire with air under a pressure of five atmospheres?

9. If a diver is working at a depth of 6 m. in sea water (sp. gr. = 1.023), what pressure ($\frac{\text{kg.}}{\text{cm.}^2}$) is he under?

10. A receiver is exhausted by an air pump when the atmospheric pressure is 760 mm. If the air still remaining in the receiver exerts a pressure of 4 mm., what fraction of the air at first present has been removed?

11. If the capacity of an inflated automobile tire is 10 liters, and air is pumped into it until the pressure becomes equal to five atmospheres, what is the increase in weight of the tire?

12. The height of the mercury in a Torricellian tube is 75 cm. Air is then introduced into the tube until the mercury column is 50 cm. high. If the volume of the air above the mercury is 15 cm.³, what was its volume before it was put into the tube?

CHAPTER V

FORCE AND MOTION

UNIFORMLY ACCELERATED MOTION

86. Motion. — Motion is a continuous change of position. As the location of a body in space can be determined only by specifying its direction and distance from some other body or from some point of reference, a body in moving must change either its direction or its distance with reference to some other body or some point. All motions known to us must therefore be relative. A body may at the same instant be at rest with reference to certain objects and in motion with reference to certain others. Thus a passenger in a railroad car may be at rest relatively to the car and the other passengers, but in motion relatively to outside objects. We mean ordinarily by the expression "a body at rest" that the body does not change its position with reference to that which supports it, as, for example, the deck of a ship or the surface of the earth. Obviously this is all we can mean, for the earth itself is spinning upon its axis as well as revolving round the sun. Such a thing as absolute rest or motion does not come within the realm of our experience.

87. Position in a Plane. — Chicago is in $41^{\circ} 53'$ north latitude and $87^{\circ} 38'$ west longitude. That is to say, the city is at certain fixed distances (measured in degrees) north of the equator and west of the meridian passing through Greenwich. Its position is therefore described

with respect to a point of reference, *viz.*, the point of intersection of the equator with the Greenwich meridian. All places on the earth's surface may be located in this manner.

A similar method of describing the position of a point in a plane is used in physics. The point of reference, called the *origin*, is at the intersection of two perpendicular lines of reference called *axes* or *coördinates*. The horizontal line is called the *axis of abscissas* or *axis of X* (sometimes

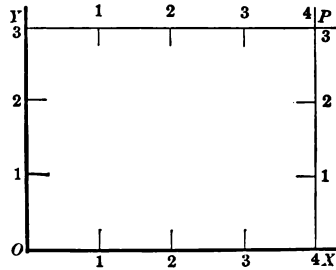


FIG. 51. — Rectangular Coördinates.

x-axis), and the vertical line, the *axis of ordinates*, or *axis of Y* (sometimes *y-axis*). Lines parallel to the axis of X are called *abscissas*; those parallel to the axis of Y , *ordinates*. Thus the point P (Fig. 51) is three units of length from the x -axis and four units from the y -axis. Its position with respect to these axes is therefore completely described.

88. Position in Space. — Just as a more complete description of the position of a body on the earth may be given by stating its height above sea level (a *plane of reference*), so, in physics, the position of a point in space may be determined first in a certain plane, and then the position of this plane may be located with respect to a plane of reference.

89. Translation and Rotation. — When a body so changes its position that all its points move at the same rate in the same direction, and hence describe parallel lines, the motion is said to be one of *translation*. But when all the points of the body describe arcs of circles around some point or axis, the motion is one of *rotation*. An example of pure translation is the motion of an elevator; of pure rotation, the spinning of a top. By combinations of trans-

lations and rotations, all possible varieties of motions may be produced.

90. Speed.—Suppose that a ship sails 40 mi. in 5 hr. This is at the average rate of 8 mi. an hour. It is quite unlikely that the ship has gone exactly 8 mi. during any one of the 5 hr.; it may have gone 10 mi. during one of the hours and 6 mi. during another, but on an average its time rate of passing from one position to another 40 mi. distant is 8 mi. per hour. *Speed* is the name given to *the time rate of change of position*.

$$\text{Speed} = \frac{\text{Change of position}}{\text{Time taken}}.$$

The measurement of speed involves measurements of distance and of time, and is expressed in units of distance per unit of time, as miles or kilometers per hour, yards or meters per minute, feet or centimeters per second, etc. A convenient notation for expressing speeds is as follows: 12 centimeters per second is written $12 \frac{\text{cm.}}{\text{sec.}}$; 50 miles per hour, $50 \frac{\text{mi.}}{\text{hr.}}$.

91. Uniform Motion and Speed.—A body is in uniform motion and has uniform speed when it goes over equal distances in equal intervals of time. Thus, if l denote the distance gone over in the time interval t , the defining equation for speed is

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}; \quad s = \frac{l}{t} \quad (1)$$

From this equation are derived

$$l = st, \quad (2)$$

and

$$t = \frac{l}{s} \quad (3).$$

92. Applications. — The question is frequently asked in everyday life: How long will it take to go from one place to another? The intelligent answerer thinks of the distance between the two places and the speed of the conveyance available, and, dividing the former by the latter, deduces the time. But this is a direct application of the third equation.

Distances may be stated in terms of time. Thus a certain place may be said to be "two hours away" from another. Such an expression assumes that the speed of travel is known, which, when multiplied by the time, gives the distance, as is shown by the second equation to be generally true.

PROBLEMS

1. What is the average speed of a train that goes 90 miles in 3 hr.?
2. How far from Chicago is New York if a train with an average speed of $40 \frac{\text{mi.}}{\text{hr.}}$ takes 24 hr. to run from one city to the other?
3. How long will it take a man walking at the average rate of $3 \frac{\text{mi.}}{\text{hr.}}$ to go nine miles?

93. Average Velocity. — When the direction of a motion is stated or implied, it is customary to substitute the term *velocity* for speed. Thus, when we speak of or have in mind the sailing of a ship in some particular direction, as east or north, we call the time rate of its change of position its velocity. Velocity and speed have the same numerical values, but with velocity is associated the notion of direction: *velocity is directed speed*.

Suppose that our ship sails due north 6 mi. between two and three o'clock, and 10 mi. between three and four o'clock. The total distance traveled from two to four o'clock is 16 mi., and the ship's average velocity for the 2 hr. is $8 \frac{\text{mi.}}{\text{hr.}}$. Let, in general, V denote this average velocity of a moving body, V_0 its velocity when first observed, and V_1 its velocity when last observed.* Then,

* V_0, V_1, V_2, V_3 , etc., to V_n are read V -oh, V -one, V -two, V -three, etc., to V - n , or V -sub one, V -sub two, etc. The figures or letters thus written are called *inferior*

as the average of two observations is equal to half their sum ;

$$V = \frac{V_0 + V_1}{2}. \quad (4)$$

PROBLEMS

1. A train has a velocity of $50 \frac{\text{mi.}}{\text{hr.}}$ at the bottom of an incline and $30 \frac{\text{mi.}}{\text{hr.}}$ at its top. What is its average speed along the incline ?

2. A bullet going at the rate of $400 \frac{\text{m.}}{\text{sec.}}$ strikes a plank, and after penetrating it, has a speed of $20 \frac{\text{m.}}{\text{sec.}}$. What is its average speed while piercing the plank ?

94. Distance and Average Velocity.—As by definition,

$$s = V = \frac{l}{t} = \frac{V_0 + V_1}{2}, \quad (5)$$

then

$$l = \frac{V_0 + V_1}{2} t. \quad (6)$$

Suppose that our ship now sails more and more slowly so that its velocity during 4 hr. falls at a uniform rate from $10 \frac{\text{mi.}}{\text{hr.}}$ to $2 \frac{\text{mi.}}{\text{hr.}}$. Its average velocity is $\frac{(10 + 2)}{2} = 6 \frac{\text{mi.}}{\text{hr.}}$ and at this average velocity, the ship will sail 24 mi. during the 4 hr., for

$$l = \left(\frac{10 + 2}{2} \right) \times 4 = 24.$$

PROBLEMS

1. How long does it take an automobile with an average speed of $30 \frac{\text{ft.}}{\text{sec.}}$ to come to rest within a distance of 300 ft., when the power is shut off ?

figures or letters (also subscripts), while those written like exponents, as V^2 , V^3 , V^n , etc., are called *superior figures* or *letters*, because they are placed below and above, respectively, the line. The subscript "0" does not necessarily mean that the value of the quantity it distinguishes is zero, but merely that the quantity is a certain one of a series.

2. A bullet is stopped .02 sec. after penetrating 6 cm. of a plank. What was its speed when it struck the plank?

3. A bicyclist goes 100 ft. in 5 sec., and then has a speed of $10 \frac{\text{ft.}}{\text{sec.}}$. What was his initial speed?

95. Uniform Acceleration. — Suppose now that the wind freshens and that the velocity of our ship increases in one hour from $2 \frac{\text{mi.}}{\text{hr.}}$ to $5 \frac{\text{mi.}}{\text{hr.}}$, thus gaining in velocity, $3 \frac{\text{mi.}}{\text{hr.}}$. Suppose further that the ship sails faster and faster until in 4 hr. it is going at the rate of $17 \frac{\text{mi.}}{\text{hr.}}$. The increase in velocity is now 12 miles in 4 hr., an average gain of $3 \frac{\text{mi.}}{\text{hr.}}$. Gain in velocity may be denoted by

$$V_1 - V_0,$$

and gain in velocity per unit of time by

$$\frac{V_1 - V_0}{t}.$$

Acceleration is the name given to the gain of velocity per unit of time, and its symbol is a . In algebra then

$$a = \frac{V_1 - V_0}{t}, \quad (7)$$

and in words: *Acceleration is the time rate of change of velocity.* It is assumed in the above discussion that the acceleration is uniform, *i.e.* that there are equal gains in velocity in equal intervals of time; the motion is uniformly accelerated.

As velocity is the time rate of change of position, and acceleration is the time rate of change of velocity, it follows that acceleration is the time rate of the time rate of change of position, the time thus receiving double consideration. The difference in the velocities of our ship during the first

and the fourth hours is $12 \frac{\text{mi.}}{\text{hr.}}$, and the acceleration is therefore $12 \frac{\text{mi.}}{\text{hr.}}$ divided by 4 hr.; that is, $3 \frac{\text{mi.}}{\text{hr.}^2}$, the hr.^2 indicating that the distance is divided by the time twice. With this notation, $40 \frac{\text{ft.}}{\text{sec.}^2}$, for example, means an acceleration of 40 ft. per second *per second*, although it is customary to omit the last "per second" when it is understood that an acceleration is meant.

96. Uniform Retardation. — When there is a decrease in velocity, the motion is said to be retarded. Statements similar to those for uniform acceleration apply also to uniform retardation. As retardation may be regarded as negative acceleration, the terms in the formulas for uniformly accelerated motion need only to have their signs changed to apply to uniformly retarded motion.

97. Formulas for Uniform Acceleration. — Formulas (6) and (7) are fundamental, and by their help any problems involving uniformly accelerated motion may be solved. By algebraic methods they may be transformed into other formulas which prove more convenient in certain applications. Thus, if the two equations are multiplied together, we have

$$at = \left(\frac{V_1 + V_0}{2} \right) t \left(\frac{V_1 - V_0}{t} \right); \quad (8)$$

from which, after canceling the t 's, multiplying quantities in parenthesis, and clearing of fractions, we obtain

$$2 at = V_1^2 - V_0^2. \quad (9)$$

And, as from equation (7), we find that

$$V_1 = V_0 + at, \quad (10)$$

by substitution of this value in equation (6), we have

$$l = \frac{1}{2} at^2 + V_0 t. \quad (11)$$

98. Simplified Formulas for Uniform Acceleration. — If it is assumed that a body starts from a state of rest, and therefore has no initial velocity, the value of V_0 is zero, so that V_0 disappears from the above equations. We then find by making

that $V_1 - V_0 = v$,
 $V_1 - V_0 = at$ becomes $v = at$; (12)

$l = \frac{1}{2} at^2 + V_0 t$ becomes $l = \frac{1}{2} at^2$; (13)

$V_1^2 - V_0^2 = 2 al$ becomes $v^2 = 2 al$. (14)

Notice that these three equations all contain a , and that each one contains two of the three quantities v , l , and t . When a body moves with uniform acceleration, Equation (12) shows the relationship between velocity and time, Equation (13) the relationship between distance and time, and Equation (14) the relationship between velocity and distance.

PROBLEMS

1. What is the speed gained by a train in ten seconds after leaving a station, if its acceleration has been uniformly $40 \frac{\text{ft.}}{\text{sec.}^2}$?
2. What distance will the above train go in the ten seconds?
3. What will be the speed gained by a train that has gone 2000 ft., with a uniform acceleration of $40 \frac{\text{ft.}}{\text{sec.}^2}$?

LAWS OF MOTION

99. Force and Acceleration. — When unbalanced forces act upon a body, motion ensues, and the longer the action endures, the faster the body moves. Thus the pull of an engine on the cars causes a train to move faster and faster, although a limit to the acceleration is reached when other forces, such as the resistance of the air and the friction

of the rails, equal the pull of the engine. When the steam is shut off, the train goes more and more slowly by reason of the retarding forces. Constantly acting unbalanced forces produce either accelerated or retarded motion according to their direction.

100. Inertia.—It is often brought to our attention that bodies offer a certain resistance, as it were, to any change in their condition of motion or rest. Thus, sitting in a car, we are thrown backward if the car starts suddenly, and forward if it stops suddenly. Again, when we see that a car is about to turn a corner, we brace ourselves for the jolt due to the change of direction. When our bodies have become, so to say, accustomed to one kind of motion, and then its rate or its direction is changed, time is needed for us to adjust ourselves to the change. *Bodies at rest tend to remain at rest, and bodies in motion tend to keep in motion.* We say that this behavior is a general property of matter and assign to it the name *inertia*.

101. Newton's First Law of Motion.—There are three relationships between force and motion, which were first formulated by Sir Isaac Newton in 1686. The first of these laws is: *Every body continues in its state of rest or of uniform motion in a straight line, unless compelled by some external force to change that state.* This law is but another statement of the definition of inertia. It asserts that matter has inertia, and that because of inertia, force is required to change a body's condition of rest or motion.

No direct experimental proof of this law can be given, for the reason that it is impossible to eliminate all the forces acting upon a body; friction, for example, is always present. As we find, however, that the more these dis-

turbing forces are diminished or eliminated, the more nearly does the motion correspond to that which is prescribed by the law, and that all legitimate conclusions drawn from it are in accord with fact, we cannot but be convinced of its universal validity.

102. Momentum.—It is well known that the speed which a given force will impart to a body depends upon the mass of the body. Thus a well-directed kick will send a football flying, if it is blown up with air, but will hardly budge it, if it is filled with sand. The vigor of the kick may be the same, but the resulting speeds are very different. The amount of acceleration alone then is not a sufficient measure of the magnitude of a force. Account must be taken also of the amount of matter moved. The quantity or amount of motion of a body depends upon both its speed and its mass, and is called its *momentum*. Momentum is measured by the product of the numbers representing the mass and the velocity of a given body:

$$\text{Momentum} = \text{Mass} \times \text{Velocity}.$$

$$M = m \times v. \quad (15)$$

Newton's first law affirms that the motion a body may have remains the same so long as it is free from the action of external forces. Inasmuch as the mass of a body is independent of its motion, the first law may also be stated thus: *A moving body not acted upon by external forces has a constant momentum:*

$$M = mv = a \text{ constant}. \quad (16)$$

103. Newton's Second Law of Motion.—The greater the force acting upon a body of invariable mass, the greater will be the acceleration given to it. The harder the kick

on the same football, the farther it flies. The momentum is therefore greater, and a change of momentum must be associated with a change of force. Universal experience shows also that the motion is always in the same direction as that of the force producing it. Newton's second law is the general statement of these facts: *Change of momentum is proportional to the force and takes place in the direction in which the force acts.* As the momentum of a given body is the product of its velocity and its mass, a change in its momentum must be the product of its acceleration and its mass; for its mass must remain the same, and change of velocity is nothing but acceleration. Since the change of momentum is a measure of the force applied to the body:

$$\text{Force} = \text{Mass} \times \text{Acceleration.}$$

$$F = m \times a. \quad (17)$$

104. Force Defined.—Formula (12) may be transformed into

$$a = \frac{v}{t},$$

and the substitution of this value of a in formula (17) gives

$$F = \frac{mv}{t}. \quad (18)$$

Force may then be defined as *the time rate of change of momentum.*

Our definition of force is now adequate and precise. It is something more than mere muscular sensation and exertion (§ 22), or that which produces or tends to produce motion. It is that which imparts to a definite mass a definite acceleration; it is that which gives a

definite momentum in a definite time. Acceleration and force are inseparably associated in thought. Whenever a body is moving faster and faster or more and more slowly, one force at least is acting constantly upon it.

105. The Unit of Force, the Dyne.—The unit force is that force which changes the momentum of a body at the rate of one unit per unit of time (Equation 18), or it is that force which imparts to the unit of mass the unit of velocity for each unit of time it acts (Equation 17). The physicist's units of length, mass, and time being the centimeter, the gram, and the second, the unit force in the Centimeter-Gram-Second system is the force which *imparts to a gram-mass an acceleration of one centimeter per second per second*. This unit is called the *absolute C. G. S. unit of force* and has received the name of *dyne* (from the Greek word *dunamis* = force).

106. The Gram and the Dyne.—Until now we have been using the earth pull upon a mass of one gram, pound, etc., for our unit of force. A gram-force imparts to a gram-mass an acceleration of nearly $1000 \frac{\text{cm.}}{\text{sec.}^2}$; hence a gram-force is nearly 1000 times greater than a dyne; at Chicago, it is 980.26 times greater. To convert dynes into grams, divide by 980; to convert grams into dynes, multiply by 980.

107. Action and Reaction.—When you press your hand upon the table, the hand exerts an *action* on the table, while the table exerts a *reaction* upon the hand. In the case of a body suspended by a string, the action is the earth pull (weight), and the reaction is the upward pull of the string on the body. A close scrutiny of every force

acting between two bodies shows that it always has two aspects. When both these aspects are taken into account, we speak of the force as a *stress* (§ 25), its two aspects being called *action* and *reaction*.

Newton was the first to recognize this dual nature of forces, and expressed his conviction in his third law of motion: *Action and reaction are equal and opposite in direction*. Another way of expressing the same idea is: *Every action between two bodies is a stress*.

108. Impact and Recoil.—*Impact* is the name applied to an *extremely brief action between bodies*, as in the case of batting a ball. By *recoil* is meant the *sudden change in direction of a body after impact with another*. If a body *A* strikes another *B*, the action of *A* on *B* is equal and opposite to the reaction of *B* on *A*, and the duration of the action is of course the same as that of the reaction. The force and the time being the same, the changes of momentum must be equal, for force is the time rate of change of momentum. As momentum is measured by the product of mass and velocity, bodies may have equal momenta and yet unequal masses and velocities. Thus a body with a mass of 2 g. and a velocity of a hundred centimeters per second, has the same momentum as one of a hundred grams and a velocity of $2 \frac{\text{cm.}}{\text{sec.}}$; for the products of mass and velocity are in both instances the same.

109. Application.—When the powder in a gun is exploded, both bullet and gun are acted upon by the same force, that of the suddenly generated gases. The bullet moves forward and the gun backward, and we speak of the recoil or kick of the gun. The momentum of the bullet is equal to that of the gun. As the mass of the gun is many times greater than that of the bullet, in order that the momenta of both

may be the same, the velocity of the bullet must be as many times greater than that of the gun. The bullet is pointed so as to increase its pressure per unit area (§ 37), that is, its "penetrative power," while the stock of the gun is made broad so as to decrease its pressure per unit area. In other words, the total pressure exerted by the bullet is concentrated in a small area, while that exerted by the gun is spread over a large area.

110. Colliding Bodies.—The phenomena of impact are modified by the elasticity of the colliding bodies. The usual apparatus for investigating the laws of impact consists of balls suspended by strings from a common support (Fig. 52). Suppose the balls to be of equal mass and to be made of some elastic material, such as glass or ivory, and let them hang side by side. Draw one aside and let it swing against the other. The first ball after impact remains at rest, while the second swings as far as the first would have swung had there been no collision. All the momentum of the first ball is given up to the second. But if the balls are made of some inelastic material, such as putty or clay, when one is drawn aside and let fall against the other, both swing out together, but only half as far as the first would have done, had no collision occurred. The momentum of the first is shared with the second, and as the mass is thereby doubled, the velocity, and hence the extent of the swing, is halved.

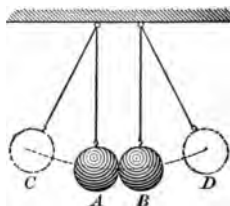


FIG. 52.—Colliding Bodies.

111. Reflected Motion.—Every one knows that if a rubber ball be thrown perpendicularly against a plane surface, it will rebound and retrace its path; also, that if it be thrown at an oblique angle, it will glance off at about an equal

angle. The angle formed by the lines representing the path of the ball before the rebound and a normal to the

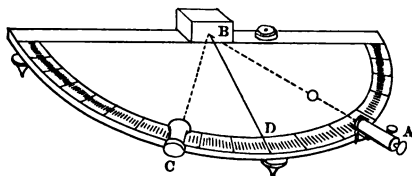


FIG. 53.—Reflected Motion. A marble shot from *A* is reflected from *B* and caught at *C*, so that the angle *ABD* = the angle *CBD*.

reflecting surface is called the *angle of incidence* (Fig. 53), and that formed by the path of the ball after the rebound and the normal is the *angle of reflection*. Experiments with perfectly

elastic bodies have proved that *the angle of incidence is equal to the angle of reflection and lies in the same plane*. If the bodies are not elastic, the angle of reflection is found to be greater than the angle of incidence.

EXERCISES

1. Why is a running jump longer than a standing one?
2. Why is it that one can jump farther with a springboard than without?
3. How does dodging help a hare to escape a hound?
4. How is it that a circus rider leaps directly upward from a horse's back, and yet passes forward through the hoop?
5. While a bullet shot from a rifle makes only a small hole in a window-pane, it shatters the glass to pieces if thrown by the hand. Account for this difference.
6. How is the head of a hammer fixed more firmly to the handle, by tapping the head or the end of the handle on a table? Explain the principle involved.
7. What condition must be met in order that equal forces may produce equal accelerations?
8. Which law of motion is illustrated by the kick of a gun? Why doesn't the gun hurt as much as the bullet?
9. What success would a man have in propelling a boat forward, if he blew air against the sail by means of a big bellows fixed in the stern?

10. Two boats are being drawn towards each other, and the mass of one boat is twice that of the other. Resistances being disregarded, how do the momenta and the velocities of the boats compare at any time?

11. What kind of motion is produced by the action of a single constant force? What kind of motion when the force ceases to act?

12. A constant force of 100 dynes acts upon a mass of 20 g. and gives it a final speed of $400 \frac{\text{cm.}}{\text{sec.}}$. How many seconds does it act?

13. What force in dynes is needed to impart to a mass of 20 g. an acceleration of $10 \frac{\text{cm.}}{\text{sec.}^2}$?

14. What is the mass of a body moving with an acceleration of $30 \frac{\text{cm.}}{\text{sec.}^2}$ under the action of a force amounting to 50 dynes?

15. If a body with a mass of 40 g. is moved by a constant force of 29 dynes, what acceleration is imparted to the body?

16. An electric car starts from a state of rest and in a minute has acquired a speed of $10 \frac{\text{mi.}}{\text{hr.}}$. What is its average acceleration, expressed in feet per second per second?

17. Two balls have equal momenta. One weighs 100 g. and moves over 100 cm. in 5 sec. The other moves over 200 cm. in 8 sec. What is the mass of the second ball?

18. An elastic ball weighing 100 g. is moving at the rate of $50 \frac{\text{cm.}}{\text{sec.}}$ when it strikes another ball weighing 500 g. at rest. If the first ball flies back with a speed of $5 \frac{\text{cm.}}{\text{sec.}}$, find the speed of the second ball.

19. Two inelastic bodies collide while moving in opposite directions. If one has a mass of 200 g. and a velocity of $25 \frac{\text{cm.}}{\text{sec.}}$, and the other has a mass of 100 g. and a velocity of $20 \frac{\text{cm.}}{\text{sec.}}$, what is (a) the direction; (b) the momentum; (c) the velocity of the combined bodies after the collision?

20. A gun weighs 10 lb. and its bullet, half an ounce. What is the speed of recoil of the gun, if the bullet leaves the muzzle of the gun with a speed of $1000 \frac{\text{ft.}}{\text{sec.}}$?

CHAPTER VI

COMPOSITION AND RESOLUTION OF FORCES

112. Composition of Motions.—If a man on board a ship which is sailing eastward is walking from west to east, the velocity of the ship and that of his walking will have to be added together to give the velocity with which he is moving eastward. If he turns about and walks from east to west, however, the rate at which he is traveling eastward is equal to the difference between his velocity and that of the ship. In the latter case the result may be positive, *i.e.* he is really going eastward; negative, *i.e.* he is really going westward; or zero, *i.e.* the ship carries him eastward just as fast as he walks westward, so that in reality he has no movement at all with respect to the water, but is, as it were, merely marking time in the same place.

If a steamer travels toward the east and a man walks across its deck toward the south, the path he passes over

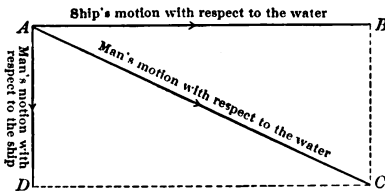


FIG. 54.—Composition of Motions.

with respect to the water is along a line that is the diagonal of a rectangle (Fig. 54), the sides of which represent the eastward and southward velocities. Similar considerations apply to any two

simultaneous motions of constant speed and direction which may be imparted to a body. The result of their concur

rence is a single motion with a single speed and direction. This single motion is called the *resultant* of the *component* motions.

113. Parallelogram of Velocities.—The resultant of two simultaneous velocities (its components) at an angle is the diagonal of a parallelogram, the adjacent sides of which represent the two velocities. If the lines AB and AC (Fig. 55) represent the directions and, on any chosen scale, the magnitudes of the velocities simultaneously imparted to a body at A , the body will move along the diagonal AD to the point D in the same time that, with the velocity AB alone, it would have taken to reach B , or with the velocity of AC alone, to reach the point C .

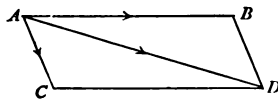


FIG. 55.—Parallelogram of Velocities.

114. Triangle of Velocities.—As the diagonal of a parallelogram divides it into two equal triangles, a simpler method of finding the resultant of two velocities is illustrated as follows. Let AC (Fig. 56) represent the magnitude and direction of one velocity, and CD the magnitude and direction of another velocity. Then the line AD completing the triangle ACD is the resultant of the two velocities.

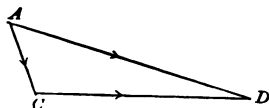


FIG. 56.—Triangle of Velocities.

115. Resolution of Velocities.—The resolution of velocities is the converse of their composition. The resultant velocity is given, and it is required to find its components. The directions of the components must be specified, for a given line may form the diagonal of any number of different parallelograms. Suppose a man to be climbing a hill at the rate of $2 \frac{\text{mi.}}{\text{hr.}}$, and it is required to find his vertical

and horizontal component velocities. Draw the line AB (Fig. 57) to represent the grade of the hill and velocity of

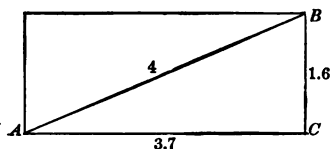


FIG. 57.—Resolution of Velocities.

the motion on a scale of 2 cm. = $1 \frac{\text{mi.}}{\text{hr.}}$ (any other convenient scale would do as well), and make it the diagonal of a parallelogram (a rectangle in this case), the sides being drawn on the same scale. The lines AC and BC measure 3.7 cm. and 1.6 cm., respectively. Hence the horizontal component is $1.85 \frac{\text{mi.}}{\text{hr.}}$, and the vertical component is $.8 \frac{\text{mi.}}{\text{hr.}}$.

116. Composition and Resolution of Forces.—Every force has a *point of application*, a *direction*, and a *magnitude*. These attributes may be represented by a line, one end standing for the point of application, and the length and direction of the line standing for the magnitude and direction of the force. Forces, then, like velocities, may be represented by lines, and their composition or resolution found out as in the case of velocities. Lines are drawn to represent, on some convenient scale, the forces; and each of the geometric propositions established with reference to velocities has its counterpart in a proposition relating to forces.

117. Directed Quantities or Vectors.—Motions, velocities, accelerations, and forces, all have a definite direction, expressed or understood. A simple statement of their magnitudes alone does not describe them completely; their directions also must be known. Geometric methods of description are therefore applicable to such directed quantities, or *vectors*, as they are called.

118. Rules for Finding Resultants of Directed Quantities.

—I. *The resultant of two vectors (motions, velocities, or forces) in the same straight line is their algebraic sum. If they are in the same direction, they are to be added; if in opposite directions, they are to be subtracted, the direction of the resultant being that of the larger component.*

II. *If two vectors not in the same straight line, and not parallel, are represented on some convenient scale by the adjacent sides of a parallelogram, their resultant will be represented by the diagonal drawn from the point of intersection of the two lines, its direction being between the directions of the component vectors.*

III. *The resultant of more than two vectors may be found by repeated applications of the above rules. The resultant of any two of the vectors is found, and then the resultant of this first resultant and another vector, and so on, until a single line representing the final resultant is obtained.*

119. Resolution of Forces.—In studying the effects of a force, it may be found useful to regard it as replaced by two or more forces which together produce the same effects. The resolution of a force into its components is accomplished in the same way as the resolution of a velocity. While a given force may be resolved into components forming any angle with one another, the examples given have to do only with rectangular components.

120. Examples of Resolution of Forces.

—I. *Inclined Plane.* IND (Fig. 58) represents an inclined plane on which a

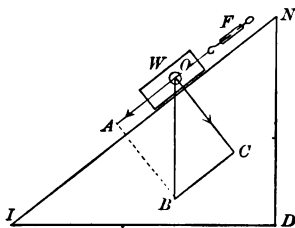


FIG. 58.—Inclined Plane.

block, having the weight W , is supported by a spring balance with a reading F . Suppose W and F to act at the same point O , which, as will be shown later, may be considered to be the point where the weight of the block is concentrated. Resolve the weight into components, of which one OC is perpendicular to the plane, and the other OA is parallel to the plane. The component OC merely presses against the plane and is balanced by the resisting force of the plane. The component OA tends to pull the block down the plane, and is balanced by the force exerted by the spring scale. The length OA is therefore proportional to F . As the triangles AOB and DNI are equiangular, they are similar, and from the proportionality of their sides, it follows that

$$\frac{F}{W} = \frac{OA}{OB} = \frac{DN}{IN}.$$

This shows that the force which is required to hold a body in place on an inclined plane (friction being disregarded) bears the same ratio to the weight of the body as the height of the plane bears to its length.

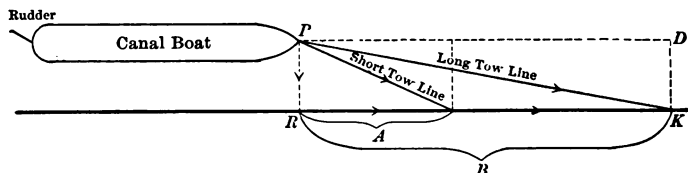


FIG. 59.—Canal Boat Towing.

II. *Canal Boat Towing.* The towline pulls the boat toward land as well as forward, but the rudder is so held that it keeps the keel of the boat parallel with the banks. The force applied to the towline may be resolved into two components, one perpendicular to the towpath, the other parallel to it. The diagram (Fig. 59) shows that the longer the towline, the greater is the parallel component urging the boat forward.

III. *Boat Sailing.* How is it that boats may sail in opposite directions with the same wind? Suppose a boat to be sailing northward with the wind from the west (Fig. 60). The wind force WO may be resolved into the two components, WR , parallel to the plane of the sail SL , and RO , perpendicular to SL . The parallel component slips by the sail and is not effective in moving the boat forward. The perpendicular component, which tends to move the ship towards the northeast, may in turn be

resolved into the components RT , perpendicular to the keel line of the boat, and TO , in line with the keel. The first component tends to drive the boat sidewise, this tendency being opposed by the broadside, deep keel, or centerboard, while the second pushes the boat forward.

When the boat is turned around it may be made to sail southward, still with a west wind.

The resolutions of the forces to this end are shown in Figure 61, in

which the lettering and explanation are the same as for Figure 60, when south is substituted for north.

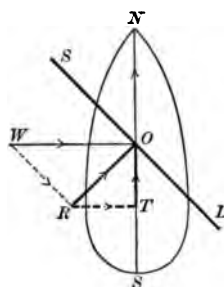


FIG. 60.—Boat sailing Northward.

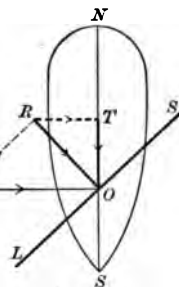


FIG. 61.—Boat sailing Southward.

121. The Aeroplane. — A kite flies because the force of the wind has a component that is directed upward, the string holding the kite against the wind. An aeroplane (Fig. 62) may be regarded as a self-

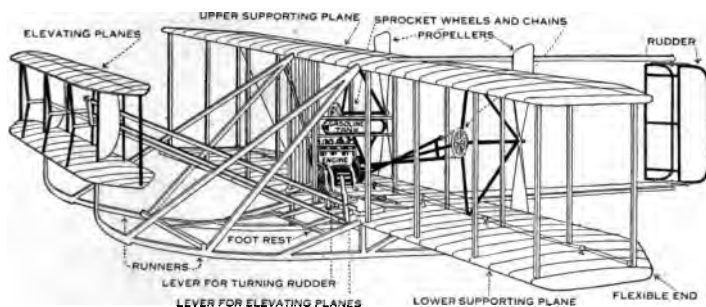


FIG. 62.—The Wright Aeroplane.

propelled kite. It is driven through the air at such speed that the upward component of the air's pressure is sufficient to sustain it. By means of elevating planes on a horizontal axis an aeroplane may be steered up and down, while by the aid of a rudder on a vertical axis and of the flexible ends, it may be steered sidewise.

122. Equilibrant.—A resultant force is a single force that may be substituted for two or more forces, and produce the same effect that all these forces acting simultaneously can produce. An *equilibrant* of two or more forces is a single force that is equal in magnitude to their resultant, but acts in the opposite direction. Resultant and equilibrant are but the two aspects of a stress (§ 25). When an equilibrant acts upon a body, the result is rest, for the forces are balanced, and balanced forces cause no motion.

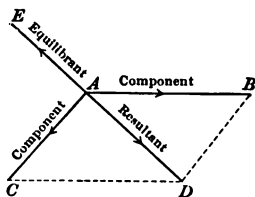


FIG. 63. — Concurring Forces.

Thus, the resultant of the forces (Fig. 63) AB and AC is AD , which is balanced by its equilibrant AE .

Of the three forces in equilibrium (Fig. 63) any one may be considered as the equilibrant of the other two. Thus the equilibrant of AE and AC is AB , and that of AE and AB is AC . Since it is easier to use apparatus at rest rather than in motion, the statements in regard to resultants can be more readily proved experimentally by the study of equilibrants. As an equilibrant has the same point of application and magnitude as the resultant, whatever is found true of an equilibrant is also true in the opposite direction of the resultant.

123. Composition of Parallel Forces.—Suppose that two forces A and B (Fig. 64) are parallel and acting upon a rigid bar of negligible weight; and that a third parallel force E also acts upon the bar so as to produce equilibrium. This third equilibrating force has to be applied between the two component forces, and is equal and opposite to their resultant. Experiment shows:

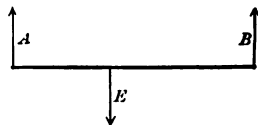


FIG. 64. — Parallel Forces.

I. *The magnitude of the equilibrant (and hence also of the resultant) is equal to the sum of the magnitudes of the two forces.*

II. *The distances of the points of application of the two forces from the point of application of the equilibrant (or resultant) vary inversely as the magnitudes of the components.* Thus, if the larger component is 15 g., and the smaller is 5 g., then, as $\frac{15}{5} = \frac{3}{1}$, the point of application of the equilibrant will be three times as far from the point of application of the smaller force as it is from the point of application of the larger force.

When more than three parallel forces act upon a body, the equilibrant of any two of them can be found, and its resultant ascertained. The equilibrant of this resultant and another force may then be determined, and from this, the resultant of the three component forces that are under consideration. By continuing this process, the resultant of any number of parallel forces is obtained.

124. The Lever.—A rigid bar or rod which can turn round a fixed axis, called a *fulcrum* or *prop*, constitutes a *lever* (Fig. 65). Forces may be applied to the bar at each side of the fulcrum, the perpendicular distances between the line of action of the fulcrum and the lines of direction of the forces being termed the *arms of the lever*. One of the forces may be called the *effort*, and the other the *resistance*, whence we have the self-explanatory terms *effort arm* and *resistance arm*. A lever presents a case of three parallel forces, the fulcrum acting in one direction and the effort and resistance in the opposite direction.

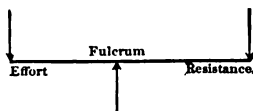


FIG. 65.—The Lever.

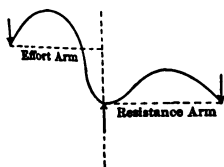


FIG. 66.—The Arms of a Crooked Lever.

the combination of three parallel forces shown in Fig. 64. It must be borne in mind that, whatever the shape of the lever itself, the arms are the perpendicular distances from the fulcrum to the effort and resistance, respectively (Fig. 66).

125. Moment of Force.—When a lever is balanced on its fulcrum, the application of more force on either side causes the lever to rotate about the fulcrum. Experiment shows that this turning effect depends not only upon the magnitude of the added force, but also upon the perpendicular distance of its line of direction from the axis of rotation. Both the force and the lever arm have their influence (are of *moment*) upon the tendency to turn. To denote this composite influence, the term *moment of force* is used, and *its measure is the product of the magnitude of the force by the perpendicular distance from the axis of rotation to the line of direction of the force*. The two factors of a moment—force and distance—are of like importance in determining the turning effect; doubling either doubles the turning effect, and doubling both quadruples it. If several forces act upon the lever, the moment of each is to be found independently of the others. If the moments of all the forces tending to turn the lever in one direction be added, their sum will be the total moment for that direction.

126. Equilibrium of Moments. Law of the Lever.—The moment of a force is said to be positive when it tends to produce rotation in the same direction as the hands of a clock (*clockwise*); and negative, when it tends to produce rotation in a *counter-clockwise* direction. No rotation is produced, *i.e.* the moments are in equilibrium, when the sum of the positive moments equals the sum of the negative moments; or, in other words, when the algebraic sum of all the moments is zero.

This statement holds good also for the equilibrium of the lever, and the Law of the Lever may be expressed thus: *The product of the effort by its arm equals the product of the resistance by its arm, when the lever is in equilibrium.*

127. Balances or Scales.—A familiar form of lever is the common beam balance (Fig. 67), in which the arms are of equal length and are horizontal when the weights in the scale pans are equal. For special uses balances with unequal arms are employed. In the *Roman Steelyard*



FIG. 67.—Beam Balance.

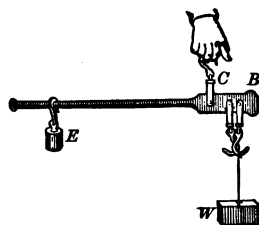


FIG. 68.—Roman Steelyard.



FIG. 69.—Danish Steelyard.

(Fig. 68) a known weight is moved along the graduated longer arm until it is in equilibrium with the weight of the object hung from the shorter arm, while in the *Danish Steelyard* (Fig. 69), the position of the fulcrum is shifted until equilibrium is found between the permanent weight at one end and the weight of the object hung at the other end. The beams of both steelyards are so graduated as to permit of direct readings of the weights of the attached articles.

128. Couples.—Consider the forces engaged in turning a key. The thumb and finger act in opposite directions upon the key with equal forces, and the motion produced is one of rotation. Such a pair of equal forces acting upon a body in parallel and opposite directions, not, however, in the same straight line, constitutes what is called a *couple* (Fig. 70).

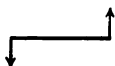


FIG. 70.—Couple.

EXERCISES

1. Why does a sportsman aim a little ahead of a duck flying rapidly across his field of view ?

2. A steamer moves away from a wharf at the rate of $3 \frac{\text{mi.}}{\text{hr.}}$. A deck hand walks toward the stern holding one end of a tightly stretched rope, the other end of which is tied to the wharf. What is his speed with reference to (1) the wharf; (2) the water; (3) the steamer ?

3. A brakeman walking forward at the uniform rate of $2 \frac{\text{mi.}}{\text{hr.}}$ on a moving train notes that the train takes 2 min. to pass from one milepost to another. What is his speed with reference to the ground ?

4. A boat takes 2 hr. to pass over a distance of 8 mi. on a straight river, going against the current which flows at the rate of $2 \frac{\text{mi.}}{\text{hr.}}$. What is the speed of the boat with reference to (1) the banks ? (2) the water ?

5. If a north wind makes a boat drift $3 \frac{\text{mi.}}{\text{hr.}}$ southward, and the boat is rowed eastward at the rate of $4 \frac{\text{mi.}}{\text{hr.}}$, what is the path and velocity of the boat ?

6. A boy dives straight down into a river flowing $3 \frac{\text{mi.}}{\text{hr.}}$. If he swims downstream under water for 30 sec. at the independent rate of $2 \frac{\text{ft.}}{\text{sec.}}$, how far will he be from the diving place when he comes up ?

7. An airship moving horizontally with a uniform velocity of $20 \frac{\text{km.}}{\text{hr.}}$ meets with a wind that has a velocity of $10 \frac{\text{km.}}{\text{hr.}}$, the direction of which makes an angle of 180° with that of the ship. What is the resultant velocity of the ship ?

8. A balloon rising uniformly at the rate of $20 \frac{\text{ft.}}{\text{sec.}}$ is carried in 40 sec. by the wind 600 ft. to one side of the vertical line passing through its starting place. What is the actual path and velocity of the balloon ?

9. A man weighing 60 kg. sits in a hammock so that one rope *A* makes an angle of 30° with a vertical line, while the other rope *B* makes an angle of 60° . What is the pull on each supporting rope ?

10. Resolve a force of 10 g. into two components acting at right angles to each other, one of which is three fourths as great as the other.

11. Two parallel forces of 50 g. and 100 g., respectively, are applied in opposite directions to a straight inflexible rod at points 90 cm. apart. What is the magnitude, direction, and point of application of a third force that balances the first two forces ?

12. Two boys carry a string of fish weighing 25 lb. suspended from a pole 10 ft. long. If each boy holds the pole at one foot from his end, where must the string of fish be attached so that one boy may carry 10 lb. and the other boy the rest of the weight?

13. A limb of a tree extends horizontally from the trunk and would break if a weight of 20 kg. were hung on it 500 cm. from the trunk. How far out on the branch can a boy weighing 50 kg. venture with safety?

14. An elastic ball moving N.E. with a velocity of $50 \frac{\text{cm.}}{\text{sec.}}$ strikes an elastic surface extending due N. and S. What is the direction and velocity of the ball after impact?

CENTER OF GRAVITY AND EQUILIBRIUM

129. **Center of Gravity.**—Terrestrial bodies have all their particles attracted toward the center of the earth. The sum of all these pulls on the particles composing a body makes up its weight or gravity as a whole. The earth's radius is so large in comparison with the size of most objects on its surface that the directions of the earth pulls on the individual particles may be considered as practically parallel. Now it is possible to find for any system of parallel forces a resultant that may take the place of all the component forces. The point of application of this resultant in the case of a body acted upon by gravity is known as its *center of gravity*; and it is at this point that the whole weight of the body may be considered to act (Fig. 71). It may also be shown that the mass of a body may for certain purposes be regarded as concentrated in a single point—its *center of mass*. The centers of gravity and of mass occupy the same position in a body.



FIG. 71. — The earth's pull on a body is not usually regarded as a myriad of little pulls, but as a single pull applied at the center of gravity.

130. Center of Figure.—When the material of a body is homogeneous, that is, when any one portion has the same properties as any other portion, and the body has regular geometrical dimensions, its center of gravity coincides with its center of figure. Thus, the center of gravity of a sphere is located at its geometrical center; that of a cylinder, at the middle point of its axis; that of a parallelopiped, at the intersection of its diagonals.

131. Methods of Locating the Center of Gravity.—Whatever the form of a body may be, if it is suspended from any one of its points, as o (Fig. 72), its center of gravity must be located somewhere in a vertical line passing through the point of suspension; for the force required to hold the body suspended is the equilibrant of all the forces acting downward upon it, and a resultant is in the same straight line with its equilibrant. By suspending the body from another point, as p (Fig. 73), a new vertical line passing through the center of gravity is found, and the center of gravity must be at the intersection of these lines.

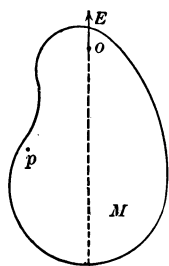


FIG. 72.—Locating the Center of Gravity.

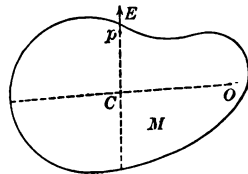


FIG. 73.—Locating the Center of Gravity.

When the center of gravity has thus been located by means of suspensions from two points, the vertical lines drawn through all other points of suspension will pass through the same point. The center of gravity does not necessarily lie within the material of which the body is composed, as, for instance, in the case of a ring or bottle. In case the center of gravity does fall within the material composing a body, however, if it be supported, the whole body is supported. Hence, by balancing a body on a pointed object, the center of gravity will be exactly above or below the point. If the body is in the form of a bar or rod, it may be shifted along a supporting knife-edge until it maintains a horizontal position. The center of gravity is then vertically above the supporting base line.

132. Weight of Lever.—The weight of a body may be considered as concentrated at its center of gravity. Thus, if a uniform rod ROD (Fig. 74) weighing 10 g. and 30 cm. long is balanced on a prism O , the supporting line comes directly below the center of gravity at the middle

of the rod. The arrangement constitutes a lever with the fulcrum half-way between the ends. Suppose a weight of 5 g. to be suspended at R . The equilibrium is thereby disturbed, but can be regained by shifting the fulcrum 5 cm. to the left of O . The moment of the force on the left side of the fulcrum is then $5 \times 10 = 50$, and the right-hand moment must also have this value. The weight of the lever itself is the force tending to produce rotation contrary to that of the 5 g. added, and is applied at the center of gravity of the lever, which is 5 cm. from the fulcrum. The right-hand moment is therefore $10 \times 5 = 50$ also.

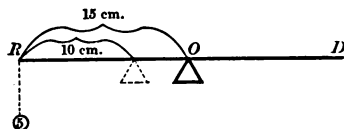


FIG. 74. — Weight of Lever.

133. Line of Direction; Base. — The *line of direction* of

a body is the vertical passing through its center of gravity. The *base* of a body is that portion of it which is in contact with a supporting surface; or it is the



FIG. 75. — The base of a body is the area included by lines drawn around its outside points of support.

area included by the straight lines connecting its outer points of support (Fig. 75).

134. Equilibrium. — A brick lying upon a table on any one of its faces is in equilibrium with respect to gravity

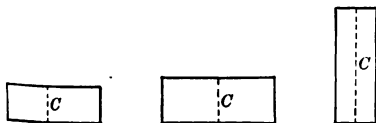


FIG. 76. — Brick in Different Positions of Equilibrium.

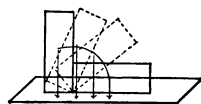


FIG. 77. — Overturning of Brick.

(Fig. 76). Clearly it can be more easily overturned when on end than when on its side. The line of direction in any

one of the three positions falls within the base. If the brick when on end be tilted up until the line of direction falls outside its base, it will then tumble over on its side (Fig. 77). In this tilting, the center of gravity of the brick is raised, and when it tumbles over, the center of gravity is lowered. The higher the center of gravity and the narrower the base, the more readily is an object overturned.

135. Kinds of Equilibrium. — I. *Stable.* When a body

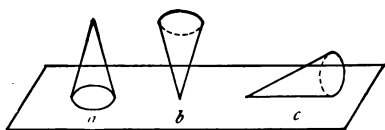


FIG. 78. — Kinds of Equilibrium.

is placed upon a supporting surface so that any movement tipping it raises its center of gravity, it is said to be in stable equilibrium (Fig.

78 *a*). Its line of direction falls within its base.

II. *Unstable.* When a body is supported on a surface so that any movement to tip it will lower its center of gravity, its equilibrium is unstable (Fig. 78 *b*). Its line of direction falls without its base.

III. *Neutral or Indifferent.* When a body placed upon a supporting surface has its center of gravity neither raised nor lowered by tipping it or rolling it over, it is in neutral or indifferent equilibrium (Fig. 78 *c*).

When a body is balanced on a point (Fig. 79), the equilibrium is stable, if the supporting point is above the center of gravity; unstable, if it is below; and neutral, if it is at the center of gravity.



FIG. 79. — Center of Gravity below Point of Support.

136. Equilibrium of Position. — Given three similar rhombohedral blocks (Fig. 80). A single one placed on a table is in stable equilibrium, since its line of direction falls within its base. When two are

fastened together as shown in the figure, the line of direction falls on an edge of the base so that the combination is in unstable equilibrium. And the line of direction of three attached blocks falls without the base so that equilibrium of any kind is impossible; the combination at once tumbles over. By attaching a piece of lead, however, to one side of the lowest block, the center of gravity is shifted, with the result that the line of direction falls within the base, and a state of stable equilibrium is attained.

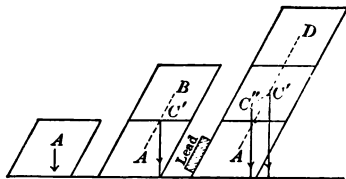


FIG. 80.—Equilibrium of Position.

EXERCISES

1. Why is it difficult to walk on a tight rope?
2. Why does a person lean forward on rising from a chair?
3. Why does a person lean over to one side when carrying a heavy load in one hand?
4. Why does a feeble person use a cane in walking?
5. Why does a boat tip over more easily when its occupants are standing than when they are sitting?
6. Why is it impossible to stand with your heels against a wall and lean forward without falling?
7. Two spherical bodies weighing 120 g. and 30 g., respectively, are connected by a rigid rod of negligible weight in such a manner that their centers are 100 cm. apart. Where is the common center of gravity of the joined bodies?
8. Given an oar, a 10-lb. anchor, a tapeline, and some cord, how can you find the weight of a string of fish?
9. A boy weighing 50 lb. wishes to seesaw alone on a plank 15 ft. long and weighing 75 lb. If his weight is regarded as applied at one end of the plank, how far from the other end must the support be placed?
10. A train going $40 \frac{\text{mi.}}{\text{hr.}}$ northward meets a west wind having a velocity of $30 \frac{\text{mi.}}{\text{hr.}}$. From what direction does the wind seem to come? What is its velocity with respect to the train?

CHAPTER VII

MOLECULAR MOTIONS AND FORCES

137. Molecular Forces. — We have thus far considered force only as acting upon a body as a whole, and have disregarded any action of forces between the molecules of the body. These molecular forces, however, give rise to many important phenomena, some of which will now be described.

138. Kinetic Theory of Matter. — Matter is supposed to be made up of molecules. Do these molecules remain stationary, or are they moving about within a body? And if they move, what are the conditions and characteristics of their motions? In an attempt to answer such questions as these, the kinetic theory has been advanced. Applied at first to gases, it has been found to give, as well, plausible explanations of certain phenomena of liquids and solids.

139. Kinetic Theory of Gases. — The space actually occupied by the molecules is supposed to be small in comparison with that occupied by the gas as a whole. This is in accordance with the fact that the volume of a given mass of a substance is much greater when the substance is in the gaseous state than when it is in the liquid state. For example, a cubic inch of water expands to nearly a cubic foot of steam. Furthermore, the molecules of a gas are supposed neither to attract nor to repel one another, but to lead independent existences. The molecules are also assumed to be in constant motion. This is illustrated by the fact that if a little illuminating gas escapes from a cock,

its odor soon becomes apparent in all parts of the room. Each molecule is thought of as moving in a straight line with great velocity until it collides with another molecule or with the molecules of the wall of the containing vessel.* The pressure of a gas is ascribed to these molecular bombardments. Just as a jet of water played against a wall exerts a continuous pressure on it, so do the blows of the myriads of molecules of a gas striking against a restraining wall exercise a continuous pressure upon it. The denser the gas is, the more frequent are the bombardments, and therefore the greater is the pressure. This conclusion is quite in accord with the fact that the pressure of a gas is proportional to density. If a gas is placed in a vacuous space, its molecules at once begin to move out and occupy this space.

140. Diffusion of Gases. — If a wide-mouthed bottle is filled with the lightest known gas, hydrogen, and is placed inverted upon another bottle of similar shape and size filled with air (Fig. 81), it will be found in a few minutes that, although air is more than fourteen times heavier than hydrogen, yet it has risen and mixed with the hydrogen which, in turn, has moved down and mixed with the air. This is done in spite of the force of gravity, which tends to keep the heavier gas in the lower position. Such a process of intermixing or *diffusing* continues until every portion of the gaseous mixture has the same composition. All gases exhibit this property of diffusion.



FIG. 81. —
Diffusion
of Gases.

* It may assist to a clearer image of the supposed condition of a gas if the appearance of a swarm of insects as seen in the air on a summer day is called to mind. The swarm as a whole may remain almost stationary, but each insect is moving rapidly about.

The kinetic theory offers a simple explanation of diffusion. The molecules of each gas, being in a state of perpetual agitation, move across the boundary of the two gases, and very soon * their constant motions result in a perfect mixing.

141. Osmose of Gases. — An unglazed earthenware cup contains a multitude of tiny holes; it is porous. Let one

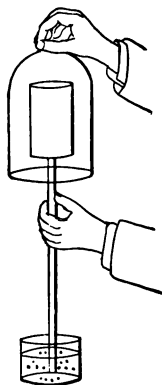


FIG. 82. — Osmose of Gases.

end of a glass tube (Fig. 82) be connected with such a cup, and the lower end placed in water. When a jar filled with hydrogen is brought down over the cup, bubbles of gas issue from the tube through the water, thus showing an increase of pressure within the cup, while, when the jar is removed, the water rises in the tube, indicating a decrease of pressure. How does the kinetic theory explain this phenomenon?

The hydrogen molecules not only move more rapidly than the air molecules, but they are also smaller. They can therefore traverse the pores of the earthenware more quickly than can the air molecules. They enter the cup more swiftly than the air molecules can leave it; and this increase in the quantity of gas within the cup results in a corresponding increase of pressure. When the jar is removed, the hydrogen escapes more speedily than the air reenters, the gas within the cup becomes more rarefied than that outside, and the pressure is therefore decreased.

142. Molecular Motions in Liquids. — It is a familiar fact that a liquid left in an open vessel sooner or later disap-

* Hydrogen molecules move at the rate of about $1700 \frac{\text{m.}}{\text{sec.}}$.

pears; we say that it *evaporates*. Now the molecules of liquids are so much closer together than those of gases that they exercise mutual attractions. How then can evaporation take place? If we assume that the molecules of the liquid are in motion, it seems plausible that now and then a molecule may move with such speed toward the surface that it will shoot beyond the surface in spite of the attraction of the other molecules. The evaporated molecules are then in the gaseous state, and mixing with the air pass from the liquid.

143. Diffusion of Liquids.— If into the bottom of a glass cylinder partly filled with water a strong solution of copper sulphate is introduced by means of a funnel tube (Fig. 83), the plane between the two liquids of different color and density is at first distinctly marked. But after a day or so this plane becomes indistinct, the heavier blue liquid rising and the water sinking, until in the course of a few weeks, the two liquids will have become thoroughly mixed.

The phenomenon is similar to the diffusion of gases, although the time of complete mixing is much longer for liquids than for gases. According to the kinetic theory, some of the copper sulphate molecules move in such a manner as to break away from the other molecules and pass up into the water. It is the constant motion of the molecules that causes them to diffuse.



FIG. 83. — Diffusion of Liquids.

144. Solution of Liquids.— There are many liquids that do not mix when brought together. Such is the case with oil and water. And there are other liquids which mix only

to a certain degree, as ether and water. Liquids which, when placed in contact, diffuse into each other, and finally produce a homogeneous mixture, are said to be *soluble*. All gases are miscible with one another in all proportions; but this is not true of many pairs of liquids.

145. Solution of Solids.—Certain solids when placed in appropriate liquids waste away and in the course of time become incorporated with the liquid so as to form a homogeneous whole. This act of solution may be hastened by agitating the solid and liquid so as to bring fresh portions of the liquid in contact with the solid, as well as by powdering and heating the solid. Most solids dissolve only to a limited extent in liquids, and the proportion of a *solute* (as the dissolved substance is called) in a definite amount of the *solvent* is termed its *solubility*. Thus the solubility of salt in water is 32 %; that is to say, if a mixture of salt and water be shaken together until no more of the salt enters into solution, every hundred parts of the solution will contain 68 parts of water and 32 parts of salt by weight.

Why a certain solid will dissolve in one liquid and not in another, we do not know. The action between the liquid and the solid is confined to the surface separating them. As soon as a layer of molecules separates itself from the solid, the molecules diffuse into the liquid.

146. Kinetic Theory of Solutions.—The molecules of a dissolved substance behave in much the same fashion as the molecules of a gas. They move about with a certain independence, the result being that the solvent is at last uniformly filled with them, just as a vacuous space is uniformly filled with any gas placed in it. The molecules of

the dissolved substance must, because of their motions, exert a pressure upon the bounding surfaces of the solvent. This pressure can be demonstrated if the solution be separated from some of the pure solvent by a porous partition (semi-permeable membrane), through which the molecules of the solvent can pass freely, but which does not permit the passage of the molecules of the solute. The molecules of the solute therefore bombard this partition, and the amount of pressure produced is capable of measurement.* This *osmotic pressure* has been found to be proportional to the concentration of the solution; that is, the more molecules of the solute there are in a given volume of the solution, the greater is the osmotic pressure. Now as the concentration of a solution is analogous to the density of a gas, it follows that Boyle's law is applicable to solutions.

147. Cohesion and Adhesion. — The attraction between the molecules of the same substance is called *cohesion*. While gases have practically no cohesion, its effects are very apparent in liquids and solids. The attraction between the molecules of different substances is called *adhesion*. Adhesion is often stronger than cohesion; boards, for instance, may be glued together so that the joint is stronger than the wood itself.

148. Cohesion in Solids. — As cohesion is due to molecular forces, it can act only through molecular distances.

* "To illustrate how such measurements might be possible, let us imagine a fish seine with which fishermen have succeeded in encircling a school of mackerel. Water passes freely through the net and exerts no pressure, but the fish are too large to pass through the meshes. In their efforts to escape they strike against the net and thus exert a pressure upon it. Similarly, if a mesh can be prepared so fine that molecules of a substance dissolved in water cannot pass through it, but still so large that water molecules may readily pass, it would furnish a means of measuring osmotic pressure." — TALBOT and BLANCHARD.

When a solid is divided, its parts cannot be made to cohere again unless they are brought very close together, and to accomplish this, enormous pressures must be used. Thus the graphite, which is the "lead" of our pencils, was first in powdered form, and was then, by means of great pressure exerted by a hydraulic press, molded into the little sticks which are fitted into the wooden cases. Two bright and smooth surfaces of lead may be made to cohere when pressed together with the hands.

149. Elasticity.—Elasticity has already been discussed in connection with the measurement of forces (§ 26). When a stress is applied to a solid, the relative positions or motions of the molecules are altered, but after the stress has been removed, their attractions cause them to return to their original condition. But if the elastic limit has been passed, the molecular condition is permanently changed. The degree of elasticity of different materials is found by measuring the forces required to produce equal distortions of articles of like dimensions made of these materials. One substance is said to be twice as elastic as another when twice as much force is required to produce in it the same change of size or shape. Measured in this way, the elasticity of India rubber is very small, for but a small force is needed to distort it considerably. Indeed, the ordinary meaning of the word *elasticity* is different from that given it in physics. When the physicist says that rubber is elastic, he means that a large distortion is possible before it reaches its elastic limit.

150. Tenacity or Tensile Strength.—A match, although readily broken by bending, cannot be pulled apart except by exerting great force. The *tenacity of a substance is the greatest resistance that it can offer to a longitudinal stress, i.e.* one tending to pull it apart. The

molecules, when pulled away from one another, refuse to cohere after the pull reaches a certain limit, and the substance breaks. The tensile strengths of various materials differ greatly. In the case of metallic wires, it is independent of their length, and is directly proportional to their cross-sectional areas.

151. Hardness.—Lead can be readily scratched with the finger nail, but not copper. Copper can be scratched with steel, but not so the diamond. We say that certain substances are harder than others when they cannot be scratched or abraded by them. The cohesion between the molecules of one substance may be less than that between the molecules of another. The cohesion constituting hardness seems to be different from that conferring strength; for glass, while very hard, is also very fragile.

152. Ductility and Malleability.—Such substances as can be drawn out into fine wire are said to be *ductile*. Such substances as can be hammered out into thin sheets are said to be *malleable*. Usually ductile substances are also malleable, but there are many exceptions.

153. Factor of Safety.—Enough material must always be used in building operations to make the strength of every part so great that no force to which it will be subjected can give it a permanent strain (*a set*). It is good practice to employ from about four times (in the case of metals) to about 20 times (in the case of wood) as much material as is necessary to withstand the stress. When a beam breaks at five times the load it is designed to support, it is said to have a *factor of safety of five*.

154. Shape of Beams and Girders.—The prime requisites of floor beams and bridge girders are that they be light and yet so stiff that they bend but slightly under the greatest loads that they may have to support. A minimum of material with a maximum of stiffness is demanded. When a beam is bent downward, its upper side is compressed and its lower side lengthened (Fig. 84). Between these two sides there must be a portion of the material which is practically under no stress at all. As, then, the greatest resistance offered to bending stress is at its upper and lower sides, most of the material of properly

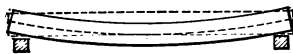


FIG. 84.—Bending Beam.

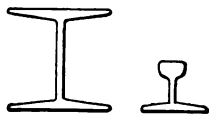


FIG. 85. — Shapes of Beams and Rails.

constructed beams is placed at these parts and comparatively little at the middle. Accordingly, when the material permits, as in the case of metals, beams are made with flanges at the top and bottom, connected by a thin web (Fig. 85).

155. Cohesion in Liquids.— If a perfectly clean plate of glass or metal be suspended from one arm of a balance in a horizontal position, counterpoised, and then a vessel of water lifted up until its surface just comes in contact with the plate, it will be found that a definite force must be applied to lift the plate from the water (Fig. 86). That this force is due to the cohesion of the liquid is proved by the fact that with plates of the same size but different materials, the force required is the same, and that some water adheres to the plate; the molecules of the water are separated from one another, and the force needed to do this is a measure of the liquid's cohesion. The values of the cohesive forces of different liquids are different, and much smaller than those of most solids.

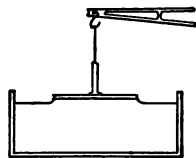


FIG. 86. — Cohesion in Liquids.

156. Condition of the Surface of a Liquid.— The molecules in the interior of a liquid attract one another in all directions, while those in the surface of the liquid are attracted towards its center (Fig. 87). The effect of this one-sided attraction is to subject the molecules at the surface to a great pressure directed inward. Because of this surface pressure the molecules in the sur-

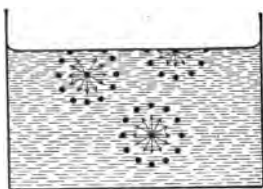


FIG. 87. — The surface of a liquid is in a state of tension.

face are forced closer together, and the liquid is, as it were, surrounded by a membrane made up of a layer of tightly packed molecules. That the superficial film of the liquid is in quite a different condition from that of the interior is shown by the following experiment.

Hold a needle horizontally and lay it gently upon the surface of some water. The needle, although made of steel that is nearly eight times as heavy as water, nevertheless floats, and is seen to lie in a depression (Fig. 88) in the surface. If the needle be forced through the surface, it promptly falls to the bottom of the vessel.



FIG. 88.—A Floating Needle.

157. Surface Tension. — Because of the attraction of the molecules of a liquid, the boundary layer is subjected to a definite tension, called the *surface tension* of the liquid. With rise of temperature, surface tension diminishes.

If a liquid is placed in another liquid which has the same specific gravity and which does not dissolve it, for example, olive oil and dilute alcohol, the first liquid is practically freed from the force of gravity, and is acted upon by its own molecular forces only. It then assumes a

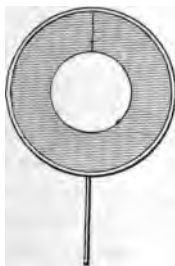


FIG. 89.—Surface Tension of a Film.

spherical form, for the reason that its surface tension makes its surface as small as possible, and of all geometrical forms the one that has the least surface for a given volume is the sphere. It is likewise this surface tension that makes drops and bubbles spherical. The contractile force of the film in a soap bubble is so great that it expels the air from the pipe with sufficient

speed to deflect and even to blow out a small candle flame against which it is directed. Let a thin and long rubber band be floated upon the surface of water, and let the surface tension within the

632792

band be lessened by adding a drop of alcohol. The result is that the tension of the film outside the band makes it take a more or less circular shape. Again, let a wire ring be dipped into a soap solution, then withdrawn with a soap film upon it (Fig. 89), and a loop of fine silk thread laid over the film. If the film within the loop is broken by touching it with a hot needle, the thread is drawn into a circular form. A lump of gum camphor when placed on clean warm water begins to dissolve. Since its solution has a smaller surface tension than has water, the floating lump is drawn away from its solution, and as usually now one part and then another dissolves faster than the others, it flies about on the surface of the water. A trace of fat even so slight as that given off by the finger when touched to the water, lessens the surface tension enough to stop the camphor's motions. If a drop of alcohol is placed in a very thin layer of water on a clean plate, as the surface tension of the mixture of water and alcohol is less than that of the water, the mixed portions of the two liquids are drawn away on all sides, leaving a bare spot in the middle of the plate.

158. Surface Viscosity.—The surface of a liquid seems to be tough or sticky, and because of this viscosity, bubbles are readily formed, as in foam and froth. Surface viscosity is independent of surface tension as to magnitude; that is, a liquid with a large surface tension may have a small viscosity, and vice versa. If the surface tension is greater than the surface viscosity, as is the case with perfectly pure water, bubbles and froth form with difficulty. The toughness of this superficial film permits insects to walk upon water, and is instrumental in the smoothing of waves by the pouring of oil upon them; the film of oil has a small tension but considerable viscosity, so that it is not easily broken by the water pressing against it, the formation of surf being thus lessened.

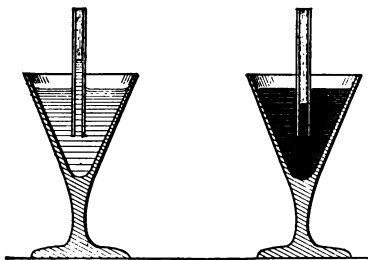


FIG. 90.—Capillary Tubes.

159. Capillary Phenomena.—When a fine glass tube is wet and set upright in water (Fig. 90), the water rises and stands at a higher level within the

tube than without. If a similar tube be placed in mercury, which does not adhere to glass, the mercury within the tube is at a lower level than that without. Such phenomena as these are said to be due to *capillary** action, because they take place in tubes with bores of hair-like thickness. In the case of water and glass the free surface is concave, while in the case of mercury and glass it is convex.

Capillary phenomena result from the combined action of the cohesion of the liquid and its adhesion for the material of the tube. It is because of capillary action that blotting paper absorbs ink, oil rises in a lampwick, and water passes from one end of a towel to the other. In short, capillary phenomena are manifest wherever a liquid comes in contact with a solid.

160. Laws of Capillary Action.— Experiment shows that: *Liquids are raised in capillary tubes made of a material which they wet, and are depressed in tubes the material of which they do not wet.*

The amount of the elevation or depression varies inversely as the diameter of the tube.

A rise of temperature diminishes the amount of elevation or depression.

161. Absorption.— At the surface of most solids is exercised an attractive force by reason of which thin films of liquids or gases are condensed upon it; they are said to be *absorbed*. When the solids are porous or are powdered, they have exceedingly large surfaces, so that a correspondingly large amount of condensation takes place upon them. There seems to be a sort of selective absorption exercised in addition to the surface action. Thus charcoal, which is highly porous, can absorb ninety times its own volume of ammonia, forty times its volume of carbonic acid gas, four times its volume of air, and twice its volume of hydrogen. The deodorizing action of charcoal is due to its high absorptive powers.

* From the Latin word *capillus*, meaning hair.

162. Vacua produced by Absorption.— The absorptive force of charcoal for some gases is so great that practically all of a gas may be removed from a confined space by its action. Thus, if a piece of recently heated charcoal is dropped into a tube filled with bromine vapor, and the opening of the tube sealed together, a vacuum equal to that furnished by the best exhausting pumps is soon obtained.

EXERCISES

1. A drop of water introduced into a tapering tube tends to move to the narrowest part of the tube (Fig. 91), while a drop of mercury tends to move to the widest part. Show how surface tension accounts for this behavior.



2. Why is it that drops of water do not spread out and cover greasy surfaces?



FIG. 91.

3. Why is it that the sharp edges of a glass rod or a stick of sealing wax become rounded off when the glass or wax is heated until it is soft?

4. Why cannot pure water bubbles be blown like soap bubbles?

5. Why does a slender stream of water often break up into drops?

6. Grease may be removed from cloth by covering it with blotting paper and passing a hot flatiron over the paper. Account for this.

7. Why is it that a shallow dish with a bottom made of wire cloth having a very fine mesh can be floated upon water?

8. A copper wire breaks under a tensile stress of 10 kg., while another one breaks under a stress of 15 kg. What is the ratio between the diameters of the wires?

9. If a liquid rises 10 cm. in a tube .01 mm. in diameter, how high will it rise in a tube .05 mm. in diameter?

10. If a 4 per cent solution of sugar in water has an osmotic pressure of 208 cm. of mercury, what is the osmotic pressure of a 1 per cent solution?

CHAPTER VIII

GRAVITATION

FREELY FALLING BODIES

163. Falling Bodies. — Which falls the faster, a heavy or a light body? Everyday experience teaches that light bodies, such as leaves, feathers, and the like take more time to fall than do heavy bodies, such as stones or bullets, and the conclusion commonly drawn from these observations is that heavy bodies fall the faster. This opinion was unchallenged until Galileo in 1638 performed some experiments that showed it to be wrong.

164. Galileo's Experiments. — The leaning tower of Pisa was used by Galileo as the place of experimentation, and his experiments consisted in dropping various materials from one of the upper galleries, and noting the time they took to reach the ground. He found that large and small bodies of any material whatsoever fall at the same rate if they are of such a shape as to offer about the same proportional resistance to the air. Bullets fall faster than feathers because they offer less surface to the action of the air, which retards their time of falling. Heavy metals, such as gold, when beaten out into foil or leaf, flutter slowly downward instead of falling rapidly like a compact chunk of the metal.

165. Influence of the Air. — The air pump was unknown in Galileo's time, but when it was invented by von Guericke about 1650, one of the first uses to which it was put was a

confirmation of Galileo's view, that it is the resistance of the air which prevents all bodies from falling at the same rate. This classic experiment goes by the name of the "Guinea and Feather Experiment," and is performed in this wise. A long glass tube with a stopcock at one end, and closed at the other end, contains a coin (guinea) and a feather (Fig. 92). When the air is pumped out of the tube, and when the tube is suddenly inverted, the coin and feather take the same time to fall the length of the tube.



FIG. 92.—Guinea and Feather Tube.

166. Weight a Constant Force. — The weight of a body is due to the action of the earth upon it, and the amount of this gravitation stress depends upon the masses of the earth and of the body. Weight is a constant force at any place and is always directed downward. Now a body acted upon by a constant force moves with uniformly accelerated motion, and the acceleration due to gravity has the same value at any given place. The weight w of a body is equal to the product of its mass m and the acceleration due to gravity g ; $w = mg$. But this is nothing but a special case of the general formula for force: $F = ma$, as weight is force, and g , acceleration. The relationships established in Chapter V for uniform acceleration apply to the case of freely falling bodies. Indeed, these general relationships were first established by Galileo through his investigations on falling bodies.

167. Measurements on Falling Bodies. — Freely falling bodies move so rapidly that the measurement of their time

of fall for a convenient distance cannot be made with any degree of accuracy by ordinary methods. Among the numerous devices for showing the relationships between the time, distance, velocity, and acceleration of falling bodies, the following may be mentioned.

168. Galileo's Method of the Inclined Plane.— When a ball rolls with very little friction down an inclined plane (Fig. 93), the same final

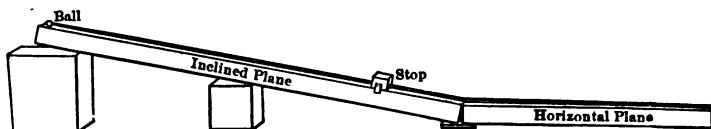


FIG. 93. — Galileo's Apparatus for studying Falling Bodies.

velocity is attained as when it falls vertically through a distance equal to the height of the plane. It takes a longer time to acquire this final velocity, however, on the plane than in the case of direct fall. It may be proved that the ratio of the time for a body to move down a plane to the time of direct fall is equal to the ratio of the length of the plane to its height. Thus, if the length of the plane is ten times its height, the body will take ten times as long to acquire the velocity that it gains in falling straight down. The time of fall may therefore be increased enough to permit of accurate and convenient measurements, the nature of the fall, however, not being essentially changed.

169. Atwood's Method.— Suppose that two bodies are hung by a thread passing over a frictionless pulley (Fig. 94), and that one body *A* has a mass of 21 g., and the other *B* a mass of 23 g. *A* will rise as fast as *B* falls. But while the total mass set in motion is $21 + 23 = 44$ grams of mass, the force causing the motion amounts to only $23 - 21 = 2$ grams of force. The acceleration of the bodies is therefore only $\frac{2}{44} = \frac{1}{22}$ of what it would be in the case of free fall. The motion is accord-

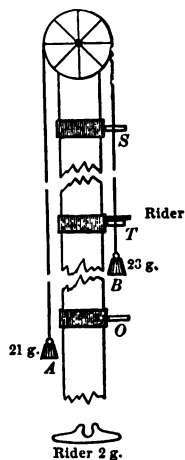


FIG. 94. — Atwood's Machine. The stop *T* removes the rider so that the masses of *A* and *B* become equal.

ingly made slow enough to allow of accurate measurements of the time intervals.

170. Time and Distance of Fall.—The measurements of time and distance made by any of the methods employed in the study of the fall of bodies may be put into the following form.

<i>Times:</i>	1	2	3	4	5	6	7	...	t
<i>Distances traversed:</i>	l	$4l$	$9l$	$16l$	$25l$	$36l$	$49l$	...	lt^2

During the first two time intervals the body traverses four times as great a distance as during the first; during the first three time intervals, nine times as great a distance, and so on, the distances increasing as the squares of the time intervals. The distance fallen during any single time interval is found by subtraction.

<i>Time intervals:</i>	1	2	3	4	5	6	7	...	t

Distances traversed during the corresponding time intervals:

$$l \quad 3l \quad 5l \quad 7l \quad 9l \quad 11l \quad 13l \dots (2t-1)l$$

An inspection of these values shows that there is a constant gain in the distance traversed in each succeeding time interval equal to $2l$; the acceleration is therefore equal to $2l$. If a be substituted for $2l$, then $l = \frac{1}{2}a$, and the following table is obtained.

<i>Times:</i>	1	2	3	4	5	6	7	...	t

Distances traversed:

$$1 \times \frac{1}{2}a \quad 4 \times \frac{1}{2}a \quad 9 \times \frac{1}{2}a \quad 16 \times \frac{1}{2}a \quad 25 \times \frac{1}{2}a \quad 36 \times \frac{1}{2}a \quad 49 \times \frac{1}{2}a \dots \frac{1}{2}at^2$$

Distance traversed in any one time interval:

$$1 \times \frac{1}{2}a \quad 3 \times \frac{1}{2}a \quad 5 \times \frac{1}{2}a \quad 7 \times \frac{1}{2}a \quad 9 \times \frac{1}{2}a \quad 11 \times \frac{1}{2}a \quad 13 \times \frac{1}{2}a \dots \frac{1}{2}a(2t-1)$$

171. Velocity and Time of Fall.—If at the bottom of Galileo's inclined plane a horizontal plane be attached, the moving body will, as soon as it reaches this plane,

move on with uniform velocity, — that which it has at the instant of leaving the incline. Again, if in Atwood's machine the body B be made in two parts, one with a mass of 21 g. and the other with a mass of 2 g., and a device provided for removing the smaller mass (rider) at any given point, the two equal masses after its removal will move with uniform velocity which is the same as that they have at the instant of the removal. Measurements can thus be made of the velocities of falling bodies at the expiration of any time interval. A tabulation of the general results obtained in such a manner follows.

Times:

1 2 3 4 5 6 ... t .

Velocities at the end of any time:

$2l = a$ $4l = 2a$ $6l = 3a$ $8l = 4a$ $10l = 5a$ $12l = 6a$... at .

172. Formulas for Falling Bodies. — The formulas derived from experiments on falling bodies are the same as those derived for bodies moving with uniform acceleration (§ 98). The acceleration due to gravity g at Chicago is $32.16 \frac{\text{ft.}}{\text{sec.}^2}$, or $980.26 \frac{\text{cm.}}{\text{sec.}^2}$. (In problems these numbers may be rounded off to 32 and 980.) The algebraic formulas for the laws governing the free fall of bodies are, if t denote the time, v , the velocity, l , the total distance traversed in a given time, and l' , the distance traversed during any single time interval,

$$v = gt, \quad v = 32 t, \quad v = 980 t, \quad (1)$$

$$l = \frac{1}{2}gt^2, \quad l = 16 t^2, \quad l = 490 t^2, \quad (2)$$

$$l' = \frac{1}{2}g(2t - 1), \quad l' = 16(2t - 1), \quad l' = 490(2t - 1), \quad (3)$$

$$v^2 = 2gl. \quad v^2 = 64 l; \quad v^2 = 1960 l. \quad (4)$$

$$\text{or } v = 8\sqrt{l}.$$

would fall straight down, and the distances traversed in equal times would be equal to half the product of the acceleration due to gravity, by the square of the time. Let these values also be plotted on the same scale in a vertical line. The positions actually occupied by the projectile are at the intersections of the perpendiculars erected to the horizontal and vertical lines at the points marking the distances gone over. A smooth curve drawn through these points gives the path of the projectile.

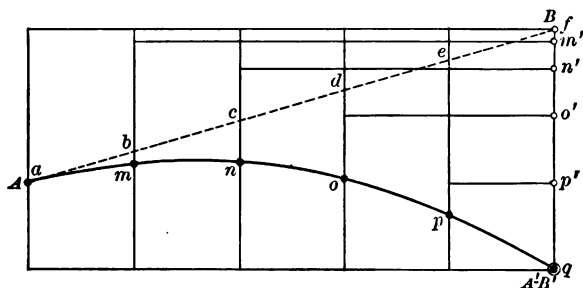


FIG. 96. — Projectile with Initial Velocity Oblique.

III. **Initial Velocity Oblique.** — A construction similar to the preceding will give the path of the projectile (Fig. 96).

174. Ballistic Curves. — In the discussion of the path of projectiles no account has been taken of the resistance of the air. While the curves traced for projectiles are all varieties of the curve known as the parabola, the resistance of the air modifies this parabolic path, especially when the speed is great. The properties of a parabola show that a projectile will go farthest if it is started in a path making an angle of 45° with the horizon. In gunnery practice, however, it is found that the elevation of the gun must be greater to get the longest range.

175. Range Finding. — The sights of a gun are placed upon its barrel, one near the stock and the other near the muzzle (Fig. 97). The gun is aimed by placing the eye and the gun in such positions that a line from the point aimed at to the eye passes along the sights. If the velocity of the bullet after leaving the gun be $1000 \frac{\text{ft.}}{\text{sec.}}$, it will take the bullet one second to reach an object 1000 ft. away. If the line of sight be horizontal, the bullet will fall 16 ft. during the first second of its flight and will strike 16 ft. below the point. If, however, the rear

sight be elevated, the gun will be tilted a little in aiming, so that allowance can thus be made for the falling of the bullet. The rear sight is so constructed that it can be adjusted for various ranges, as 100, 200, 300, etc., yards. The marksman estimates the distance between himself and the object and then adjusts the sights accordingly.

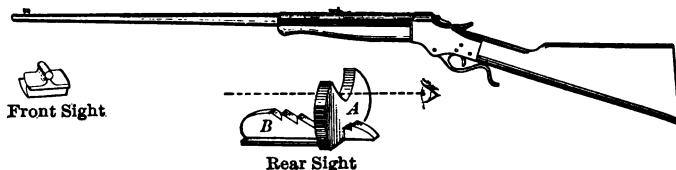


FIG. 97.—The Sights of a Gun. By setting *A* in the different notches of *B*, different elevations of the gun barrel are obtained.

176. Newton's Law of Universal Gravitation.—If the planets and the sun had no attraction for one another, they would move in straight lines. The very fact that the planets move around the sun in approximately circular orbits shows that there must be some mutual force acting upon them. Apparently the earth moves round the sun, but in reality both move round their common center of gravity or mass, which is located within the sun because the mass of the sun is so much greater than that of the earth. The fact of attraction being evident, the question is: How is this attraction exerted? In 1687 Newton published the following answer: *Every body in the universe attracts every other body with a force directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers of mass.*

Denote the masses of two bodies by m and m' , the distance between their centers of mass by d , and the gravitational force by F . The algebraic expression for this law then is

$$F = \frac{mm'}{d^2}. \quad (5)$$

EXAMPLE. — Suppose that two bodies have masses of 100 g. and 500 g., respectively, and that their centers of mass are 10 cm. apart. The attraction between them is

$$\frac{100 \times 500}{100} = 500;$$

that is, it is 500 times as great as the attraction between two one-gram masses, having their centers of mass one centimeter apart.

177. Weight and Acceleration due to Gravity. — Weight and gravity are almost synonymous terms for gravitation stress between the earth and terrestrial bodies. While the mass of a body does not alter anywhere on the earth's surface, there is a variation in the distance it falls in a given time. Thus at the equator a body falls 489 cm. during the first second, at Chicago it falls 490 cm., and at the poles it is calculated to fall 491 cm. The value of g at the equator is, therefore, $978 \frac{\text{cm.}}{\text{sec.}^2}$, at Chicago, $980 \frac{\text{cm.}}{\text{sec.}^2}$, and at the poles, $982 \frac{\text{cm.}}{\text{sec.}^2}$. The weight of the body must also change in the same proportion, for weight is a force, and force is proportional to the acceleration it produces.

178. Weight above the Earth's Surface. — The higher above the earth's surface a body is, the farther removed it is from the earth's center, and therefore the less is the gravitational stress. If d and d' represent the distances from the earth's center to two bodies that are above its surface, and w and w' their weights, then, by the law of gravitation,

$$\frac{w}{w'} = \frac{d'^2}{d^2}. \quad (6)$$

EXAMPLE. — How much would a body weighing 10 lb. at the earth's surface weigh 2000 mi. above the surface?

Solution. The two distances from the earth's center are 4000 mi. (the

earth's radius) and $4000 + 2000 = 6000$ mi. Substituting these values in the equation, we have

$$w' = \frac{6000^2}{4000^2} = \frac{36 \times 10^6}{16 \times 10^6} = \frac{9}{4};$$

whence

$$w' = \frac{10 \times 4}{9} = 4.4 \text{ lb.}$$

EXERCISES

1. Which moves eastward with the greater velocity — the top or the bottom of a tall tower?
2. Why is it that a ball dropped in still air on the south side of a tall and massive tower strikes the ground very slightly north and quite perceptibly east of the vertical line?
3. What will be the acceleration in $\frac{\text{cm.}}{\text{sec.}^2}$ of a ball rolling down an inclined plane having a height one tenth as long as its length?
4. If the two masses of an Atwood's machine are 30 g. each, what acceleration will a rider with a mass of 2 g. added to one of the masses impart?
5. How many meters deep is a well if it takes a stone 3.5 sec. to reach the bottom?
6. If the velocity of a falling body at the instant it reaches the ground is $490 \frac{\text{cm.}}{\text{sec.}}$, how long was it in falling?
7. If a ball rolling down an inclined plane goes 20 cm. during the first second, how far will it go during the fifth second, and what velocity will it have at the end of the seventh second?
8. With what velocity must water be forced from the nozzle of a hose in order to reach the roof of a building 64 ft. from the ground?
9. A stone weighing 10 g. on the earth's surface would weigh about 270 g. on the sun's surface. How far would it fall in 5 sec. if it were on the surface of the sun?
10. The mass of the moon is .012 that of the earth and its diameter is .273 that of the earth. If a certain body stretches a spiral spring 200 mm. on the surface of the earth, how far would it stretch the same spring on the surface of the moon?
11. A stone is thrown vertically upward with an initial velocity of $20 \frac{\text{m.}}{\text{sec.}}$. How long will it be before it returns to its starting point?

12. A stone is thrown horizontally from a cliff with a velocity of $400 \frac{\text{cm.}}{\text{sec.}}$, and strikes the ground in 4.32 sec. How high is the cliff? How far from the base of the cliff does the stone strike? Choose convenient scales and trace its path graphically.

13. At what angle should an arrow leave a bow to go the greatest horizontal distance, the resistance of the air being neglected?

CURVILINEAR MOTION

179. **Motion in a Circle.**—Consider the case of a stone whirled in a sling. The stone may be made to move with *uniform speed in a circular path* with a radius of the length of the string (Fig. 98). If the speed is increased, the tug on the hand is increased proportionately, and if the cord breaks or is let go, the stone will begin to move along a tangent to the circle. To keep the stone in a circular path, a constant force directed toward the hand must be exerted. The constant force in such cases has the effect of changing the direction of the motion.

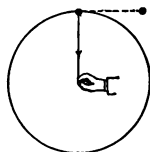


FIG. 98.—Motion in a Circle.

180. **Centripetal and Centrifugal Force.**—When a body is moving in a circle, some force must be acting crosswise to the motion in such a way as to draw the body continually toward the center of the circle; this force is called the *centripetal* (center-seeking) *force*. Its magnitude may be constant, but its direction changes continuously, being always at right angles to the direction of the motion. And for the reason that it is at right angles to the path of the moving body, it can produce no acceleration along the line of that motion (*linear acceleration*). When a body is whirled round by a string, as in the case of a sling, the tension in the string is really a stress. Viewed with

respect to the *action* of the hand on the whirling body, it is a center-seeking force, but viewed with respect to the *reaction* of the body on the hand, it is a center-fleeing (*centrifugal*) force. Centripetal and centrifugal force are then terms expressing two ways of regarding one and the same stress. It simplifies matters to have but one name for both, such as that of *central force*.

181. Inertia of a Rotating Body.—When a stone attached to a string is whirled round in a circle, if the string breaks or is let loose, the stone begins to move with uniform speed in the direction of a tangent to the circle. The stone moves along the tangent, not because there is any special “centrifugal force,” but solely because it has inertia. If no force should act upon it after it leaves its circular path, it would go on forever in a straight line, with the speed it had at the instant it left the circular path. As a matter of fact, both the force of gravity and the resistance of the air act upon it, so that the path described is a modified parabola.

182. Value of Central Force.—Experiment shows that the force which keeps a body moving in a circle is directly proportional to (1) its mass m , (2) the square of its speed s , and inversely proportional to the radius r of its path:

$$f = m \frac{s^2}{r}. \quad (7)$$

183. The Pendulum.—Any body so supported that it can swing about a point or axis constitutes a *pendulum*. The essentials of a pendulum are its *center of suspension* P (Fig. 99), its *center of oscillation* O , and its *length*, i.e. the distance between the centers of suspension and oscillation. If a sphere of some dense substance, lead, for instance, is

suspended by a fine thread, the center of oscillation is very nearly at the center of gravity, a close approximation thus being made to a *mathematical* or *simple pendulum*, which is defined to be a *single material point attached to its center of suspension by a massless cord*. Real pendulums, which are known as *physical* or *compound pendulums*, cannot, of course, fulfill these ideal conditions.

184. Pendular Motion.—If the bob of a pendulum be drawn to one side of the position (Fig. 99) it assumes when at rest, and then let go, it will move along a circular path with accelerated motion. It does not stop at its lowest position, however; for its inertia carries it against gravity in a circular path up on the other side with retarded motion, until it is nearly as high as it was at the beginning of the motion. It then falls back and repeats the swing in the reverse order. Such motions are repeated until friction brings the pendulum to rest in its position of most stable equilibrium.

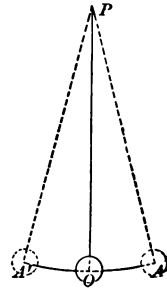


FIG. 99.—A Simple Pendulum.

185. Definitions.—The *period* of a pendulum is the time in seconds between its successive passages through its lowest position. The *frequency* is the number of vibrations or swings in either direction made during one second. The greater the period, the less the frequency. The *amplitude* is the wideness of the swing, *i.e.* the distance from the position of rest to the farthest position it reaches on either side.

186. Laws of the Pendulum.—Newton compared the periods of two pendulums of equal length, having as bobs

wooden boxes of the same size, so that the resistance of the air might affect them equally. He filled the boxes with various substances of different densities, such as sand, wheat, and shot, and found that the times of vibration were the same. He accordingly drew the conclusion that *the period of a pendulum is independent of its mass or its material*. Experiments have also proved that *the period (1) is approximately independent of the amplitude; (2) varies directly as the square root of the length; (3) varies inversely as the square root of the acceleration due to gravity*.

187. The Pendulum Formula. — The laws of the pendulum are included in the following equation, t denoting the period and l the length :

$$t = \pi \sqrt{\frac{l}{g}}. \quad (8)$$

Suppose that two pendulums having the lengths l and l' and the periods t and t' are compared at the same place, in order that g may be treated as a constant. Then

$$t = \pi \sqrt{\frac{l}{g}} \text{ and } t' = \pi \sqrt{\frac{l'}{g}}; \quad (9)$$

whence, by division,

$$\frac{t}{t'} = \pi \sqrt{\frac{l}{g}} \div \pi \sqrt{\frac{l'}{g}}, \quad (10)$$

or,

$$\frac{t}{t'} = \sqrt{\frac{l}{l'}}. \quad (11)$$

the algebraic form of the law showing the relation between the period and the length.

If a pendulum with a period t at a locality where the acceleration of gravity is g is taken without change of length to a locality where the acceleration is g' and the period there found to be t' , then, as

$$t = \pi \sqrt{\frac{l}{g}} \text{ and } t' = \pi \sqrt{\frac{l}{g'}}, \quad (12)$$

$$\frac{t}{t'} = \sqrt{\frac{g'}{g}}, \quad (13)$$

the algebraic form of the law showing the relation between the period and the acceleration due to gravity.

It is to be noted that neither the mass nor the amplitude figures in the formulas. In this way is shown their lack of influence upon the period.

188. The Compound Pendulum.— Every actual pendulum is compound; since all its particles are subject to gravity, each one would act like a simple pendulum if it were sundered from its fellow particles. The particles near the lower part of the compound pendulum would form longer simple pendulums than those near the upper end; the former would therefore vibrate more slowly than the latter. But there must be some intermediate particle which would form a simple pendulum having a period equal to that of the undivided compound pendulum. This particle is at the center of oscillation.

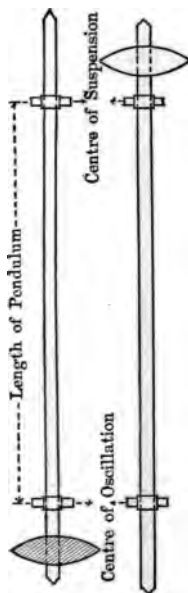


FIG. 100. — Reversion Pendulum.

189. Center of Oscillation.— The location of the center of oscillation may be found in a swinging body by applying the law discovered by Huygens in 1673 as to the reversibility of the centers of suspension and oscillation. In the *reversion pendulum* invented by Kater in 1818 there are two adjustable knife edges about either of which the pendulum can be made to swing. The pendulum is suspended on either knife edge and its period determined (Fig. 100). It is then turned upside down, supported on the other knife edge, and the period again determined. The knife edges are shifted on the bar until the two periods are the same. The distance between the knife edges is then the true length of the pendulum, the center of suspension being at one knife edge and the center of oscillation at the other.

The approximate position of the center of oscillation may be ascertained by suspending a nearly

simple pendulum (bullet attached to a fine thread) to the axis of suspension of any compound pendulum, and adjusting the length of the simple pendulum until it swings in the same time as the compound pendulum. A point on a level with the center of the bullet is approximately the center of oscillation, and if this be made the center of suspension, the period will be the same as before.

190. Center of Percussion. — The center of percussion of a body is that point in it where a blow has the least effect as observed at the center of suspension. It occupies the same position in the body as the center of oscillation. When, in striking a ball, the ball comes into contact with the center of percussion of the bat, the ball is sent farther and the hands feel less of a shock or stinging than is the case when the contact between ball and bat is not at the center of percussion.

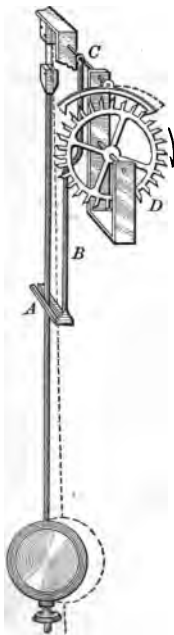


FIG. 101. — A Clock Pendulum.

191. The Clock Pendulum. — The rod extends between the prongs of a fork *A* (Fig. 101) in such a way as to make the rod *B* swing with it. At the end of the horizontal axis of *B* is a curved strip, the *escapement*, having a projection at each end. As the pendulum swings to and fro, these projections catch alternately in the teeth of the *escapement wheel D*, which is the last of a train of wheels that are driven by the clock's weight or spring. The escapement allows only one of the wheel's teeth to pass by it during each double swing of the pendulum. The clock's wheels and hands are thereby made to move through equal distances in equal times. The projections and teeth are so shaped that at each swing a slight impulse is given to the pendulum, which is sufficient to keep up its motion.

192. Metronome. — The pendulum in this instrument has its bob above the axis of suspension (Fig. 102). The bob slides over the rod, which is marked in divisions corresponding to vibration rates of from 40 to 208 per minute. The clockworks which drive the pendulum make a loud tick at each vibration, the sense of hearing as well as of sight being thus appealed to. The

metronome is designed to indicate the time according to which a musical composition is to be played. Thus, if ($\text{♩} = 60$) is placed above the first bar of the composition, 60 quarter notes or their equivalent are to be played per minute.

193. Determination of the Acceleration due to Gravity.—The pendulum formula may be written in this way :

$$g = \pi^2 \frac{l}{t^2}. \quad (14)$$

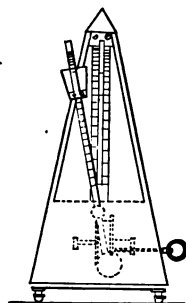


FIG. 102. — A Metronome.

As the length of a pendulum and its period can be measured with great accuracy, the value of g at different localities can be best determined by means of a pendulum. If g is known at any place, the length of a second's pendulum there is found from the equation

$$t = 1 = \pi \sqrt{\frac{l}{g}}; \quad (15)$$

whence

$$l = \frac{g}{\pi^2}. \quad (16)$$

EXERCISES

1. A body with a mass of 100 g. is whirled in a circle of 20 cm. radius by means of a cord which breaks under a stress of 8000 g. What velocity must be imparted to the body in order to break the cord ?
2. If a ball with a mass of 16 g. is whirled in a circle by a cord 40 cm. long, at the rate of $50 \frac{\text{cm.}}{\text{sec.}}$, what is the central force in (a) grams ? (b) dynes ?
3. With the aid of a diagram resolve the force of gravity acting upon a pendulum into two components, one in the direction of the cord, and the other in the direction of the motion. State the effect of each component.
4. If a pendulum 30 cm. long vibrates three times as fast as another, what is the length of the other pendulum ?

5. A boy in a tree wishes to find out how high he is above the ground. Tying his knife to a string, he makes it vibrate as a pendulum bob near the ground. If the value of g is $32 \frac{\text{ft.}}{\text{sec.}^2}$, what is the length of the string when it takes 4.4 sec. for the stone to make a single vibration?

6. A simple pendulum is pulled aside until its bob is raised 4 in., and then let go. What is its velocity when it passes through the lowest point of its arc, if $g = 32 \frac{\text{ft.}}{\text{sec.}^2}$?

7. If the pendulum of a clock which loses 30 sec. a day is 25 cm. long, what change must be made in its length in order to make the clock keep correct time?

8. Richer, in 1672, found that a clock, the pendulum of which beat seconds at Paris, lost 2.5 min. daily at Cayenne, French Guiana. If the value of g at Paris is $981 \frac{\text{cm.}}{\text{sec.}^2}$, what is its value at Cayenne?

9. If the pendulum of a clock vibrates 3600 times an hour at Chicago, where $g = 980.264 \frac{\text{cm.}}{\text{sec.}^2}$, how many times an hour will it vibrate if taken to Denver, where $g = 979.595 \frac{\text{cm.}}{\text{sec.}^2}$?

10. What is the length of a pendulum beating seconds at Chicago? at Denver? ($\pi = 3.14159$.)

CHAPTER IX

WORK, ENERGY, AND MACHINES

WORK AND ACTIVITY

194. Work. — To drive a nail into wood, to pump water, to push a cart—in short, in every physical action that involves movement work must be done. Before any motion will take place, sufficient force must be exerted to overcome any opposing force. In driving the nail, the force that holds the particles of wood together must be overcome; in pumping the water, the weight of the liquid must be overcome; and in pushing the cart, the friction of the wheels must be overcome. The general name of *resistance* is given to all such opposing forces, and work may be defined as the *overcoming of resistance*. Work is done only when a force actually produces motion, and is measured as the product of a force by the distance through which it acts. The defining equation for work is

$$\begin{aligned} \text{Work} &= \text{Force} \times \text{Distance}, \\ W &= F \times L. \end{aligned} \tag{1}$$

195. Units of Work. — As work is the product of a force by a distance, the unit of work is the product of unit force by unit distance. The physicist's unit of force and of distance being the dyne and the centimeter, his unit of work is the amount of work done when a dyne acts through a distance of one centimeter. Instead of calling this unit a *dyne centimeter*, it has been given the special name of *erg*,

which is merely the Anglicized form of the Greek word for work. As this unit is very small, another unit ten million times greater, called the *joule*, is frequently employed.

As the dyne is equal to $\frac{1}{1000000}$ of a gram, the erg may also be defined as the work done when $\frac{1}{1000000}$ of a gram is raised through the vertical height of one centimeter. Indeed, so much of practical work consists in moving bodies upward against gravity, that weight or gravity units are more frequently used than absolute units. The English-speaking engineer uses almost exclusively a unit of work based upon the foot and the pound. This is called the *foot pound*, and is defined to be the work done in raising one pound one foot high. Engineers in metric countries make use of a gravity unit based upon the kilogram and the meter, their *kilogrammeter* being the amount of work needed to elevate one kilogram one meter. These engineering units, while convenient, change in value from place to place because of the variations in the intensity of gravity. Inasmuch as forces acting in any direction may be measured in pounds or kilograms, so may foot pounds or kilogrammeters be used to measure work done in any direction.

If the products of different forces causing motion and the respective distances through which they act are equal, the same amount of work is done, no matter how the magnitudes of the forces and distances vary. Thus, 100 lb. raised 1 ft., 1 lb. raised 100 ft., 50 lb. raised 2 ft., 2 lb. raised 50 ft., 4 lb. raised 25 ft., 20 lb. raised 5 ft., etc., all represent equal amounts of work. In general, then, to lift m lb. h ft. requires mh foot pounds of work.

196. Activity or Power. — The amount of work done is independent of the time taken to do it. Thus it requires a certain definite amount of work to erect a building, which is the same, however long the building is in course of erection. But the wages of the workmen, however, are paid

usually by the day or hour, and are regulated mainly by the average amount of work a man can do in a day. The more active or powerful a man, the more work he can do in a given time. Hence, in general, *activity or power is the ratio of the work done to the time taken*; it is the *time rate of doing work*.

197. Units of Activity. — The unit of activity is the unit of work done in the unit of time. When men of science and affairs in the latter part of the eighteenth century began to experience the desirability of having units in terms of which to compare the activities of various agencies, the horse was the chief source of power. As the result of a series of experiments, James Watt found that on an average a horse could do 33,000 foot pounds a minute (550 foot pounds per second). This rate of doing work is called a *horse power* (H.P.) and is in common use by American and British engineers. Any source of power may be rated as so many horse power, as a steam engine of 100 H.P.

In the C. G. S. system the unit of activity is an erg per second. As this unit is for most purposes inconveniently small, a ten million times larger one has been adopted, and has been called the *watt*. A horse power is equal to 746 watts. Electrical engineers also employ a unit a thousand times larger than the watt, calling it the *kilowatt*.

EXERCISES

1. How much work is done in kicking a football weighing 14 oz. to a height of 32 ft.?
2. A man is walking against a wind exerting a pressure of $30 \frac{\text{lb.}}{\text{ft.}^2}$. If the surface of his body exposed to the wind is 5 ft.², how many foot pounds of work will he do in walking a mile against the wind?
3. How much work is done when a force of 100 dynes acts through 25 cm.?

4. If 75 ergs of work were done in moving a body 15 cm., what force was applied?

5. If a force of 50 dynes was applied to a body and 200 ergs of work were done, through what distance did the body move?

6. A block of granite, $60 \times 180 \times 300$ cm., is to be raised 900 cm. If the density of granite is 2.65, how many kilogrammeters of work must be done?

7. A steel rail of uniform cross section and homogeneous structure is 20 ft. long and weighs 100 lb. How much work is required to lift it from a horizontal to a vertical position, one end being kept always upon the ground?

8. If a carpenter exerts upon a saw a force equivalent to 30 lb. downward and 40 lb. forward, how many foot pounds of work does he do in 20 forward strokes of the saw, each stroke being 18 in. long?

9. A laborer weighing 150 lb. carries a hod and mortar weighing 75 lb. up a ladder 25 ft. high once every 20 min. How much work does he do in 8 hr.?

10. What horse power must an engine have to raise 10^6 lb., 10^8 ft. in 1 hr.?

11. If a boy weighing 110 lb. runs upstairs at such a rate as to gain 5 ft. in vertical height every second, what is his activity?

12. If a man weighing 89 kg. lifts his body 5 cm. with every step, what is his activity, if his steps are each 75 cm. long, and he walks at the rate of $6 \frac{\text{km.}}{\text{hr.}}$?

ENERGY

198. Energy. — When a force acts upon a body, some effect must of necessity be produced; the body is either moved or undergoes a change in size or shape. Work is done upon the body, and consequently the body may itself be put into a condition to do work. Thus work is done in raising a body, and the body, if allowed to fall, does work. Work also is done in winding a spring, and the wound-up spring is capable of doing work when it unwinds.

The work done upon a body may be, so to say, stored up in the body. Such stored-up work is called *energy*, this name being derived from the Greek word meaning *work within*. A body possessing energy has the power of doing

work; energy may therefore be defined as the *ability to do work*.

199. Units of Energy. — As energy expended is nothing but work done, the units for measuring energy are the same as those for measuring work. Thus we may speak of foot pounds or ergs of energy with the same propriety as of foot pounds or ergs of work.

200. Forms of Energy. — When work is done by a force acting through a distance, *mechanical energy* is said to be expended. We shall learn later that work may also be done by the expenditure of other forms of energy, such as *heat*, *electrical*, and *light energy*. As all these forms of energy can more or less perfectly be converted into mechanical energy, one set of units serves to measure them all.

201. Conservation of Energy. — Whenever work is done, one body or system of bodies loses energy, while another gains it; work consists in a transfer of energy. If strict account is kept of all the energy that is transferred when a definite amount of work is done, it is always found that as much energy is gained by one system as is lost by another; there is neither creation nor annihilation of energy. Energy cannot be created nor destroyed by any agency under the control of man; the total amount of energy in the universe remains constant or is conserved. All that man and the forces of nature can do is to transfer or transform energy.*

* "Energy is protean in form, intangible, yet measurable in magnitude, and all its changes are by definite equivalent amount and value. There is a most rigid system of bookkeeping in the transactions of the physical universe. You may have anything you like in the way of energy served out to you, but the amount of it is debited to your account immediately, and the bill has to be discharged by paying an equivalent in some other form of energy before you can remove the goods from the counter." — FLEMING.

202. Types of Energy.—When work is done upon a body, it is either moved or strained. A moving body or one that is strained therefore possesses energy. A body may be moving and not exert any force; but if it encounters a resistance, work is done in overcoming this. Or, a body may be at rest and merely exert a pressure upon another body that is supporting it; but if the supporting body be taken away, the first body will move and do work. The energy that a body possesses because of its motion is called its *kinetic energy*; that which it possesses because of its position or its strained condition is called its *potential energy*, or *energy of stress*, since stress is the cause of strain. Whenever work is done, there is an incessant interchange of these two types of energy. Consider the case of a stone thrown vertically upward. When it leaves the hand, its energy is of the kinetic or moving type; but as it rises, it moves more and more slowly and its kinetic energy decreases continuously and becomes nothing at the instant the stone begins to fall. As the stone rises, however, its potential energy is on the increase and attains its maximum value when the stone is at its highest position. The total energy possessed by the stone at any time is equal to the sum of its kinetic and potential energies. Another example of the change in type of energy is the motion of a pendulum. A pendulum loses kinetic but gains potential energy as it swings out to the end of its path; and as it swings back, it loses potential and gains kinetic energy. At either end of its path it is farthest from the earth and has zero velocity; and its potential energy is at a maximum. At the middle of its path the bob is nearest the earth and has its greatest velocity; its kinetic energy is therefore at its maximum value and its potential at its minimum.

203. Measure of Potential Energy. — For the present we shall limit our discussions of potential energy to the form exhibited by the stress between the earth and an elevated body. To raise a body, work must be done upon it, and to keep a raised body stationary, a support strong enough to withstand the pressure (weight) of the body must be provided. If the support is removed, the body falls. Letting mg denote the weight of the body and l the vertical distance through which it could fall, then $mg l$ measures the amount of energy due to the relative positions of the body and the earth. Its value is equal to the amount of work expended upon the body to raise it, and to the amount of work that can be done by the body when it falls back to its original position.

204. Measure of Kinetic Energy. — If a body has the potential energy $mg l$ because of its having been raised through the distance l , and if it is then allowed to fall, the potential energy will change into kinetic energy. When the body arrives at the lower limit of the distance, its potential energy will have all changed into kinetic energy, and it will be moving with the velocity v . From the relation between the velocity and the distance of a freely falling body (Equation (4), p. 107) we have

$$l = \frac{v^2}{2g}; \quad (2)$$

and, since the kinetic energy $K.E.$ is equal to the potential energy $P.E.$,

$$K.E. = P.E. = mg l = mg \frac{v^2}{2g} = \frac{1}{2} m v^2. \quad (3)$$

The kinetic energy of a moving body is equal to half the product of its mass by the square of its velocity. The amount

of energy must be expressed in ergs, if the C. G. S. system is used; for the formula does not contain any term or quantity the value of which depends upon gravity. If gravity units are to be used, since $m = \frac{w}{g}$ (§ 166), the formula becomes

$$K.E. = \frac{wv^2}{2g} \quad (3a)$$

The erg may, according to Equation (3), be given a new definition: it is *half the energy of a gram mass moving with a velocity of a centimeter a second.*

205. Kinetic Energy Formulas. — The formulas for kinetic energy enable us to compute from measurements of mass (or weight) and velocity, the number of units of work that have been expended upon a body to set it in motion, and also the amount of work that must be done upon the moving body in the opposite direction in order to stop its motion. When one ball of perfectly elastic material collides with another ball of equal mass and similar material, the first stops, its motion being communicated to the second; but the energy of the first ball has likewise been transferred to the second. Whenever there is a change in the velocity of a moving body, there must be a transference of energy.

206. Dissipation of Energy. — Amounts of energy are always relative. No body can be absolutely devoid of energy. Thus a stone falling to the ground loses some of its potential energy, but not all; for if the ground were removed from beneath it, the stone would fall still farther. But the energy the stone possesses when lying on the ground confers upon it no practical power of doing work; for it cannot fall farther unless a great deal more work is done in digging a hole beneath it. Even if the body should fall to the center of the earth, where its potential energy with respect to the earth would be reduced to nothing, it would still have some potential energy with respect to the

sun and other heavenly bodies. A body may then have potential energy, and yet not be able to do work of any value; *its energy is not available.*

Whenever energy is transferred from one body to another, some of the energy seems to be lost. But this is only apparent, for a certain fraction of it is converted into heat energy. With every transfer of energy there is a decrease in availability. This general tendency of energy to assume less available forms is called the *dissipation of energy*. Energy as a whole becomes less and less available; if the availability of a certain amount is raised, that of a greater amount is lowered. Potential energy tends to change into molecular kinetic energy, *i.e.* heat, which can never be made to give back the full amount of potential energy from which it sprang. The main cause for the dissipation of energy is the ever present force of friction.

EXERCISES

1. Why can a stone be thrown much farther than a cork of the same size?
2. If the speed of a bullet is such that it can just pierce a plank, how many such planks can it pierce if its speed is doubled? trebled?
3. How much kinetic energy must be imparted to a body with a mass of 50 g. in order to make it rise vertically for 5 sec.?
4. From what height (in centimeters) must a body fall ($g = 980 \frac{\text{cm.}}{\text{sec.}^2}$) in order to have, when it reaches the ground, the same velocity as a body with a mass of 20 g. and a kinetic energy of 196×10^3 ergs?
5. By how much is the potential energy of a body changed, if its mass is 100 g., and it is raised 50 cm. at a place where the acceleration due to gravity is $980 \frac{\text{cm.}}{\text{sec.}^2}$?

FRICTION

207. Friction. — A body moving in a medium, such as air or water, has to displace the medium as it moves; and the motion is opposed by the force of friction. A solid rolling or sliding over another encounters a resistance —

the opposing force of friction. Friction tends to retard the motion of a body, and to transform its energy into less available forms, notably heat. The amount of work accomplished in overcoming friction is equal to the product of the frictional force and the distance through which it acts. To avoid complicating matters, we have assumed, in most of the foregoing discussions on the motions of bodies, that the force of friction may be made so slight as to be negligible. In all actual motions, however, the force of friction, no matter how small, cannot be ignored, and in many of them it plays an important rôle.

208. Sliding Friction. — It is impossible to render any surface perfectly smooth; the smoothest surface that can be made, when viewed through a microscope, shows tiny projections and cavities. When bodies are placed in contact, the ridges and depressions of the surface of one fit into those of the other, so that more or less force is required to make the surfaces slide past each other. If certain substances, such as oil or graphite, are placed between the sliding surfaces, the depressions are in a measure filled up with the lubricant, and the force of friction is thereby diminished. The inequalities in the surfaces of various substances may be of different orders of magnitude so that the ridges of one do not fit so snugly into the depressions of another, as in the case of surfaces of the same substance. Hence the sliding friction between the surfaces of two unlike substances may be less than that between the surfaces of the same substance. Steel shafts are found to turn with less friction in brass or Babbitt metal journals than in steel ones.

209. Coefficient of Friction. — Let a smooth block of wood be drawn along a horizontal board, and let the

drawing force be measured by a spring balance. The pressure of the block down upon the board is equal to its weight together with that of any load that it bears. It is observed that the force required to start the block to sliding is greater than that needed to keep it in motion with uniform speed. The ratio of the force needed to start the motion to the pressure between the two surfaces is called the *coefficient of starting friction*, and the ratio of the force required to maintain uniform motion to the pressure is the *coefficient of sliding friction*. Thus, if a force of 50 g. is required to start a weight of 100 g. sliding on a surface, and but 40 g. are required to keep it in motion, the coefficient of starting friction is $\frac{50}{100} = .5$, and that of sliding friction is $\frac{40}{100} = .4$. The reason why the friction in starting the motion is greater than the friction during motion is, perhaps, the following. When the surfaces are relatively at rest, their ridges and depressions have time to fit more or less snugly into one another, while, when the surfaces are moving past each other, the ridges jump, so to speak, over the depressions, the motion having much the same effect as a lubricant.

210. Laws of Sliding Friction.—It has been demonstrated experimentally that the value of starting and of sliding friction (1) depends upon the nature of the substances in contact; (2) varies directly as the pressure; (3) is independent of the rate of motion, provided the differences in the rates are not too great; (4) is independent of the area of contact, provided that the differences are not too large; (5) is lessened by smoothing the surfaces as well as by smearing them with lubricants.

211. Friction in Fluids.—When solids move through fluids, such as air or water, the resistance they encounter is found to vary approxi-

mately as the square of the speed; but for very high speeds, the rate of increase of frictional force is much greater. The resistance that air opposes to the movement of a train is small in comparison with the other resistances, such as the friction of the bearings and wheels, so long as the train moves slowly; but when the train's speed becomes great, the air resistance becomes much greater than the other resistances. This is one of the chief reasons why it is so expensive to run a fast train. In the case of power boats running at high speeds, the power required is found to increase as the cube of the speed, so that, to double the speed of a boat, its power must be made eight times as great.

212. Uses of Friction.—Friction has its advantages as well as disadvantages. Without friction, the wheels of self-propelled vehicles would not move forward but would merely turn round in the same place; brakes would have no effect; walking would be impossible; nails would not hold; and so on. A host of other instances could be cited.

The main disadvantage of friction is felt in the use of machinery, where more or less of the energy put into the machine has to be used up in overcoming the resistance offered by the moving parts of the machine. No inconsiderable proportion of the energy delivered to a machine is employed in merely keeping the machine in motion, and is lost for doing useful work.

EXERCISES

1. Mention some instances where friction is to our advantage.
2. Two horses can pull a load of 5 tons on a sled. How many horses are needed to pull 10 tons on the same sled? How many if the runners are made twice as wide?
3. A brick weighing 6 lb. did not begin to move on a table top until a force of 3 lb. had been applied to it, but it required only 2 lb. of force to keep it moving uniformly. What was the coefficient of starting and of moving friction?
4. If the coefficient of friction between two cast iron surfaces is .15, what force is needed to move the surfaces past each other, when the pressure is 50 lb.?

MACHINES

213. Definition of Machine.— A machine is a device so arranged as to furnish work from energy derived from some external source. By means of a machine forces applied at one place may be exerted at another with certain changes in direction, distance, or magnitude. The word *machine* is commonly restricted to such contrivances as serve to transfer mechanical energy. Other terms, such as steam engine, gas engine, dynamo, and motor, are applied to combinations of machinery that transform heat or electrical energy into mechanical energy or vice versa. Such machines will be described later; this section is restricted to machines transferring mechanical energy.

214. Advantages of Machines.— A machine may be regarded as a device for the advantageous application of force to resistance. A large force may be applied to one part of a machine and, while moving through a small distance, or slowly, cause another part of the machine to move through a great distance, or rapidly, while exerting a small force. Thus, in winding a clock, a considerable force is required, but the distance through which it acts is small. The hands of the clock are thereby made to move over long distances, while the force required to move them is slight. Another advantage is that a force may be made to change its direction. This also is illustrated by clock-works, or more simply by a fixed pulley. A third great advantage of machines is that use may be made of the forces exerted by animals, wind, water, and steam.

215. Mechanical Advantage.— The force that makes a machine go is called the *effort* or *driving force*; that which is exerted by the machine, the *resistance* or *load*. The

effort is applied to the machine, the resistance is derived from it. The ratio of the distance through which the effort moves to the distance through which the resistance moves in the same time is the *velocity* or *speed ratio*. The ratio of the magnitude of the resistance to the magnitude of the effort is the *mechanical advantage*; it represents the advantage to be gained from the use of the machine.

216. Efficiency. — A machine transmits and often transforms energy. Energy, definite in kind and quantity, is delivered to a machine; some of it is used in overcoming the friction of the moving parts; the rest is often given out by the machine in quite a different form. For example, the mechanical energy due to the translatory motion of a waterfall may be changed by a waterwheel having a rotatory motion into another form of mechanical energy, which then by the agency of a dynamo may be converted into electrical energy, and the electrical energy transformed into light energy by means of electric lamps, or into heat energy by means of electric stoves.

The energy used in making one part of a machine move with respect to another is looked upon as wasted, and in the construction of most machines much attention is devoted to devices intended to reduce this waste. A perfect machine would be one in which no wasted work would be possible, and as a machine approaches the more nearly to perfection the less is the work wasted. The ratio of the utilizable work obtained from a machine to the total amount of energy given to it is called its *efficiency*. A perfect machine would have an efficiency of unity, or of 100% (efficiencies being usually expressed in terms of percentages). If, for instance, 10,000 ergs of energy were delivered to a machine, and 5000 ergs obtained from it, it is

manifest that the other 5000 ergs were used up in making the machine go. The efficiency of such a machine is therefore $\frac{5000}{10000} = \frac{1}{2} = 50\%$. To simplify problems, we shall assume in our discussions of machines, unless otherwise stated, that their efficiency is 100%; or, in other words, that there are no losses due to friction.

217. Law of Machines. — The energy put into a machine is equal to the work done upon the machine merely to make it go, added to the work obtained from the machine and available for the purpose in hand. But work and energy are measured by the product of a force and a distance. If, then, F denotes the force applied to a machine, l , the distance through which it acts, r , the work done to overcome friction, R , the force exerted by the machine, and l' , the distance through which it acts, we have

$$Fl = Rl' + r. \quad (4)$$

For a perfect machine, r reduces to zero, and

$$Fl = Rl'. \quad (5)$$

The product of the effort by the distance through which it acts equals the product of the resistance by the distance through which it acts. Inasmuch as the distances traveled by the effort and by the resistance in the same time give their respective speeds, it follows that *the effort multiplied by its speed equals the resistance multiplied by its speed.*

From Equation (5) we have

$$\frac{F}{R} = \frac{l'}{l}; \quad (6)$$

that is, *the ratio of the effort to the resistance is the same as the ratio of the distance traversed by the resistance to that traversed by the effort.* And by inversion,

$$\frac{R}{F} = \frac{l}{l'}. \quad (7)$$

But $\frac{R}{F}$ is the mechanical advantage, and $\frac{l}{l'}$ is the velocity ratio; hence *the mechanical advantage is numerically equal to the velocity ratio.*

218. Perpetual Motion. — The amount of energy received by one system of bodies must always equal the amount lost by another system; no increase in energy can ever be effected by a machine. Yet innumerable attempts have been made to invent a machine that, when furnished with a definite quantity of energy, would go on forever giving it out. Such "perpetual motion machines" have always resulted in failure, as their would-be inventors have not fully recognized the fact that all one can do by means of a machine is to vary the two factors, force and distance, of a given amount of energy.

219. The Lever as a Machine. — The lever in connection with parallel forces has already been considered (§ 126). There, however, the forces were regarded as in equilibrium



FIG. 103. — The Lever as a Machine.

and stationary; now they should be regarded as moving and thereby doing work.

Suppose that while a driving force F is acting at the distance l from the fulcrum (Fig. 103), it moves along the arc (ab) , and causes the resisting force R at the distance l' to move at the same time along the arc (cd) . Since the vertical angles at O are equal, the lengths of the arcs are proportional to the lengths of the radii (the arms of the lever), so that

$$\frac{l}{l'} = \frac{(ab)}{(cd)}. \quad (8)$$

Now the work done by the force F , acting through the distance (ab) may be represented by $F(ab)$, and that done by R , acting through the distance (cd) , by $R(cd)$. Assuming that no work is required to overcome friction, we have by the law of machines:

$$F(ab) = R(cd); \quad (9)$$

whence

$$\frac{R}{F} = \frac{(ab)}{(cd)} = \frac{l}{l'}. \quad (10)$$

As $\frac{R}{F}$ is the mechanical advantage, its value may be calculated for any lever from measurements of the lengths of its arms. The law of the lever (§ 126) therefore may also be stated as follows: *A given effort can balance a resistance as many times greater than itself as the effort arm is times longer than the resistance arm.**

220. Classes of Levers.—It is a time-honored custom to divide levers into three classes, the basis of classification being the relative positions of fulcrum, effort, and resistance. When the fulcrum is between the effort and the resistance, the lever is of the *first class*; when the resistance is between the effort and the fulcrum, it is of the *second class*; and when the effort is between the fulcrum and the resistance, it is of the *third class*.

221. The Wheel and Axle.—The motion of the lever, as commonly used, is limited. A crowbar, for instance, cannot move an object very far before its effort strikes the ground. The fulcrum may indeed be raised after a movement of the lever, and a load thus lifted by a succession of intermittent actions. But when a continuous motion is desired, the wheel and axle may be substituted for the lever.

* To make the lever do work the effort must, of course, be greater than that needed merely to hold the resistance in equilibrium.

The wheel and axle consists of two cylinders of different diameters fastened together and rotating on a common axis (Fig. 104). The larger cylinder

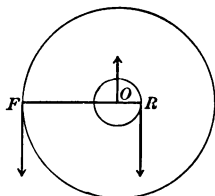


FIG. 104.—The Wheel and Axle.

is called the *wheel*, and the smaller, the *axle*. The entire wheel is not always used, the effort sometimes being applied to one or more spokes of it. Thus the windlass (Fig. 105) has a crank which forms a single spoke of the wheel, the distance between the axis of the crank handle and the axis of rota-

tion being the length of the wheel's radius.

222. The Wheel and Axle, a Modified Lever.—The fulcrum is at the axis O (Fig. 104); the effort F and the resistance R are applied at the circumferences of the wheel and axle, respectively. The effort arm and the resistance arm are the radii of the wheel and axle, respectively. The me-



FIG. 105.—One Form of Windlass.

chanical advantage of the wheel and axle is manifestly the same as that of the lever of which it is a modification. As the ratio of the circumferences or diameters of circles is the same as that of their radii, it follows that *a given effort applied on the wheel can support a load on the axle as many times greater than itself as the radius, diameter, or circumference of the wheel is times greater than the corresponding dimension of the axle.*

223. The Pulley.—The pulley consists of a wheel, called a *sheave*, with an axle supported in a *block*, and a

cord passing over its grooved rim. To the ends of this cord are attached the effort and the resistance. The cord is the essential thing; for it is what is *pulled*, the purpose of the sheave being to diminish friction due to the bearings and the stiffness of the rope. With a frictionless and perfectly flexible cord a sheave would not be necessary. The cord must, of course, aside from the effects of friction and stiffness, be under the same tension in all its parts.

224. Fixed Pulleys.—A fixed pulley is one in which the block remains stationary when the cord is pulled and the sheave turns (Fig. 106). The load moves as far as does the driving force in the same interval of time; and the magnitudes of effort and resistance are equal. No mechanical advantage

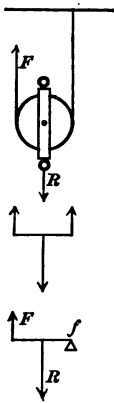


FIG. 107. —
Single Movable Pulley.

accrues from the use of fixed pulleys, but they are convenient to use in changing the direction of forces. A fixed pulley may be looked upon as an equal-armed lever of the first class, the fulcrum being at the axis, and the effort and resistance being applied at opposite sides of the sheave.

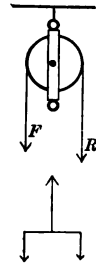


FIG. 106. —
A Fixed Pulley.

225. Movable Pulleys.—With the load attached to the block, both load and block are moved when one end of the cord passing round the sheave is pulled, the other end being attached to a rigid support. With a single movable pulley (Fig. 107), the block, and hence also the load, moves just half as far and as fast as the driving force. With every sheave added to a movable block, the magnitude of the effort and that of the distance

traveled by the resistance is halved. In the commonest form of the pulley the cord passes round each of the movable sheaves and an equal number of fixed sheaves (to

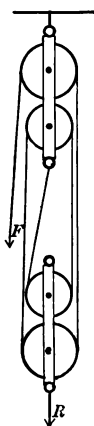


FIG. 108.—
Block and
Tackle.

change the direction), and thus connects the movable and the fixed blocks by two parts or strands of the rope for every sheave in the movable block. Suppose that four strands pass between the fixed and movable blocks (Fig. 108). When the resistance R attached to the movable block is moved through the distance s , the distance between the blocks is diminished by s ; and since four parts of the rope pass between the blocks, the whole length of the cord which is between the blocks is shortened by a distance equal to $4s$. This is also the length of the rope that is pulled

through the fixed block by the driving force F . The work done on the system is therefore $F \times 4s$, which is equal to Rs . The mechanical advantage of such pulley systems depends then upon the number of sheaves in the movable block, each movable sheave doubling it; or, as there are two strands of the rope for each movable sheave,* *a given effort can balance a resistance as many times greater than itself as there are parts of the rope passing between the fixed and movable blocks.*

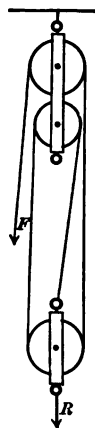


FIG. 109.—Double
Fixed and Sin-
gle Movable
Pulley.

* If the resistance end of the rope is attached to the movable block (Fig. 109), the parts of the cord are one more than twice the number of the movable sheaves; if it is attached to the fixed block, the number of strands is twice the number of movable sheaves.

226. A Movable Pulley, a Modified Lever. — A movable pulley may be regarded as a lever of the second class, the fulcrum being at the point where the part of the rope supporting the load comes in contact with the wheel on the side opposite the driving force, and the load being suspended from the axis (Fig. 107). The effort arm is twice as long as the resistance arm, thus giving a mechanical advantage of 2.

227. The Differential Pulley. — A differential pulley consists of a fixed block with two sheaves of unequal size, and an endless chain passing round the two sheaves and round a movable pulley (Fig. 110). When a force F causes the fixed sheaves to make one whole turn, the chain between the upper and lower blocks shortens by a length equal to the difference between the circumferences of the sheaves of the fixed block; for the chain has been pulled up a distance equal to the circumference of the larger sheave and let down a distance equal to the circumference of the smaller sheave. The movable block and its attached load have therefore been moved through a distance equal to half (*half* because of the movable pulley) the difference between the circumferences of the fixed sheaves. The mechanical advantage, being equal to the ratio of the distances traversed by the driving force and by the resistance, is therefore found by dividing the circumference of the larger fixed sheave by half the difference between the circumferences of the fixed sheaves. As this difference can be made slight, a small downward force may be made to raise very great loads.

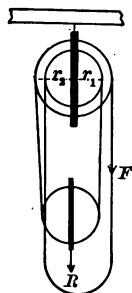


FIG. 110. — The Differential Pulley.

EXERCISES

1. What is the location of effort, resistance, and fulcrum in the following applications of the lever: door? rudder of a boat? spade? tongs? beam balance? wheelbarrow? nutcracker?
2. Why do tinner's shears (snips) have the handles longer than the blades, while tailor's shears have the blades longer than the handles?
3. What effect would the placing of the axle of a movable pulley not at the center of the sheave have upon its action?
4. How does the mechanical advantage of a system of pulleys, arranged as shown in Fig. 111, compare with the number of fixed and movable sheaves?

5. What is the relationship between the magnitudes of effort and resistance in the system of pulleys shown in Fig. 112?

6. If the handles of a wirecutter (Fig. 113) are 24 cm. long and the wire is placed 1.2 cm. from the pin or bolt, what force must be exerted to cut a wire offering a resistance of 50 lb.?

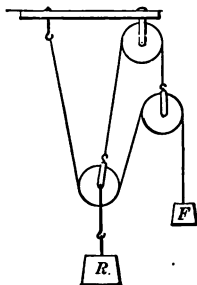


FIG. 111.

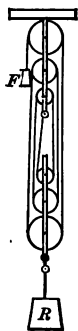


FIG. 112.



FIG. 113.



FIG. 114.

7. If the handle of a claw hammer (Fig. 114) is 30 cm. long and the claw is 3 cm. long, what effort must be applied at the end of the handle in order to pull out a nail that requires a force equal to 20 kg. to start it?

8. A rigid bar 9 ft. long is supported at its ends *A* and *B*, and a weight of 90 lb. is suspended at a point 4 ft. from the end *B*. Disregarding the weight of the bar itself, what is the pressure on each support?

9. Three men are carrying a uniform stick of timber 24 ft. long. One man holds one extremity, and the other two hold the ends of a crossbar, the middle of which is under the stick. How far from the end of the timber should the crossbar be, in order that the men may all carry equal loads?

10. A uniform bar 10 cm. long weighs 40 g. If a weight of 60 g. is hung from one extremity of the bar, what should be the position of the fulcrum in order to make the lever balance?

11. A uniform bar is 300 cm. long, and each centimeter of its length weighs 50 g. It rests upon a horizontal table top with one end to which is attached a load of 500 g. projecting. How far can the bar project over the table before it begins to turn on the edge of the table as an axis?

12. If the axle of a windlass is 8 in. in diameter and it is worked by a crank 2 ft. long, what force on the crank can support a weight of 300 lb. on the axle?

13. The sprocket wheel of a bicycle has 30 teeth and the hub of the rear wheel has 10 teeth. How far does the bicycle go with one revolution of the pedals if the rear wheel is 28 in. in diameter? How many turns will the pedals make while the bicycle is going a mile?

14. The diameter of a wheel is 100 cm. and that of the axle is 20 cm. To move 500 g. attached to the axle, 160 g. has to be applied to the wheel. What is the efficiency of this wheel and axle?

15. If the diameter of the steering wheel of a ship is 60 in., and the diameter of the axle is 15 in., what is the greatest resistance of the rudder that a man applying a force of 150 lb. to the wheel can overcome?

16. What load can a man weighing 160 lb. hold up by means of a system of two fixed and two movable pulleys, if the friction is equivalent to an addition of one fourth to the load?

17. In a system of four movable pulleys with the weight end of the rope attached to the fixed block, a force of 100 lb. is required to raise a weight of 700 lb. What is the efficiency of this system?

18. Sketch a system of five movable pulleys and the needed number of fixed pulleys with one end of the rope attached (*a*) to the fixed block; (*b*) attached to the movable block. What effort is required to raise 1000 g. in each case?

19. What effort has to be applied in order to hold in equilibrium a weight of 100 lb., when (*a*) one, (*b*) two, (*c*) three, (*d*) four, and (*e*) five movable pulleys together with the requisite number of fixed pulleys and a single cord are employed? Sketch the arrangements of the pulleys.

228. The Inclined Plane as a Machine. — The resolution of a weight supported upon an inclined plane into two components, one parallel to the plane and the other perpendicular to it, has already been considered (§ 120). The inclined plane is a machine, for the reason that it supports a part of the weight bearing down upon it, while the driving force supports the rest. The work done in moving without friction a body up an inclined plane is equal to the weight of the body multiplied by the height of the plane. The effort applied is smaller than the resistance, but has to move through a proportionately longer distance; the

steeper the plane, the greater will be the effort. Let R represent the weight of a body (Fig. 115) drawn from the bottom to the top of the inclined plane by the effort F . If friction is neglected, the effort does work equal to the product of its magnitude and the distance of the action, that is, $F \times AB$; and the work done on the resistance is equal to the product of its magnitude and the distance of action, that is, $R \times BC$. By the general law of machines

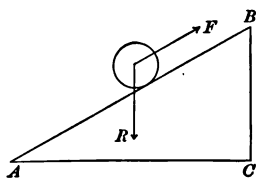


FIG. 115. — Inclined Plane.

$$F \times AB = R \times BC, \quad (11)$$

or,

$$\frac{R}{F} = \frac{AB}{BC}. \quad (12)$$

This shows that the mechanical advantage $\frac{R}{F}$ is the ratio of the length of the plane to its height. *A given effort can support a resistance as many times greater than itself as the length of the plane is times greater than its height, provided that the line of action is parallel to the plane.*

229. Friction on an Inclined Plane. — Suppose a smooth block (Fig. 116) to be placed upon a board that is then gradually tilted upward until the block slides slowly downward. The force of friction in that case equals the force that would have to be applied upward, in a direction parallel to the board, in order to check the motion. The weight of the block may be resolved into two components, one perpendicular and the other parallel to the board. The perpendicular component OR represents the pressure be-

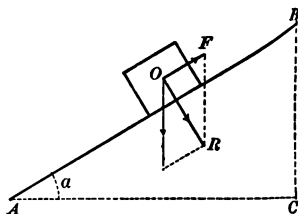


FIG. 116. — Friction on an Inclined Plane.

tween the block and the board ; and the parallel component OF , the force of friction. Since the triangle ROF is similar to the triangle ABC ,

$$\frac{OF}{OR} = \frac{BC}{AC}. \quad (13)$$

The coefficient of sliding friction is defined to be the ratio of the force required to slide two surfaces past each other to the pressure between the surfaces. Hence the coefficient of friction $= \frac{OF}{OR}$.

The angle a is called the *limiting angle of friction*, and is the greatest angle at which a plane can be inclined to the horizontal without another plane sliding on it. The coefficient of friction between small pieces of the same material determines the angle presented by heaps of them, as is seen in piles of coal, of grain, etc.

230. The Wedge.—If instead of moving a load along an inclined plane, the plane itself is moved beneath the load, the plane is then called a *wedge* (Fig. 117). While this name is usually restricted to pieces of wood or metal driven by blows into wood to split it (Fig. 118), or into a joint to tighten it, all cutting or piercing instruments,

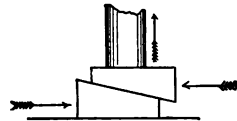


FIG. 117. — The Wedge.



FIG. 118. — A Common Wedge.

such as knives, chisels, axes, needles, and nails are but varieties of wedges. The direction of the driving force is usually along the base of the inclined plane or planes composing the wedge. The wedge may be moved by continuous pressure or by blows, and its usefulness depends largely upon friction. Thus, when a nail is driven into wood, it is held in place by friction; but for this the nail would recoil after each blow, thus frustrating the purpose of the labor spent. Because

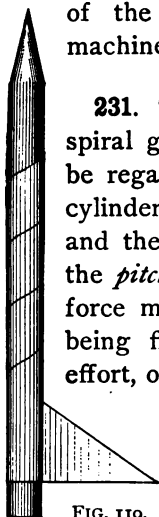


FIG. 119.
The screw is a spiral
inclined plane.

of the great variability of friction, the laws of machines cannot well be applied to wedges.

231. The Screw. — The screw is a cylinder with a spiral groove cut round its circumference. It may be regarded as an inclined plane wrapped round a cylinder (Fig. 119). The spiral is called the *thread*, and the distance between two consecutive threads, the *pitch*. The screw turns in a *nut*. The driving force may be applied to either nut or screw, these being fitted with a lever or wheel to multiply the effort, or the head of the screw may be slotted for a screw-driver, or made square or hexagonal for a monkey wrench. When the effort F turns the screw once round, thus going a distance equal to $2\pi l$ (l is the length of the lever), the resistance R is moved a distance equal to the pitch d of the screw. The general law of machines as applied to the screw then becomes

$$F \times 2\pi l = Rd; \quad (14)$$

whence

$$\frac{R}{F} = \frac{2\pi l}{d}.$$

A given effort will support a resistance as many times greater than itself as the circumference of the circle described by the effort is times greater than the pitch of the screw.

232. Water Power. — Wherever water runs to a lower level, it may be made to turn a wheel. The potential energy of the water at the higher level is in part converted into the kinetic energy of the rotating wheel. The forms of water wheels depend upon the nature of the water

supply and the work to be done. Where a small amount of water with a considerable fall is available, the *overshot wheel* is employed (Fig. 120). This consists of a wheel mounted on a horizontal axis, on the circumference of which are fastened buckets. The weight of the water that fills the buckets on one side makes the wheel turn. Overshot wheels have been made of immense size, and as they are turned by the force of gravity rather than by the speed of the current, they are frequently called *gravity wheels*. Their efficiency sometimes runs as high as 85 %. The *breast wheel* (Fig. 121) and the *undershot wheel* (Fig. 122) depend for their action mainly upon the force of the current, and are used where the fall of water is slight; their

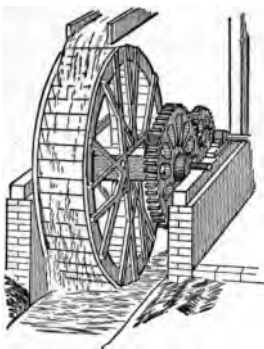


FIG. 120. — Overshot Wheel.

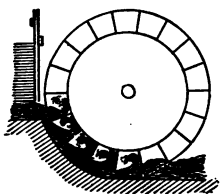


FIG. 121. — Breast Wheel.

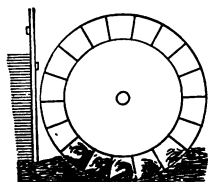


FIG. 122. — Undershot Wheel.

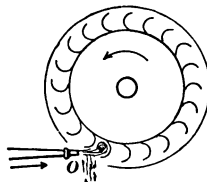


FIG. 123. — Pelton Wheel.

efficiency is usually small. A modified form of the undershot wheel, however, called the *tangential* or *Pelton wheel*, has a very high efficiency. It consists of a wheel with buckets of peculiar shape fastened to its rim (Fig. 123), against which is thrown a jet of water. This can exert an immense pressure if the water comes from a considerable

elevation above the wheel. Such wheels are used extensively in mountainous regions, where the needed elevations occur. Another water wheel that has an efficiency of nearly 80 % is the *turbine* (Fig. 124). It turns on a verti-

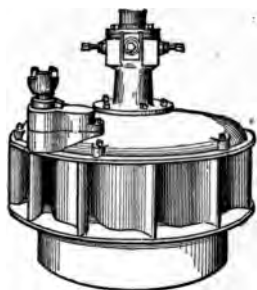


FIG. 124. — Turbine Wheel.

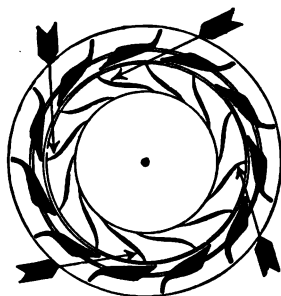


FIG. 125. — Arrangement of Vanes and Guides in a Turbine Wheel.

cal axis and is placed at the bottom of a deep pit. To cause the falling water to strike the vanes of the wheel most advantageously, the inclosing case has guides which direct the water at the proper angle against the vanes (Fig. 125).

233. The Hydraulic Ram.—When a faucet from which water is flowing is suddenly closed, a thump is felt and heard, due to the inertia of the moving water. The violence of the thump is diminished by

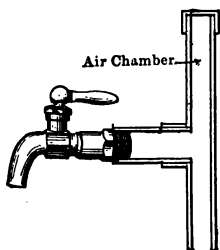


FIG. 126. — Faucet with Air Chamber.

means of an air chamber near the faucet (Fig. 126), the air acting as a cushion. The hydraulic ram acts on the same principle. Water flows into the ram (Fig. 127) and escapes through an opening *O*, which can be closed by a valve *V* moving vertically. When the speed of flow reaches a certain value, pressure enough is developed to raise the valve and suddenly check the flow. The kinetic energy of the water makes it lift another valve *U* leading to the air chamber. The water enters the chamber and compresses the air therein. As soon as

the speed of the flow is somewhat reduced, the pressure of the water no longer suffices to hold up the first mentioned valve, which drops and again allows the water to escape. The flowing water again soon acquires sufficient velocity to close valve V , and more water is made to enter the air chamber. The steady pressure of the compressed air forces the entrapped water through the pipe S to a height considerably above the ram. Hydraulic rams are therefore used for obtaining a water supply from an abundant source at a lower level, much of the water running to waste.

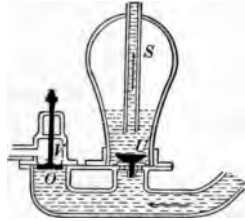


FIG. 127.—Hydraulic Ram.

234. Centrifugal Pumps.—If a turbine wheel be made to rotate in still water, not by means of flowing water, but by some outside power, such as that of a steam engine, the water will be thrown away from the center of the wheel toward its circumference (Fig. 128). By admitting water from below at the center of the wheel and by having the wheel inclosed in a case with a single opening at the circumference, the water drawn in at the center is expelled through this opening.

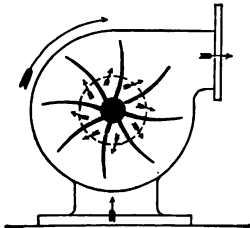


FIG. 128.—Centrifugal Pump.

Such an arrangement is called a *centrifugal pump*. Centrifugal pumps deliver large amounts of water; and, having no valves, are not liable to get out of order. While they cannot draw water from any great depth or force it to any great height, they are extensively used in engineering operations, where rapid service through comparatively small heights is required. Air fans for ventilation or other purposes illustrate the manner in which centrifugal pumps act.

EXERCISES

1. What reasons can you give for believing that perpetual motion is impossible?
2. What force applied parallel to the plane is needed to move 1280 g. up an inclined plane 800 cm. long and 500 cm. high?
3. What fraction of his weight does a man constantly lift while riding a bicycle up a well-graded hill 1 mi. long and 264 ft. high?

4. A wagon weighing 1000 lb. is resting upon an inclined plane which has a 10% rise (rises one foot in every 10 ft.). What force must a man exert in order to keep the wagon from rolling downhill?

5. What force is needed to move a 1000-lb. wagon up a hill of 3% grade?

6. If a man can exert a force of 80 kg. and wishes to lift a body weighing 400 kg. into a wagon 100 cm. high, what is the least length of the inclined plane he can employ?

7. While it takes two men to lift a box weighing 200 lb. into a wagon 36 in. high, one can place it in the wagon by using an inclined plane 72 in. long. How do the forces exerted in the two cases compare? How do the amounts of work compare?

8. How many times longer than its height must an inclined plane be in order that a man exerting 200 lb. of force may move a body of 400 lb. up the plane?

9. How many jackscrews (Fig. 129), with a pitch of half an inch and handles 20 in. long, have to be used to raise a house weighing 100 tons, if a force of 200 lb. is applied to each jack? If the house is raised 4 ft., how many foot pounds of work are done in all?

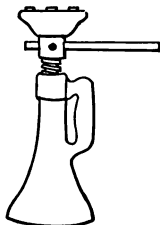


FIG. 129. — Jack-screw.

10. If the pitch of a screw is a quarter inch, and a force of 1 lb. is applied at the end of a lever 3 ft. long, what weight can be moved?

11. What mechanical advantage can be obtained by the use of a copying press of the following dimensions: radius of arms = 12 in.; pitch of screw = $\frac{1}{8}$ in.?

12. A 5000-lb. safe is to be moved into a building along an inclined plane 25 ft. long and 60 in. high. If a system of three movable sheaves be used to haul up the safe, what power must be exerted, if the friction is equivalent to a 30% addition to the weight of the safe?

13. What is the horse power delivered by a water wheel with an efficiency of 70%, if 6000 kg. of water under a head of 7.61 m. are delivered to it each minute?

14. In the worm wheel (Fig. 130), if l is the length of the crank arm C , n the number of

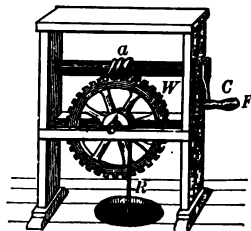


FIG. 130. — Worm Wheel.

teeth in the cogwheel W , and r the radius of the axle a , the mechanical advantage is

$$\frac{R}{F} = \frac{2\pi l n}{2\pi r} = n \frac{l}{r}.$$

How many turns must the handle make while the cogwheel is making one?

15. If in the geared windlass (Fig. 131) the crank handle is 60 cm. long and the wheel a has a diameter of 6 cm., and if there are 100 teeth in the large cogwheel and 10 in the smaller one, what is the mechanical advantage? If a force of 50 kg. is needed to raise a load of 2500 kg., what is the efficiency of the machine?

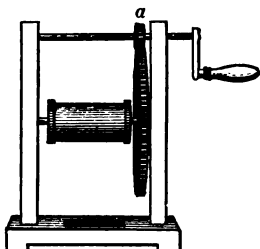


FIG. 131.—Geared Windlass.

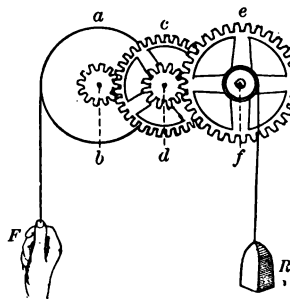


FIG. 132.—Train of Gear Wheels.

16. Show by applying the principle of work that in the train of gear wheels (Fig. 132) the mechanical advantage is given by

$$\begin{aligned} \frac{R}{F} &= \frac{\text{circumference of } a}{\text{circumference of } f} \times \frac{\text{No. of cogs in } c}{\text{No. of cogs in } b} \\ &\quad \times \frac{\text{No. of cogs in } e}{\text{No. of cogs in } d}. \end{aligned}$$

17. What is the mechanical advantage of a hydraulic crane (Fig. 133) if the area of the water pipe a is one hundredth that of the cylinder b , in which the piston moves, the pulleys being connected as shown? What is the ratio between the speeds of the piston and of the load R ?

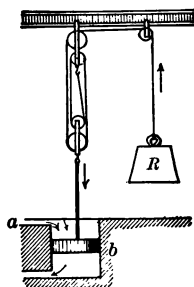


FIG. 133.—Hydraulic Crane.

CHAPTER X

NATURE, PRODUCTION, AND TRANSMISSION OF HEAT

235. Theories concerning the Nature of Heat. — Two rival hypotheses in regard to the nature of heat contended for supremacy from the dawn of science to the middle of the last century. According to one, heat is due to the rapid vibrations of the molecules of a body; according to the other, heat is a subtle, weightless fluid which permeates the pores of all substances. Heat was supposed to be either molecular motion or a peculiar substance called *caloric*. Both these views were gained from a superficial experimental knowledge, and in consequence the explanations of heat phenomena based upon either one of them were often crude and even fantastic.

236. Heat developed by Friction, Impact, and Compression. — Every one knows that friction generates heat. The rubbing of the hands together and the rubbing of a coin against the coat sleeve are familiar instances. Repeated and rapid blows of a hammer upon a nail laid on an anvil will soon make the nail too hot to handle. This is a case of heat developed by *impact*. In inflating the tires of a bicycle, the pump after a few strokes feels warm from the repeated *compression* of the air forced into it. In such actions as these mechanical energy is being converted into heat energy.

237. Rumford's and Davy's Experiments. — Rumford, in 1798, while engaged in boring cannon at the Munich

arsenal, was astonished at the enormous amount of heat developed as compared with the small amount of borings. To investigate the matter, he inclosed a cannon in a tank of water, in such a way that it could be rotated by horse power when pressed tightly against a blunt borer. He found that in two and a half hours the water in the tank was actually boiling.

In meditating over the results of all these experiments, Rumford asks: "What is Heat?—Is there any such thing as an igneous fluid? . . . In reasoning on this subject we must not forget that most remarkable circumstance, that the source of the heat generated by friction in these experiments appeared evidently to be *inexhaustible*. . . . It is hardly necessary to add that anything which any *insulated* body or system of bodies can continue to furnish without *limitation* cannot possibly be a *material substance*; and it appears to me to be extremely difficult, if not quite impossible, to form any distinct idea of anything capable of being excited and communicated in the manner in which heat was excited and communicated in these experiments except it be *motion*."

The fatal blow, however, to the caloric theory was delivered by Davy who, in 1799, showed that two pieces of ice can be liquefied at the freezing point merely by rubbing them together. "The immediate cause of the phenomena of heat," writes Davy, "is motion."

238. Heat a Form of Energy. — The notions of physicists at the end of the eighteenth century concerning energy were not at all clearly defined. They used the word "motion" in much the same way that the word "energy" is used to-day. Hence the conclusion to be drawn from Rumford's and from Davy's experiments is that *heat is a form of energy*. This conclusion, however, was not accepted by most physicists until after Joule, in 1847, had shown by long series of carefully planned and skillfully

executed experiments that *a definite amount of mechanical energy always yields the same amount of heat energy.*

239. The Kinetic Theory and Heat. — Heat energy is supposed to be molecular kinetic energy. The more heat energy a body has, the more rapid are the motions of its molecules. To make the molecules move faster, work must be done on the body. If a body in motion as a whole is stopped by another body, the kinetic energy of the bodies is largely transformed into the kinetic energy of their molecules. As a consequence both bodies become heated. Although these molecular motions cannot be seen, many phenomena are satisfactorily explained by assuming them to take place, and this theory of heat has proved of great value. When the kinetic energy of a body as a whole is transformed into molecular kinetic energy, and the amounts of mechanical and of heat energy are compared, it is found that the one differs from the other by a constant factor the value of which depends upon the units of measurement. This mechanical equivalent of heat will be discussed in detail in Chapter XIII.

240. Sources of Heat. — Besides friction, impact, and compression, the principal sources of heat are: (1) the interior of the earth; (2) the sun; (3) chemical action, and (4) electricity.

Interior of the Earth. — The temperature rises as we penetrate into the earth, the amount of change varying with the locality. Hot springs, geysers, and volcanic phenomena all testify that certain portions, at least, of the interior of the earth are at a very high temperature.

The Sun. — Practically all the heat produced upon the surface of the earth can be traced back to the sun as its source.

Chemical Action. — When a substance burns, *i.e.* combines with the oxygen of the air, heat is given out, and this heat of combustion is the source of the power that is developed by steam engines and gas engines—

Most chemical actions besides combustion are also accompanied by a development of heat. A familiar instance is the slaking of lime; both the lime and the water may be very cold, but when they are mixed, much heat is given off.

Electricity.—Electricity in passing through a conductor produces heat, the electrical energy of the current being in part transformed into heat energy.

241. Effects of Heating.—It is well known that ice, when exposed to an adequate source of heat, changes into water, and the water into steam. One effect, therefore, of imparting heat energy to a substance is to change it from a solid into a liquid or from a liquid into a gas. It is also well known that heating causes an increase in the dimensions of a body, familiar in the operations of fitting tires to wagon wheels and of riveting joints in structural iron, the tires and rivets being applied when hot, so that their shrinkage on cooling may cause a tight fit. And the fact that bodies get hotter or colder as they are exposed to or screened from sources of heat is so familiar as scarcely to need mention. The chief effects of giving heat energy to substances are then: (1) they get hotter; (2) their volume increases; (3) their state changes.

242. Intensity of Heat.—The terms *hot*, *warm*, *cool*, *cold*, express relative degrees of heat. These terms are vague even when modified by other words, as *red-hot*, *lukewarm*, *ice-cold*. By such words we endeavor to express the effects of varying heat intensity upon our senses, especially the sense localized in the skin, which is stimulated by a gain or loss in heat energy. But our senses often give conflicting testimony as to the intensity of a heat source. To illustrate, place one hand in hot water and the other in cold water for a few minutes. Then place them both promptly in tepid water. To the hand that was in the cold water,

the tepid water will seem warm, while to the hand that was in the hot water, the tepid water will feel cold. Evidently then, our senses cannot always be trusted to give reliable indications as to the heat intensity of substances. Everyday observation goes to show that, when heat passes of itself from one body to another, the transference is always in the direction from higher intensity to lower; it proceeds the more rapidly, the greater the difference in heat intensity, and it ceases when the intensities are equalized.

243. Temperature. — *Temperature is the name applied to intensity of heat.* A body is said to be at a higher temperature than another when it can impart heat to the other. Two bodies are at the same temperature when each receives from the other just as much heat as it gives to the other. The more heat put into a body, the higher is its temperature; quantity of heat, however, is not at all the same thing as temperature. Thus a cup of water taken from a bucket has the same temperature as the water in the bucket, but a smaller amount of heat; but the cup of water can contain as much heat as the water in the bucket provided that its temperature can be raised sufficiently. In the light of the kinetic theory, a rise of temperature is due to the increased speed of the molecules of a body caused by the energy imparted to them. Bodies are at the same temperature when their molecules have on an average the same kinetic energy.

244. Transfer of Heat. — There is a universal tendency in nature toward an equalization of temperatures. When two bodies at different temperatures are so situated that heat may pass from one to the other, the one loses and the other gains heat until their temperatures are the same. The rate of transfer of heat energy between the bodies

depends upon their difference in temperature, their nature, and the kind of medium separating them.

245. Heat Transmission.—Heat may be transferred from one body to another in three ways :

I. *Conduction*, by which heat passes through a body without any of the parts of the body appearing to move. Thus, if one end of a short piece of copper wire be held in the fingers and the other end be placed in a flame, the wire will soon become too hot to be held with comfort.

II. *Convection*, by which heat is conveyed by matter that is itself in motion. Thus, if the hand is held above a flame, a rising current of hot gases is felt. The heat energy of the flame is communicated to the molecules of the combustion products by which it is carried to the hand.

III. *Radiation*, by which heat is propagated by a wave motion in the ether. Thus, when the hand is held beside a flame or below a heated body, a sensation of warmth is felt, although there is no solid conducting nor fluid conveying the heat.

246. Conduction.—Suppose a thick bar of some metal to have holes bored in it at equal distances along its length, in which the bulbs of thermometers are placed (Fig. 134).

When one end of the bar is heated, the thermometers show that the portions of the bar with which they are in contact

are receiving heat and rising in temperature. After a time, the reading of each thermometer becomes constant ; the bar is then losing just as much heat as it is receiving ; it is in the *stationary state*. The bar has not changed except in temperature, and it certainly has not moved as a whole.

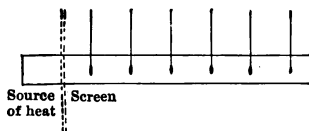


FIG. 134. — Conduction of Heat.

The heat has been passed along from molecule to molecule by some process the exact nature of which we do not fully know.

Experiment shows that *the rate at which heat passes through a substance depends upon its material, is directly proportional to its cross-sectional area and to the difference of temperature between its extremities, and is inversely proportional to its length.*

247. Conductivity.—Substances vary greatly in their power to conduct heat. One of the best conductors is

copper and one of the poorest is air. Metals conduct

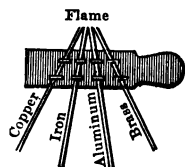


FIG. 135.—Conductivity of Metals. A match head slid slowly along a wire takes fire at a certain distance from the flame. The greater this distance is, the greater is the conductivity.

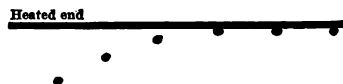


FIG. 136.—Speed of Flow of Heat. Shot are fastened with wax along bars of different metals. The greater the conductivity, the faster the shot fall off, when one end of a bar is heated.

heat best, although among themselves there is considerable difference in conductivity (Figs. 135 and 136). Glass, brick,

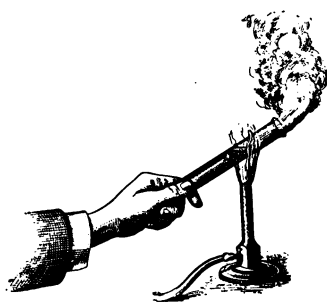


FIG. 137.—Conductivity of Liquids. Water can be boiled in the upper part of a test tube, while the lower part is held in the hand.

stone, and the like are poorer conductors than the metals, but still are much better than wood, wool, feathers, fur, and similar substances. Liquids, with the exception of mercury, have very little conducting power (Fig. 137), and gases, with the exception of hydrogen, hardly any. The feeble conductivity of such substances as fur and feathers is chiefly due to their

imprisoning large amounts of air, and for a similar reason, snow and powdered substances in general are very poor conductors of heat.

A vacuum does not conduct heat at all. Hence in work with liquefied gases, double-walled vessels are used, the space between the walls being exhausted of air, and thus shielding the liquid from outside heat.

248. Davy Lamp.—If a piece of wire gauze be held down over a small flame, the flame will not pass through it (Fig. 138). If the flame

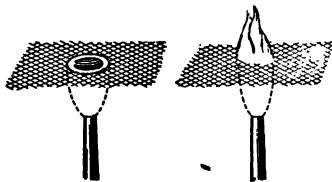


FIG. 138.—Flame and Wire Gauze.

is extinguished while the gas is still turned on, it may be set on fire above the gauze, but its flame will still refuse to go down to the burner. The metallic gauze conducts the heat away from the flame so rapidly that the gas on the opposite side is not raised to a temperature high enough to cause combustion.

The Davy Lamp (Fig. 139) consists of an oil lamp, the flame of which is inclosed by wire gauze. While any inflammable gases encountered in a mine may pass freely through the gauze and burn quietly within, the flame cannot pass through the gauze and set the gas outside on fire so as to cause an explosion.

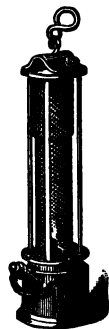


FIG. 139.
Davy Lamp.

249. Sensation and Conductivity.—Different objects that have been lying side by side, and hence must have the same temperature, often give rise to different sensations when touched. Thus iron may feel cold while the wood in contact with it may feel warm. The reason for this is that the sensation of heat or cold depends upon the rate at which heat is abstracted from the hand; and as iron conducts heat better than wood, more heat in a given time is taken from the hand by the iron than by the wood. If,

however, the temperature of the iron and wood be higher than that of the hand, the iron will give out its heat faster to the hand than the wood will, the consequence being that the iron will feel hotter than the wood.

250. Convection.—When a portion of a fluid is heated, it expands; its density is thereby diminished and the heated part is pushed upward by the adjacent colder and heavier portions. Accordingly, when heat is applied beneath a fluid, the lower portions, being heated first, rise while the upper portions sink. A vertical circulation of the fluid is set up whereby the whole is put in motion. The *convection currents* distribute the heat received from the source to all bodies with which they come in contact. As convection consists in the motions of the parts of a body, it is clear that it can occur only in fluids, as solids are too rigid for such motions to take place.

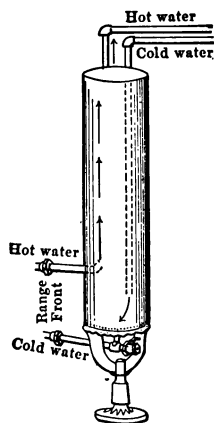


FIG. 140. — Convection Currents of Kitchen Boiler.

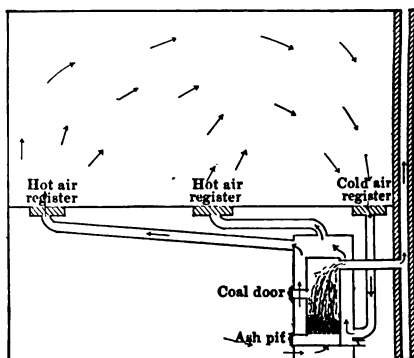


FIG. 141. — Convection Currents of Hot Air Furnace.

251. Applications.—By convection the water in the kitchen boiler connected with the range or heater is heated (Fig. 140), and in a

similar fashion but on a larger scale, houses are heated by hot water (Fig. 142). The hot air furnace (Fig. 141) and the steam radiator depend upon convection currents to carry their heat through the rooms of a house, and methods of ventilation are based upon convection currents.

In nature also convection plays a most important part. The unequal heating of the air above land and sea creates the land and sea breezes. The air in the tropics becoming heated is lifted up by the colder air rushing in from higher latitudes, and this action in conjunction with the earth's rotation causes those steady movements of the air known as trade winds. The great ocean currents, such as the Gulf Stream, are also due in a large measure to convection.

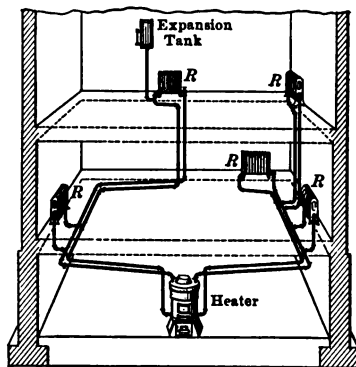


FIG. 142.—Convection Currents of Hot Water Plant.

252. Ventilation.—To secure good ventilation, provision must be made not only for the entrance of fresh air, but also for the escape of the foul air. In steam-heated buildings no special arrangement is usually made for ventilation, it being assumed that the cracks in the windows and doors will suffice for inlets and outlets. If more ventilation is desired, the best way is to open the windows slightly at the top to let out the heated air, and at the bottom to let in the fresh air. Hot air furnaces not infrequently fail to heat a house uniformly because there is no way for the air in certain rooms to escape so as to permit fresh and heated air to enter. The best results in heating large schools and other public buildings are obtained by forcing into the rooms by means of fans either warm or cold fresh air, ventilators being provided to carry off the impure air.

EXERCISES

1. How are safes made fireproof?
2. In what way does the use of double windows save fuel?
3. How is the hand protected from the heat of a poker or a flatiron?

4. Mention as many substances as you can which although at the same temperature do not when touched cause like sensations of temperature.

5. Why, in making ice cream, is the freezing mixture put into a wooden pail, and the cream into a metal one ?

6. Why is a substance like felt or hair a poorer conductor when loosely packed than when firmly packed ?

7. In the heating of a room by means of a fireplace, which of the methods of heat transference is most active ?

8. What are the essential points in the proper construction of a good ice box ? of a fireless cooker ?

9. Why is the ordinary "soldering iron" made of copper ?

10. Describe the convection current that is set up when a door between a cold and a warm room is left ajar.

11. Describe in detail how a room is heated (1) by fireplaces, (2) by hot air furnaces, (3) by steam, (4) by stoves, (5) by hot water, (6) by forced draft.

12. In an air-cooled gas engine, such as that of a motor cycle, the cylinder is cast with numerous projections on its outside. What is the purpose and action of these projections ?

13. Which is better — a good or a poor conductor — for keeping a body warm ? For keeping it cool ?

14. Why does a morning frost on a board walk disappear first around the heads of the nails ?

CHAPTER XI

TEMPERATURE AND CHANGE OF SIZE

THERMOMETRY

253. Measurement of Temperature. — Temperature cannot be measured directly, since we know of no method of measuring the velocities of molecules directly. Recourse then must be had to some property of substances that changes continuously with the temperature; measurements of this property can serve as measurements of temperature.

As the volume of most substances increases or decreases continuously when their temperature rises or falls, respectively, the amount of expansion may be taken as a measure of the change of temperature that causes it. Fluids as a rule expand more for the same change of temperature than do solids. If a bulb be blown at one end of a fine glass tube and then filled with a fluid, such as air, alcohol, or mercury, both the fluid and the solid will expand when heated. As the volume of the fluid increases more rapidly than does the capacity of the bulb, the fluid will fill more and more of the tube as the temperature rises. Such an instrument as this when provided with a properly graduated scale constitutes a *thermometer* or temperature measurer.

A good thermometer should be readily portable, should always give the same reading when exposed to the same temperature, should permit of easy testing of the correctness of its graduations, and should be such a shape and size as to assume quickly the temperature of a body with which it is in contact. Of the various kinds proposed

those made of glass and containing mercury have proved to have the greatest all-round usefulness (Fig. 143).

254. Graduation of Thermometers. — A thermometer measures primarily changes of volume, and to make it measure changes of temperature, relationships must be established between the volume changes and the temperature changes causing them. If the inside diameter of the tube is uniform and the expansion of the materials composing the thermometer is regular, for equal changes in volume the end of the mercury thread moves through equal distances, and the temperature changes thus indicated are equal. The scale on a thermometer therefore consists of equal divisions. With the same thermometer, then, equal temperature changes cause equal volume changes. But how is it with different thermometers having different lengths of scale divisions?

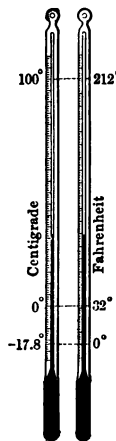


FIG. 143. —
Thermometer.

How can readings on one thermometer be made to indicate the same temperature as readings on another thermometer? Evidently the only way is to compare thermometers at certain definite temperatures, and thus find out the relationships of their scale readings. Indeed, by observing at what points on the stem the mercury stands for two definite temperatures, the distance between these two *fixed points* may be divided into a certain number of equal parts, and the scale divisions, the length of which is thus determined, may be extended above and below the fixed points.

255. The Fixed Points. — When a substance is made to pass from one state to another, the temperature will re-

main the same during the transformation. Thus ice melts and water freezes at the same temperature, and water boils and steam condenses at another temperature fixed and definite as long as the pressure remains constant. The temperature of melting ice is taken as one fixed point (freezing or melting point) and that of water boiling under a pressure of 760 mm. of mercury as another (boiling or condensing point). The temperature interval between these fixed points is called the *fundamental interval*.

256. Freezing Point. — The freezing point is determined by packing the thermometer in finely crushed and melting ice or snow up to the top of the mercury column, and marking its position on the stem.

257. Boiling Point. — The boiling point is found by suspending the bulb and stem in steam, and marking on the stem the position of the top of the mercury column. If the pressure of the atmosphere is not 760 mm. of mercury, certain corrections have to be applied.

258. Graduating the Stem. — If the bore of the tube be uniform, equal changes of temperature will make the mercury move equal distances in any part of the stem. When the scale according to which the thermometer is to be graduated, has been decided upon, the distance between the fixed points is divided into a number of equal parts called *degrees*, and the same value for a degree is continued above and below the fixed points.

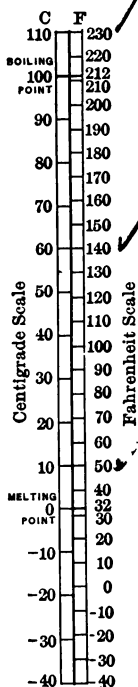
259. Thermometer Scales. — The scale in everyday use in English-speaking countries is the *Fahrenheit*, which has the freezing point at 32° , and the boiling point at 212° ; the fundamental interval consists of 180° , and the zero is 32° below the freezing point. The *Centigrade* scale (the hundred-degree scale of Celsius) is in common use in most European countries, and in all countries it is the scale that

is almost exclusively used in scientific work. This scale puts the freezing point at 0° , and the boiling point at 100° , the fundamental interval being therefore 100° . With both scales, readings below zero are distinguished by the minus sign, as -20° (twenty degrees below zero).

260. Comparison of Scales. — Since the Fahrenheit scale has 180 divisions between the fixed points and the Centigrade scale has 100 divisions, $1^{\circ} \text{C.} = \frac{9}{5}^{\circ} \text{F.}$, and $1^{\circ} \text{F.} = \frac{5}{9}^{\circ} \text{C.}$ When the equivalent of a reading on one scale is to be found on the other, the differing positions of the zero points must be taken into account (Fig. 144). The following formulas are of service:

$$\text{C.} = \frac{5}{9}(\text{F.} - 32). \quad (1)$$

$$\text{F.} = \frac{9}{5}\text{C.} + 32. \quad (2)$$



261. The Reaumur Scale. — Mention should also be made of the Reaumur scale in popular use in Germany and Russia. This scale is also used in some manufacturing operations, notably brewing. Its freezing point is at 0° and its boiling point at 80° .

262. Displacement of the Fixed Points. — Glass that has been heated does not, after cooling, revert at once to its original volume; months and even years are required. For this reason the freezing point, determined immediately after the thermometer has been heated to the boiling point, is found to be slightly lower than it is after the glass has been cold for some time. The best thermometers are stored for a year or more before being graduated, because of this slow change in the glass. Thermometers should have their

FIG. 144. — Thermometer Scales.

fixed points occasionally verified so as to keep track of any changes in them and to make the necessary corrections.

263. Limitations of Mercury-in-glass Thermometers.—As mercury solidifies at -39°C ., mercury-in-glass thermometers cannot be used below this temperature. For temperatures lower than -39° , alcohol or toluene is substituted for mercury. Mercury boils under a pressure of 760 mm. at 357°C . Mercury-in-glass thermometers can, however, be used up to about 500°C ., if the stem is filled with nitrogen, for the pressure of the gas keeps the mercury from boiling.

264. Pyrometers.—A *pyrometer* is a thermometer for measuring higher temperatures than can be found with a mercury-in-glass thermometer. The expansion of bars of metal or clay, the fusion of substances of known melting points, the change in electric resistance of platinum, the generation of a current of electricity by a *thermopile*, and the change of color of heated substances, may be mentioned as means of ascertaining temperatures from measurable changes in the substances heated, the amount of change being proportional to the temperature.

EXERCISES

1. Specify three objections to a water-in-glass thermometer.
2. How should a mercury-in-glass thermometer be made so that a degree would occupy a centimeter or more of the stem? What effect would such a construction have on the minuteness of the change of temperature detectable by it?
3. Change -50°C . to the equivalent Fahrenheit reading.
4. What is the Centigrade reading corresponding to 70°F .?
5. What on the Fahrenheit scale corresponds to -273°C .?
6. Mercury boils under a pressure of one atmosphere at 357°C . What is this temperature in Fahrenheit degrees?
7. If a change of 27 mm. of mercury in barometric pressure makes a corresponding change of 1°C . in the boiling point of water, at what temperature will water boil when the barometer reads 745 mm.?
8. At what temperature are the readings on the Centigrade and Fahrenheit scale the same?
9. If the reading on a Fahrenheit thermometer is twice that of a Centigrade thermometer having the same temperature, what is the temperature on both scales?

EXPANSION OF SOLIDS AND LIQUIDS

265. Linear Expansion of Solids. — With but few exceptions, solids expand when heated, and if they are not of a certain crystalline structure, they expand equally in all directions. The increase of one of the dimensions of a solid due to a rise of temperature is called *linear expansion*. Because of the enormous force called into activity by this expansion, provision must be made in all structures consisting of long beams or girders for one piece to move freely upon another; otherwise, the changes of temperature from season to season would warp the structure out of shape. One end of long iron bridges is therefore placed on rollers so as to allow for the expansion, and the rails of a track are laid so that they will just touch in warm weather, their contraction in cold weather leaving little gaps between their ends.

266. Coefficient of Linear Expansion. — The amount of expansion for equal temperature changes varies, each substance possessing its own expansive individuality. The amount of expansion varies also for the same substance for different ranges of temperature. Experiment has shown that if the length of a solid at the temperature t be denoted by l , and at the higher temperature t' by l' , the relationship between these quantities may be given by the formula :

$$l' = l[1 + c(t' - t)]. \quad (3)$$

The quantity c represents a number, called the *coefficient of linear expansion*, which, for a limited range of temperature, has a constant value in the case of a given substance. By rearrangement of the above formula, the defining equation of c becomes :

$$c = \frac{l' - l}{l(t' - t)}. \quad (4)$$

The coefficient of linear expansion may therefore be determined experimentally by measuring the lengths of a solid at two different temperatures; and it may be defined as *the increase in length which the unit of length of any material undergoes when its temperature is raised one degree.*

267. Coefficient of Surface and of Cubical Expansion.—Let a non-crystalline cube that occupies a volume of 1 cm.³ at 0° be heated one degree. Each edge will lengthen to $(1 + c)$ cm., where c is the coefficient of linear expansion. The cube's surfaces will each increase to $(1 + c)^2$ cm.², or to $(1 + 2c + c^2)$ cm.², and the volume of the cube will be $(1 + c)^3 = 1 + 3c + 3c^2 + c^3$. But as c is always a very small fraction, c^2 and c^3 become so small as to be negligible for all practical purposes,* and the formulas may be simplified:

$$(1 + c)^2 = 1 + 2c,$$

and

$$(1 + c)^3 = 1 + 3c.$$

The coefficients of surface and of cubical expansion are accordingly two and three times, respectively, as large as the coefficient of linear expansion.

268. Coefficient of Cubical Expansion of Fluids.—While the coefficient of cubical expansion of a solid may thus be found from its linear coefficient, that of a liquid and gas must be found by direct volume measurements. Let v and v' represent the volumes of a fluid at the temperatures t and t' , respectively. The coefficient of cubical expansion is

$$c = \frac{v' - v}{v(t' - t)}. \quad (5)$$

The coefficient of cubical expansion is that fraction of itself by which a given volume of a substance increases when heated one degree.

* If the coefficient of linear expansion of a certain substance is .002, then $(1 + c)^3 = 1.006012008$. It is manifest that all the figures after the 6 can be neglected. The coefficient of cubical expansion is therefore .006, which is thrice .002.

269. Compensated Pendulums.—A clock cannot keep perfect time unless the length of its pendulum remains constant. Variations of

temperature, however, cause variations in length, to compensate for which, two forms of pendulums have been devised. In the *gridiron pendulum* (Fig. 145) the bob is hung from a system of steel and brass rods. The coefficient of linear expansion of steel is very nearly two thirds that of brass. Hence, if the sum of the lengths of the brass rods be two thirds the sum of the lengths of the steel rods and if the system be arranged in such a way that the brass rods are free to expand in one direction and the steel rods in the opposite direction, the expansion of the two metals will make the length of the pendulum remain constant, whatever the change in temperature. In the *mercury pendulum* (Fig. 146) the bob consists of cylindrical cups filled with mercury. When the tempera-



FIG. 145.—
Gridiron
Pendulum.

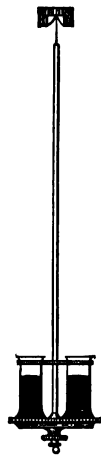


FIG. 146.—
Mercury
Pendulum.

and thus counteracts the linear expansion of the supporting rod that would lower the bob.

270. Expansion of Composite Bar.—

When two strips of different metals, such as brass and iron, are riveted together at ordinary temperatures, the composite bar thus formed bends, when heated, in such a manner that the more expansible metal is on the outside of the curve (Fig. 147).

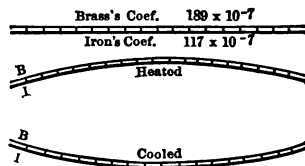


FIG. 147.—Composite Bar.

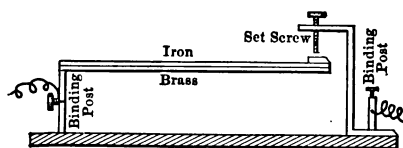


FIG. 148.—Alarm Thermometer.

271. Alarm Thermometer.—

One end of a composite bar is fastened to a post set in a base (Fig. 148) with its other end opposite a set screw supported by a second post. Wires lead from the

posts to an electric bell and battery. When the temperature of the bar rises, it bends in such a fashion as to bring its free end into contact with the set screw. An electric current then passes and rings the bell. By adjusting the set screw, the bell may be made to ring at any temperature desired.

272. Compensated Balance Wheel.—In the balance wheel of a watch, which serves the same purpose as a pendulum in a clock, most of the mass lies in the rim (Fig. 149). If the temperature changes, the length of the spokes changes, thereby causing a variation in the rate of vibration. To offset this expansion, the rim of the wheel is composed of two pieces, each piece consisting of two strips of metal fastened together, the one with the greater coefficient of expansion being on the outside. One end of these composite strips is attached to a spoke of the wheel, and the other end left free. A rise of temperature, while it lengthens the spokes, at the same time causes the ends of the composite bar to bend inward, the rate of vibration being thereby rendered independent of the temperature.

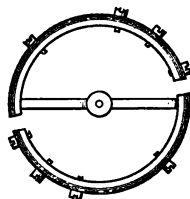


FIG. 149. — Compensated Balance Wheel.

273. Metallic Thermometer.—This consists of a flat, cylindrical case with dial and pointer protected by a glass front (Fig. 150).

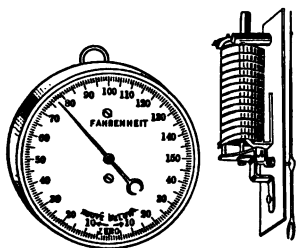


FIG. 150. — Metallic Thermometer.

Fixed to the back of the dial is one end of a spiral composed of different metals soldered together. The other end is attached to a mechanism for multiplying the motion of the metal strips and communicating it to the index. The metals expand unequally when subjected to changes of temperature, the result being that the spiral winds or unwinds a little. The index is thereby made to move around the dial, which is so graduated as to give direct readings of temperature.

274. Expansion of Liquids.—Inasmuch as liquids have no shape of their own they cannot be said to have coeffi-

cients of linear expansion, but only of cubical expansion. Liquids vary quite as much as solids in the values of their coefficients of expansion, those of liquids being, as a rule, greater than those of solids. As the liquids are contained in solid vessels, measurements of their expansions likewise involve measurements of the expansions of the containing vessels.

275. Irregular Expansion of Water. — While most liquids expand when heated, water within a certain range of temperature presents a remarkable exception. It has the property of possessing the same density at two different temperatures, that is to say, it presents a maximum of density 4° C. above its freezing point. The density of ice is less than that of water at its freezing point. An important consequence of this behavior of water is that the formation of ice on bodies of water is confined to the surface. The temperature of the deep water in fresh-water lakes is about 4° C. the year round.

EXERCISES

1. Why is it that a beaker (which is made of thin glass) can be heated over a Bunsen flame with comparative safety, while a tumbler will break?
2. Why is a pendulum rod made of wood preferable to one of metal?
3. How is it that a glass stopper sticking in the neck of a bottle may often be loosened by heating the neck?
4. The steel rails of a car line are each 30 ft. long. What space must be left between the ends of the rails in order to allow for the expansion from -10° C. to 40° C.? (Coefficient for iron and steel is 122×10^{-7}).
5. If an iron steam pipe is 40 ft. long at 0° C., how long is it when steam at 105° C. is passing through it?
6. If a certain scale is 99.981 cm. long at 10° C., and 100.015 cm. long at 40° C., at what temperature will the scale be just 100 cm. long?

7. An iron ring which at 20° refuses to slip over a rod 10,000 cm. in diameter, when heated to 100° just fits the rod still kept at 20° . What is the internal diameter of the ring at 20° ?

8. If a brass chain is 246.8 cm. long at 20° , what will be its length at 50° ? (Coefficient for brass is 192×10^{-7}).

9. If a certain steel wire 16 ft. long stretches .1 in. by a force of 2 lb., what weight can be raised by it while cooling from 90° to 0° ?

10. Why must provision be made for emptying water pipes to protect them against frost?

EXPANSION OF GASES

276. Expansion of Gases.—Experiment shows that all gases have practically the same coefficient of expansion, the same uniformity being exhibited as was shown in the change of volume brought about by a change in pressure. From the standpoint of the kinetic theory this is perhaps to be expected, as the gas molecules are relatively so far removed from one another that any specific action due to difference of substance becomes vanishingly small. The heat energy delivered to a gas causes increased molecular agitation, which results in a striving toward an increase in the intermolecular distances; and if the volume is kept constant, an increase in pressure due to increased frequency of molecular bombardments takes place.

277. Charles's Law.—If the vessel containing a gas does not change in volume, the effect of heating the gas is to produce an increased pressure on the walls of the vessel, the volume remaining constant. If, on the other hand, the pressure is maintained constant, a rise of temperature produces an increase in volume. Now it has been found that if the volume of a given mass of gas at 0° is v_0 , its volume, when heated under constant pressure through one degree, is increased to $v_0 + \frac{1}{273} v_0$, and when

heated through t° , its volume becomes $v_0 + \frac{t}{273}v_0$. Denoting the volume at t° by v_t , we have

$$v_t = v_0 + \frac{t}{273}v_0, \quad (6)$$

$$= v_0 \left(1 + \frac{t}{273} \right). \quad (7)$$

Furthermore, if the pressure of a given mass of gas at 0° is p_0 , its pressure p_t , when it is heated at constant volume to t° , becomes

$$p_t = p_0 + \frac{t}{273}p_0, \quad (8)$$

$$= p_0 \left(1 + \frac{t}{273} \right). \quad (9)$$

The fraction $\frac{1}{273} = .00367$ is the *coefficient of expansion of gases*, and has nearly the same value for all gases, whether heated under constant volume or pressure. This general truth was first noted by Charles in 1787, was more thoroughly studied by Dalton in 1801, and was subjected to a sufficient experimental verification in 1802 by Gay-Lussac; it is usually referred to as Charles's law. Using the abbreviation "pressure coefficient" for coefficient of expansion at constant volume, and "volume coefficient" for coefficient of expansion at constant pressure, we may state Charles's law as follows: *The pressure coefficients of all gases are the same and are equal to their volume coefficients.*

278. Air Thermometers.—The first thermometers contained air as the expansive substance (Fig. 151). While

for ordinary purposes such thermometers have been replaced by liquid-in-glass thermometers, yet thermometers having bulbs filled with air or hydrogen are used in physical laboratories for very accurate work. Gas thermometers are standard instruments with the readings of which the readings of other thermometers are to be compared.

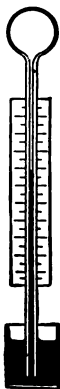


FIG. 151.—
Air Thermometer.

279. The Standard Hydrogen Thermometer.

— In this thermometer (Fig. 152) the volume of the gas is kept constant and the change of pressure due to change of temperature measured. The glass bulb has a long capillary stem which is connected by means of a rubber tube with a vertical glass tube placed before a scale. When the bulb is heated, the hydrogen expands and forces down the level of the mercury in the nearer branch.

By raising the mercury in the other branch, however, the gas is subjected to an increased pressure and made to occupy the same volume as before. The increase in pressure is equal to the difference in the levels of the mercury in the two branches. From Equation (9) we have

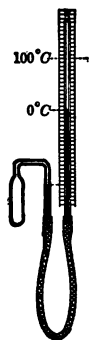


FIG. 152.—
Hydrogen Thermometer.

$$t = \frac{273 p_t}{p_0} - 273. \quad (10)$$

Hence, by measuring the different pressures of the gas, the corresponding temperatures may be ascertained. To express these in Centigrade degrees, the pressures when the bulb is immersed in melting ice and in steam from boiling water must be measured. One hundredth of the difference in these pressures represents one degree Centigrade.

280. The Absolute Zero and Scale.—Factoring out $\frac{1}{273}$ from Equations (7) and (9),

we have
$$v_t = \frac{v_0}{273}(t + 273), \quad (11)$$

and
$$p_t = \frac{p_0}{273}(t + 273). \quad (12)$$

Inasmuch as $\frac{v_0}{273}$ and $\frac{p_0}{273}$ are constants, it follows that the volume or pressure of a gas is directly proportional to its temperature on the Centigrade scale plus 273° C. By adopting a zero 273° below the Centigrade zero, the volume or pressure is made directly proportional to the temperature on this new scale.

Suppose a mass of gas occupying a volume of 273 cm.^3 at 0° C. to be cooled to -1° C. It will contract $\frac{1}{273}$ of its volume, i.e. $\frac{1}{273}$ of 273 , and will occupy 272 cm.^3 . For each degree that its temperature is lowered, its volume will decrease $\frac{1}{273}$ of the original volume, and at -273° , the contraction would amount to $\frac{273}{273}$ of 273 or 273 cm.^3 . At -273° C., therefore, the gas would have no volume at all. In reality the gas condenses into a liquid before this temperature is reached, and Charles's law applies only to gases. There are several reasons for believing that -273° is the lowest limit of possible temperatures. It is therefore called the *absolute zero*, and a scale based on this zero is termed the *absolute scale*. The freezing point of water on this absolute scale is 273° C. To convert Centigrade readings into absolute readings, add 273 ; to convert absolute readings into Centigrade readings, subtract 273 . Charles's law may now be stated as follows: *With constant pressure, the volume, and with constant volume, the pressure of a given mass of gas is directly proportional to its absolute temperature.*

281. The Gas Equation. — Since both pressure and volume of a gas are directly proportional to the absolute temperature, their product is also directly proportional to the absolute temperature. If then P , V , and T denote the pressure, volume, and absolute temperature of a given mass of

gas, and R a constant, the value of which depends upon the units of measurement, the *gas equation* is

$$PV = RT. \quad (13)$$

If by means of subscripts different values of P , V , and T for the same mass of gas are distinguished, then

$$P_1 V_1 = RT_1, \quad (14)$$

$$\text{and} \quad P_2 V_2 = RT_2; \quad (15)$$

whence, since R is constant,

$$\frac{P_1 V_1}{P_2 V_2} = \frac{T_1}{T_2}. \quad (16)$$

This form is convenient in the solution of problems.

EXAMPLE.—If 156 cm.³ is the volume of a gas at 20° and 78 cm. barometric pressure, what would be its volume at 0° and 76 cm. pressure, *i.e.* under so-called *standard conditions*?

Solution. Changing Centigrade temperatures to absolute, and making proper substitutions in Equation (16), we have

$$\frac{76 \times V_1}{78 \times 156} = \frac{273}{273 + 20};$$

whence

$$V_1 = 149 \text{ cm.}^3$$

EXERCISES

1. Prove that at constant pressure the density of a gas is inversely proportional to its absolute temperature.
2. How many degrees above 0° C. must the temperature of a gas be raised to double its volume, the pressure remaining the same?
3. What will be the volume at 60° of a certain mass of gas which occupies 100 cm.³ at 20° C?
4. An open bottle is of such size as to contain one liter of air at 0° C. weighing 1.293 g. What weight of air escapes when the whole is heated to 546° C.?
5. A porcelain vessel able to withstand a pressure of 10.333 $\frac{\text{kg.}}{\text{cm.}^2}$ is sealed when full of gas at 0° C. and under a pressure of one atmosphere. To what temperature must the vessel be heated in order to break it?

CHAPTER XII

QUANTITY OF HEAT AND CHANGE OF STATE

SPECIFIC HEAT

282. Factors of Heat Quantity. — When a body is exposed to a source of heat, *i.e.* put in thermal communication with another body at a higher temperature, the temperature of the first body rises while that of the second falls; there is a transfer of heat energy from the second body to the first. It is manifest that the greater the difference in the temperatures, the greater must be the amount of heat transferred in a given time; we associate quantity of heat with change of temperature. We know also that the same source of heat takes longer to raise the temperature of a large amount of a substance than it does that of a small amount; we associate quantity of heat with quantity of matter, the mass. Quantity of heat depends then upon at least two factors, mass and temperature change. This we know from everyday observation. But there is also another factor, due to the nature of the substances, which is perhaps not so familiar. This factor is called the *thermal* or *heat capacity* of a given substance.

283. Heat Capacity. — When equal masses of any substance, water, for example, are mixed at different temperatures, the resulting temperature lies halfway between the two original temperatures. The hotter portion of the body has lost as much heat as the cooler portion has received. When equal masses of different substances at different

temperatures, however, are mixed, the common temperature is ordinarily not halfway between the original temperatures. Thus, if equal masses of water and copper are mixed, the common temperature is nearer that of the water than that of the copper. When, for instance, 100 g. of copper at 100° are immersed in 100 g. of water at 0° , the temperature of the resulting mixture is found to be about 8.5° . It takes $\frac{8.5}{91.5} = .093$ as much heat to raise

the temperature of copper through a given interval as it does to raise through the same interval an equal mass of water. Copper is able to hold only about .093 as much heat as water under like conditions; or, in other words, the heat capacity of copper is only .093 that of water. Every substance has a heat capacity of its own, the value of which has to be found experimentally. Work involving the measurement of quantities of heat is referred to as *calorimetric*, the topic being known as *calorimetry*.

284. Unit of Heat the Calorie. — To measure a quantity of heat requires the specification of three items: (1) the substance; (2) its mass; (3) the temperature interval through which the substance is cooled or heated. The choice of a heat unit involves, then, the selection of (1) a standard substance; (2) a unit of mass; (3) a unit of temperature change. For various reasons the standard substance chosen is water. The unit of mass for the physicist is the gram, and the unit of temperature interval is one degree on the Centigradé scale. The physicist's unit of heat, called the *calorie*, is therefore *the quantity of heat taken in or given out when the temperature of one gram of water is raised or lowered one degree Centigrade*.*

* Strictly speaking, the exact temperature of the water should be specified, as the heat capacity of water at different temperatures is slightly different. This dif-

285. Specific Heat. — From the very fact that water has been selected as the standard substance in the measurement of quantities of heat, any change in the quantity of heat of other substances may be reduced to an equivalent process of changing the temperature of water. Thus it takes $\frac{1}{11}$ as much heat in the case of copper, and $\frac{1}{80}$ as much heat in the case of mercury to bring about a given change of temperature as it does to effect the same temperature change for an equal mass of water. Such numbers as $\frac{1}{11}$ ($= .09$) and $\frac{1}{80}$ ($= .03$) represent then the ratio of *the quantity of heat required to produce a given change of temperature in a given mass of any substance to the quantity of heat required to produce the same change of temperature in an equal mass of water*. This ratio is called the *specific heat* of the substance under examination. If we know the specific heat s of a substance, we can find how great a quantity of heat H is required to change the temperature of a given mass m of it from t' to t , for it is manifestly the product of the three factors of heat quantity.

$$H = ms(t' - t). \quad (1)$$

286. Water Equivalent. — The product of the weight and the specific heat of a substance gives the weight of water which is equivalent, as regards quantity of heat, to the weight of the substance. Thus 100 g. of copper require the same amount of heat to raise their temperature through a given interval as do 9 g. of water, for $.09 \times 100 = 9$. If, then, water is heated in a vessel of known mass and specific heat, by multiplying these two values together, a number is obtained, which, as far as heating effects are concerned, is the equivalent, in terms

ference is so small, however, that it may be neglected except in the most refined work.

of water alone, of the material and mass of the vessel. In calorimetric measurements based on the method of mixtures described below, it is customary to find the water equivalents of all the pieces of apparatus used, such as the containing vessel, the thermometer, and the stirrer, and to add them to the mass of the water.

287. Method of Mixtures.—The method usually employed in determining the specific heat of a substance consists in heating m_1 g. of the substance to the temperature t_1 and then introducing it into a mass m_2 of water at the lower temperature t_2 . If this operation is properly conducted, practically all the heat lost by the substance is gained by the water when the two substances come to the common temperature t . If we denote by s_1 the specific heat of the substance under examination, and the fall of temperature by $(t_1 - t)$, then the quantity of heat given out is $m_1 s_1 (t_1 - t)$. In similar fashion, the heat gained by the water equals $m_2 \times 1 \times (t - t_2)$, the specific heat of water being unity. As these two quantities of heat are by the experimental conditions of the problem equal, we have

$$m_1 s_1 (t_1 - t) = m_2 (t - t_2). \quad (2)$$

This equation contains six quantities, and if any five of them are known, the sixth may be readily calculated. Usually it is either the specific heat or the temperature of the hot substance that is to be determined. The equation may then for convenience be transformed into these:

$$s_1 = \frac{m_2 (t - t_2)}{m_1 (t_1 - t)}, \quad (3)$$

$$t_1 = \frac{m_2 (t - t_2) + m_1 s_1 t}{m_1 s_1}. \quad (4)$$

It has been assumed, to simplify matters, that the water alone receives the heat of the hot body; but in reality, the vessel holding the water (the calorimeter), the thermometer, and the stirrer in the water are also heated from t_2 to t . Account may be taken of the heat that these articles receive by calculating their water equivalents and adding them to the mass of the water.

EXAMPLE. — If 100 g. of mercury at 100° are mixed with 100 g. of water at 0° , the resulting mixture will have a temperature of 3.2° . The change of temperature of the mercury is $(100 - 3.2) = 96.8^\circ$; that of the water is $(3.2 - 0) = 3.2^\circ$. The water has gained $100 \times 3.2 = 320$ cal., and the mercury has lost an equal number of calories. Since 100 g. of mercury lose 320 cal. in cooling through 96.8° , 1 g. in cooling through 1° would lose .033 cal.; and since 1 g. of water loses 1 cal. in cooling through 1° , the specific heat of mercury is .033.

EXERCISES

1. What is meant by saying that the specific heat of water is twice that of ice?
2. What will be the temperature resulting from the mixing of 100 g. of water at 90° with 200 g. of water at 10° ?
3. How many grams of ice water must be poured into 50 g. of water at 80° in order to lower the temperature to 10° ?
4. If 70 g. of water at 25° are mixed with 30 g. of water at 75° , what will be the temperature of the mixture?
5. 100 g. of mercury at 100° are mixed with 100 g. of water at 20° . The resulting temperature is 22.6° . Find the specific heat of mercury.
6. 100 g. of aluminium (sp. ht. = .22) at 80° are placed in 150 g. of water at 10° . What is the temperature of the mixture?
7. 500 g. of water at 20° are placed in a platinum calorimeter weighing 148 g. and 500 g. of mercury at 99.5° are then added. If the final temperature is 22.5° , what is the specific heat of mercury? (sp. ht. of platinum = .032.)
8. A piece of nickel at 100° is placed in an equal mass of water at 0° . If the final temperature is 9.8° , what is the specific heat of the nickel?
9. 100 g. of water at 60° are poured into a copper vessel of mass 140 g. and temperature 10° . The common temperature is 54° . What is the specific heat of the copper?

CHANGE OF STATE

288. Melting Point. — The temperature of a mixture of ice and water remains the same no matter what their relative proportions may be, provided the mixture is kept well stirred so that full opportunity is given for an interchange of heat. This is also true of every other pure and crystallizable substance; the change from the solid to the liquid state and vice versa takes place at constant temperature. This temperature is called the *melting point* (also *fusing point* or *point of fusion*). The converse of melting is *freezing*, *congealing*, or *solidifying*. While a pure, crystalline solid begins to melt at a perfectly well-defined temperature, the melting point is lowered when some substance soluble in the melted substance is present. Such substances as glass, wax, and steel are in reality mixtures of two or more substances, and have no sharply defined melting points. These substances become soft and plastic through a certain range of temperature, the transition from solid to liquid being gradual. It is because of this property that glass can be bent and molded, and iron forged and welded.

289. Freezing Point. — There is no definite temperature at which a liquid begins to solidify. It may be cooled several degrees below its melting point if care be taken not to disturb the vessel containing it and not to allow any of the solid substance to come in contact with the cooling liquid. A slight jarring of the vessel or the addition of ever so minute a fragment of the solid substance, however, sets up the congealing process, and the temperature promptly rises to that of the melting point of the

290. Latent Heat. — When a solid at its melting point is exposed to a source of heat, such as a flame, the thermometer continues to register the same temperature until the solid is entirely melted; then only does its temperature rise. This phenomenon was first studied when the caloric theory was in good standing, and, in accordance with the theory, the terms *sensible heat* and *latent heat* were introduced, the former denoting heat that changes the temperature of a substance, and the latter, heat that changes its state without affecting its temperature. To the adherents of the caloric theory the heat which did not manifest itself by a change of temperature so as to affect the senses was concealed or hidden from the thermometer, and was therefore termed *latent* from a Latin word meaning *hidden*. We no longer believe in the existence of caloric, and retain the name more from habit than from necessity. We now know that heat is a form of energy which, to effect the transformation of a solid into a liquid, must do work on the molecules of the substance.

291. Heat of Fusion. — To change a definite mass of a solid into a liquid requires a definite amount of heat; and conversely, when a liquid freezes, the amount of heat given out equals that needed for liquefaction. The *heat of fusion* is the number of calories needed to melt without change of temperature one gram of a substance. Hot water of known mass and temperature furnishes a convenient source of heat, the amount of which can be readily measured. Thus, if m g. of ice at $-t_2$ be introduced into m_1 g. of water at t_1 , and the common temperature after the ice has melted is t , the amount of heat given up by the water to the ice is $m_1(t_1 - t)$ cal. This heat has done three things: (1) it has warmed the ice from $-t_2$ to 0° ;

(2) it has melted the ice; (3) it has warmed the m g. of melted ice (water) from 0° to t . The first amount of heat is equal to $.5 mt_2$ (the specific heat of ice being .5). The amount of heat required to melt the ice is mL , where L denotes the latent heat of fusion; and the heat needed to raise the temperature of the melted ice to t is mt .

Equating, we have

$$m_1(t_1 - t) = .5 mt_2 + mL + mt; \quad (5)$$

whence
$$L = \frac{m_1(t_1 - t) - .5 mt_2 - mt}{m}. \quad (6)$$

If the ice is at 0° when it is introduced into the water, the term $.5 mt_2$ drops out, so that

$$L = \frac{m_1(t_1 - t) - mt}{m}. \quad (7)$$

EXAMPLE. 150 g. of ice at 0° were placed in 500 g. of water at 80° , and the temperature after the ice melted was 43.1° . What is the heat of fusion of ice?

Solution. The heat given out by the water was

$$500(80 - 43.1) \times 1 = 18450 \text{ cal.}$$

The heat required to melt the ice was $(150 \times L)$ cal., and that to raise the melted ice from 0° to 43.1° was

$$43.1 \times 150 \times 1 = 6465 \text{ cal.}$$

Then

$$18450 = 6465 + 150 L,$$

and

$$L = \frac{18450 - 6465}{150} = 79.9.$$

This means that it takes as many calories to melt one gram of ice as it does to raise the temperature of one gram of water through nearly 80° .

292, Change of Volume in Melting. — Most substances expand on melting, *i.e.* they occupy a larger volume when

liquid than when solid. But water and type metal are notable exceptions to the rule. Ice is therefore lighter than water, and type metal makes sharp castings. If water were not heavier than ice, our ponds and lakes would become solid masses during the winter, and during the summer but a thin layer at the top would be melted. The effect upon climate can be imagined.

293. The Ice Calorimeter.— Suppose two blocks of ice with plane surfaces to be fitted snugly the one upon the other (Fig. 153), and the

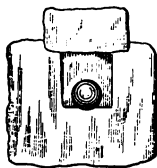


FIG. 153.— Ice Calorimeter.

lower block to have a cavity hollowed out in it. Let a piece of metal heated to a high temperature be dropped into the cavity that has been previously wiped perfectly dry, and let the cover block be immediately fitted on. The metal in cooling down to 0° melts some of the ice, and the water resulting can be absorbed by a sponge and weighed. Suppose that m g. of a metal with a specific heat s is heated to t° and that when placed in an ice calorimeter it melts m' g. of ice. If the amount of heat needed to melt 1 g. of ice at 0° be denoted by L , then $m'L$ cal. will have been given up by the metal in cooling from t° to 0° . But all of the heat given out by the metal in cooling, which is equal to mst , is used in melting the ice, the blocks being thick enough to prevent the escape of any appreciable amount of heat. Hence

$$m'L = mst. \quad (8)$$

Five quantities appear in this equation, and if any four of them are measured, the fifth may be computed. While the above form of calorimeter is rather primitive and cumbersome, the principle involved is used in another form due to Bunsen. In his form, instead of weighing the ice melted, the change in volume due to the change of state is measured (Fig. 154).

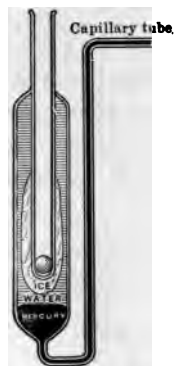


FIG. 154.— Bunsen's Ice Calorimeter. When a hot body is placed in the inner tube, some of the ice is melted. The volume is thereby diminished, and the mercury in the capillary tube moves to the left.

294. Influence of Pressure on Melting Point. — Solids that expand as they liquefy have their melting points raised by an increase of pressure, while if they contract, their melting points are lowered by pressure. So slight are the effects of pressure on the melting point that we should not consider them but for the case of melting ice. When two pieces of ice at 0° are pressed firmly together, they may be made to stick together even under warm water. The phenomenon is called *regelation*, and is explained by the fact that the melting point of ice is lowered by pressure. When fragments of ice at 0° are pressed together a little of the ice between the pressed surfaces melts, the heat required being derived from the ice itself. As soon as the pressure is lessened, the film of water freezes again, thus cementing the two pieces firmly together. In making snowballs, regelation is brought into play; the small particles of ice constituting snow, when pressed together at 0° , cohere (Fig. 155).

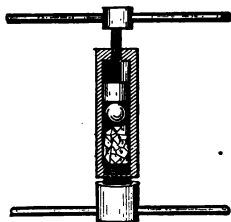


FIG. 155. — Pressure and Melting Point. A ball is placed on top of some ice in the cylinder. After pressure has been applied by means of the screw and the cylinder then opened, the ball is found below the ice. The ice melts under pressure and the ball sinks. When the pressure is removed, the water freezes again.

295. Heat of Solution. — Just as heat is absorbed in the liquefaction of a solid, so is it absorbed when a solid dissolves, unless there be some chemical action, when it may happen that heat is evolved. But whereas melting can take place at one temperature only, solution can go on through a wide range of temperature. The heat required for solution is taken from the liquid and solids themselves, and from the surrounding vessels and objects. This behavior is utilized in freezing mixtures, which are commonly composed of ice and salt. The source of heat for the melting of the ice and consequent solution of the salt is mainly the cream in the freezer, which, as it gives up heat, is frozen.

296. Freezing Point of Solutions. — The freezing point of a solution is lower than that of the pure liquid. Salt sprinkled on ice on sidewalks in winter causes the ice to melt and form a solution that congeals only when the temperature falls to about -20°C . Where this temperature is not attained, ice may be removed by the application of salt. Should the temperature, however, fall below -21° , the solution would freeze hard and conditions would be even worse than before.

297. Vaporization. — When a dish of water is left standing in the air, the water gradually disappears; it changes into vapor that mixes with the air and passes off. If the dish is gently heated, the change into vapor takes place more and more rapidly, but is confined to the surface of the liquid until a temperature of about 100° is reached; bubbles then appear within the liquid and the water is said to boil. This passing of a substance from the liquid into the gaseous state is called *vaporization*; the converse process is called *condensation*. When, in the case of liquids, the change takes place quietly and only at the surface, it is called *evaporation*. When a solid changes directly into the gaseous state without passing through the intermediate liquid state, it is said to *sublime*.

298. Sublimation. — The fact that solids evaporate, or rather *sublime*, does not obtrude itself very often upon our attention; but familiar instances are at hand. The corners of a sharply cut block of ice gradually become rounded off even if the ice is kept on the shady side of a building during the coldest weather. This could happen only through the sublimation of the ice. Similarly, a lump of camphor exposed to the air slowly wastes away without melting.

299. Vapor Pressure. — In the open air evaporation will continue until all the liquid is converted into vapor, which will diffuse away into the air. But if the liquid is confined in a gas-tight vessel, the process of evaporation finally comes to a standstill. If the vessel be provided with :

manometer, it will be seen that at every different temperature the vapor exercises a different pressure. This *vapor tension* or *pressure* increases as the temperature rises (Fig. 156).

300. Dalton's Law. — If there was air above the liquid when it was inclosed in the gas-tight vessel, the pressure of the vapor alone is observed to be the same as if no air were present. The pressure of the vapor merely adds itself to that of the air. The same is true of any number of gases or vapors, provided they do not act chemically upon one another. This general fact goes by the name of Dalton's law: *Each gas or vapor present in a mixture of gases or vapors exerts the same pressure as it would if it alone were present; or, in other words, the pressure of a gaseous mixture is the sum of the partial pressures of its component gases.*

301. Boiling. — The temperature of water heated in an open vessel rises to its boiling point, at which the temperature of both steam and water becomes stationary and continues so until the water is all boiled away. If the water is inclosed in a gas-tight vessel, it may be heated to a much higher temperature before it boils. The water in an open vessel is subjected to the pressure of the atmosphere; hence, variations in barometric pressure call forth variations in the boiling point. If the water merely evaporates, the molecules of its vapor have time to diffuse through the molecules of the air without disturbing the air as a whole. But



FIG. 156. — Vapor Tension. A liquid introduced into a Torricellian tube changes into vapor, which pushes the mercury down.

when water boils, the formation of vapor is so rapid that the atmosphere as a whole is thrust aside to make room for the steam. *The boiling point accordingly is the highest temperature to which a liquid can be raised in an open vessel; it is the temperature at which the vapor pressure of a liquid becomes equal to that of the atmosphere.*

As, in boiling a liquid, the heat is usually applied at the bottom of the vessel, the lower layers are at a higher temperature than the upper, and bubbles of vapor are formed first in the lower portions of the liquid. As the bubbles rise into the cooler portions of the liquid, they collapse, and produce the well-known "singing" familiar in the case of the teakettle. When all the water is "boiling hot," the bubbles reach the surface and the water boils freely.

302. Heating in Vacuo. — A round-bottomed flask is filled half full of water, the water is boiled until its steam has driven out all the air, and the flask is then closed with a rubber stopper, at once removed from the flame,

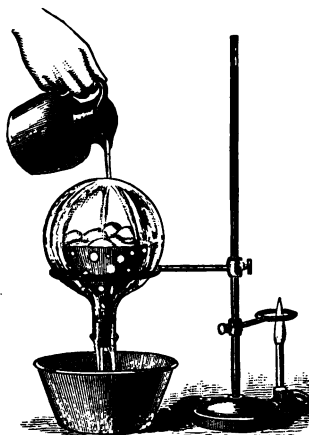


FIG. 157. — Franklin's Experiment. The boiling point depends upon the pressure.

and inverted (Fig. 157). The temperature soon falls below the boiling point under atmospheric pressure, but if cold water be poured upon the flask, the water within the flask will again boil vigorously for a little while. By cooling with ice water, the water will boil even when its temperature is as low as 15° . The cold water poured upon the flask condenses some of the vapor so that the tension of the remaining vapor is lessened. More vapor promptly forms, however, and the liquid boils under reduced pressure. Application is made of this fact in several industrial operations. Thus less heat is required and the cost of fuel made correspondingly less by conducting evaporations under

diminished pressure. Besides, some substances, notably sugar, are subject to undesirable changes when heated up to 100° , and hence their solutions are concentrated by driving off the water under diminished pressure.

303. Dew Point. — Water vapor is a normal although variable constituent of the atmosphere, and the variation in its amount is largely responsible for the fluctuations of the barometer. Air confined in a gas-tight vessel at a constant temperature over water soon becomes permeated with aqueous vapor which exerts the same pressure that it would have if the air were not also present. The pressure of the water vapor adds itself to that of the air so that the pressure as observed on a manometer is equal to the sum of the partial pressures of air and vapor. Air is said to be saturated with water vapor when at a given temperature no more vapor mixes with it, even though the air be confined over water; at temperatures above this definite temperature of saturation, the air is said to be unsaturated. When the temperature falls below the point of saturation, some of the vapor condenses, forming *dew* if deposited as a liquid film on solid objects, or *cloud*, *mist*, or *fog*, if remaining as tiny droplets in the air. The temperature at which this condensation takes place is called the *dew point*. In dry air, the dew point is low; in damp air, it is high.

304. Rate of Evaporation. — The amount of a liquid that evaporates in a stated time is the greater: (1) as the temperature is higher, for, as the temperature is higher, the speed of the molecules of the liquid increases, and therefore a larger number can in a given time make their way through the free surface and pass off as vapor; (2) as the free surface of the liquid is larger, for an increase of free surface offers more chances for the escape of the

molecules; (3) as the pressure upon the liquid is less, for by removing some of the air or vapor above the free surface, the molecules that escape are not hindered so much in their motions and can move rapidly away from the liquid and make room for other molecules; (4) as the air above the liquid is changed more rapidly, for, as soon as the air above the free surface becomes saturated, evaporation ceases, because the gaseous molecules will strike the surface and pass into the liquid just as fast as the molecules within the liquid escape through its surface; evaporation and condensation in that case go on at the same rate, so that the mass of the liquid does not alter.

305. Laws of Boiling. — The following general facts with respect to the boiling of liquids have been experimentally established.

I. Every liquid boils at a definite temperature which is always the same under like conditions.

II. When the boiling point is reached, the temperature remains constant until the liquid is all vaporized.

III. The pressure of the vapor given off during boiling just exceeds the pressure of the atmosphere.

IV. The boiling point of a liquid is raised when another substance, much less volatile, is dissolved in it.

306. Altitude and Boiling Point. — Since atmospheric pressure decreases with the altitude and since a diminution of pressure lowers the boiling point, the height of a place above sea level may be found from boiling point determinations. A change of elevation of 295 m. makes a difference of one degree in the boiling point. Thus, at Quito, water boils on an average at 90° . Hence its elevation above sea level is $295 \times (100 - 90) = 2950$ m.; the elevation thus found corresponds closely to that determined by other methods.

307. Distillation. — The process known as *distillation* consists (1) in boiling a liquid, (2) in separating the vapor

from the liquid, and (3) in condensing the vapor. The cooling is done in a vessel called a *still*. The vapor rises to the *still head*, and passes into the *condenser*, a tube or pipe around which cold water is made to circulate in such a way as to take heat from the vapor and convert it again into a liquid. The condenser may have a worm-

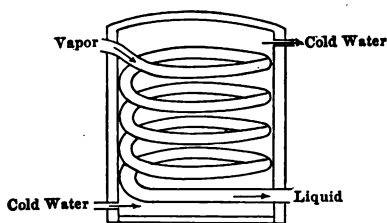


FIG. 158. — Worm Condenser.

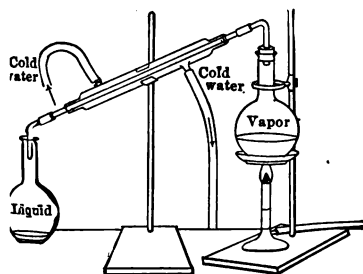


FIG. 159. — Liebig's Condenser.

thermometers are distilled in order to get rid of certain impurities. The gold and silver that in certain mining operations are obtained dissolved in mercury are saved as a residue after distilling off the mercury. Distillation differs from evaporation and boiling only in that precautions are taken to save either the distillate or the residue, or both.

308. Boiling Points of Mixtures. — The boiling point of a mixture of two liquids usually lies between those of its constituents, and depends on the proportions of these constituents. During distillation the liquid with the lower boiling point as a rule passes over in greater abundance than the liquid with the higher boiling point. The "rectification of spirituous liquors" consists in distilling a mixture of alcohol (boiling

shape, whence the word "worm" often applied to such a condenser (Fig. 158), or it may be straight as in the case of the so-called *Liebig's condensers* (Fig. 159).

Distillation is employed in the separation of non-volatile substances from the liquid in which they are dissolved. This is done either to purify the liquid or to obtain the residue. For example, the water and the mercury used in barometers and

point = 78°) and water prepared by the fermentation of grains or fruits, the distillate containing a larger proportion of alcohol than the original mixture.

309. Heat of Vaporization. — That heat is required to change a liquid into a gas is shown by the fact that, although heat is constantly supplied to a boiling liquid, its temperature remains the same. The heat supplied becomes latent. *The heat of vaporization of a liquid is the number of calories required to convert a gram of it at its boiling point into vapor at the same temperature.* The *heat of condensation* of a vapor is *the number of calories given out by one gram of it when it condenses at the boiling point of the liquid.* As condensation is the opposite of vaporization, the two amounts of heat for the same substance are equal.

310. Volume Changes during Vaporization. — The changes of volume in vaporization are under ordinary circumstances much greater than in the case of fusion. For example, ice occupies about one ninth more space, and steam about 1700 times more than does an equal weight of water. More heat energy is required to vaporize than to melt most substances, and, therefore, as a rule, heats of vaporization are larger than heats of fusion.

311. Ice Machines. — The action of an ice machine depends upon the fact that heat is required to vaporize a substance. If a vessel of water

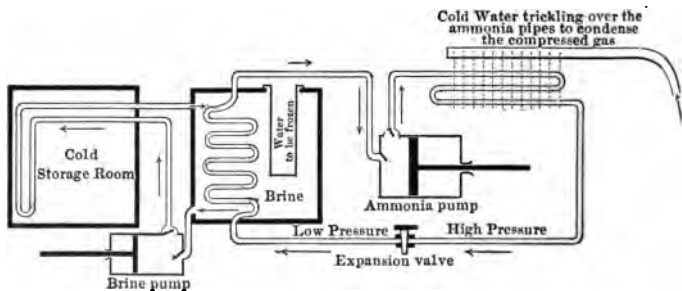


FIG. 160. — Artificial Ice and Cold Storage Plant. The ammonia cools the brine, which in turn abstracts the heat from the water in the tank, or from the air in the cold storage room.

be placed in a liquid such as ammonia that has a boiling point twenty or thirty degrees below the freezing point of water, the heat required to vaporize the ammonia will be supplied by the water which is thereby converted into ice. In practice, provision is made for the recovery of the ammonia gas which is then liquefied by compressing it by means of powerful pumps (Fig. 160).

312. Method of Mixtures.—A common method of determining the heat of condensation of steam consists in passing steam into a known quantity of cold water, and noting the weight of the steam condensed and the temperature changes. Suppose that m g. of steam at 100° are condensed in m_1 g. of water at t_1 , and that the resulting temperature is t . Then $m_1(t - t_1)$ represents the amount of heat received by the water. The m g. of steam at 100° in turning into m g. of water also at 100° will give out l cal., l standing for the heat of condensation of one gram of steam. The m g. of water at 100° will also give out $m(100 - t)$ cal. in cooling down to the common temperature t . As the experiment must be so conducted that practically all of the heat of the steam is used in warming the water, we have

$$m_1(t - t_1) = ml + m(100 - t); \quad (9)$$

$$\text{whence} \quad l = \frac{m_1(t - t_1) - m(100 - t)}{m}. \quad (10)$$

In accurate work, the water equivalents of the calorimeter, thermometer, etc., must be added to the weight of the water.

EXAMPLE.—17.6 g. of steam at 100° were condensed in 500 g. of water at 18° , the resulting temperature becoming 39° . Find the latent heat of condensation of steam.

Solution. To warm 500 g. of water from 18° to 39° , a temperature interval of 21° , requires, since the specific heat of water is unity, $500 \times 21 \times 1 = 10500$ cal. When 17.6 g. of steam at 100° condense into 17.6 g. of water at 100° , the amount of heat given out is 17.6 l cal. And when 17.6 g. of water cool

from 100° to 39° , the number of calories evolved equals $17.6 \times 61 \times 1 = 1073.6$. The amount of heat received by the cool water is equal to the amounts of heat given out by the steam by its condensation and by the hot water it yields.

Therefore,

$$500 \times (39 - 18) = 17.6 l + (100 - 18) \times 17.6;$$

whence

$$l = 536.$$

313. The Steam Calorimeter.— Suppose that a piece of metallic gauze at room temperature, say t° , the weight and specific heat of which are m g. and s , respectively, is introduced into a vessel kept filled with steam at 100° . The gauze condenses and retains, because of capillary attraction, a weight m' of condensed steam (water), which furnishes by its condensation enough heat to raise the temperature of the gauze to 100° . If l denote the heat of condensation, then lm' represents the amount of heat given to the metal by the condensing steam. The amount of heat needed to raise the temperature of the metal from t to 100° is $ms(100 - t)$ cal. As the heat of condensation is employed solely in warming the metal to 100° , these two quantities of heat are equal, and

$$lm' = ms(100 - t). \quad (11)$$

This method of measuring heat can be applied to finding either the heat of condensation of steam or the specific heat of the metal. Thus, by transforming Equation (11) we get

$$l = \frac{ms(100 - t)}{m'}, \quad (12)$$

and

$$s = \frac{lm'}{m(100 - t)}. \quad (13)$$

After determining the weights of the metal and condensed vapor, as well as the change of temperature, we can, knowing the heat of condensation, compute the specific heat, or knowing the specific heat, we can find the heat of condensation.

314. The Graph for Change of State and of Temperature.— During the melting or the boiling of a substance, the temperature remains the same. If, therefore, the temperature of a substance is changing, its state is not changing. With change of state there is no change of temperature; and with change of temperature there is no change of state. The specific heat of the same substance has a different value for each

state. For example, the specific heat of ice is .5, that of water, unity, and that of steam is .48.

In Fig. 161 equal distances marked on one line represent calories, and those on the line perpendicular to it, temperatures. Suppose one gram of ice to be warmed from -20° to 0° . Its temperature will rise, and an amount of heat will be required, equal to $1 \times 20 \times .5 = 10$ cal. The slant line AB will represent, then, both the fact that the temperature rises, and that the amount of heat in the ice increases. At 0° the ice will melt, and its temperature will remain the same until it is all water.

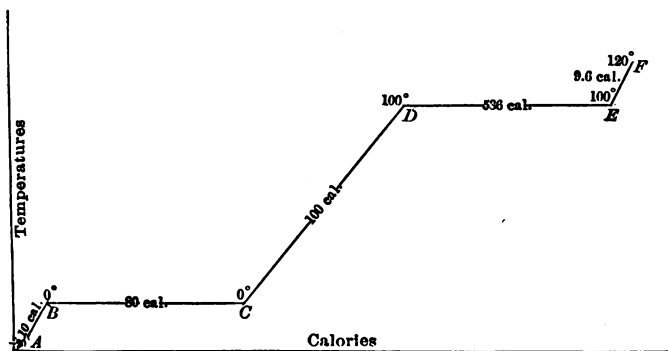


FIG. 161. — Graph for Change of State and of Temperature.

The line BC shows by its remaining parallel to the "calories line" that there is no change in temperature, and its length of 80 units shows that 80 cal. are required to melt one gram of ice. The slant line CD shows both rise of temperature and increase of heat, the former being 100° and the latter 100 cal. The parallel line DE shows that no change of temperature occurs until the water at 100° is all changed into steam at 100° ; and its length is 536 units, the number expressing the latent heat of vaporization of water. When the steam is heated, say to 120° , an amount of heat equal to $(1 \times 20 \times .48)$ cal. is required, and this change of temperature, as well as of heat content, is shown by the slant line EF .

EXERCISES

1. Why do clothes dry faster on a windy day than on a calm day?
2. Why is it that frost forms on the inside but not on the outside of a window pane?

3. Will a piece of solid type metal float or sink when placed in melted type metal?

4. Gold, silver, nickel, and copper occupy a smaller volume when solid than when melted, while the opposite is true of type metal. Why, then, are coins made by stamping the metal in dies, while type is made by casting in molds?

5. What valuable property is given to glass by the circumstance that it does not have a definite melting point?

6. When steam issues from the spout of a teakettle containing boiling water, it is invisible. The cloud of vapor which appears at a little distance from the spout soon dissipates. How are these facts accounted for?

7. At the top of Pike's Peak, water boils at 86° . What is the altitude of the peak?

8. Black, in 1757, took two similar flasks and thermometers, filled one flask with water at 0° and the other with an equal weight of ice at 0° . He found that after leaving them side by side in a room at 20° , the temperature of the water rose to 16° in the course of an hour, while it took the ice five hours to melt. What is the latent heat of fusion of ice as deduced from this experiment?

9. Equal weights of melting ice and hot water were mixed, and the temperature of the resulting water was 0° . What was the temperature of the hot water?

10. How much water at 40° will melt 500 g. of ice at 0° ?

11. What weight of water at 70° must be mixed with 100 g. of ice at 0° in order to melt the ice and raise the final temperature to 20° ?

12. How much is the waste water in an ice chest increased if 5000 g. of water at 25° are placed in the chest and its temperature falls to 5° ?

13. It is found that a piece of aluminum with a mass of 250 g., when heated to 100° and then at once placed in an ice calorimeter, melts 68.1 g. of ice. What is the specific heat of aluminum, the latent heat of fusion of ice being 80 cal.?

14. 100 g. of water were distilled at 100° , being condensed by passing through a tin "worm" weighing 250 g. which was surrounded by 5480 g. of water. If the temperature of the water in the condenser was 6° at the beginning of the distillation, what was its final temperature? (Latent heat of vaporization = 536 cal.; sp. ht. of tin = .056.)

CHAPTER XIII

HEAT AND WORK

MECHANICAL EQUIVALENT OF HEAT

315. Heat Energy. — We have already considered how heat came to be regarded as a form of energy. It is, so to say, the lowest form of energy because into it all other forms tend to pass. As there is a tendency in nature toward an equalization of temperatures, it follows that ultimately all forms of energy will be reduced to heat energy, and that all bodies will be at the same temperature. This conclusion becomes all the more convincing when it is considered that a given quantity of heat energy can only in part be converted into other energy forms, some of it always remaining as heat energy at a lower temperature. The availability of energy as a whole is becoming less and less because of this incessant transformation into heat.

316. The Mechanical Equivalent of Heat. — The apparatus that Joule used to determine the precise relationship between mechanical work and heat consisted of a vessel of water (Fig. 162) in which were paddles that could be rotated by means of a falling body. The heat produced by the churning of the water was measured, as

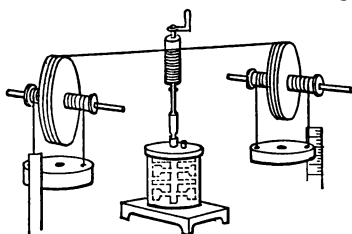


FIG. 162. — Joule's Apparatus.

was also the work done by the falling body. To move a body of mass m vertically upward through the distance h , the work W is done, and, if g denote the value of the acceleration due to gravity, then

$$W = mgh; \quad (1)$$

and this amount of work, when corrected for the friction of the bearings, is also that done on the stirrer by the body in falling. If M denote the mass of the water stirred, together with the water equivalents of the stirrer, thermometer, and vessel, and $t_1 - t$ the rise of temperature due to the transformation of mechanical energy into heat energy, then the amount of heat H is

$$H = M(t_1 - t). \quad (2)$$

Now what Joule found out in his numerous experiments and what all subsequent experimenters along this line have found out is, that for every unit of mechanical energy which disappears as the body falls, a definite and constant quantity of heat appears in the water stirred. The ratio of W to H is always the same, so that

$$\frac{W}{H} = \frac{mgh \text{ (measured in ergs)}}{M(t_1 - t) \text{ (measured in cal.)}} = a \text{ constant.} \quad (3)$$

The value of this constant has been found to be 4.19×10^7 . This means that 4.19×10^7 ergs or 4.19 joules of mechanical energy when transformed into heat energy warm one gram of water one degree Centigrade.

317. Values of the Mechanical Equivalent of Heat in Other Units.—

In the English system of measures, the unit of heat is the amount of heat required to raise the temperature of one pound of water one degree Fahrenheit. The English engineer's unit of work is the foot pound. The mechanical equivalent of heat in these units is 774 ft. lb., *i.e.* it

requires as much energy in the form of heat to raise the temperature of one pound of water 1° F. as it does of mechanical energy to raise a body weighing one pound 774 ft. high.

The engineers of metric countries measure work in kilogrammeters, and they use kilogram-calories instead of gram-calories. The mechanical equivalent of heat in these units is 427; that is to say, it takes as much work to elevate one kilogram 427 m. high as it does to raise the temperature of one kilogram of water 1° C.

EXERCISES

1. Account for the fact, first observed by Rumford in 1788, that the barrel of a musket fired without a bullet becomes hotter than when a bullet is discharged.

2. Two blocks of ice at 0° C. are rubbed together in a vacuum. How many ergs of work must be done to melt 10 g. of ice by means of friction alone?

3. A piece of iron having a mass of 100 g. is cut in two by 100 forward strokes of a hack saw. If the average force applied is 419×10^4 dynes and the length of the stroke is 10 cm., what is the change of temperature of the iron, on the assumption that all the heat is used in warming the iron alone? (sp. ht. of iron = .112.)

4. With what velocity must a 20-g. lead bullet (sp. ht. = .033) strike a target in order to raise the temperature of the bullet through 100° ?

5. A lead bullet with a mass of 56 g., a velocity of $30 \times 10^3 \frac{\text{cm.}}{\text{sec.}}$, and a temperature of 20° strikes a target with the result that half of the energy is spent in heating the bullet. What temperature does it attain?

6. From what height must a gram of ice at 0° fall in order to generate enough heat by impact to melt itself?

7. A kilogram of sand is dropped from a height of 1000 cm. into a kilogram of water. What is the change of temperature of the sand and the water, their temperatures before mixing being identical? (sp. ht. of sand = .16 and $g. = 980 \frac{\text{cm.}}{\text{sec.}^2}$)

8. A certain room contains 50 kg. of air (sp. ht. = .24) at 10° . Steam at 100° passes into a radiator in the room, and the water resulting from the condensation of the steam leaves it with a temperature of 36° . How many grams of steam are used in warming the room to 20° ?

HEAT ENGINES

318. Changes of Temperature and Volume in Gases.—

When a gas is being compressed, its temperature rises (Fig. 163), and when it is expanding, its temperature falls. The reason why changes of temperature accompany changes of volume is readily explained by the kinetic theory. Consider a piston compressing air in the cylinder of a bicycle pump. The molecules of the gas rebound from the advancing piston with a greater speed than they do when the piston is stationary, and since temperature depends upon molecular speed, the temperature rises. When the compressed gas expands, the speed of the molecules is diminished because the surface from which they rebound is receding from them.

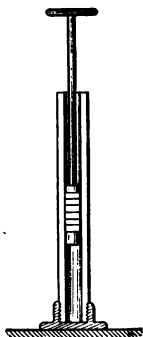


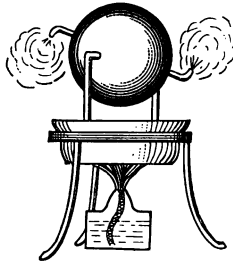
FIG. 163.—Fire Syringe. A bit of a readily inflammable substance, such as gun cotton, may be ignited by rapidly pushing the piston downward.

319. Energy of Compressed Gases. —To diminish the volume of a gas, force must be exerted and work thereby done. Because of the work done, a compressed gas has energy, which by the aid of suitable devices may be

made to do work. If the gas is confined in a vessel with a movable side, the amount of work done is measured by the product of the force exerted upon the movable side by the distance through which it moves. In nearly all cases a compressed gas from which work is to be obtained is confined in a cylindrical vessel closed at one end and provided with a piston. If, for example, the pressure of a gas inclosed in such a cylinder is 10 dynes per cm^2 , and the area of the piston is 10 cm^2 , the total pressure upon the piston is $10 \text{ dynes per cm}^2 \times 10 \text{ cm}^2 = 100$

dynes. If the piston moves 10 cm., the work done is $100 \text{ dynes} \times 10 \text{ cm.} = 1000 \text{ dyne centimeters} = 1000 \text{ ergs}$. Since the distance the piston moves, multiplied by its area, gives the change of volume of the gas, the work done by an expanding gas is equal to the product of its change in volume by its average pressure per unit of area.

320. Engines run by Compressed Gases. — While almost any gas when compressed may be used to do work, the number actually employed for such a purpose is small. Thus we have *compressed air* engines, *gas* or *gasoline* engines, and *steam engines*. In most of the forms given to these engines, the pressure of the gas is increased by raising its temperature. Numerous as are the varieties of engines they are all reducible to two types. In the *reciprocating engine* type the essential feature is a cylinder in which moves a piston. The expanding gas is made to act upon this piston in such a way as to move it to and fro, and by means of a crank this to-and-fro motion is changed into rotary motion. In the *turbine type* the compressed gas is directed against blades or paddles set in the rim of a wheel, which is thereby made to rotate, just as wind or water turns wind-mills or water wheels.



321. The Steam Engine. — Attempts were early made to get work performed by the expansive force of steam (Fig. 164). Success did not crown these efforts, however, in any great measure, until towards the end

FIG. 164. — Hero's Engine. Steam is generated in the boiler heated by the lamp and passes through the hollow columns into the hollow sphere. The reaction of the air upon the escaping jets of steam makes the sphere rotate.

of the eighteenth century, when James Watt invented the form of steam engine which without essential modification has endured to the present day. Watt's engine is of the reciprocating type and its essential parts are: (1)

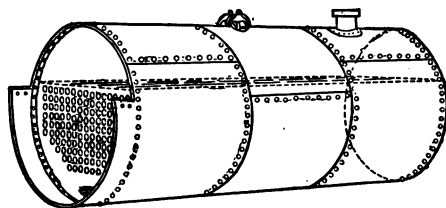


FIG. 165. — Tubular Boiler. The flames pass through the tubes around which the water circulates.

the *boiler*; (2) the *cylinder and piston*; (3) the *steam chest* with its *slide valves* worked by the *eccentric*; (4) the *crank and connecting rod*; and (5) the *flywheel*.

The energy of the steam engine is derived from the energy of the burning fuel, and is stored up in the compressed steam.

322. The Boiler. — Water is converted into steam in the boiler, and as it is generated faster than it is used, its pressure becomes greater than that of the atmosphere because of superheating (Fig. 165). Since the strength of the boiler is limited, a *safety valve* (Fig. 166) is provided to discharge the steam automatically if its pressure should become dangerously high.

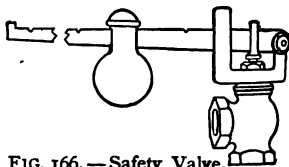


FIG. 166. — Safety Valve.

323. The Steam Engine in Action. (Fig. 167.) — To start the engine, steam is admitted into the steam chest whence it passes through a slide valve and a port to one side of the piston. The piston moves before the expanding steam until it nears one end of the cylinder, when the slide valves are shifted automatically by means of the eccentric. The steam then passes from the steam chest through the other slide valve and port to the other side of the piston, and forces it to move back. The steam already admitted into the cylinder escapes from the exhaust pipe during the

turn of the piston to its first position. The admission of steam, now on this side and now on that side of the piston, makes it move to and fro, its reciprocating motion being converted by means of the crank and connecting rod into the rotary motion of the flywheel. The flywheel is made large and massive in order that its kinetic energy may be sufficient to carry the piston over the *dead points*, these being close to the ends of the cylinder where the direction of the motion of the piston reverses. The inertia of the flywheel makes the rotation of the shaft more uniform.

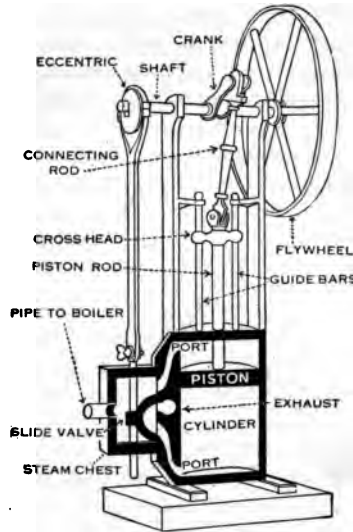


FIG. 167.—Vertical Steam Engine.

324. Efficiency of Steam Engines.—Since heat energy is supplied to a steam engine and mechanical work is delivered by it, its efficiency is the ratio of the amount of work done in a given time to the amount of heat energy furnished. Engineers commonly express the amount of work done in *horse-power hours*. As one horse power = $746 \times 10^7 \frac{\text{ergs}}{\text{sec.}}$ and one hour contains 3600 sec., a horse-power hour = $3600 \times 746 \times 10^7 = 26856 \times 10^9 = 269 \times 10^{11}$ ergs. Now suppose an engine consumes 1000 g. of fuel for every horse power it delivers for an hour, and that 1000 g. of this fuel when burned yield 800×10^4 cal. Multiplying this last number of calories by the mechanical equivalent of heat, we have $800 \times 10^4 \times 4.19 \times 10^7 = 33.52 \times 10^{18} = 335 \times 10^{12}$ ergs, which is the amount of energy supplied the engine per horse-power hour. Since efficiency equals the ratio of work done to energy supplied:

$$\frac{\text{Work done}}{\text{Energy supplied}} = \frac{269 \times 10^{11}}{335 \times 10^{12}} = \frac{26.9}{335} = .0803 = 8\%.$$

This result means that of all the energy furnished by the burning coal, 92% is used up in making the engine go, and only 8% is available for doing useful work. Different efficiencies are of course found for different engines. Locomotives seldom show an efficiency as high as 8%, and even the best of stationary engines have efficiencies not much exceeding 20%. The efficiency is increased by augmenting the range of temperature through which the engine works. This may be done by raising the temperature in the boiler and lowering the temperature of the exhaust steam. Rarely, however, can large boilers withstand a pressure of more than 15 atmospheres, corresponding to a temperature of about 200° ; and the temperature of the exhaust cannot without undue expense be lowered beyond about 35° . If an engine could be built to work between these temperature limits, it would have an efficiency of nearly 35%.

325. Back Pressure. — When the piston starts on its return stroke, the cylinder is filled with steam, and as the piston moves back, it has to push the exhaust steam out of the cylinder against the pressure of the atmosphere. A *back pressure* equal to that of the atmosphere is therefore exerted against the pressure of the fresh steam so that the actual pressure of the fresh steam against the piston is equal to the pressure in the boiler minus that of this back pressure.

326. Condensers. — If the steam exhausted into a vacuum, this back pressure would be eliminated, and the efficiency of the engine correspondingly increased. By the use of a *condenser* an approximation to a vacuum is obtained. The condenser consists essentially of a coil of pipes empty of air and surrounded by cold water. The exhaust steam on entering these pipes condenses into water so that the back pressure is reduced to that of the water vapor at the temperature of the condenser. While the condensed water has to be pumped out of the condenser against atmospheric pressure, its volume is much less than that of the steam yielding it, and the wasted work done is correspondingly less.

327. Compound Engines. — The efficiency of a steam engine is increased by using steam under high pressure and by expanding the steam in the cylinder enough to bring its pressure down to that of the water in the condenser. But this causes such large changes of pressure that very severe and trying strains are put upon the parts of the engine, and great variations of temperature in the cylinder prove wasteful and lower the efficiency. To prevent these bad effects and yet have as large

a difference as possible between the temperature of the steam in the boiler and in the condenser, the expansion is carried out in two, three, and even four cylinders; that is, the exhaust steam from the first cylinder passes into the second cylinder, the exhaust from the second cylinder into the third cylinder, and so on, until the steam reaches the condenser. Such engines are called *double-expansion*, *triple-expansion*, and *quadruple-expansion* engines.

328. The Steam Turbine.—Although the first heat engine (Fig. 164) was of the turbine type, the practical

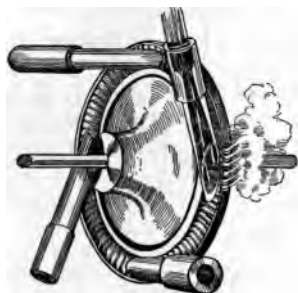


FIG. 168. — De Laval Steam Turbine.

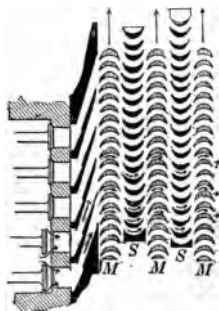


FIG. 169. — Arrangement of Blades in a Turbine.
S, stationary blades;
M, revolving blades.

difficulties in the construction and operation of steam turbines have been surmounted only within the last few years (Fig. 168). Large turbines work through a range of temperature corresponding to a range of pressure from about 15 atmospheres to $\frac{1}{80}$ of an atmosphere, and hence have higher efficiencies than reciprocating engines. They are free from the jarring which is so trying to reciprocating engines, and work with comparatively little friction. Because of these and other advantages turbines are supplanting reciprocating engines in steamships and for running dynamos.

329. Internal Combustion or Gas Engines. — When a gas or vapor burns, the products of the combustion occupy a much greater volume than does the mixture of unburned gas and air, mainly because the products are heated to a very high temperature. By making the combustion take place in a comparatively restricted space, the increase in volume can be but small. Hence a correspondingly large pressure is produced, and an explosion results when the combustion takes place rapidly. In internal combustion engines this explosion occurs in a cylinder, and the force of the explosion acts upon one side of a piston. The piston is thereby moved, and by means of a crank its motion is made to turn a flywheel, the shaft of which is connected with the particular machinery that is to be driven. To prevent overheating, the cylinder is either surrounded with a water jacket through which a current of water is made to flow, or has corrugated projections round it to present a large surface to the air, which then takes up and carries off the surplus heat.

As the temperature of the burning gases in the cylinder may be very high, the range of temperature through which the engine works is correspondingly great, and a high efficiency results. Good gas engines should show an efficiency of 25 % and more.

In the action of every form of gas engine there are four operations to be accomplished, the completion of which is called a *cycle*.

I. *Admission of charge* (a mixture of air with the proper proportion of combustible gas or vapor).

II. *Compression of charge.* (By strongly compressing the charge before firing it, greater efficiency is secured.)

III. *Ignition and expansion of ignited charge.* (During this expansion the heat energy turns into mechanical energy; work is done on the piston.)

IV. *Exhaustion, or expulsion of the burnt gases.*

To complete the above four events constituting a cycle, many forms of engines have been devised. All practical forms, however, are either of the *two-cycle* or the *four-cycle* types.

330. Action of Two-cycle Engine. — When the piston is in its lowest position (Fig. 170 *b*), the inlet and the exhaust ports are both uncovered; but when it has risen a little (Fig. 170 *a*), it covers these ports, and a partial vacuum is created within the gas-tight crank case. The pressure of the atmosphere opens the admission port valve, and air charged

with vapor by passing through the carbureter rushes into the crank case. During the downward stroke of the piston the admission port valve closes, as the compressed gas in the crank case exerts a pressure greater than that of the atmosphere. When the inlet port is open, the compressed mixture passes around the piston through the by-pass into the cylinder, and is directed upward by the deflector. At the same time any gas already within the cylinder escapes through the exhaust port. Before any of the charge admitted, however, can escape from the exhaust port, the piston has begun its upward stroke and closed both ports. During the rest of the upward stroke the mixture is compressed

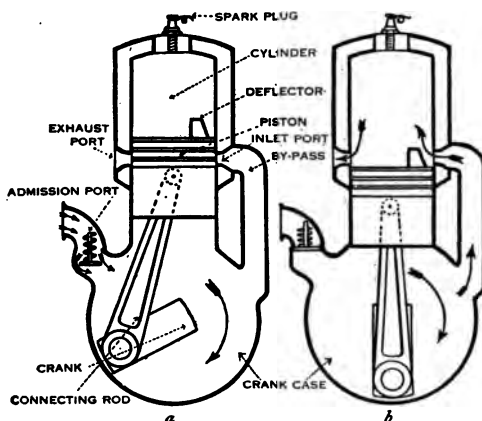


FIG. 170. — *a* and *b*. Two-cycle Gas Engine.

in the cylinder, and is ignited by means of an electric spark when the piston is near its highest position. The piston is driven down by the expanding gases and when near its lowest position, opens the exhaust port so that part of the burnt gases can escape; and a moment later the inlet port is uncovered and a fresh charge passes from crank case into cylinder, sweeping out the remainder of the spent gases. While this fresh charge is being compressed during the upward stroke, a new supply of the mixture is admitted into the crank case. The cycle is completed every two strokes of the piston, whence the name of *two-cycle engine*.

331. Action of Four-cycle Engine. — The four-cycle engine resembles in general appearance the two-cycle type, but is provided with valves

worked by cams that are operated by gears on the shaft (Fig. 171). Suppose the piston to be at the top of its stroke and to be moving downward; the inlet valve opens and a charge passes into the cylinder. This valve closes when the piston reaches its lowest position, and the imprisoned charge is compressed during the upward stroke. Ignition occurs when the piston arrives at the end of the stroke, and the force of the expansion drives it down again. The exhaust valve then

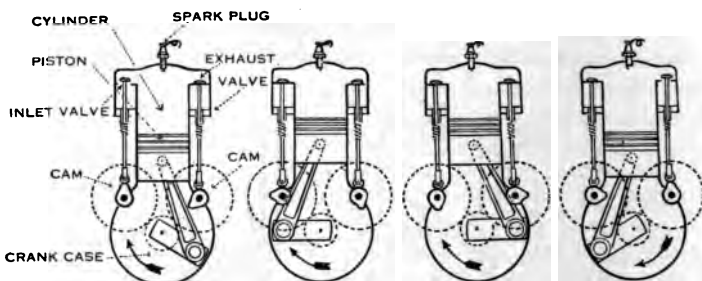


FIG. 171. — Four-cycle Gas Engine.

opens so that the spent gases may escape during the second upward stroke. The cycle is accomplished in four strokes of the piston, whence the name of *four-cycle engine*.

EXERCISES

1. Why is it that when the valve of an inflated pneumatic tire is opened, the escaping air feels cold?
2. Why is it that steam frequently escapes from the safety valve of a locomotive when it is stopping at a station?
3. Hold the palm of your hand close to your mouth and breathe gently upon it. Does the breath feel warm or cool? Blow as hard as you can against your palm held farther away. Account for the difference of sensation which you experience.
4. Steam engines are sometimes called *external combustion engines* to distinguish them from gas engines, which are called *internal combustion engines*. What is the appropriateness of this classification?
5. Why are locomotives not provided with condensers, when these are always used on the large engines of steamboats?

CHAPTER XIV

WAVE MOTION

332. Vibratory Motion. — We have already studied an important case of vibratory motion, namely, that presented by the pendulum (§ 184), and have learned that the characteristic feature of such pendular motion is its regularity in repeating itself, *i.e.* its periodicity. Two other cases of periodic motion are possible. To illustrate, let some heavy body* be hung on one end of a spiral spring suspended from a rigid support. Draw the heavy body (bob) to one side without stretching the spring, and let go. The spring and bob swing to and fro like any pendulum. Such vibrations are *crosswise* to the length of the vibrating body, and hence are called *transverse vibrations*. Let the spring come to rest in a vertical position, pull down the bob a little, and release it. The bob thereupon moves up and down, as the spring contracts and stretches, in *lengthwise* or *longitudinal vibrations*. Again, let the spring come to rest, turn the bob round a vertical axis so as to twist the spring a little, and let go. The spring now twists and untwists, turning the bob regularly now this way and now that. Such *twistwise* periodic motions are called *torsional vibrations*. Of these three kinds of vibrations, the first two are going to occupy much of our attention because of their bearing upon many of the phenomena of Sound,

*A heavy body is not really essential, as the spring alone suffices. Still, by loading the spring, the vibrations take place more slowly, and hence are more easily observed.

Light, and Electricity. Torsional vibrations, however, need only to be mentioned in an elementary book.

333. Conditions for Vibratory Motion. — To pull aside, to stretch, or to twist the spring requires work; and by reason of the work done, the potential energy of the spring and bob is increased. When the force, twisting, stretching, or pulling aside the spring is removed, its own elastic force, or its weight, sets it moving, and its potential energy begins to change into the kinetic form. When the bob arrives at its position of equilibrium where its potential energy is at a minimum, the motion of the spring is not arrested; for the kinetic energy that it has acquired carries it on beyond this point. The kinetic energy now changes into potential, the spring comes to rest for an instant with an amount of potential energy almost equal to that originally imparted to it, and then retraces its path, thereby completing its period. This period is repeated again and again. A vibrating body is, therefore, continually changing its type of energy. The vibratory motion finally dies out as the energy given to the body at first is gradually dissipated into heat which is communicated to the surrounding air.

334. Simple Harmonic Motion. — Let a ball at the end of a cord be swung round at a uniform rate in a horizontal

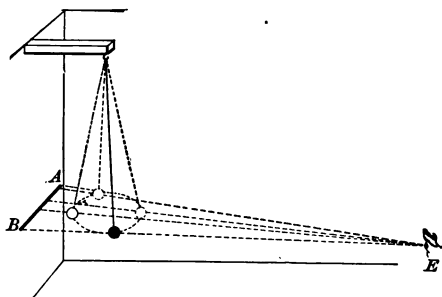


FIG. 172. — Conical Pendulum. To the eye at *E* the ball seems to move in a straight line between *A* and *B*.

circle (Fig. 172). If the eye is placed above or below the bob of such a conical pendulum, the path described by it appears to be circular or elliptical. But if the eye is placed on a level with the plane in which the bob travels, the ball

appears to move to and fro in a straight line, the illusion being the more perfect, the farther away the observer is

stationed. This motion, as it appears to an observer at a great distance, is called *simple harmonic motion* (S. H. M.).

Construction for Simple Harmonic Motion.— Let the actual circular path of the bob be represented by a circle (Fig. 173), and its apparently rectilinear path by the diameter AB . Divide the upper semicircumference into any even number of equal parts, say eight, drop perpendiculars from the points of division upon the diameter, and produce them until they meet the lower semicircumference. The points on the circumference show the positions of the bob at equal intervals of time during one of its rounds, and the points on the diameter its apparent positions.

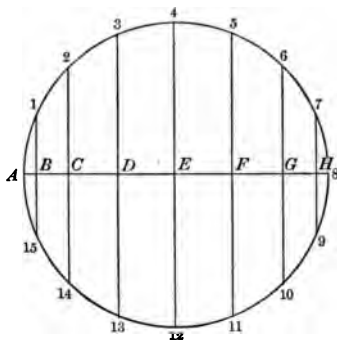


FIG. 173. — Simple Harmonic Motion.

335. Terms used in Simple Harmonic Motion Discussions.

— The *amplitude* is the distance from the middle of the apparent path to either extremity, as EA or $E8$. The *circle of reference* is the actual path traversed by the bob. The amplitude is equal to the diameter of the circle of reference. The *period* is the time interval between the passage of the bob through any point and its next passage in the same direction through the same point. The *phase* is the fraction of a period elapsing since the bob last passed in the positive direction through the middle point E . Thus the period (Fig. 173) is divided into sixteen equal parts or intervals. When the bob is moving to the right (in the positive direction), its phase at F is $\frac{1}{16}$, at G it is $\frac{2}{16}$, and at 8 it is $\frac{4}{16}$. When the bob is moving toward the left (in the negative direction), its phase at E is $\frac{8}{16}$, at D $\frac{9}{16}$, at B $\frac{11}{16}$, etc. The phase, therefore, specifies both the position and the direction in simple harmonic motion.

336. Composition of a Simple Harmonic Motion with a Uniform Straight-line Motion.

— *Experiment.* Let us use for a pendulum bob a funnel with a narrow stem (Fig. 174). When filled with sand and made to swing in the same plane, the funnel traces its path upon a piece of paper below it as a deposit of sand in a straight line. If the paper be drawn uniformly along in a direction at right angles to the plane of the swinging pendulum, the sand will be deposited in a wavy line.

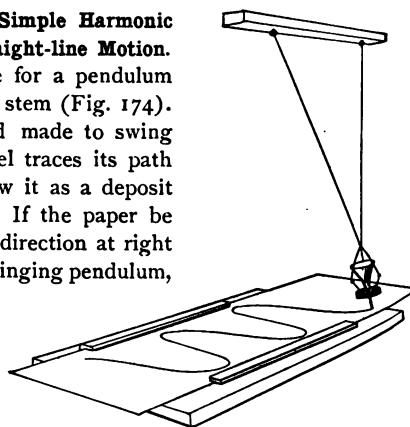


FIG. 174. — Sand Pendulum.

Construction. Let AH (Fig. 175) represent the path of a body moving in simple harmonic motion. Then AB, BC, CD , etc., to H are the distances apparently gone over in intervals of time, each equal to $\frac{1}{8}$ of the period. Now suppose the point of suspension to be moved uniformly in a direction at right angles to the path, and let Ea, ab, bc ,

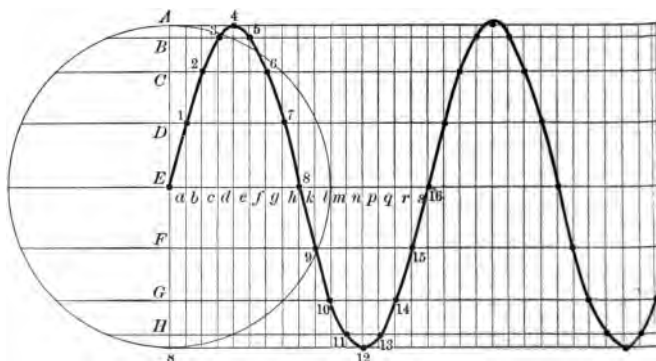


FIG. 175. — The Harmonic or Sine Curve.

cd , etc., denote the distances traveled by the point of suspension in intervals equal to $\frac{1}{8}$ of the period. Then, starting to count when the body is at E , we shall find it after the lapse of $\frac{1}{8}$ of a period at the point 1, after $\frac{2}{8}$ of a period at 2, etc. The smooth curve drawn through

these points resembles that shown by the sand. Such a curve is the resultant of a simple harmonic motion compounded with a uniform straight-line motion; it is called the *harmonic* or *sine curve*.

337. Water Waves.—The similarity between the form of the sine curve and that of the cross section of a wave of water is striking. That the particles of water themselves are executing simple harmonic motions in vertical planes can be seen by employing a trough of water with glass sides (Fig. 176), and placing in the water wax pellets loaded with sand so that they just float. By moving a paddle fitting the trough to and fro near one end, a wave is sent along the trough. The pellets are seen to move in

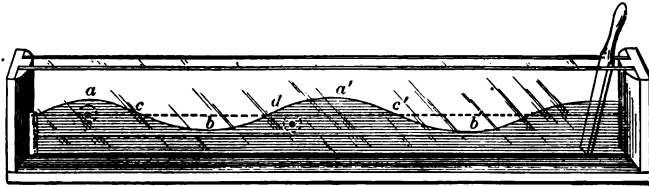


FIG. 176. — Wave Trough.

nearly circular orbits in vertical planes parallel to the direction that the wave has. The water moves up and down, but the wave moves forward. The lengths of the waves are the distances from the top of one crest to the top of a neighboring crest, as from *a* to *a'*, or from the bottom of one trough or valley to the bottom of the next trough, as from *b* to *b'*, or, in general, from any particle, as *c*, in any phase, to the next particle, as *c'*, in the same phase. Particles at the same height are not necessarily in the same phase. Thus, while the particles *c* and *d* are at the same height, they are in different phases because *c* is moving downward while *d* is moving upward. The amplitude of the wave is the vertical distance between the top of

a crest and the bottom of a valley; it is obviously equal to twice the amplitude of the corresponding simple harmonic motion.

338. Reflection of Water Waves; Stationary Waves.—

When a wave sent out from one end of the trough reaches the other end, it is seen that a crest is reflected as a crest and a trough as a trough. With a short wave trough we can start a fresh wave of such a length and at such an instant that the beginning of the advancing crest of the second wave meets the beginning of the reflected valley of the first wave at the middle of the trough. Particles at the middle will then be acted upon by equal and opposite forces tending to move them upward and downward at the same time, the result being that they remain at rest at the same height as the calm water. But particles at the ends of the trough will rise and fall alternately; they are

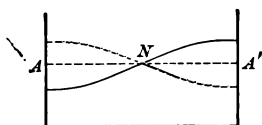


FIG. 177. — Reflection of Waves.

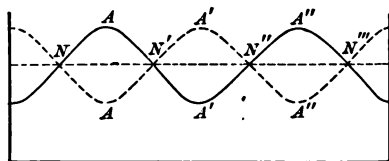


FIG. 178. — Stationary Waves.

always in opposite phases (Fig. 177), and therefore half a wave length apart. By continuing to generate waves half a wave length apart in the above fashion, the water is alternately heaped up at one end and scooped out at the other. Such systems of direct and reflected waves are called *stationary waves*. With a trough much longer than half a wave length, several stationary waves may be set up (Fig. 178).

339. Waves in Cords. — Lay a rope out straight on the ground, and give a sudden jerk upward or sidewise to one end of it held in the hand. A wave is thereby made to move along each portion of the rope, passing the jerk on to the next portion. The jerk causes a simple harmonic motion to be communicated for an instant to the rope. How the motion is passed on may be explained as follows.

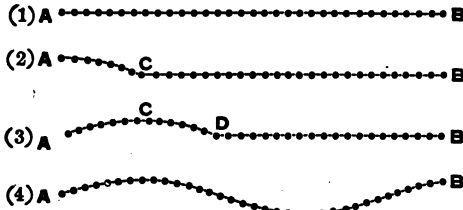


FIG. 179. — How a Wave is propagated in a Cord.

Imagine a row of particles connected by elastic bands (Fig. 179). In (1) they are at rest. In (2) an upward motion has been given to *A*, and by the time *A* has moved to the end of its path, the motion has reached the particles as far as *C*. In (3) *A* has returned to its original position, but *C* has now moved to the end of its path, and the jerk is felt as far as *D*. In (4) *A* has moved as far downward as it moved upward in (2), and the disturbance has reached *B*.

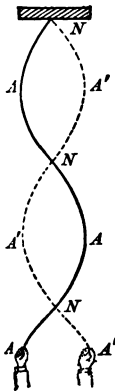


FIG. 180. — Stationary Waves in Cords.

340. Stationary Waves in Cords. — If one end of the rope is tied to a post or is hung up by one end (Fig. 180), a wave traveling through the rope is reflected on reaching the rigid support. By sending a series of waves along the rope at properly timed intervals, it can be made to show stationary waves. The points where there is the least motion are called *nodes*; the parts between the nodes are called *loops*.

or *ventral segments*; and the points halfway between the nodes where the motion is greatest are the *anti-nodes*. The wave length is defined to be the distance from any point in the vibrating rope to the next point vibrating in the same phase, as, for example, the distance between every other node or antinode. If the same rate of vibration is impressed upon a rope, the waves become longer, the tighter it is stretched; *the wave length increases with the tension*. If the same tension is maintained, and the time of the vibration, or the *period*, is increased, the waves become longer; *the wave length increases with the period*.

341. Speed of Waves. — Suppose that ten waves a second are sent out in a given medium, and that the length of each wave is one foot. When the eleventh wave starts, the first wave will be ten feet away. The wave disturbance has therefore been passed on at the rate of $10 \frac{\text{ft.}}{\text{sec.}}$, and the speed of the wave motion is $10 \frac{\text{ft.}}{\text{sec.}}$. If twenty waves a second were sent out and should travel with the same speed as the ten waves, each of the twenty waves would be half a foot long. The *frequency*, that is, the number of waves produced in a second, varies inversely as their length. The speed s of waves is equal to the product of frequency n and wave length l :

$$s = nl.$$

This relationship is true for all homogeneous media and for all kinds of waves.

342. Longitudinal Waves. — Place a number of similar elastic balls in a groove so that they touch one another (Fig. 181). Roll the end ball A against its neighbor B . None of the balls seems to be affected except the last.

which promptly begins to move. But how has the energy of the first ball been communicated to the last? Evidently it must have been transmitted by the other balls. The transmission has been effected in some such way as this: When

(A) (B) (C) (D) (E) (F) (G) (H) (I) (J)

FIG. 181. — How a Longitudinal Wave is Propagated.

A knocked against *B*, both the balls (Fig. 182) were compressed slightly for an instant. On recovering from this compression, the balls, because of their inertia, did not at once resume their original shape, but suffered momentary compressions at right angles to their first compressions. *B* therefore knocked against *C*, and in like manner *C* knocked against *D*, and so on through the line. A wave of compression travels along the balls in the direction of the line passing through their centers. The speed of this wave is so great that to the unaided eye it appears instantaneous.

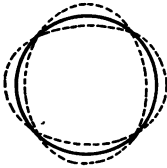


FIG. 182. — Compression of Ball.

343. Typical Wave Form. — The surface of wave-tossed water is so familiar that it is easy to see how a regular wavy line may represent the forms of all transverse waves.

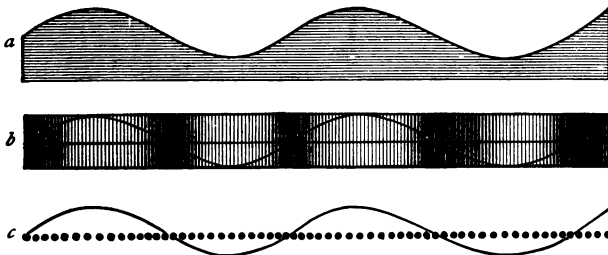


FIG. 183. — The Sine Curve is Typical. (a), Wave-tossed water; (b) and (c), Longitudinal waves.

But such a curve may also represent longitudinal waves as well. Longitudinal waves are often represented by lines or dots spaced closer together where condensations occur, and farther apart where rarefactions come (Fig. 183). But the harmonic curve can also represent this, and in describing wave motions we shall make frequent use of this sine curve. We shall represent all kinds of waves as transverse waves, and shall depend upon the context for information as to the kinds of waves under discussion.

344. Circular and Spherical Waves. — We have thus far confined our considerations to waves transmitted in a single direction. But waves are frequently transmitted in all directions on a liquid surface, giving rise to *circular waves*, as well as in all directions in space, forming *spherical waves*. Thus, when a stone is dropped into still water, it pushes aside the water where it strikes, and causes the water to rise on all sides (Fig. 184). The raised water in falling pushes up the water around it, because of the energy it has acquired, and rings of ever widening waves are produced.

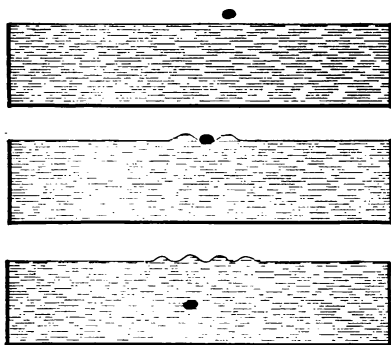
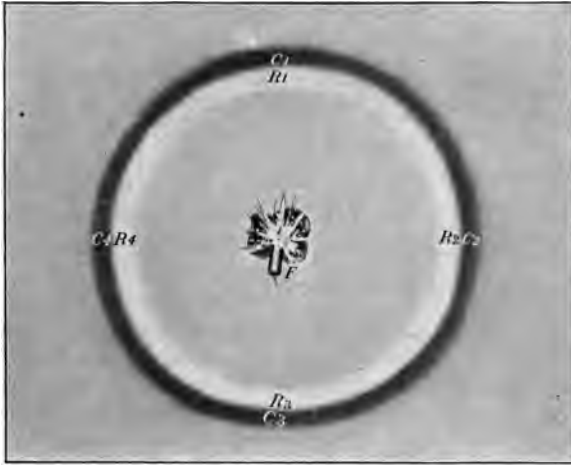


FIG. 184. — Circular Waves produced by Pebble falling into Water.

Again, if a firecracker tossed into the air explodes (Fig. 185), the surrounding layers of air are compressed, and the disturbance causes the production of spherical shells of waves spreading out uniformly in all directions. Both circular and spherical waves retain their shapes because the surrounding medium is the same, so that one part does not travel faster than another part of the waves.

345. Plane Waves. — When waves come from a source so far distant that the radii of the circles or spheres in the direction of which they

el are practically parallel, the front of the wave may be assumed to be practically straight or plane.



185. — Spherical Wave produced in the Air by the Explosion of a Firecracker. R_1, R_2, R_3, R_4 , represent rarefied air, and C_1, C_2, C_3, C_4 , compressed air.

46. Reflection of Waves. — When a train of waves strikes the surface of a substance in which they cannot move, or move at a different rate, as in the case of a pier in water or a wall in air, the train of waves changes its direction; it is *reflected*. The reflection of waves obeys the same laws as the reflection of motion (§ 111). When the direction of the wave motion is normal to the plane of obstacle, the reflected train of waves retraces its path, and the incident and reflected waves, provided their lengths are equal, combine to produce stationary waves.

47. Circular Wave reflected from a Plane Surface. — Suppose that a series of the waves sent out from the point P (Fig. 186) strike a flat surface FS , and let aOb be the wave front that first reaches the surface. If

there were no reflecting surface, this front would an instant later come into the position crd . But since reflection occurs, and since the speed

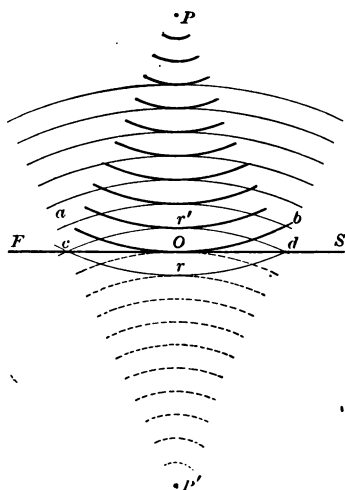


FIG. 186. — Circular Wave reflected from a Plane Surface.

of the reflected wave must be the same as that of the incident wave, there being no change in the medium, the position of the wave front after reflection is $cr'd$, inasmuch as Or and Or' are equal. The center of this reflected wave front, $cr'd$, is at P' , which is as far from the surface on one side as P is on the other side, for the arcs crd and $cr'd$ are equal, so that the circles to which they belong are equal and hence have equal radii. To an observer the reflected waves appear to come from a center directly opposite the source and as far behind the surface as the source is in front of it.

348. Plane Wave reflected from a Circular Surface. —

Imagine a train of plane waves to encounter a circular surface SE , such as a post in water (Fig. 187). Were there no reflecting surface, an instant after the wave came into the position aOb it would have passed on to the position crd . But the surface bends the wave around, so that at the instant considered the wave is reflected into the position $cr'd$, where Or' equals Or . The reflected waves are therefore circular, and appear to an observer as if they came from a point F on the inner side of the reflecting surface.

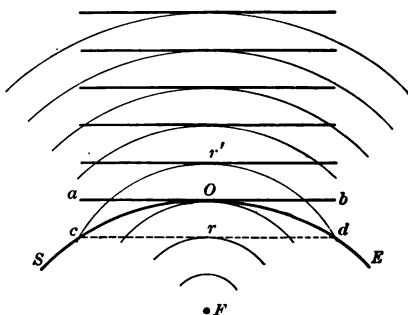


FIG. 187. — Plane Wave reflected from a Circular Surface.

349. Refraction of Waves. — When a train of waves passes at any angle but a right angle from one medium into another in which its speed is different, the direction of the train of waves is changed; the waves are *refracted*, *i.e.* bent out of their course.

ILLUSTRATIONS. — I. Suppose a regiment of soldiers to be marching across a field one half of which is smooth and the other half rough ground (Fig. 188). If all the soldiers in a rank pass from the smooth into the rough ground at the same instant, the line of march will not change, even though the speed of the march be lessened because of the rough ground. But if the line of march form an angle with the line between the smooth and rough ground, the soldiers who reach the rough ground first will march more slowly than those still on the smooth ground. The consequence will be that each rank will form a broken line while passing over the dividing line, and the line of march will swing around somewhat.

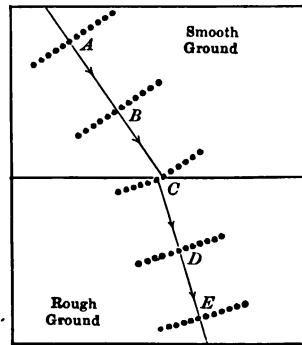


FIG. 188. — Illustration of Refraction.

II. Waves in deep water travel faster than in shallow water. This being true, refraction explains why it is that waves on a body of water, which are not parallel to the shore line, become more and more parallel as they approach the shore; for that portion of a wave which comes into shallow water first is retarded, while the other portions, not being retarded, swing around, until finally the wave breaks, almost if not entirely parallel with the shore line.

350. Diffraction of Waves. — When waves encounter an obstacle that is small in comparison with their length, they bend around the obstacle, taking a different course (Fig. 189). Such a change of direction is said to be due to *diffraction*. A train of plane waves meeting an obstacle

in which there is an opening, the dimensions of which are small in comparison with the wave lengths, also suffers

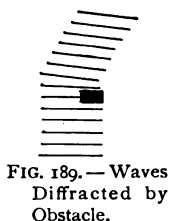


FIG. 189. — Waves Diffracted by Obstacle.

diffraction. Thus all parts of a train of waves moving toward a break-water in which there is a comparatively small hole cannot pass through the hole (Fig. 190). Such portions as do manage to pass through spread out so that the plane waves are changed into circular waves.

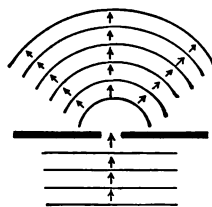


FIG. 190. — Waves Diffracted by Passing through Opening.

351. Sympathetic Vibrations. — A body may be set in vibration by the action of very feeble forces, provided they are repeatedly applied at properly timed intervals. To illustrate, let a thread be tied to a very heavy pendulum bob. Pull the thread gently and then relax it. The pendulum will be drawn aside almost imperceptibly. But pull the thread again and again, timing the pulls so that they occur only when the bob begins to swing toward the hand; *i.e.* making the time between pulls equal to the time the pendulum takes to complete a double vibration. By such repetitions the pendulum can finally be set in vigorous vibration. If, however, the time between pulls is not equal to the natural period of the pendulum, they will not succeed in making it vibrate, however numerous they are.

Each pull does a little work in moving the bob; and by repeating this work at proper intervals the amount of energy finally given to the bob becomes many times that expended at any one pull. The hand and the pendulum execute *sympathetic vibrations*, with the result that there is an accumulation of energy. In general, then, the vibra-

ions of one body are freely imparted to others if their rates of vibrations are equal.

ILLUSTRATIONS.—Two familiar illustrations of sympathetic vibrations are seen in the rocking of a heavy boat in still water caused by throwing one's weight first to one side and then to the other at regular intervals, and in the rocking of a street car on its trucks by "teetering" on the platform. A suspension bridge was once set rocking so violently by the regular step of soldiers passing over it that it was torn from its fastenings. As a consequence, soldiers are now ordered not to keep step on crossing bridges.

352. Forced Vibrations.—While a body will not vibrate freely when the vibrations of another body are impressed upon or communicated to it unless the periods of both are naturally the same, a body having a natural period different from that of another body may be forced to vibrate with it, especially if the energy of vibration of the forcing body, because of superior mass or swifter motion, is much greater than is required to set the forced body vibrating. When the periods are but slightly different, the vibrations are most easily forced.

353. Interference of Waves.—Let two similar stones be dropped into a placid pool at the same time. Each stone sets up a system of circular waves of its own, which meet and cross one another. Where two crests meet, there is an increased elevation of the water, and where two troughs come together, a deeper trough results; the amplitude of waves meeting in like phase is increased. But when the crest of one wave meets the trough of another wave having the same length and amplitude, the water of the crest just fills up the hollow of the trough and the surface becomes calm. Intermediate effects result from the *interference* of waves that are in neither like nor opposite phases, the

effect produced at any point being the algebraic sum of the effects of the separate waves.

354. Waves and Energy. — Emphasis needs to be laid upon the fact that work has to be done upon a medium in order to set it vibrating, and that the resulting wave motion serves to carry energy from one part of the medium to another. Let it be kept clearly in mind that the particles composing the medium merely vibrate, but, in so doing, they give rise to wave motions which transfer energy. Matter may indeed under certain conditions flow from one place to another so that energy may be transferred by currents of matter, and such transfers of energy are common enough. But this process is essentially different from wave motion.

EXERCISES

1. How does the wind act in causing waves on water?
2. Explain how waves are formed by a steamboat passing through still water.
3. Why are waves on a swiftly flowing river smaller than those on a lake of the same width, even though the force of the wind raising the waves is the same in both cases?
4. If the line of direction of a train of waves passing obliquely from one medium into another makes a smaller angle with the bounding surface in the first medium than in the second, in which medium is the wave speed the greater?
5. How does the length of the trough compare with the length of the stationary waves set up in the short wave trough (§ 338)?
6. If a wave travels $100 \frac{\text{cm.}}{\text{sec.}}$, and the wave length is 2.5 cm., find the frequency of a particle participating in the wave motion.
7. Compute the frequency of a wave in steel, if the wave length is 300 ft. and the speed in steel is $15,000 \frac{\text{ft.}}{\text{sec.}}$.
8. What are the lengths of the waves traversing a cord 300 cm. long when it is vibrating in (1) one loop? (2) two loops? (3) three loops? (4) four loops? (5) five loops?

CHAPTER XV

PRODUCTION AND PROPAGATION OF SOUND

355. Sound due to Vibrations. — Sound-producing bodies are in a state of vibration, as is attested by many familiar experiments. Cases of special interest in the study of sound are the following

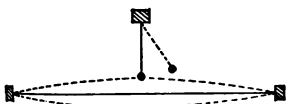


FIG. 191. — Vibrations of Strings.

I. *Transverse Vibrations of Strings.*

Stretch a wire tightly between two rigid supports (Fig. 191) and pluck it, or pluck the string of some musical instrument. A light body, such as a ball of pith or cork attached to a thread, when held close to the sounding string, is knocked backward. This proves that the string is in vibratory motion.

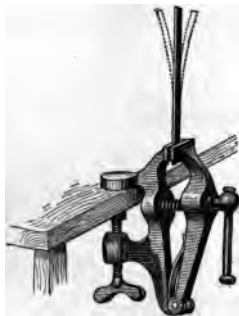


FIG. 192. — Vibrations of Rod.

II. *Transverse Vibrations of Rods or Bars.*

Let one end of some elastic material, as steel or brass, be fixed in a vise (Fig. 192). Draw the other end aside and then let it go. The rod will

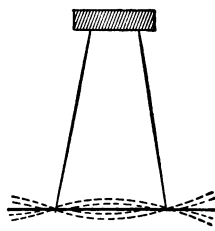


FIG. 193.

be seen to vibrate. Make the vibrating portion of the rod shorter and shorter by lowering it in the vise. While its vibrations may now be made so rapidly as not to be individually discernible by the sense of sight, the sense of hearing will be affected; the vibrating rod produces sound.

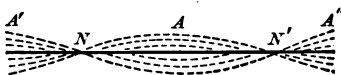


FIG. 194.

Tuning Forks.—A rod suspended, as shown in Fig. 193, vibrates with nodes and antinodes, as depicted in Fig. 194. If the rod is bent around into a two-pronged fork and a handle attached, it forms a *tuning fork* (Fig. 195), an instrument much used in the study of sound and music. The relative positions of nodes and antinodes remain the same, however, as in the straight rod, the handle being thereby made to vibrate longitudinally.



FIG. 195.—
Tuning
Fork.

III. *Longitudinal Vibrations of Rods.* Clamp the center of a long rod of elastic material fast (Fig. 196), and draw a cloth covered with powdered rosin along it near one of its ends. The rosined cloth grips the rod, and, as it is pulled along, lengthens the rod slightly. The rod then slips from the cloth, and while its elasticity causes it to spring back into its original condition, its inertia carries it slightly beyond, so that it shortens a little. A succession of such grips

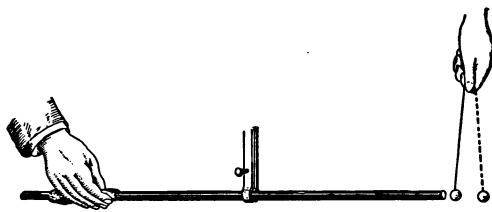


FIG. 196.—Longitudinal Vibrations.

and slips results in setting the rod into longitudinal vibration, as may be proved by bringing a pith-ball pendulum near one end. A rod thus set into longitudinal vibrations gives a shrill sound. It is evident that a node must be present at the point clamped, and antinodes at the ends (Fig. 197).

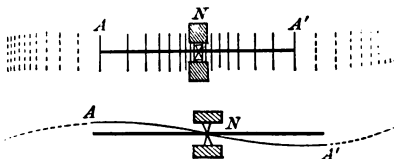


FIG. 197.—Nodes and Antinodes of a Rod Vibrating Longitudinally.

IV. *Vibrations of the Air.* Fasten to one end of a rod arranged to produce longitudinal vibrations a thin disk that fits snugly but easily within a wide glass tube (Fig. 198). Provide the glass tube with a piston, and

sprinkle evenly along the tube some fine light powder, such as cork filings. Set the rod vibrating and note that the powder is violently

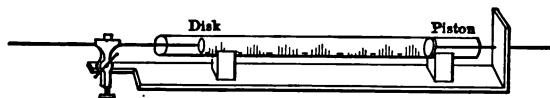


FIG. 198. — Kundt's Apparatus for Showing Vibrations in a Gas.

agitated. Move the piston to or fro until the powder arranges itself somewhat as shown. The air is set in vibration by the vibrating rod, and trains of waves are sent out, which are reflected back from the piston. When the piston is properly adjusted, the incident and reflected waves combine to form stationary waves. At the nodes of the waves there is no motion, and consequently no agitation of the powder. At the antinodes the vibratory motion of the air forces the dust particles to assume the positions illustrated.

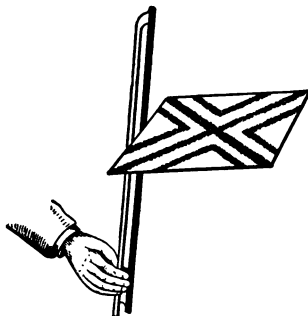


FIG. 199. — Vibrations of Plates.

V. Vibrations of Plates. Fasten a plate by its central point to a stand and strew some sand over its horizontal surface. Draw a violin bow over one edge of the plate (Fig. 199). A musical sound is heard and the sand dances about and finally settles along definite lines.

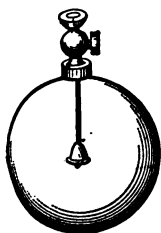


FIG. 200. — Bell in Vacuum.

356. Sound Media. — Any elastic body can propagate waves, but sound waves are carried to the ear only through the medium in which the ear is immersed, which is nearly always air. Sound-producing vibrations set up trains of waves in the air, which so act upon the ear as to give rise to the sensation of sound. An elastic medium then as well as a vibrating body is required for the hearing of a sound.

357. A Medium Necessary. — If a bell is hung in a globe (Fig. 200) from which the air is exhausted by means of an air pump, it is found that, after exhaustion, the ringing of the bell can scarcely be heard. There is no medium to carry waves from the bell to the ear, except the cord by which the bell is suspended, and this can transmit but few vibrations to the globe, and thence to the air outside. The conclusion is therefore justifiable that if no medium at all intervened, there could be no propagation of sound.

SPEED OF SOUND

358. Speed of Sound in Air. — Sound does not travel instantaneously. We see the flash of a distant gun several seconds before we hear its report. As light in the case of all terrestrial distances requires an inappreciable time for its propagation, the tardiness of the sound must be because of its comparative slowness of transmission. The following is a direct method of measuring the speed of sound. Cannon are fired alternately at two stations several miles apart, and the lapse of time between seeing their flashes and hearing their reports is measured. The distance and the time being known, the speed is calculable. The reason for placing a cannon at each station is that the disturbing effect of the wind is thereby eliminated; for the wind will hasten the speed of the report of the cannon at one station just as much as it will delay the report from the other station. As the result of a number of experiments by such direct methods, as well as of a number of experiments made by indirect methods, the average speed of sound in air has been found to be $332 \frac{\text{m.}}{\text{sec.}} = 1090 \frac{\text{ft.}}{\text{sec.}}$ at 0°C.

359. Speed of Sound in Liquids. — The speed of sound in water has been determined directly by stationing two boats at a known distance

apart, and sounding a bell under water from one of the boats at the same instant that some gunpowder is flashed. By means of an ear trumpet, partly under water, it can be determined when the sound reaches the other boat. The time between seeing the flash and hearing the sound is thus determined. The speed of sound in water is found to be about four times as great as in air.

360. Speed of Sound in Solids.—The direct method has not given very accurate results for the reason that it is difficult to get a long and continuous solid to experiment upon. Iron water pipes have been used, the difference between the times required for the transmission of a sound through the iron and through the air being measured. Indirect methods based upon measurements of the elasticity and density of solids have been frequently applied. Such solids as glass, iron, and fir transmit sound about twelve times more rapidly than air does, while others, such as lead, only about four times more rapidly.

361. Kundt's Method of Comparing Speeds of Sound in Various Media.—The apparatus described in § 355 for making visible the mode of vibration of sound waves in air can be applied in comparing the speeds of sound in solid and gaseous media.

I. Air and Solids. Since there is a node at the middle point of the rod (Fig. 197) which is vibrating longitudinally, and there are antinodes at the ends, the wave length in the material of the rod is twice as long as the rod itself. Hence measurements of the length of the rod give the length of the waves. Within the tube the average distance between the centers of two adjacent dust heaps is half a wave length in air (Fig. 198). Let l and l' represent, respectively, the wave lengths of the sound in air and in the rod, and n and n' , the respective frequencies. Then,

$$\text{Speed of sound in the air} = n\lambda. \quad (1)$$

$$\text{Speed of sound in the rod} = n'\lambda'. \quad (2)$$

Now since it is the vibrating rod which sets the air in vibration, the frequencies in both media must be equal, so that $n = n'$. Dividing (1) by (2), we have

$$\frac{\text{Speed of sound in the air}}{\text{Speed of sound in the rod}} = \frac{l}{l'}; \quad (3)$$

1

and since the speed of sound in air is known,

$$\text{Speed of sound in the rod} = \frac{l}{l'} \times 332 \frac{\text{m.}}{\text{sec.}} * \quad (4)$$

II. *Air and Gases.* The glass tube is provided with openings, one near each end so that a gas may be passed through it and displace the air. If the distance between consecutive dust heaps in air is denoted by l , and between consecutive dust heaps in some other gas by l' , then, since the vibrating rod imparts the same frequency to the waves in the gas as to the waves in the air,

$$\frac{\text{Speed of sound in the air}}{\text{Speed of sound in the gas}} = \frac{l}{l'}, \quad (5)$$

from which the speed of the sound in the gas can be computed.

362. Speed of Sound depends upon Density and Elasticity of Medium.—If s denote the speed of sound (or of any wave motion in an elastic medium), d , the density of the medium, and e , its elasticity, the following formula expresses the relations of these quantities :

$$s = \sqrt{\frac{e}{d}}. \quad (6)$$

By the application of this formula it is possible by measurements of the elasticity and density of any medium, be it solid, liquid, or gas, to determine the speed of sound in that medium.†

363. Influence of Pressure and Temperature on the Speed of Sound in Gases.—The elasticity of a gas is measured by its pressure. As the density of a gas at a given temperature is directly proportional to its pressure, it follows that the value of $\frac{e}{d}$ is a constant ; consequently *the speed of sound is independent of the pressure.* Sound travels at the same rate on mountain tops, where the pressure and the density of the air is comparatively small, as it does in diving bells, where the air is compressed and condensed.

* The temperature will change the speed of sound in air (§ 363), so that, instead of 332, the number corresponding to the temperature of the air inside the tube must be used.

† Certain corrections have to be applied because of the heating effects produced by the vibrations.

Equation (6) also shows that *the speed of sound in various gases under the same pressure is inversely proportional to the square root of their densities*. Thus air is nearly 16 times heavier than hydrogen; hence sound travels nearly four times as fast in hydrogen as in air.

A rise in temperature increases the pressure of a gas more than it decreases its density. Hence $\frac{v}{\rho}$ increases with rise of temperature. This increase has been found to be about $.6 \frac{\text{m.}}{\text{sec.}} = 2 \frac{\text{ft.}}{\text{sec.}}$ per degree Centigrade.

EXERCISES

1. Why is it that the rear of a long column of soldiers cannot march in time with the music in front?

2. Why is it that a listener at one end of a long iron pipe hears the sound of two blows one after the other, although but one is struck at the other end?

3. Why is solid wood a better transmitter of sound than sawdust?

4. How much larger is the surface of a spherical wave 10 ft. from the center or the source of the sound than it is 5 ft. from the center? How many more molecules of air are set to vibrating in the larger than in the smaller wave front? How much less energy must there be on a unit of surface of the larger wave front than on the unit of surface of the smaller wave front?

5. On June 21 and 22, 1822, at 11 P.M., two parties of scientists, appointed by the French Academy, stationed themselves on hills in view of each other, and observed the time elapsing between seeing the flashes and hearing the reports of cannon fired at intervals of 10 min. by each party. The mean time observed by one party was 54.8 sec., that observed by the other party was 54.4 sec. The temperature was 16° , and the distance between the two cannon was 18,612 m. What is the speed of sound in meters per second as deduced from these observations?

6. A man sets his watch by the striking of a clock 1 mi. away. If the speed of sound is $1120 \frac{\text{ft.}}{\text{sec.}}$, how much behind the clock is his watch?

7. Thunder is heard 3 sec. after its flash of lightning. If the temperature is 20° , how many meters away is the thunder cloud?

8. A rifle ball is fired at a target 500 m. distant, its average speed being $340 \frac{\text{m.}}{\text{sec.}}$. If the temperature is 20° , which reaches the target first — the ball or the report of the rifle?

9. What is the speed of sound at 0° in a gas with a density (a) one fourth that of air? (b) four times that of air?

10. How much must the temperature of air be raised above 0° in order to increase the speed of sound by 10%?

11. If a man places his ear near a railroad track, how much sooner will he hear the sound of a blow struck by a hammer on the track a kilometer away through the track than through the air? (Take speed of sound in steel as $5 \frac{\text{km.}}{\text{sec.}}$, and in air as $333.3 \frac{\text{m.}}{\text{sec.}}$.)

12. If in the C.G.S. system the elasticity of water is expressed by the number 196×10^8 , calculate the speed of sound (in $\frac{\text{m.}}{\text{sec.}}$) in water.

13. A brass rod used to excite a Kundt's tube was 200 cm. long, and the distance between eleven consecutive dust heaps was 100 cm. Calculate the speed of sound in brass, the temperature being 20° .

14. In a certain Kundt's tube filled with air, the distance from the first to the eleventh dust heap was 100 cm., while, when filled with coal gas, the distance between two dust heaps was 75 cm. Compute the speed of sound in coal gas, the temperature being 18° .

REFLECTION OF SOUND

364. Reflection of Sound.—When sound waves in air strike against the surface of a substance in which they travel at a different rate, a considerable number of the waves is reflected. A sound produced in a room sends out waves in all directions, which are reflected from the walls and other objects. An observer hears, then, not only the original sound, but also its reflections. The sound is thereby strengthened; and for this reason it is much easier to hear and to make one's self heard in a room than in the open air. But the sound is not only strengthened; it is also altered, as the reflections prolong the duration of the sound. This prolongation is hardly noticeable in a small room, but in large halls the words of a speaker or the tones of a singer may be so prolonged as to interfere with one another, the reflections of one sound being heard while the next is being produced. Such halls are said to

have poor acoustic properties. The remedy consists in doing away with large reflecting surfaces and in making the walls and other objects in the room of such materials as absorb sound readily, that is, are easily set in vibration themselves. This is practically accomplished by the galleries, the seats, the hangings, and other decorations, as well as by the persons composing the audience, — all of which increase the number of reflections, and absorb sound energy.

365. Whispering Galleries. — Chambers are sometimes found of such a shape that curious reflections of sound occur. Thus the dome of St. Paul's Cathedral at London is so shaped that two persons at opposite points of one of the galleries can carry on a conversation in a mere whisper, which is inaudible to persons between them. The sound is transmitted from one whisperer to the other by successive reflections along the curve of the dome. Like instances are also not infrequently found in caves and under the arches of bridges and viaducts.

366. Echoes. — The condition for the formation of an echo is a large surface, such as the side of a building or the bottom of a well, at some distance from the source of the sound. A sound emitted by an observer toward such a surface returns from it by reflection, and if the distance is sufficiently great, the reflected sound will be heard some time after the sound is uttered. Since at ordinary temperatures the speed of sound is about $340 \frac{\text{m.}}{\text{sec}}$, a speaker at a distance of 170 m. from an echoing surface will, if he takes one second to pronounce a certain word, hear its echo as soon as he finishes the word, for the sound wave has in that case had just enough time to go from the speaker to the reflecting surface and back again.

367. Multiple Echoes and Reverberation. — When a sound is produced between two parallel surfaces, as between two cliffs, it is reflected back and forth several times, thus forming a *multiple echo*. When the

distance between the reflecting surfaces is comparatively small, the multiple echoes are not distinct but blend into one another so that a rolling, confused sound is heard. This is called a *reverberation*, and may be heard by shouting into a cistern, cave, or an empty barrel. The rolling or reverberation of distant thunder is due to multiple reflections between the clouds or between the clouds and the earth.

368. Refraction of Sound.—When sound waves pass from one medium into another in which their speed is different, their direction is altered. Thus, if the lower layers of the atmosphere are warmer and accordingly less dense than those immediately above, the gradual increase of density upward causes sound produced near the earth's surface to be deflected upward, so that a sound inaudible to a person a little distance away on the surface, may be heard by one at a considerable elevation above the surface. And, on the other hand, if the lower layers are the cooler, the sound is deflected downward.

369. Diffraction of Sound.—If we are on one side of a surface of such size and material as to reflect or absorb all the sound waves impinging upon the other side, none of the waves will reach our ears; for we are in a *sound shadow*. But if such a screen is small in comparison with the length of the waves, some of them will bend around the screen, and *sound diffraction* will occur. For example, if a person interposes a screen between himself and a band performing at some distance, some of the multitude of tones emitted by the various instruments will be totally reflected, some diffracted, and the rest not affected. The screens cast sound shadows of some of the tones, so that they are not heard. The consequence is that the quality of the music is altered. Such a change in quality may be noticed when a band, marching along a street other than that on which the observer stands, turns around the corner. The buildings between the hearer and the band, when these are on different streets, intercept certain of the sounds, while others bend around them. As soon, however, as the band comes into the street where the hearer is, the reflections and diffractions are much diminished, and the consequent alteration in quality is quite noticeable.

EXERCISES

1. Why are the acoustic properties of a hall poorer when it is empty than when an audience is assembled in it?

2. How does holding the hand with the palm outstretched behind the ear aid in hearing faint sounds?

3. The echo of the report of a gun discharged in front of a cliff is heard after 4 sec. The temperature of the air being 16° , how many feet away from the gun is the cliff?

4. A cannon ball is fired and the gunner hears the crash it makes against the target 10 sec. afterwards. What is the distance between the target and the cannon, if the average velocity of the ball is $800 \frac{\text{m.}}{\text{sec.}}$, and the temperature of the air is 15° ?

5. Two loud-sounding metronomes are placed side by side and made to tick in unison twice a second. As one metronome is moved away from the other, four ticks per second are heard, until it is 560 ft. farther from the ear than the other metronome, when only two ticks are again heard. How is this accounted for? At what other distances between the metronomes would only two ticks per second be heard? How may the speed of sound be found by this "unison method"?

6. The specific gravity of carbonic acid gas referred to air as a standard is 1.5. What is the speed of sound in it at 0° ?

CHAPTER XVI

LOUDNESS AND PITCH OF SOUNDS

LOUDNESS OF SOUND

370. Loudness. — By *loudness* is meant the intensity of the sensation produced by sound waves. This intensity depends upon the amount of energy expended in producing the sound, the area of the vibrating body, the distance of the ear from the source of the sound, and the nature of the medium through which the sound waves pass.

371. Loudness and Amplitude. — Tap a tuning fork gently and the sound produced is soft. Tap it more vigorously, and the sound is proportionately louder. In general, the more work done in causing sound, the louder is the sound. But the greater the amount of energy communicated to a sounding body, the greater is the amplitude of its vibrations. Consider any particle of a vibrating body, such as one end of a rod clamped at the other end. Since the vibrations, whatever their amplitude, must be executed in equal times, it follows that the greater the amplitude of a particle, the greater is its speed. Since the energy of a moving body varies as the square of its speed, *the energy of sound waves is proportional to the square of the amplitude of the vibrations of the sounding body.*

372. Loudness and Distance. — It is daily brought to our attention that the loudness of a sound diminishes as the distance from the source increases. Sound waves in a

homogeneous medium spread out from the source in ever widening spherical shells, and the energy expended in producing the sound distributes itself over an ever increasing surface. As the particles of a medium transmit the sound wave, the same amount of energy is given to an ever increasing number of particles, and the share of this amount of energy received by the individual particles, becomes ever less. While the distances of sound waves from their source are the radii of the spheres that they form, the particles affected at one time are all in the surfaces of the spheres. Hence, since the surfaces of spheres vary as the squares of their radii, *the energy of sound waves, to which loudness is due, varies inversely as the square of the distance from the source.* For example, to a person standing 100 m. from a source of sound, its loudness would be only one fourth what it would be were he 50 m. from the source.

373. Pitch. — One sound may not only be louder or softer than another, but it may also be higher or lower; in other words, its *pitch* may be different. To what property of vibrations is pitch due? Let the following experiments tell.



FIG. 201. — Savart's Wheel.

I. *Savart's Wheel.* A toothed wheel, looking somewhat like a circular saw, is mounted on a rotator (Fig. 201). The wheel is rotated with ever increasing rapidity, and the edge of a card is held lightly against the teeth. At first, the separate clicks of the card snapping against the teeth may be distinguished; but, as the speed increases, these clicks blend and form a continuous sound, the pitch of which rises higher and higher, as the rotation becomes faster and faster.

II. *Seebeck's Siren.* A metallic disk with a row of equidistant holes near its edge is mounted on a rotator (Fig. 202). It is whirled round

with constantly increasing speed, and a jet of air is blown against the holes. At first, a series of puffs is heard; but as the rapidity of rotation increases, these coalesce into a continuous sound with a pitch that becomes higher as the disk is turned faster.

The legitimate conclusion from such experiments as these is that pitch is due to the rapidity of vibration. *The pitch of a tone depends upon its frequency, i.e. the number of vibrations a body makes in a second.*

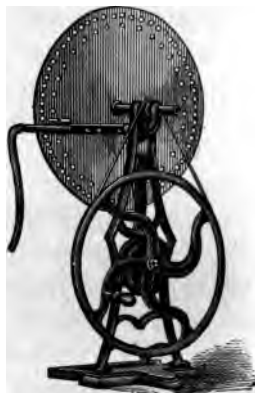


FIG. 202. — Seebeck's Siren.

374. Measurement of Pitch. — I. *Siren Method.* The siren, when provided with accessories for counting the number of rotations, can be used to find the frequency of a body producing a tone of a given pitch, such as that of a certain tuning fork. Let the siren be rotated until it gives a sound which, as well as the ear can judge, has the same pitch as that emitted by the fork. Keep the rate of rotation uniform, so that the pitch may be sustained, and by means of the counter, find out how many rotations are made in a given time. This number, multiplied by the number of holes in the disk, gives the number of vibrations in the given time, both of the sound produced by the siren and of that yielded by the fork. The number of vibrations per second is obtained by dividing by the number of seconds in the given time. This frequency or vibration number is not only that of the siren and the fork, but also that of any other instrument producing a sound of the same pitch.

II. *Graphical Method.* To one prong of a tuning fork is attached a light stylus bent in such a way as to graze a glass plate covered with a coating of soot or whiting (Fig. 203). Above the plate is suspended a pendulum with a bob that also carries a stylus just touching the glass when the bob is in its lowest position. Determine the period of the pendulum, and while it is still swinging, set the fork in vibration, and draw the plate along beneath the vibrating bodies. The stylus scratches off the soot and traces lines, as shown in Fig. 204. The number of waves traced by the stylus between any two consecutive lines

aced by the pendulum is the number of vibrations made by the fork during the time occupied by the pendulum in making a single vibration.

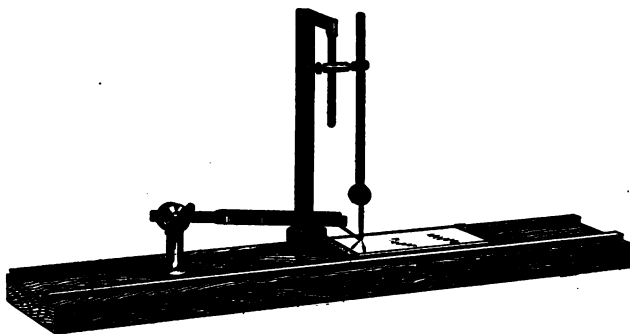


FIG. 203. — Graphical Method of Measuring Pitch.

The duration of a single swing of the pendulum being known, the vibration number of the fork can be calculated. The frequency of any other vibrating body can in like manner be measured.

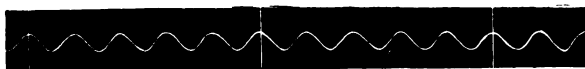


FIG. 204. — Trace from the Graphical Method.

III. *Indirect Methods.* The fundamental relation (§ 341) between wave speed, frequency, and length may be applied in measuring pitch; a measurement of wave length and speed gives the frequency, for if $s = n\lambda$, then $n = \frac{s}{\lambda}$, and pitch depends upon frequency.

375. Pitch changed by Relative Motion. — It is sometimes noticed that the pitch of a bell on a locomotive rapidly approaching the hearer rises until it reaches him, and then falls. The frequency of the sounds emitted by the bell remains the same, but its onward motion causes more sound waves to reach the ear in a given time as it approaches than would be the case if the bell were stationary. The increased number of waves striking the ear in a stated time affects the ear just as an increase in the vibration number of the bell itself would affect it, and the pitch of the sound is judged to be higher. In like manner, when the bell is receding from the observer, a smaller number of vibra-

tions reaches his ear in a given time, and the pitch of the sound seen lower. In general, the tones of sounding bodies, as they swiftly approach or recede from a hearer, become higher and higher or lower and lower, respectively. This phenomenon is sometimes referred to as *Doppler's principle*.

376. Limits of Audibility. — Not all vibrations occurring in the air can affect the sense of hearing, and the range of hearing varies with individuals. Most people can distinguish sounds arising from frequencies between 30 and 40,000 vibrations per second; while some can recognize low sounds due to a frequency of only 16, and others high sounds having as many as 50,000 vibrations per second. The lowest tone on a $7\frac{1}{3}$ -octave piano has a vibration number of $27\frac{1}{2}$, while the highest has a frequency of about 4220.

EXERCISES

1. Why does the pitch of the sound emitted by a circular saw in action become lower when the saw enters a board?

2. How can you illustrate the causes of difference in pitch and intensity of sounds by drawing the edge of a visiting card over the teeth of a fine comb?

3. A siren with 15 holes makes 1094 rotations in 30 seconds, when yielding a tone of the same pitch as a certain tuning fork. Find the frequency of the fork.

4. A siren has to revolve at the rate of 1320 times a minute in order to give a tone of the same pitch as that of an organ pipe with a frequency of 440. How many holes are in the siren?

5. A comparison of the traces of the vibrations of two tuning forks made by the graphical method showed that 9 waves of one are of the same length as 6 of the other. If the fork of lower pitch has a frequency of 256, what is the frequency of the other?

6. Two trains going $30 \frac{\text{mi.}}{\text{hr.}}$ pass each other on double tracks. If the locomotive bells vibrate 300 times per second, how many sound waves per second, given out by one of the bells, will reach the ears of a man riding on the other train, the speed of sound being $1130 \frac{\text{ft.}}{\text{sec.}}$?

7. A tuning fork vibrates 336 times a second. What must be the temperature in order that the waves in the air should be each one meter long?

8. What is the frequency of a sound with a wave length in air at 0° of 33.2 cm.?

MUSICAL SCALES

377. Musical Intervals.—The difference in pitch between two tones is called their *interval*. While in music special names are given to many intervals, in physics they are expressed as ratios of the vibration numbers of the tones. Thus the musical interval called the *octave* is expressed by the ratio 2 : 1, since the vibration number of the one of higher pitch is twice that of the tone of lower pitch. Again, two tones, the frequencies of which are as 3 : 2, give an interval known as the *fifth*. These intervals, being ratios, are independent of the actual number of vibrations of the tones composing them; the intervals are the same, whether the tones are of high or low pitch, provided that the ratios of their respective frequencies are identical.

378. Musical Scales.—Only a comparatively small number of all possible sounds is available for musical purposes, and those which are used stand in definite relationships to one another. In the music of civilized peoples the interval of an octave is divided up into certain lesser intervals, and the tones corresponding to certain of them when sounded in succession form a *scale*.* The most important of the scales is the *diatonic* scale, which consists of eight tones, each of which is definitely higher in pitch than the preceding one. The tone of lowest pitch is called the *fundamental* or *keynote*, and after the seventh tone comes the octave of the fundamental. The same intervals then form a scale an octave higher in pitch, each succeeding octave being a repetition of the same intervals.

*Scale is from the Latin word for ladder (*scala*). The German word—*Tonleiter* (tone ladder)—is very expressive.

379. Musical Notation.—In principle, the notation of music is the same as that used in finding the position of a point in a plane (§ 87). Five equidistant horizontal lines are marked off in *measures* to form the *staff*. The pitch of a tone is given by the position of its note on the staff, and its duration by the appearance of the note. By means of conventional symbols placed at the beginning of compositions and of staves, the different positions of notes in the staves as well as the number of certain kinds of notes to the measure are indicated. The lines of the staff correspond then to the axis of abscissas, and the lines perpendicular to them and marking off the measures correspond to the axis of ordinates. The notes considered with respect to their duration are distinguished as whole, half, quarter, eighth, etc., notes, while with respect to their pitch they are lettered *A, B, C, D, E, F, G*, or are given the syllable names *do, re, mi, fa, sol, la, si*.^{*} The tone that is to form the keynote is indicated by means of the number of flats (♭) or sharps (♯) placed at the left of the staff. When no flats or sharps are so placed, the keynote is *C*, and the composition is said to be in the key of *C*.

TABLE

Major Scale in the Treble Clef in the Key of C

<i>Number in scale</i>	1	2	3	4	5	6	7	8
<i>Letter names</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>	<i>A</i>	<i>B</i>	<i>C</i>
<i>Syllable names</i>	<i>do</i>	<i>re</i>	<i>mi</i>	<i>fa</i>	<i>sol</i>	<i>la</i>	<i>si</i>	<i>do</i>
<i>Vibration numbers</i>	256	288	320	341	384	427	480	512
<i>Ratio of vibrations</i>	1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	$\frac{2}{1}$

380. Standard of Pitch.—The number of vibrations corresponding to a certain tone or note is not absolutely fixed.

* The syllable names had their origin (1025) in certain accentuated syllables of a Latin hymn to St. John, which were sung in a rising scale.

"Ut queant laxis resonare fibris,
Mira gestorum famuli tuorum,
Solve polluti labii reatum,
Sancte Joannes."

The syllable *si* was added in 1684, and in Italy and England the syllable *do* was substituted for *ut*, because more easily sung.

To simplify the mathematical relationships, physicists give middle C a vibration number of 256, which is the eighth power of 2, but musicians, by international agreement, assign middle C a pitch corresponding to a vibration number of 258.7. Successive octaves above and below middle C are distinguished as C' , C'' , C''' , C'''' , etc., and C_1 , C_2 , C_3 , C_4 , etc., respectively.

Musical instruments are tuned according to the note A , which is the tone produced by the second string of the violin; its vibration number is 435.

EXERCISES

1. In what octave above middle C (frequency = 256) is the limit of audibility (40×10^8 vibrations per second)?
2. Taking the speed of sound as $1120 \frac{\text{ft.}}{\text{sec.}}$, calculate the wave length of the lowest and highest notes on a piano.
3. What note has five times as many vibrations as middle C ?
4. If a musical sound is produced by a body vibrating 300 times per second, what is the frequency of its third, fifth, and octave, respectively?
5. If the range of pitch of the human voice varies from about 80 to about 1000 vibrations per second, what is the range of the waves, when the speed of sound is $1120 \frac{\text{ft.}}{\text{sec.}}$?

RESONANCE AND INTERFERENCE

381. Reënforcement of Sound. — The sound of a vibrating tuning fork held in the air is faint; but when its stem is pressed upon a table top, the sound becomes much louder. The table top is forced to vibrate by the fork, and the sound is thereby reënforced. The fork vibrates for a longer time when held in the air than when placed upon a sounding board. A definite amount of energy is imparted to the fork to set it in vibration, and this energy is more rapidly used up when the fork makes the board vibrate

than when it is in contact only with the hand and the air. The fork comes in contact with a much smaller amount of air than does the sounding board. Hence the amount of air set in vibration when the fork is resting upon a sounding board is greater than when it is held in the hand; a correspondingly greater amount of work has then to be done, and the supply of energy furnished the fork is thereby the sooner exhausted.

382. Resonance.—A body of air contained in a vessel often reënforces certain sounds produced near it. Let a tuning fork be set vibrating and held with its prongs in a vertical plane over the mouth of a tall cylinder (Fig. 205). By pouring in water, the length of the air column above the water may be shortened, and it is found that with a certain length, there is a strong reënforcement of the sound. Forks of different pitch will require other adjustments of the lengths of the air columns before strong reënforcement takes place.

When the prong of the fork passes from *a* to *b* (Fig. 205), it starts a condensation down the jar to be reflected from the water's surface. And when the prong passes from *b* to *a*, it produces a condensation in the outside air. If the length of the air column is properly adjusted, the condensation will go to the water and return to the mouth of the jar, while the prong is moving from *a* to *b*, and hence will be just in time to combine with the condensation produced in the outside air by the prong as it passes from *b* to *a*. In similar fashion, while the prong is going from *b* to *a*, a rarefaction is sent to the water, and, after reflection, arrives at the mouth of the jar just in time to combine with the rarefaction produced in the outside air by the prong in passing from *a* to *b*. It

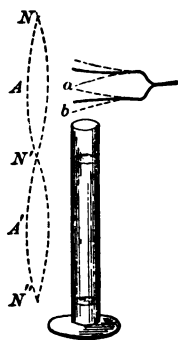


FIG. 205.—Resonating Air Column Closed at One End.

ly waves in the air column meet those in the outside air in like and the sound is made louder. But when the air column is a shorter or longer, the reflected waves do not meet the outside in like phase, and no strengthening of the sound is produced. Increments of sounds caused by the combining of direct and reflected waves are said to be due to *resonance*.

h. Wave and Resonant Air Column Lengths.—The period of a complete vibration of the prong of the tuning fork is the time it takes to move from *a* to *b* and from *b* to *a* again. While the prong is moving from *a* to *b*, it is, in effect, executing a single vibration, the condensation travels down the tube and back again,—a distance equal to twice the length of the air column. The column is therefore equal to the distance between a node and an antinode, which is one fourth the length of the wave. In general, then, *a resonant air column closed at one end is one-fourth as long as the sound wave in air which it resonates*. As there is a node at the water's surface, there must be an antinode near the mouth of the jar.

If the jar is tall enough or the wave length short enough, a second reënfacement of the sound is found half a wave length farther down the tube, and a third reënfacement after half wave length still farther down, and so on. Resonance is obtained wherever the air column is equal in length to an odd number of quarter wave lengths.

i. Resonance with Open Tubes.—Let two telescoping tubes or pipes be held in front of a vibrating tuning fork (Fig. 206) and adjusted until resonance occurs. The length of this resonating air column, open at both ends, will be found to be twice that of the resonating column closed at one end, which reënfances the same fork. *The length of an open resonating pipe is, therefore, equal to half the wave length of the sound.* There are antinodes at both the ends, and a node at the middle of the pipe.

Now resonance can take place in a tube open at both ends will be

clear when it is remembered that partial reflection always occurs when a wave passes from one place into another where the density of the medium is different. The air carrying the sound wave within the tube is less free to expand or contract at its ends than is the outside air, and when the wave leaves the tube it is free to expand in two directions, instead of one, as in the tube. It therefore behaves just as it would do in passing from a denser into a rarer medium. Part of the train of waves is reflected back into the tube, and arrives at the end nearer the fork in time to meet there in like phase fresh waves from the fork.

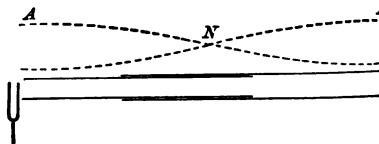


FIG. 206.—Resonating Air Column Open at Both Ends.

As the time of a complete vibration is equal to the time it takes for a wave to pass forward and backward through the pipe it is manifest that resonance should also occur when the length of the tube is equal to any number of half wave lengths. *Resonance occurs when the length of a pipe open at both ends is equal to $\frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \frac{4}{2}, \dots$ of the wave length of the tone reënforced.*

385. Resonators.—Any object containing air that reënforces some sound is called a *resonator* for that sound. Resonators absorb some of the energy of sound waves, and thereby themselves become sources of sound. The forms given resonators are those which experience has shown to be the handiest and most sensitive. If they are in the shape of tubes or pipes, the length of the waves stand in a simple relation to the lengths of the pipes, those open at both ends having lengths equal to any number of half wave lengths, and those closed at one end having lengths equal to any odd number of quarters

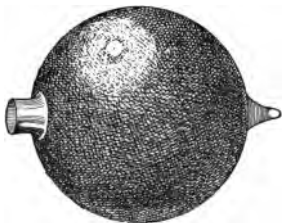


FIG. 207.—Helmholtz Resonator.

Other forms are given dimensions that most strongly reënforce certain sounds. Thus the dimensions of the resonant boxes on which tuning forks are mounted are such as have been experimentally found to be the best adapted to their purpose. The

shape Helmholtz gave his resonators (Fig. 207) was chosen because it proved extremely sensitive, gathering in at its wider end waves so feeble as to be inaudible, and concentrating them at the smaller end that is placed in the ear.

386. Interference of Sound.—Hold a vibrating tuning fork over a jar of water adjusted for resonance, and rotate it on a horizontal axis. When the fork is in any

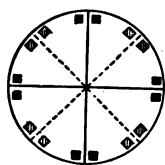


FIG. 208.—Interference of Sound.

one of the positions shown in solid black (Fig. 208), resonance occurs, but when it is in any of the positions indicated by shading, scarcely any sound can be heard. Similar effects can be produced by twirling a tuning fork held with its prongs close to the ear. The extinction of the sound is due to the interference of the waves. When the prongs move toward each other, the air between them is compressed, while the air

at their backs is rarefied; one condensation and two rarefactions are thereby simultaneously produced. Since the waves are of equal amplitude and length, interference and consequent silence will occur, if they meet in opposite phases; and this is found to occur when the edges of the prongs are turned toward the resonator. If, when the fork

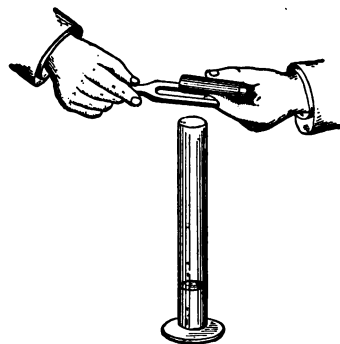


FIG. 209.—No Interference Occurs.

is adjusted for interference, a paper tube is slipped over one of the prongs (Fig. 209), most of its vibrations are prevented from entering the jar, and the tone due to the vibrations of the other prong alone is heard. Two sounds,

then, may actually produce silence, the condition therefor being that their intensities are the same and their waves made to meet in opposite phases.

387. Beats.—Place two tuning forks of the same pitch and mounted on resonant boxes (Fig. 210) near one another on the table, and set one in vibration by drawing a violin bow over its prongs. The other fork promptly begins to sound also, because of the sympathetic vibrations set up in it. Now attach a bit of wax to a prong of one fork and then make it vibrate. The sound is slightly lower than before, the load of wax making the prongs move more slowly. The other fork is not now excited by sympathetic vibrations. Excite both forks simultaneously. Peculiar throbbing sounds are heard, showing an augmentation of sound, such as is produced by resonance, and then a diminution of sound, like that due to interference. And this indeed is just what is happening—alternate resonance and interference, each lasting but for a very short time. These peculiar throbbings, due to the successive reënforcements and interferences of two tones of slightly different pitch, are called *beats*.

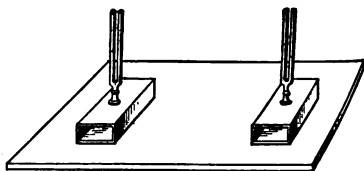


FIG. 210.—Sympathetic Vibration of Tuning Forks.

The number of beats per second is equal to the difference between the frequencies of the two sounding bodies. Thus, if a fork with a known vibration rate of 256 makes two beats per second with a tuning fork having an unknown vibration number, the second fork has a frequency of either 254 or 258. If the number of beats is decreased when a bit of wax is stuck on one prong of the first fork,

by lessening its frequency slightly, the vibration number of the second fork is 254; but if the number of beats is increased, its vibration number is 258. (Why?)

EXERCISES

1. What conditions must be fulfilled in order that the vibrations of one body may set another body vibrating freely?
2. How can beats be used to advantage in tuning two sonorous bodies to same pitch?
3. Pronounce *a* as in *father* and *e* as in *even*, and note the changes in the shape of the mouth cavity. Pronounce other vowel sounds, and note the effect on the size of the mouth cavity.
4. How do you account for the not infrequently observed fact that certain objects, such as a chandelier, a globe, a vase, and the like, rattle when particular keys are struck on a piano near by?
5. The strings of mandolins are in pairs, each pair being tuned in unison. A pair of such strings is not in perfect unison, peculiar throbbings can be felt by the finger tips resting lightly upon the sounding board of the instrument. Account for these throbbings.
6. When pumping water, a sound of gradually rising pitch is heard as the water comes up in the pipe. To what is it due, and why does the pitch change?
7. A tuning fork making 582 vibrations per second is reinforced by a tube 62.5 cm. long, when filled with hydrogen gas. Compute the speed of sound in hydrogen.
8. What is the length of the resonant air column which responds most loudly to a fork with a frequency of 512, the temperature of air being 20° ?
9. What is the length of a resonant column which reinforces most loudly the sound of a fork with a vibration number of 170, the speed of sound being $340 \frac{\text{m.}}{\text{sec.}}$?
10. A tuning fork with a vibration number of 256 is sounded at the same time as a tuning fork of less, but unknown, frequency. If 20 beats are heard in 10 sec., what is the vibration number of the second fork?

CHAPTER XVII

QUALITY OF SOUND

VIBRATIONS OF STRINGS

388. Stringed Instruments.—Probably the most important musical instruments are those consisting of strings stretched over a resonant case or sounding board. The strings are made to vibrate transversely in various ways. Thus in the violin, violincello, and other similar instruments, a bow is drawn across the strings; in the guitar, banjo, and harp, the strings are plucked with the fingers, and in the mandolin and the zither with a pick or plectrum; while in the piano, little hammers faced with felt are made to strike the wires. Sounding boards increase the volume of the sound by setting much larger masses of air into vibration than could be done by the strings alone; and resonant cases not only do this, but also make the air within them vibrate as a resonant column. The mode of excitation influences the loudness and quality of the sounds, but their pitch depends only upon the density, dimensions, and tension of the stretched strings.

389. Laws of Vibrations of Strings.—One string of an instrument can differ from another only in material, tension, thickness, and length. How do changes in these four particulars affect pitch? The following considerations of general principles will give a qualitative answer. It is readily seen that the greater the mass of a string, the greater is its inertia, and the more slowly it will move

when acted upon by a given force. The more slowly it vibrates, however, the less will be its frequency and the lower its pitch. The thicker that a string is, the lower will be the pitch of its tone; and with strings of the same thickness but of different materials, the greater the masses of equal lengths of them, *i.e.* the greater their linear densities, the lower will be the pitch. With a given mass and length, the more tightly a string is stretched, the harder it is to bend it, and the greater is the force with which it springs back after being moved aside; it accordingly vibrates more rapidly, and therefore an increase of tension raises the pitch. Since a string is excited at one point in it, the vibration takes time to spread through the whole length of the string, so that the greater its length, the longer is the time required for all the string to participate in the vibration; the frequency will accordingly be less.

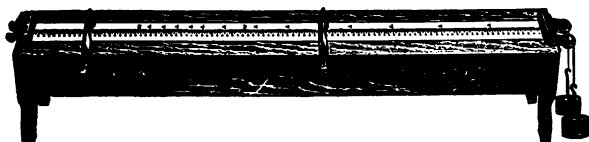


FIG. 211.—Sonometer.

To find out, however, the exact relation pitch has to the mass, tension, thickness, and length of a vibrating string,—in other words, to get a quantitative answer to the question, recourse must be had to specially devised experiments. An apparatus admirably adapted for such investigations is the *monochord* or *sonometer*. This instrument (Fig. 211) consists of a long resonant case over which strings or wires may be stretched. Near the extremities are two fixed bridges upon which the strings rest, and a scale extends from bridge to bridge. The lengths of the

strings may be shortened by placing movable bridges beneath them, and their tensions may be varied either by keys or by weights. Wires of different sizes and materials are strung over the sounding board and put under known tensions. If three of the items affecting pitch be maintained constant, the influence of the fourth item may be found. The results of experiments have established the following laws:

The frequency or pitch is inversely proportional to the length of the string; directly proportional to the square root of the tension; inversely proportional to the thickness of the string; and inversely proportional to the square root of the density of the string.

VIBRATION OF AIR IN PIPES

390. Wind Instruments.—Any musical instrument in which a vibrating column of air serves as the sonorous body is termed a *wind instrument*. Air compressed either by the



FIG. 212.—Bugle.



FIG. 213.—Cornet.

lungs or by a bellows is sent into one end of the instrument in such a fashion as to produce there a flutter or confused collection of vibrations, to one of which the tube or pipe acts as a resonator. In instruments like the pipe organ and the bugle (Fig. 212), the length of the resonant column is invariable, but in most instruments the length of the resonant column can be changed by open-

ing or closing keys placed over holes along the instrument, as in the cornet (Fig. 213), or by sliding one part of the



FIG. 214.—Slide Trombone.

instrument telescoping in another part, as in the slide trombone (Fig. 214).

391. Excitation of Wind Instruments.—*Reed Pipes.* An elastic strip of metal or bamboo, called a *reed*, is clamped at one end so as to partially obstruct the passage of a current of air blown into the pipe (Fig. 215). The reed is set into vibration so that the current of air is

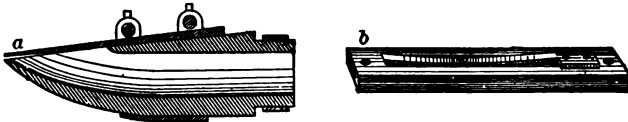


FIG. 215.—*a*, Striking Reed of Clarinet; *b*, Free Reed of Mouth Organ.

interrupted at regular intervals. Thereby is produced a series of puffs, which blend into a musical sound, this being reënforced by the air column. Where, as in the case of instruments like the clarinet, one reed has to act for a whole series of tones, it is made very light (bamboo) and loose, so as to be capable of vibrating in sympathy with any length of the air column. In instruments like the cornet, the place of the reed is taken by the lips of the performer, pressed over the mouthpiece (Fig. 216). The lips are made to vibrate sympathetically with the tone produced.

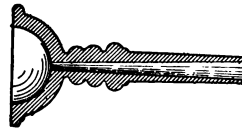


FIG. 216.—Mouthpiece of Cornet.

Flue Pipes. While the construction of the mouthpieces of instruments without reeds varies in detail, the way in which the sound is produced is essentially the same in all.* Every one knows that a tone can be produced

* It may be shown by blowing sharply against the edge of a card. In whistling, vibrations are set up by blowing through the nearly closed lips, so that a

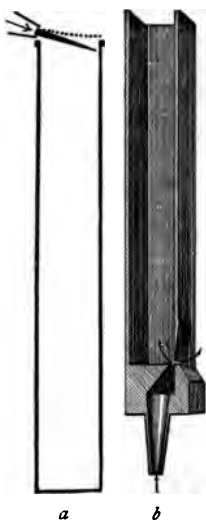


FIG. 217.—Flue Pipes.

by blowing a jet of air at an angle across the open end of a tube (Fig. 217 *a*), as in the case of a bottle or a hollow key, and the excitation of an organ pipe is accomplished in much the same way (Fig. 217 *b*). The blast compresses the air at the opening so that a wave starts down the tube. This is reflected from the bottom, and returning to the opening, presses up on the cross jet so as to deflect it upward. This upward pressure lasts for the time it takes a wave to move down and up the tube. The jet then sends down another wave which, on returning after reflection, again pushes up the cross jet. Repetitions of the operations result in the cross jet vibrating up and down regularly, the period of vibration being determined by the time a wave takes to travel down and up the tube. The length of the wave must therefore be four times that of the resonating air column. Similar considerations apply to organ pipes open at both ends, the wave length being twice that of the pipe.

EXERCISES

1. What are the two essential parts of every musical instrument?
2. Why are the strings giving the lower tones of stringed instruments wrapped with wire?
3. Why are two and even three strings tuned in unison employed for producing the high notes of the piano?
4. What are the relative advantages and disadvantages of instruments of the piano type compared with those of the violin type?
5. Point out wherein the action of a reed pipe resembles that of a siren.

blowing or hissing noise is produced. The mouth cavity corresponds to a sonorous tube, and by varying its size, tones of different pitch are reinforced, these alone being heard. Whistle as low a note as you can, and then your highest note, and observe how the tongue and cheeks act to diminish the size of the mouth cavity. In a somewhat similar fashion, the mouth acts as a resonator for such an instrument as the jew's-harp.

6. A string emitting the tone C is 36 in. long. How long must it be to give the tone D ?

7. A sonometer wire is stretched by a weight of 3 kg. What weight must be used to make it sound a tone an octave higher, its length remaining the same?

8. If, of two pieces of the same wire, one is twice as long as the other and stretched by four times as much force as the other, how do their vibration numbers compare?

9. If a certain string vibrates 200 times a second, how will its tone compare in pitch with that of a similar string, twice as long but under the same tension?

10. If a string 3 ft. long emits a tone in unison with a tuning fork which is known to vibrate 256 times a second, what will its frequency become when its length is reduced to 2 ft.?

11. Two wires of the same material are stretched by equal weights, but one wire is twice as long and half as thick as the other. How do their tones compare in pitch?

12. If the tension on a string is 16 kg., what change must be made in it so as to make it sound the next octave below?

13. If the density of brass is nine times that of catgut, how will the frequencies of strings made of brass and catgut compare, if their lengths, diameters, and tensions are equal?

14. If two strings of the same length and under the same tension weigh 36 g. and 16 g., respectively, what is the vibration number of the first, if that of the second is 360?

15. The length of the open organ pipe sounding middle C at 18° is 67.0 cm. Compute the lengths of the pipes producing the diatonic scale having middle C for its fundamental.

16. Taking the speed of sound as $1000 \frac{\text{ft.}}{\text{sec.}}$, what is the length of an open pipe producing waves 20 ft. long, and what is the frequency of the waves?

17. What is the pitch given by an open pipe 32 ft. long at the temperature of 31° ?

FUNDAMENTALS AND OVERTONES

392. Quality of Sounds.—The tones produced by one kind of musical instrument, say a violin, are readily distinguished from those furnished by a different instrument,

say a flute, even if the pitch and loudness of the tones may be the same in both instruments. Evidently, then, sounds may differ in a third particular, to which has been given the name of *quality*. Difference in quality is due to the fact that nearly all sounds are compounded of simple tones; or, in other words, that instruments emit, in addition to their fundamental tones, other tones of higher pitch, the so-called *overtones* or *partial tones*. The tones of a violin differ from those of a flute because the overtones accompanying the same fundamental of the two instruments differ in pitch and in loudness.

393. Overtones of Vibrating Strings.—The presence of overtones in a musical sound is readily detected if the attention is properly directed to them. For example, hold down the key of middle C on a piano so as to lift the damper from the corresponding wire, which is thereby free to vibrate. Strike the C_1 an octave below strongly, hold it down for a second or so, and then release it so that its damper will stop the vibrations of its wire. Middle C is now heard, proving that its wire is vibrating. This can occur only in case the lower C_1 produces vibrations that are sympathetic with middle C , besides the vibrations which correspond to its own fundamental. In a similar fashion, the presence of still higher overtones in the sound produced by the lower C_1 may be demonstrated, the G and certain other tones above middle C being made free to vibrate.

The sonometer is particularly well adapted to the study of overtones. Pluck or bow a sonometer string so as to make it vibrate as a whole (Fig. 218*a*); the sound produced is the fundamental, the lowest the string is capable of yielding. While the string is thus vibrating, touch it

lightly with the finger or a feather at its middle point. This damps the vibration at the middle, thus making a node at that point, and the string now vibrates in two segments (Fig. 218*b*). The tone heard is the octave of the fundamental. With the attention fixed on the sound of the octave, again make the string vibrate as a whole. Most ears can readily distinguish the tones of both the octave and its fundamental. Excite the string again, and touch it

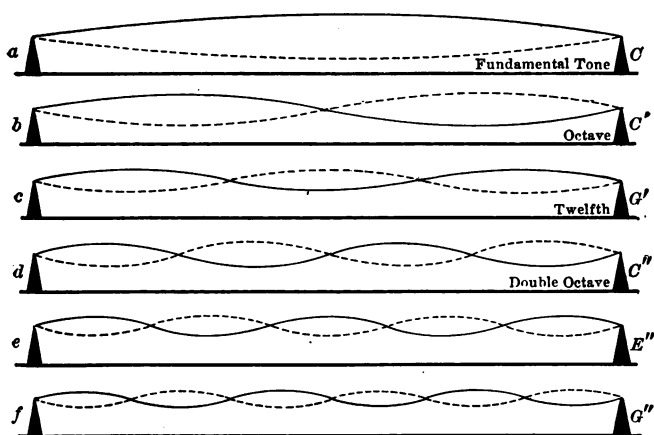


FIG. 218.—Overtones of Vibrating Strings.

at one third of its length; it divides into three segments (Fig. 218*c*), and a tone which a trained ear recognizes to be the fifth above the octave (twelfth above the fundamental) is heard. With a long string excited and then touched at a fourth, fifth, etc., of its length, additional tones may be heard. Evidently the string vibrates in a most complex manner, giving at the same time a number of tones, each of which can be distinguished by a trained ear. The partial tones are called the first, second, third, and so on, overtones, and have frequencies that are, respectively,

twice, three times, four times, and so on, that of the fundamental tone.

394. Mode of Excitation and Production of Overtones.—

When a sonometer wire is made to vibrate by plucking it at one half, one third, one fourth, etc., of the distance from one end, differences in the quality of the tones produced can be readily perceived. An antinode is always formed at the spot excited. Hence a string excited at its middle point tends to vibrate in one segment, and the tone produced is mainly that of the fundamental, the comparatively feeble overtones being those due to vibrations that also have segments at the middle of the string. By exciting the string at one fourth of its length from one bridge, it is made to vibrate in two segments, the nodes being at the middle of the string and at the two bridges. While the fundamental tone will also be heard, the loudest sound will be that of its octave accompanied feebly by such overtones as have a node at the middle of the wire. In like fashion, by exciting the string at other points, those overtones which have antinodes at the points excited will be made to sound out more loudly than the others, thus impressing a peculiar quality on the composite sound produced.

395. Quality and Overtones.—When a string or bar is made to vibrate so as to give but few and feeble overtones, the sound, as a rule, is soft and mellow, such as that produced by exciting a sonometer wire at its middle or by bowing a tuning fork. But when the vibrating body yields overtones also, especially those up to the sixth, the resultant tone is characterized by fullness and richness. With instruments of the violin class, a skillful musician can at will so use the bow as to bring out prominently certain overtones, thereby greatly enhancing the musical effect. Overtones above the sixth are mostly discordant with the fundamental, and their presence imparts a sharp, jangling quality, such as is heard when a wire is plucked very near one end, or a tuning fork is

apped with a hard object. To prevent the production of a certain discordant overtone, or *inharmonic partial*, the string is excited at a point where the node of the discordant overtone would otherwise appear.

396. Overtones of Wind Instruments.—When a gentle current of air is directed across the end of a narrow tube open at both ends, the tone produced is the lowest it is capable of yielding. By increasing the blast, a tone an octave higher, the first overtone is heard, and with still

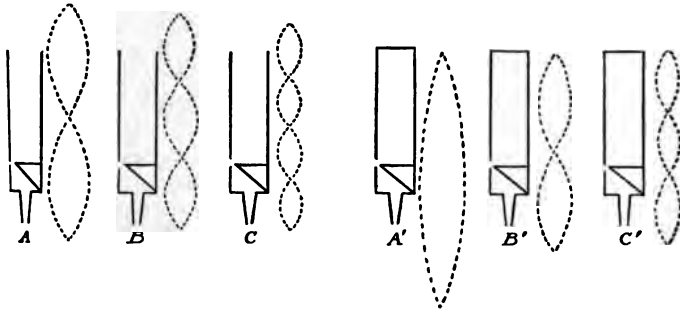


FIG. 219.—Overtones of Organ Pipes. *A, B, C* are open; *A', B', C'* are closed. The dotted lines represent the waves emitted by the pipes.

stronger air pressures, the second, third, fourth, and even fifth overtones may be produced. The vibrating air column breaks up into shorter and shorter waves (Fig. 219). By proper regulation of the jet of air, any overtone may be made to sound to the apparent exclusion of the others. The tones of a bugle, as its length is fixed, are limited to five of its overtones (the fundamental is lacking). By introducing valves into the instrument, however, like those in the cornet and similar brass instruments, tones between the overtones can be played; for in that case there can be as many groups of overtones as there are possible lengths of pipes. While with wind instruments open at both ends

all the overtones are possible, in instruments with stopped pipes only the overtones having wave lengths that are odd multiples of the wave length of the fundamental are possible. Because of this lack of overtones, the stopped pipes of an organ have a different quality from that of its open pipes.

EXERCISES

1. The seventh overtone is discordant with its fundamental. How is the hammer made to strike the strings of a piano so as to nearly obliterate this tone?
2. Almost all wind instruments are open at both ends. What objection would there be to an instrument closed at one end? Do you know of any such instruments?
3. If the second overtones of two fundamentals have the ratio $\frac{11}{10}$, what is the ratio of the fundamentals?
4. Two sounds, each consisting of a fundamental and its first overtone, reach the ear simultaneously. If one of the fundamentals has a frequency of 400 and the other of 402, how many beats per second are due to the fundamental tones and how many to the overtones?
5. What are the frequencies of the first six overtones of a stopped pipe having a fundamental with a frequency of 200?

HARMONY AND DISCORD

397. Melody and Harmony.—Any succession of tones which is agreeable to the ear may be termed a *melody*. The simultaneous sounding of two or more tones with intervals standing in what may be called a *concordant* or *consonant* relationship to one another constitutes *harmony*. The essential characteristic of melody is progression, that of harmony is rest. Melody stands in a close relationship to harmony, as will be shown in the case of one of the simplest of melodies, the major diatonic scale, and some of the simplest of harmonies.

398. Major Triads.—When any fundamental, its major third, and its fifth, are sounded together, the three tones produce a pleasing sensation; they form a harmony called a *major triad* or *chord*. Thus in the key of *C*, the major triads made up of *C - E - G*, or *F - A - C'*, or *G - B - D'* all form pleasing combinations of tones. Moreover, such

melodies as contain only the tones of the diatonic scale can be accompanied by these chords. Evidently, then, the tones of the diatonic scale stand in a close relationship to these triads, and an inspection shows that these chords contain all the tones of the diatonic scale. What is here found true of the key of *C* is likewise true of every other major scale.

399. Major Triads and the Diatonic Scale.—Since musical intervals are defined as ratios, let us, to simplify the numerical relationships, assume that the fundamental tone *C* vibrates 24 times a second. Then the vibration numbers of the three tones of the triad *C*—*E*—*G* are $24 \times 1 = 24$, $24 \times \frac{3}{2} = 36$, and $24 \times \frac{5}{4} = 30$. The vibration numbers of the triad *G*—*B*—*D'* are $36 \times 1 = 36$, $36 \times \frac{3}{2} = 54$, and $36 \times \frac{5}{4} = 45$. Since the third tone of the triad *F*—*A*—*C'* is the octave of the fundamental, its vibration number is $24 \times 2 = 48$. And since the interval between *F* and *C'* is a fifth, the vibration number of *F* is $48 \times \frac{2}{3} = 32$. Furthermore, since the interval between *F* and *A* is a major third, the vibration number of *A* is $32 \times \frac{4}{3} = 40$. To get *D'* within the compass of the octave having as fundamental *C* = 24, transpose *D'* an octave lower so that $D = 54 \times \frac{1}{2} = 27$. The tones composing the three triads when arranged in the order of their vibration numbers give the series:

Notes	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>	<i>A</i>	<i>B</i>	<i>C'</i>
Vibration numbers	24	27	30	32	36	40	45	48

By doubling these vibration numbers we obtain those of the tones of the diatonic scale pitched an octave higher:

Notes	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>	<i>A</i>	<i>B</i>	<i>C'</i>
Vibration numbers	48	54	60	64	72	80	90	96

Let us now find the vibration numbers of the tones making up the triad based on *D* = 27. Its major third has the vibration number, $27 \times \frac{4}{3} = 36$, and its fifth the vibration number, $27 \times \frac{3}{2} = 40\frac{1}{2}$. Neither of these tones is contained in the series based on *C*. $33\frac{3}{4}$ falls nearly halfway between 32 and 36, and by introducing an intermediate tone, and calling it, as is done in music, *F* sharp (*F*♯), that difficulty is surmounted. But $40\frac{1}{2}$ comes so near to 40 that either an intermediate tone must be introduced or the two tones must be recognized as practically identical.

Again, the major triad based on *E* = 30 has for its third, $30 \times \frac{4}{3} = 40$, and for its fifth, $30 \times \frac{3}{2} = 45$. While the tone represented by the vibra-

tion number 45 exists in the first scale, the tone corresponding to $37\frac{1}{2}$ falls between $G = 36$ and $A = 40$. Another intermediate tone, G sharp ($G\sharp$), can be introduced to meet this emergency.

The continuation of such calculations as these to find the vibration numbers of the triads based on all the tones of all the scales used in music would show that about seventy different tones would have to be comprised within the compass of each octave. Any keyed instruments, such as the piano or organ, would therefore need to have a correspondingly large number of keys. This would, of course, increase enormously the difficulty of executing musical compositions. Instruments of the violin or slide trombone class could be played in, and the human voice could sing in *pure intonation*, as it is called, for they are capable of producing tones of any frequency within their compass. When, however, about two centuries ago, the importance of the piano and the organ began to be appreciated, an expedient was devised to obviate these difficulties.

400. The Tempered Scale. — Instead of using a scale in which all the tones demanded by the mathematical relationships of the intervals are contained, musicians have agreed to compromise matters by adopting a scale, which although full of slight discords, yet meets the exigencies of the situation admirably. The only interval maintaining the correct mathematical ratio is the octave, and every octave is divided up into twelve equal intervals, each of which has a vibration number 1.059 times greater than its predecessor. This scale is called the *tempered* or *chromatic scale*, and its intervals are *semitones*. Tones having vibration numbers as nearly equal as 40 and $40\frac{1}{2}$, for instance, are both represented by a tone with a vibration number equal to their average, $40\frac{1}{4}$. Although the triads are out of tune when made up of these tempered tones, the discord is so slight, and we have become so accustomed to it, that the æsthetic side of music cannot be said to have suffered by the adoption of the tempered scale, while its practical possibilities have been extraordinarily enhanced.

401. Cause of Discord. — I. *Beats.* When two tones which give less than four beats per second are sounded simultaneously, the combined sound does not seem unpleasant. The swelling and falling in the intensity of the resultant tone is quite noticeable, but does not seem dis-

agreeable. But when the number of beats surpasses four or second, the sound begins to be unpleasant, affecting our sense of hearing much as a flickering light affects our sense of sight. When the beats reach thirty per second, the disagreeable sensation begins to wear off, and tones beating more frequently may, but need not necessarily, affect our auditory sense agreeably. "Sound flickers" are not particularly disagreeable when they are slow enough to count; nor are they so when they attain a certain rapidity. It is only when they have intermediate rates that they give rise to disagreeable sensations.

TABLE OF OVERTONES

Frequencies of the First Six Overtones of the Intervals Based on $C = 24$

Tone Combination	Interval	Frequencies of Overtones					
<i>C</i>	$\frac{8}{8}$	24	48	72	96	120	144
<i>D</i>	$\frac{9}{8}$	27	54	81	108	135	162
<i>C</i>	$\frac{4}{4}$	24	48	72	96	120	144
<i>E</i>	$\frac{5}{4}$	30	60	90	120	150	180
<i>C</i>	$\frac{3}{2}$	24	48	72	96	120	144
<i>F</i>	$\frac{4}{3}$	32	64	96	128	160	192
<i>C</i>	$\frac{2}{1}$	24	48	72	96	120	144
<i>G</i>	$\frac{3}{2}$	36	72	108	144	180	216
<i>C</i>	$\frac{3}{2}$	24	48	72	96	120	144
<i>A</i>	$\frac{5}{3}$	40	80	120	160	200	240
<i>C</i>	$\frac{8}{6}$	24	48	72	96	140	144
<i>B</i>	$\frac{15}{8}$	45	90	135	180	225	270
<i>C</i>	$\frac{1}{1}$	24	48	72	96	120	144
<i>C'</i>	$\frac{2}{1}$	48	96	144	192	240	288

II. *Overtones.* A second cause of discord is the absence of common overtones in the tones, so that disagree-

able beats occur, not only between the fundamentals, but also between some of their overtones. The relationship between tones, overtones, and concordance is illustrated in the Table of Overtones, an inspection of which shows that there are three overtones common to the tones forming an octave, two common to a fifth, one common to a third, fourth, and sixth, respectively, and none common to the second nor to the seventh. Now musicians recognize the interval of the octave as being the most concordant, the fifth as standing next in concordance, the third, fourth, and sixth as less concordant, and the second and seventh as being discordant.

402. Analysis of Sounds. — Since nearly every sound is made up of different tones, how may the number of these tones, as well as the intensity and pitch of each of them, be ascertained? In other words, how may sounds be analyzed? Several methods have been devised, the most important of which are the following:

I. *Helmholtz's Resonator Method.* Suppose that we have at hand a large supply of resonators, each tuned to a different tone. By applying them in succession to the ear, the individual simple tones of any sound may be picked out, and not only their pitch but also their intensity ascertained.

II. *Koenig's Manometric-flame Method.* This method requires the use of a so-called *manometric capsule*. This consists of a small box one side of which is covered with a thin and elastic membrane (Fig. 220). Illuminating gas enters the capsule at an opening in one side and escapes in a jet tube on the same side opposite the membrane. When gas is passed through such a capsule and lighted at the jet, the flame burns steadily as long as the pressure is steady. But if the membrane is pushed inward suddenly, the gas is compressed, more of it passes through the jet, and as a consequence, the flame flares up. Also, the flame shortens when the pressure within the capsule is diminished by the bulging out of the membrane. Such a flame is remarkably sensitive to

extremely slight motions of the membrane, so that by directing waves from a sounding body against it by means of an inverted trumpet, characteristic flickerings of the flame are produced by vibrations of the membrane. By viewing the image of the flame

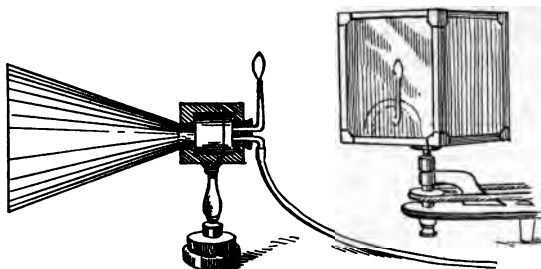


FIG. 220. — Manometric-flame Apparatus.

rotating mirror, the arrangement is made still more delicate. If no sound waves impinge upon the membrane, the pressure of air within the capsule is uniform, and a ribbon of light is observed by the rotating mirror. But let sound waves strike against the membrane, and the ribbon becomes indented

(Fig. 221). The size and shape of these indentations vary with the loudness, pitch, and quality of the sound. Thus the image produced by a certain tuning fork appeared

as shown in Fig. 221 *a*, while a tuning fork of a higher frequency produced an image as shown in Fig. 221 *b*. The number of indentations produced by the second fork is twice the number produced by the first in the same

time. The ratio of the numbers of indentations is the same as the ratio of the frequencies of the two forks. If two whistles connected by a Y-tube are substituted for the single trumpet, and the

whistles are sounded simultaneously, one before each trumpet, their sounds blend before reaching the capsule, and the image produced is shown in Fig. 221 *c*, which is seen to exhibit the characteristics of the simple tones. In similar fashion, the simple tones

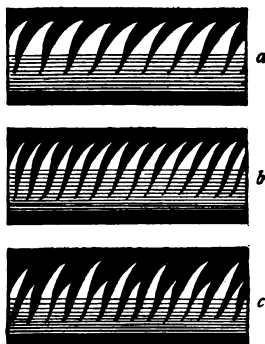


FIG. 221. — Analysis of Sound.

belonging to any sound may be ascertained, the results of the analysis being made to appeal directly to the eye.

III. *Combined Resonator and Optical Method.* If, instead of applying the resonators to the ear, they are connected with manometric capsules, the flame of each resonator will be affected only when the sound under analysis contains simple tones that are in tune with that resonator. Appeal is thereby made to the eye instead of to the ear.

403. Synthesis of Sounds. — When the pitch and loudness of the simple tones constituting any sound are known, it is possible by blending these simple tones properly to reproduce the compound sound. Although the experimental difficulties are great, it has been found possible to reproduce even such complex sounds as those of the human voice.

404. Vibrations of Plates. — A thin plate of metal clamped at its middle point to a stand (Fig. 199) produces, when bowed along its edge, a clear musical tone. Sand strewn over its surface dances about and finally collects at certain portions where there is the least vibration. The sand is thus made to mark the positions of the nodal lines. If the plate is producing its fundamental tone, these nodal lines divide it into four segments. If the plate be bowed at a different place or in a different manner, or if the finger be touched at certain points in the plate, higher sounds will be produced, and the sand marks out correspondingly different nodal lines. Various sounds corresponding to the different overtones producible may be evoked from the plate by skillful manipulation.

405. Musical Instruments containing Plates or Membranes. — *Bells.* By bending a plate or disk into a cup shape, a bell is formed. The mode of vibration of a bell is essentially the same as that of a plate. As the sounds of bells are unusually rich in the higher overtones, their vibrating segments are numerous and complex in arrangement, although the division into the four segments giving the fundamental is always the most pronounced. Because of the high and prominent inharmonic overtones, bells, however well tuned, give rise to displeasing jangles when sounded simultaneously.

Musical Glasses. A glass goblet or tumbler forms what may be considered to be an inverted bell; when partly filled with a liquid, and set in vibration by bowing or rubbing with a wetted finger, a musical sound is produced, and the ripples of the water disclose two nodal lines at right angles to each other (Fig. 222). With goblets of different sizes or of the same size but filled to different depths with water, the tones of the scale may be produced, and with such a set of musical glasses simple melodies may be played.



FIG. 222. — Musical Glass.

Cymbals and gongs also consist of plates set into vibration by striking them together or with a stick.

Drums consist of membranes stretched tightly over a resonant case, and set into vibration by beating with a stick. *Tympani* or *kettledrums* are of different sizes and are tuned by tightening or loosening the membrane.

EXERCISES

1. The *do* of a certain musical scale is produced by a string 15 in. long stretched over a sonometer. Where must the movable bridge be placed to give *sol* of the same scale?
2. Write out the major triad based on the tone *B* (480), and calculate the frequencies of the tones.
3. Three tones have frequency ratios of 4 : 5 : 6. If the vibration number of the highest tone is 300, what are the vibration numbers of the other two?
4. If a tone is due to 288 vibrations per second, to how many vibrations per second will its major third, fifth, and octave, respectively, be due?
5. Calculate the frequencies of the notes of a major scale, the frequency of the keynote being 261.
6. The sound of a bee's buzzing was estimated to be a major third below *A* (435). What was the note and the vibration number of the buzzing?
7. Why is it that chimes of bells are used to play melodies only? Why are not two or more of the bells sounded simultaneously so as to produce harmonies?

CHAPTER XVIII

RADIANT ENERGY

406. The Ether.—We learn of matter through our senses. We experience a certain sensation and attribute it to the action of some form of matter. The continued exercise of our senses results in our acquiring a fund of knowledge in regard to the aspects and actions of matter in its various forms and conditions. We can know that matter exists only by observing the phenomena to which it gives rise. If, then, many phenomena can receive an explanation satisfactory to our reason only on the assumption that there exists an unfamiliar form of matter through the agency of which these phenomena take place, we have as good a right to believe in its existence as we have in the existence of any form of matter. Now the phenomena of light are satisfactorily explained when they are ascribed to wave motions in a form of matter that is quite different from the more familiar form. So also many phenomena of heat, magnetism, and electricity receive a simple explanation when it is assumed that they are due to the vibrations of a single form of matter of remarkably simple properties. This form of matter is called the *ether*, sometimes the *luminiferous ether*, and has been characterized (by Lodge) as “a perfectly continuous, subtle, incompressible substance pervading all space and penetrating the molecules of all ordinary matter, which are embedded in it and connected with one another by its means.” This ether is quivering with incessant vibration. Although

the conception of the ether may seem strange at first, yet a study of its properties and behavior gradually forces us to the conviction that, after all, it is not imaginary, but has a real existence ; and we are finally led to believe that it is such a simple form of matter that we can find out more about it than about ordinary matter.

407. Ether Waves. — The vibrations of the molecules or the electrons (§ 5) of ordinary matter can set the all-pervading ether vibrating. Wave motion is thus set up in the ether, and these ether waves, when they impinge on other molecules of ordinary matter, can set them vibrating. A transfer of energy is thereby affected through the medium of ether waves, and it is by a study of these energy transferences that we get a knowledge of the ether itself. The name of *radiation* is given to this process of energy transference by ether waves, and energy so transferred is designated as *radiant energy*.

Ether waves are transverse, and their speed depends upon whether the medium is ether alone, or, if not, upon what substance is embedded in the ether, varying from about 100,000 mi. to 186,000 mi. per second. The length of the waves varies from a million miles (in the case of certain electrical waves) to millionths of a centimeter, those produced by molecular vibrations ranging from about 13×10^{-5} cm. to 61×10^{-7} cm. These shorter waves must have, since their speed is so great, very great frequencies ; they range from ten million millions to one thousand million millions per second.

408. Kinds of Radiant Energy.—When waves in the ether come into contact with ordinary matter, part of the energy of the wave motion is transformed into heat energy

or *radiant heat*. If the frequencies fall between 392×10^{12} and 757×10^{12} vibrations per second, they have the power of affecting the sense of sight, and the phenomena produced by such waves are ascribed to the action of *light*. When the waves have a frequency above 757×10^{12} per second, they become invisible; but they still retain, however, the power of affecting chemical changes in certain substances, such, for instance, as are in a photographic plate; such waves are called *chemical* or *actinic waves*.

The lengths of the ether waves therefore determine the kind of radiant energy. While waves of all lengths may produce heat, the shortest will also produce chemical effects, and the intermediate will produce light. We have then to do with long, so-called *dark heat rays*, medium *light rays*, and short *chemical rays*.* As we have a special sense for recognizing light waves, the phenomena of light earliest received attention. We shall therefore in our study of radiant energy confine ourselves mainly to light, keeping in mind, however, that what is found true of light waves is typical, and can, with proper modifications, be applied to other ether waves as well.

409. Color due to Frequency.—Just as differences in the frequencies of vibrations in air affect the ear differently, so do differences in the frequencies of vibrations in ether affect the eye differently; pitch in sound is analogous to color in light. Further, just as the range of frequency of air vibrations that can affect the ear is limited, so is the range of frequency of ether vibrations that can affect the eye also limited. The lowest frequency that affects the eye is about 392×10^{12} per second, and the sensation to which it gives rise is described as a red color. Increased

* Electrical waves are usually much longer than other kinds of waves.

frequency gives rise to different color sensations, running through the so-called prismatic colors, up to a violet color, due to a frequency of about 757×10^{12} . So-called *white light* is due to vibrations of all frequencies within the range of visibility, which, to borrow a musical term, is not quite an octave. When light is due to a single frequency of vibration, it is said to be *monochromatic* (*mono* = *one*, and *chroma* = *color*). The phenomena of color will be treated in Chapter XX, and until then, we shall, unless otherwise specified, consider white light alone, since the phenomena to which it gives rise, may with suitable modifications be applied to monochromatic light as well.

410. Propagation of Light.—Light waves travel in straight lines, so long, at least, as there is no change in the medium. If a luminous point—a *point source*—is considered (Fig. 223), it will

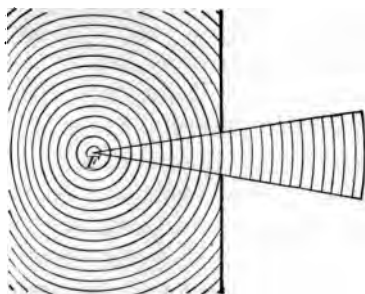


FIG. 223. — Propagation of Light.

be seen that the rays of light pass out in all directions like the radii of a sphere, the center of which is the luminous point; the wave fronts are spherical surfaces. When the rays separate as they advance, they are said to *diverge*; and all rays from a point

source must be diverging. *Converging rays* approach one another as they proceed. Both converging and diverging rays form *pencils* or *cones of illumination*. The point of a pencil of rays, *i.e.* the center of the disturbance which sets up the waves, is called the *focus*. Parallel rays all have the same direction, and the name of *beam* is given to a

bundle of them; a beam of light has a plane wave front.

411. Sources of Light. — While everything visible may be regarded as a source of light, an object may be visible of itself or may depend for its visibility upon another object or objects. Thus the sun is self-luminous, but the moon depends upon the sun for its light; moonlight is reflected sunlight. *Luminous bodies* are such as give out light independently of other bodies, and *illuminated bodies* are such as are visible only through the agency of luminous bodies. Luminous bodies generate radiant energy; illuminated bodies reflect it.

412. Artificial Light. — When the temperature of a body is raised high enough, its molecules are made to vibrate so rapidly as to set up trains of waves in the ether of the right length to affect the eye. Some incandescent substances give vibrations of numerous frequencies, their light therefore being more or less white, while others give rise to vibrations of a few frequencies only or even of a single frequency, the light produced being monochromatic. Compare, for illustrations, the colors of the lights of incandescent carbon, as produced in most flames, in the arc light, and in the incandescent lamp with the color produced by incandescent sodium, as obtained by heating a piece of glass.

413. The Emission or Corpuscular Theory of Light. — All of the phenomena of light known up to the end of the eighteenth century could be satisfactorily explained on the hypothesis that they were due to streams of particles or *corpuscles* emitted by the luminous body, in some such fashion as the odor of a body is due to the emission of particles from the odorous body. The wave theory was indeed suggested as early as 1678 by Huyghens, but, as it required the assumption of the ether and did not apparently offer a satisfactory explanation of straight-line propagation, it was not generally accepted. But when the phenomenon of interference, for which the emission theory cannot account, was found to occur in light, the wave theory was revived,

and careful study soon revealed the fact that it does in reality satisfactorily explain rectilinear propagation.

414. Young's Experiment.—In 1801 Young devised the following experiment. Two tiny holes are cut very close together in a screen which is then strongly illuminated on one side with light of one color, say red. From the two holes issue pencils of light, which overlap each other (Fig. 224). When a white screen is held in front of these overlapping cones of light, alternate red and black bands are seen upon it. If one of the holes is covered, the black bands vanish, and the screen is illuminated uniformly with red light. The black bands indicate the absence of illumination, and must be due to the interference of the two trains of light waves issuing from the holes.

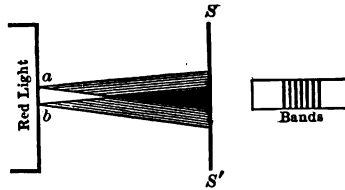


FIG. 224.—Interference of Light.

415. Diffraction of Ether Waves.—The chief objection raised against the wave theory of light was that it fails to account for the apparently true fact that light travels in straight lines, and does not, like water and sound waves, bend around obstacles in its path. This objection was sustained for more than a score of years after Young made the discovery of interference, or, until it was shown definitely that diffraction of light waves actually does occur. The reason that it is or was so hard to demonstrate the diffraction of light waves is to be ascribed to their extremely short length, because of which the amount of deviation from a rectilinear course is very hard to detect. Now, however, it is possible to produce ether waves which differ from light waves only in their greater length, and these

longer waves may be observed to bend around corners very noticeably.

416. Shadows. — When an opaque object is brought near a luminous point, it absorbs or reflects the light impinging upon it, and a dark space is left behind the object. This unilluminated space is called a *shadow* (Fig. 225). Any screen placed across it shows a section of the shadow. If a second luminous point is also used, some of its light will illuminate the shadow of the first, so

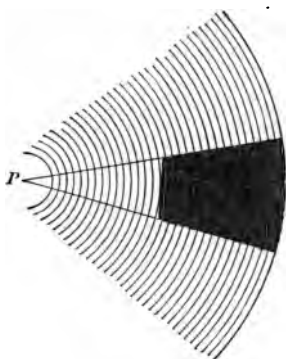


FIG. 225 — Full Shadow or Umbra.

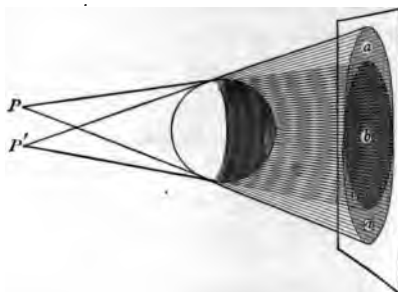


FIG. 226. — *a*, Penumbra; *b*, Umbra.

that a section of it caught on a screen will have portions less dark than others (Fig. 226). The name of *umbra* or *full shadow* is given the part of a shadow which is not at all illuminated by the source, and that of *penumbra* or *partial shadow* to the part which is only partly shaded from the light.

417. Speed of Light. — It was at one time the prevailing opinion that rays of light pass from one place to another instantaneously; but in 1675, Roemer found it necessary to assume that the passage of light is not instantaneous, in order to get a satisfactory explanation for some observa-

tions he had made of the times elapsing between successive eclipses of the most prominent moon of Jupiter. If Jupiter and the earth at a certain date are in conjunction, *i.e.* are both on the same side of the sun, and in the same straight line with it, some six and a half months later they will be in opposition, *i.e.* on opposite sides of the sun, and in the same straight line with it. Jupiter's moon revolves once in every $42\frac{1}{2}$ hours, part of the time being behind Jupiter and invisible from the earth (Fig. 227). Suppose that the time between two successive eclipses has been observed when the earth and Jupiter are in conjunction, and from this observation, the time for

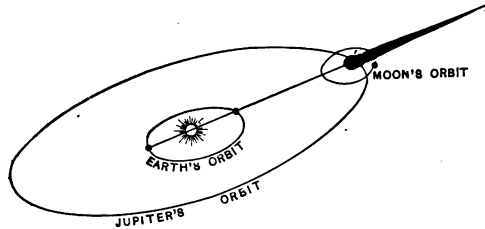


FIG. 227. — Speed of Light by the Astronomical Method.

following eclipses calculated. It is found that the eclipses keep occurring slightly later than the calculated time, the delay gradually increasing until the two planets are in opposition, when the difference between the calculated and observed times amounts to 998 sec. The delay then decreases until the planets are again in conjunction, when the observed time is found to coincide with that calculated. Roemer interpreted this behavior to mean that light requires time to traverse space. The eclipse occurs earlier than it is seen by an observer on the earth because of the time light takes to pass through a distance equal to that between Jupiter's satellite and the earth. Inasmuch as this distance is greater by a distance equal to the diameter of the earth's orbit when the planets are in opposition than when they are in conjunction, the maximum delay of 998 sec. must

be the time that light takes to travel across the orbit of the earth. This makes the speed of light in ether $186,000 \frac{\text{mi.}}{\text{sec.}}$ $= 3 \times 10^{10} \frac{\text{cm.}}{\text{sec.}}$, a result quite in accordance with the results of later measurements made by other methods.

418. Speed of Light in Different Media.—When light passes from the ether into transparent substances, its speed is diminished. While the amount of diminution is scarcely perceptible in gases, it is considerable in many liquids and solids.

419. Pinhole Camera.—If the shades be drawn so as to darken a room, a pinhole pricked in one shade, and a screen of white or translucent (oiled) paper held a foot or so back of it, an inverted and diminished image* of the scenery

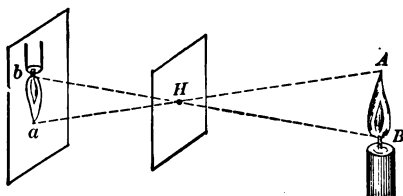


FIG. 228.—Diagram Illustrating the Principle of the Pinhole Camera.

outside may be seen on the screen (Fig. 228). If the hole is enlarged, the outlines of the image become less and less distinct, until finally they assume the shape of the hole itself, and the image of the scenery disappears. The illuminated or luminous objects outside send out light rays, some of which reach the window shade, but comparatively few pass through the hole. Those which do, however, keep their straight-line course until they reach and are reflected from the screen. If the hole is enlarged, the image will be made brighter; but so many more rays from points of the object will pass through the hole that the images will overlap, and the outlines will become blurred.

* When a luminous or illuminated body seems to be in any position in space other than that which it actually occupies, the appearance is called an *image*.

9. The Visual Angle.— If a pinhole be pricked in a card, the card close to the eye, and a pin brought slowly from arm's length toward it, its dimensions will appear to increase as its distance to the eye decreases. The apparent magnitude of the pin depends upon the extent of the retina of the eye which its image covers. It is clear that the nearer the object is, the larger the image, and therefore the larger the object seems.

The size of the image depends upon the size of the angle, termed *visual angle*, formed by the rays from the extremities of the object at the eye. The size of this visual angle is approximately inversely proportional to the distance of the object from the eye. Hence the distance of an object from us has to do with our estimation of its actual size. The image on the retina produced by seeing a man 10 ft. away is 10 times larger than when he is 100 ft. distant. But we have learned to make allowance for the effect of distance, so that we do not interpret a larger image as meaning a larger man. But if the distance of a remote object is unknown, our conception of its size is liable to be quite erroneous, as is illustrated in trying to estimate the size of the moon.

10. Illumination.— The illumination of any area is the amount of light falling upon that area. It depends upon the distance between the source and the illuminated surface as well as upon the intensity of the source.

Effect of Distance. A given amount of light constantly sent out from a point source spreads over surfaces of ever increasing area, and consequently, the intensity of illumination decreases as the illuminated surface is farther from the source (Fig. 229). Any surfaces of equal area and placed at right angles to the rays will have equal illuminations, provided they are equally distant from the source, or unequal illuminations if they are at unequal distances from the source. As the areas of corresponding parts of spherical surfaces are to each other as the squares of the radii, *the illumination of a given surface due to a source is inversely proportional to the square of its distance from the source.*

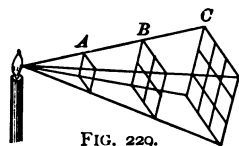


FIG. 229.
Distance and Illumination.

Effect of Illuminating Power. The illuminating power of a source is the total amount of light continuously emitted by that source. The

unit of illuminating power is the *candle power*, and is the amount of light given out by a standard candle made of sperm that is $\frac{1}{4}$ in. in diameter, and that burns at the rate of 120 grains ($= 7.78$ g.) per hour. A 16 candle-power incandescent electric lamp therefore gives out sixteen times as much light as a standard candle. *The illuminations produced by different sources of light at equal distances from an illuminated surface are directly proportional to their illuminating powers.*

Combined Effect of Distance and Illuminating Power. Denoting by P the illuminating power of a source, by I , the illumination, and by d , the distance between source and surface, we have

$$I \propto \frac{P}{d^2}.$$

422. Photometry. — Photometry (*photos* = light) has to do with the measurement of the illuminating powers of sources of light. While it is difficult to make a reliable estimate of the comparative brightness or darkness of two unequally illuminated surfaces, it is possible to judge with considerable accuracy whether or not two adjacent parts of a surface are equally bright or dark. If two sources of light are placed at such distances from a surface that they appear to illuminate it equally, then, denoting by P_1 and P_2 , the candle powers of the sources, and by d_1 and d_2 , their respective distances from the illuminated surface, we have

$$I = \frac{P_1}{(d_1)^2} = \frac{P_2}{(d_2)^2};$$

the candle powers of two sources of light are directly proportional to the squares of their respective distances from the surface which they illuminate equally.

423. Photometers. — Photometers are instruments for measuring the illuminating powers of sources of light. They must be used in dark rooms so that any light other than those compared may be excluded.

424. Bunsen's Photometer.— If a translucent spot be made in a piece of paper by oil or paraffin, and the two sides of the paper illuminated, the spot will appear darker than the paper when viewed from the side more strongly illuminated. When, however, the illuminations on both sides are equal, the spot becomes almost indistinguishable from the surrounding surface. Bunsen's photometer (Fig. 230) consists of a graduated rod supported horizontally, upon which slide bases carrying the lights to be compared and between them an oil-spot screen. The screen

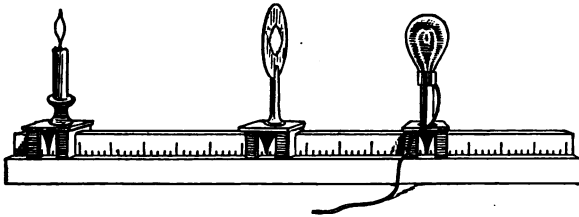


FIG. 230. — Bunsen's Photometer.

is moved between the two sources of light until the oil spot is equally illuminated on both sides. Then the ratio of the illuminating powers of the sources is equal to the ratio of the squares of the distances of the sources from the screen.

EXERCISES

1. How is it that a carpenter by "sighting" along the edge of a board can tell whether it is straight?
2. What does the fact that the shadows of opaque objects are geometrically similar to the objects prove in regard to the propagation of light?
3. Why is it that the spots of light seen on the ground when the sun is shining on a tree of not too dense foliage are always circular or slightly oval?
4. Why does an arc lamp cast shadows practically free from a penumbra?
5. Why do the parallel tracks of a railroad appear to approach at a distance?
6. What determines the ratio of the size of the image of a pinhole camera to the size of the object?
7. If the brightest star in the heavens, Sirius or the Dog Star, were suddenly annihilated, we should still see its light for 16.7 yr. after its destruction. How far away is the star?

8. When the sun's rays make an angle of 45° with the horizon, how long is the shadow cast by a vertical pole 10 m. long?

9. Show by a figure what effect it has upon the size of a shadow to remove the object twice as far from the source of light, the screen remaining in the same position.

10. At what distance from the eye will a quarter of a dollar subtend the same visual angle as a nickel held 35 cm. from the eye, if the diameters of the two coins are 25 mm. and 19 mm., respectively?

11. A silver dollar held 20 cm. from the eye hides an area of 500 cm.² on a wall. How far away is the wall?

12. A pencil 6 in. long when held vertically 2.5 ft. from the eye just covers the height of a tree 200 ft. away. Find the height of the tree.

13. If the light per unit of surface received from the sun by a certain planet is $\frac{1}{25}$ of that received by the earth, how much farther from the sun is that planet than the earth?

14. Compare the illuminations produced by a lamp of 100 candle power at a distance of 15 in. and a lamp of 250 candle power at a distance of 10 in.

15. A 2000 candle-power arc light is 1000 ft. from a 100 candle-power incandescent light. At what point between them do they produce equal illuminating effects?

16. How does the illumination upon a photographic plate held 50 cm. from an arc light compare with the illumination when it is held 80 cm. from the light?

17. How much farther from a screen must a 16 candle-power electric bulb be placed than a 10 candle-power bulb to produce equal illuminations?

18. A hole is made in a window shade in a darkened room so that the image of a tree 60 m. away is caught on a vertical screen placed 20 cm. from the hole. If the image is 25 cm. high, what is the height of the tree?

CHAPTER XIX

REFLECTION AND REFRACTION OF LIGHT

REFLECTION AT PLANE SURFACES

425. Law of Reflection. — When a sunbeam is caught upon a reflecting plane surface, it may be reflected directly back upon itself by holding the surface so that it is perpendicular to the direction of the beam. By rotating the mirror the reflected beam is seen to move through an angle that is always twice as

large as the angle through which the mirror is rotated. The angle ION (Fig. 231), the *angle of incidence*, between the incident beam IO and the normal NO ,

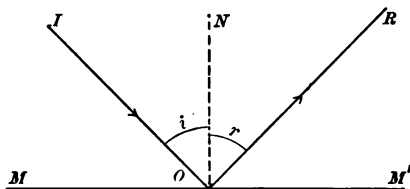


FIG. 231. — Reflection of Light.

is always equal to the angle RON , the *angle of reflection*, between the reflected beam OR and the normal NO ; and both incident and reflected beams are in the same plane. *The angle of incidence is equal to the angle of reflection, and both lie in the same plane.*

426. Regular and Irregular Reflection. — When a beam of light falls upon a reflecting surface, the angle of reflection of the whole beam will not equal the angle of incidence, unless the surface is perfectly smooth (Fig. 232). With a rough surface some of the rays composing the beam will

be reflected in different directions, and while there will be equality between the angles of incidence and reflection for each ray, the reflected rays will not, like the incident rays, make a regular beam, but will be scattered (Fig. 233).

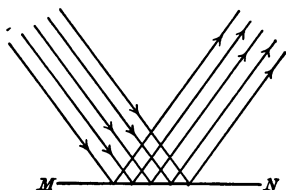


FIG. 232. — Regular Reflection.

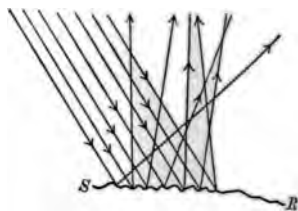


FIG. 233. — Irregular Reflection.

427. Mirrors. — Any surface that reflects regularly a large portion of the light it receives is called a *mirror*. Mirrors must have surfaces so smooth that their inequalities are small in comparison with the lengths of light waves, otherwise the light incident upon them will be scattered. Polished metals, such as silver, tin, or steel, form excellent mirrors, as do also films of silver or tin amalgam (a mixture of tin and mercury) attached to plate glass. In the last kind, a little light is reflected from the front surface of the glass; but the main reflection comes from the metallic film on its back.

428. Diffused Light. — Any surface that is not perfectly smooth causes the light incident upon it to be more or less irregularly reflected or *diffused*, and a portion of such a surface large enough to be seen reflects light in so many directions at once as to behave like a luminous body. Every point of such an illuminated surface acts therefore as a source of light and sends out reflected waves in many directions so as to produce much the same effects as a self-luminous body. Sunshine before reaching the ex-

is intercepted by myriads of particles of dust in the atmosphere, each one of which reflects the light. The sunshine is thereby diffused to a large degree. Objects on the earth also diffuse the sunlight, and by the repeated reflections of this diffused light, objects not in direct sunshine are rendered visible. The same result occurs, whatever the source of light, be it natural or artificial.

429. Images.—Whenever an object appears to be in a position other than that which it actually occupies, this seeming object, this optical counterpart, is called an *image*. The image of a point is termed a *focus*, and the image of an object is made up of the images of all the points of the object. When the rays of light actually pass through the position occupied by the image, the image is said to be *real*, but when they only seem to pass through that position, the image is said to be *virtual*. Thus, by means of a burning-glass, an image of the sun may be formed at its focus, and such an image is real, as may be tested by putting the finger on it; but the image seen in a looking-glass is virtual, for no light really comes from behind the glass where the image is. To describe an image fully with reference to its corresponding object, six items must be stated. These *characteristics of an image* are:

- (1) *Situation*—whether image is on the same or opposite side of the mirror or lens.
- (2) *Kind*—whether real or virtual.
- (3) *Distance*—from the mirror or lens.
- (4) *Size*—compared with the size of the object.
- (5) *Position, laterally*—whether reversed, *i.e.* right and left sides interchanged.
- (6) *Position, vertically*—whether erect or inverted.

430. Image of a Point in a Plane Mirror.—*Wave Construction.* The construction is identical with that for a

circular wave reflected from a plane surface (Fig. 186). The point source sends out waves with spherical fronts that on reflection appear to come from behind the mirror.

Ray Construction. Let MM' (Fig. 234) represent a plane mirror receiving light only from the luminous point P . The rays of light that stream out from P and fall upon the mirror will be for the most part reflected, and after reflection will diverge. Those that

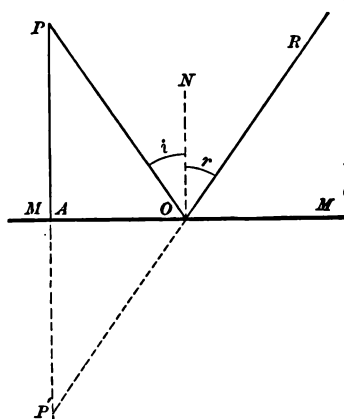


FIG. 234.—Image of a Point.

reach the eye will appear to come from a point behind the mirror, as P' , which is the image of P . The direction of the reflected rays may be found as follows. The ray PA , being normal to the mirror, is reflected back upon itself, since the angle of incidence (in this case zero) equals the angle of reflection. Hence the image of P lies in the line PA or its prolonga-

tion. Any other ray, as PO , will be reflected in the direction OR so as to make the angles PON and RON equal. Hence the image of P must also lie on OR or its prolongation. The image must therefore be located at the intersection of the prolongations of the lines PA and OR ; and since they diverge in front of the mirror, they must intersect behind the mirror at P' . NOA is a right angle, and since $RON = PON$, then $POA = ROM'$. But $P'OA = ROM'$. Therefore $POA = P'OA$, and the triangles PAO and $P'AO$ are equal, whence $PA = P'A$. *The image of a point in a plane mirror is virtual, and lies as far back of the reflecting surface as the point is in front.*

431. Image of an Object in a Plane Mirror. — Since the image of an object consists of the images of its points, the image of an object can be found by constructing the images of its points. An arrow is a simple object to construct, and if the images of its two ends are located, those of its intermediate points fall in the line connecting the extremities. Let AW (Fig. 235) represent an arrow in front of the mirror MM' . The normals at O and O' are produced behind the mirror until MM' divides the lines AA' and WW' into two equal parts. Then $A'W'$ is the image of AW . *An image in a plane mirror is erect and virtual;*

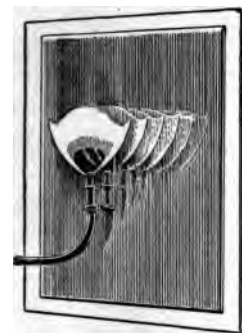


FIG. 236.—Multiple Reflection in a Thick Looking-Glass.

it is of the same size as the object and as far behind the mirror as the object is in front of it.

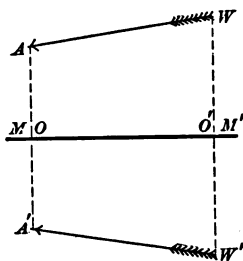


FIG. 235.—Image of an Object.

432. Multiple Images. — If an object be so placed that light from it falls upon more than one reflecting surface, there will be more than one image formed. Thus several images may be seen in a thick plate-glass mirror, a bright one from the metallic backing, and a series of others of decreasing brightness due to repeated reflections from the front and back surfaces of the glass (Fig. 236).

If an object is set halfway between two plane mirrors at right angles to each other, three images are seen when the eye

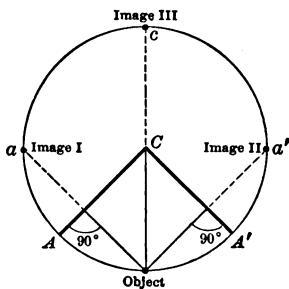


FIG. 237.

is placed near the object (Fig. 237); and if an object is set between two mirrors making an angle of 60° with each other, five images are seen (Fig. 238), situated at equal distances around the circumference of

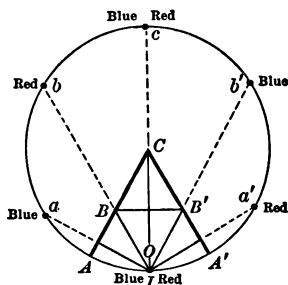


FIG. 238.—One side of the object is red and the other side blue.

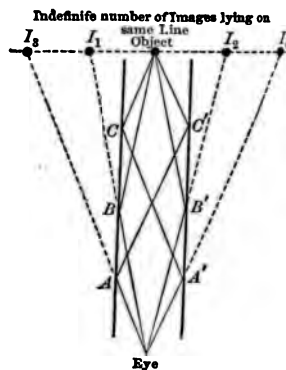


FIG. 239.

a circle. Parallel mirrors give rise to an indefinite number of images all on the same line passing through the object (Fig. 239).

EXERCISES

1. Why is a perfect mirror invisible?
2. Why is a perfectly transparent body invisible?
3. Pictures in glass-fronted frames are often obscured from view by a glare when looked at from certain angles. How is this glare accounted for?
4. Why does the sky appear much darker to an observer at high altitudes than at lower?
5. Write your name on a piece of paper, and before the ink is dry, blot it with a clean blotter. How do you account for the difference in the image seen in a plane mirror of the original writing and of its copy?
6. How does an image seen in a plane mirror differ from one observed on the screen of a pinhole camera?
7. If a man moves toward a plane mirror at the rate of $\frac{50 \text{ cm.}}{\text{sec.}}$, how fast does he approach his image?
8. A capital letter W is drawn on a piece of paper lying on the table in front of a vertical mirror. Draw a diagram to show the image of the letter in the mirror.

9. A plane mirror is inclined to the floor at an angle of 45° . What is the position of the image of a man standing upright before it?

10. What is the angle between a plane mirror and an incident ray, when the angle between the incident and reflected rays is 60° ?

REFRACTION AT PLANE SURFACES

433. Refraction of Light.—A stick partly in water and partly in air may appear to be broken (Fig. 240); and water in a pond seems to be shallower than it actually is. If a coin be put into a cup and the eye so placed that it can just see the edge of the coin over the rim (Fig. 241),

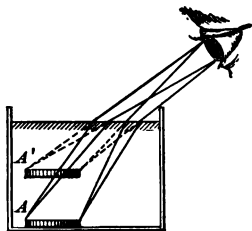


FIG. 241.—The coin appears to be raised.

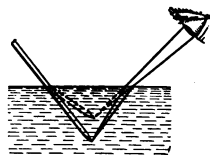


FIG. 240.—The stick seems to be broken.

the coin will apparently be lifted into full view when the cup is filled with water. Such observations as these go to show that rays of light change their course abruptly when they pass from one transparent substance into another.

When light waves arrive at the surface separating two media, those that are not reflected pass into the second substance, where some may be absorbed, *i.e.*, changed into heat, while the rest are transmitted. If an opaque object is in the second medium, some of the transmitted rays or waves are reflected from it and sent back into the first medium; the object is illuminated and acts like a source of light. The directions of the waves being changed in passing from medium to medium, any object in one medium, water, for example, when viewed with the eye in another medium, as air, is seen,

not where it actually is, but in another position, the location of which depends upon the speed of light in the two media.

434. Terms. — Let IO (Fig. 242) represent a light ray in air reaching the surface of water at O , where it is refracted and made to pursue the different direction OR . IO is the *incident ray* and OR the *refracted ray*. The angle between the incident ray and the normal is the *angle of incidence*, ION , and that between the refracted ray and the normal, the *angle of refraction*, RON' . The angle ROD shows how much the direction of the incident ray has been changed, and is called the *angle of deviation*.

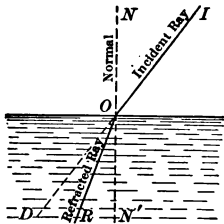


FIG. 242.—Refraction of Light.

435. Optical Density and Wave Speed. — A medium that refracts light more than another is said to be *optically denser*, while if it refracts less, it is *optically rarer*. A few pairs of substances have the same optical density; rays of light are therefore not refracted at all in passing from one into the other, no matter how oblique the angle. As a rule, the heavier a substance is, the greater is its optical density, although specific gravity and optical density do not stand in any close relationship to each other.

The greater the optical density of a substance, the less is the speed of light in it. Thus suppose waves to start from some point P (Fig. 243) within an optically denser medium, such as water. When the front of a wave reaches the surface SS' and passes out into the optically rarer substance, air, in which it can travel faster than in water, the curvature of the wave becomes greater because the

middle portions of the wave enter the air first. Hence the wave front bulges upward, changing from waw' to wbw' . The curvature of the waves in air being greater, their radius of curvature is shorter, and to the eye the center of the wave disturbance seems to come from a point P' located nearer the surface than the point P . The direction of the waves undergoes an abrupt change in passing from one medium into the other, except when normal to the surface. Now were there no change in medium, the wave

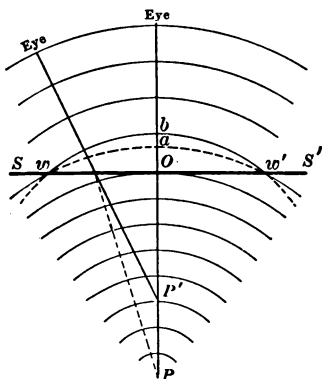


FIG. 243.—A Wave passing from Water into Air.

would travel a distance equal to Oa in the same time that it travels through the distance Ob because of the change in medium. Hence the ratio of the speeds of light in water and in air is expressed by $\frac{Oa}{Ob}$, which is equal to $\frac{OP'}{OP}$. This ratio is called the *index of refraction* from water into air.

436. Snell's Law.—*When a ray of light passes from one medium into another that is optically denser, the angle of refraction is less than the angle of incidence, and the ray is bent toward the normal; and when it passes from a more dense into a less dense medium, the angle of refraction is greater than the angle of incidence, and the ray is bent from the normal.* The optical density therefore determines which way light is refracted in passing from one medium into another. The difference between the angle of refraction and the angle of incidence may be found as follows:

Let SS' (Fig. 244) represent any plane surface separating two media, as air and water. Describe a circle of convenient size about a point O in this surface as a center. Let IO be a ray incident at O . Snell, in 1620, showed how to trace the direction of the refracted ray. Suppose that in the case of a certain pair of media the values of the angles of incidence are such that when they are constructed, the length of the perpendicular IA from I upon the normal is twice the length of the perpendicular RB from R upon

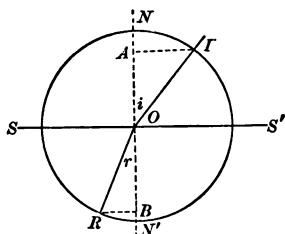


FIG. 244. — Snell's Law.

the normal. Then, whatever the magnitude of the angle of incidence, this same ratio of 2:1, found in a single case for the two perpendiculars, holds good; so that after constructing any angle of incidence, the angle of refraction may be found by making BR half of IA . The longer

these perpendiculars are, the larger the angles become; there must then be a definite relation between the magnitudes of the lines and the angles. Furthermore, the longer the radius of the circle is, the longer are the perpendiculars, even if the angles remain the same. To save words, the name of *sine* is given to the ratios $\frac{IA}{NO}$ and $\frac{RB}{N'O}$, so

that if the radius is made of unit length, IA is the sine of the angle i ($\sin i$) and RB is the sine of the angle r ($\sin r$). With the adoption of these terms Snell's law is: *The ratio of the sine of the angle of incidence to the sine of the angle of refraction has a constant value, provided the media remain the same:*

$$\frac{\sin i}{\sin r} = n \text{ (a constant).}$$

This constant ratio is called the *index of refraction* or the *refractive index*, and its value serves as a sign or index of the amount of refraction. Suppose, for example, that 4 is $.8$ as long as the radius ON , and that RB is $.6$ as long. Then the sine of the angle of incidence is $\frac{.8}{1} = .8$, and the sine of the angle of refraction is $\frac{.6}{1} = .6$. Their ratio is $\frac{.8}{.6} = 1.33$, and this number represents the index of refraction from the one medium into the other. It also shows how much faster light can travel in one medium (in this case air) than in another (water). Snell was also the first to record that *the incident ray, the refracted ray, and the normal all lie in the same plane.*

437. Indexes of Refraction. — The index of refraction for light passing from air into water is about $\frac{4}{3}$ ($= 1.33$) and from water into air about $\frac{3}{4}$; the one number is the reciprocal of the other. Light travels only about $\frac{3}{4}$ as fast in water as in air. When light passes from a vacuum into any transparent medium, the index of refraction is always greater than unity, and is called the *absolute refractive index*. The *relative index of refraction* is the index for the refraction of light passing from one medium into another; it is equal to the quotient of the absolute index of refraction of one medium divided by that of the other. As air has a small optical density, the relative index of refraction from air into other media differs but slightly from the absolute index and may be substituted for it.

438. Construction for the Refracted Ray. — It is required to find the direction of the refracted ray RO (Fig. 245) when the incident ray IO passes from air into water.

$$\text{Index of refraction} = \frac{4}{3} = \frac{\text{Speed in air}}{\text{Speed in water}} = \frac{\sin i}{\sin r} = \frac{IN}{RN'}.$$

Draw from some point on the incident ray, as I , the line IN perpendicular to the normal. It is well to select this point so that IN is readily divisible into four equal parts.

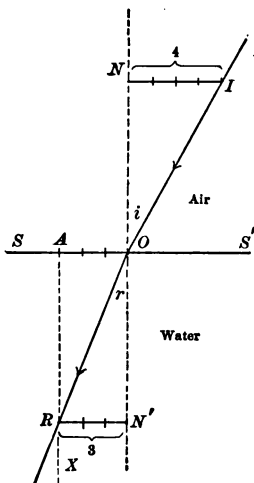


FIG. 245.—Construction for the Refracted Ray.

off a distance OA equal to $\frac{1}{4}$ of IN draw the indefinite line AX normal to surface. With a radius equal to OI scribe from O as a center an arc cutting at R . Then OR is the refracted ray, i is the angle of incidence and r is the angle of refraction.

A variation of the preceding construction is the following: Draw from some point in the incident ray (Fig. 246), as I , the perpendicular IN'' to the normal, and divide it into four equal parts.

With O as a center and OI as a radius strike an arc and draw a line parallel to the normal so as to make RN' equal to $\frac{1}{4}$ of IN'' . The downward continuation of RO gives the direction of the refracted ray OR' .

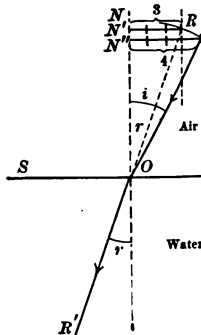


FIG. 246.—Construction for the Refracted Ray.

RELATIVE INDEXES OF REFRACTION FROM AIR INTO VARIOUS MEDIA

Water	1.33 ($\frac{4}{3}$) *	Crown glass	1.51 ($\frac{3}{2}$) *
Alcohol	1.36	Flint glass	1.62 ($\frac{8}{5}$) *
Carbon bisulphide	1.64	Diamond	2.47 ($\frac{5}{2}$) *

439. Refraction through a Plate with Parallel Sides
When a ray of light strikes obliquely one side of a transparent plate with parallel sides, it is bent toward the normal, and when it emerges, it is bent an equal amount

* The values when expressed as common fractions are more convenient in constructions.

the normal. The incident and emergent rays are therefore parallel (Fig. 247). An object viewed obliquely through a transparent plate, such as a window pane, is seen at one side of its actual position.

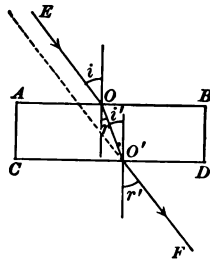


FIG. 247. — Refraction through a Plate.

440. Refraction in Prisms. — When the surfaces of a medium through which light passes are planes inclined at an angle to each other, the medium is called a *prism* (Fig. 248). The angle between the two planes is called the *refracting angle*. When a ray EF falls upon one face of the prism, it is bent toward the normal along FG , and it is bent away from the normal when it leaves the prism, taking the direction GH . It is thus bent twice around in such a way that an eye placed at H would see an image of E at L . The angle between the incident and emergent rays,

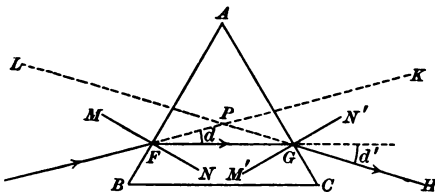


FIG. 248. — Refraction through a Prism.

$\angle PH = \angle LPE$, is called the *angle of deviation*. It is equal to the sum of the deviations d and d' , and its magnitude varies with the magnitudes of the refracting and incident angles as well as with the index of refraction of the medium, and has its minimum value when the angles of incidence EFM and of emergence HGN' are equal.

441. Internal Reflection. — If some object, such as a coin, is placed on the bottom of a tumbler partially filled with water, an image of the coin can be seen suspended in

mid-air above the water's surface when the tumbler is held up so that the under side of the water's surface is visible. Some of the light from the illuminated object is reflected from the under surface. Whenever light passes from an optically denser medium into one that is rarer, in a direction normal to the dividing surface, practically none of it is reflected internally; but as the angle of incidence increases, more and more of the rays suffer *internal reflection* (Fig. 249). As under these conditions the angle of refraction is

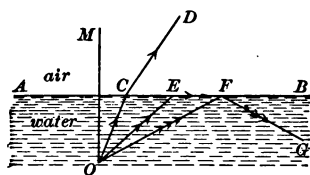


FIG. 249. — Internal Reflection.

always greater than that of incidence, a steady increase in the angle of incidence will be accompanied by a corresponding increase in the angle of refraction, until this becomes equal to a right angle. In that case the refracted ray *EB* will just skim along the surface of the liquid. With an incident angle larger than this, the refracted ray *FG* will fail to pass out of the denser medium; it will be *totally reflected*. *Critical angle* is the special name given to the angle of incidence *MOE* when the corresponding angle of refraction equals 90° . The greater the index of refraction, the smaller will be the critical angle.

442. Construction for the Critical Angle.—Since the angle of refraction corresponding to the critical angle is 90° , and since the sine of an angle of 90° is unity, the sine of the critical angle is equal to the index of refraction; for

$$\text{Index of refraction} = \frac{\sin i}{\sin r} = \frac{\sin i}{1} = \sin i.$$

In the case of light passing from water into air, the index of refraction is $\frac{3}{4} = .75$, and the angle having a sine of $.75$ is 48.5° . Hence the critical angle for water is 48.5° .

To construct the critical angle, lay off on the line representing the surface between the two media a distance OC equal to the numerator of the index of refraction (Fig. 250). With O as a center and a radius OI equal to the denominator of the index of refraction, strike an arc to intercept a normal CI to the surface. The direction of the incident ray is then IO and that of the refracted ray is OR .

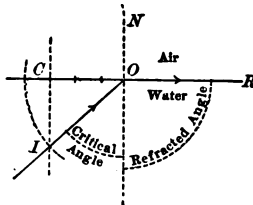


FIG. 250.—Construction for the Critical Angle.

443. Total Reflection.—When a beam of light is directed against the under surface of water, it will suffer total reflection, provided that the angle of incidence is greater than 48.5° . If a vessel half full of water be filled up with benzine so as to form two layers with a distinct surface separating them, a beam of light sent obliquely downward will, when properly adjusted, suffer total reflection at the surface of the water, because the optical density of benzine is greater than that of water. The brilliancy of the diamond, which has a high refractive index, is enhanced by cutting it so that its surface consists of a number of planes (facets), the adjacent ones making with one another angles greater than the critical angle. Internal reflection thereby takes place, and although the total amount of light emitted by the diamond cannot be greater than it receives, yet the light that enters through several facets may be so reflected internally as to emerge through only one with correspondingly increased intensity.

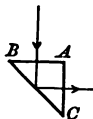


FIG. 251.—Total Reflection in a Prism.

444. Total Reflection in Prisms.—When a beam of light is incident normally on either face of a prism the cross section of which is a right-angled isosceles triangle, it is not refracted. It passes into the glass, strikes the adjacent face at an angle of 45° (Fig. 251) and is totally reflected. After reflection, it strikes the

third face normally, and therefore does not suffer any change in direction. By passing a beam of light through such a prism, its direction is changed by 90° . This fact is utilized in the construction of certain optical instruments.

445. Atmospheric Refraction.—The denser air is, the more slowly light travels through it. Hence the upper and less dense layers of the atmosphere have a smaller index of refraction than the lower and denser layers; and heated air refracts less than cold air. An object viewed over a bonfire or hot stove appears to vary irregularly in size and shape. This distortion is due to the currents of heated air which

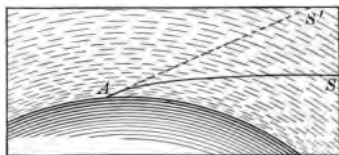


FIG. 252. — Atmospheric Refraction.
(Much exaggerated.)

refract the light coming from the object. The twinkling of stars is also due to the refraction of their light by currents of air of varying density. Besides this irregular refraction that makes starlight twinkle, there is a regular refraction due to the regular increase of the density of the atmosphere downward. Be-

cause of this regular refraction, light coming to the eye from a star near the horizon, or from the setting or rising sun, is refracted in such a way that its path is curved, the star appearing at a higher altitude than it actually is, and the sun being still visible (Fig. 252) when it is really below the horizon.

EXERCISES

1. Why do stars near the horizon twinkle more than those directly overhead?
2. When a ray of light passes obliquely from carbon bisulphide into air, it is bent from its course more than when it passes from alcohol into air. In which of these two liquids, then, is the speed of light the greater?
3. Knowing the critical angle for light passing between two media, how can you find their relative index of refraction?
4. Trace a ray of light from water through a crown glass plate into air.
5. If the index of refraction for a ray of light passing from air into diamond is $\frac{3}{2}$, what is the refractive index for a ray passing from diamond into air?

6. A ray of light strikes one face of a piece of plate glass (refractive index = 1.5) 5 cm. thick at an angle of 20° with the normal. Construct to scale the path of the ray through and out of the glass.

7. If the index of refraction from air into glass is $\frac{3}{2}$, what is the angle of refraction of a ray of light striking a glass plate at an angle of 45° ?

8. A ray of light strikes one face of a glass (index of refraction = 1.5) prism at an angle of 45° . What is the deviation of the ray if the refracting angle is 60° ?

9. Taking the speed of light in a vacuum, as $186,000 \frac{\text{mi.}}{\text{sec.}}$, find the speed of light in (a) water; (b) crown glass.

REFLECTION AT CURVED SURFACES

446. Curved Mirrors.—Any reflecting curved surface forms a curved mirror. The simplest and commonest ones are portions of a spherical surface, their cross sections being arcs of circles. Curved surfaces may be considered as made up of a large number of planes, and the normal to the surface at any point is the perpendicular to the plane passing through that point (Fig. 253). When a curved mirror is viewed from the inside, it is said to be *concave* and from the outside, it is *convex*.

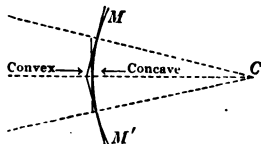


FIG. 253. — Curved Mirrors.

447. Spherical Mirrors.—Let MM' (Fig. 254) represent a spherical mirror. C is the center of the sphere, and is called the *center of curvature*. The middle point V of the

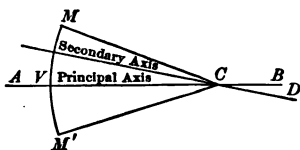


FIG. 254. — Spherical Mirror.

mirror is its *pole* or *vertex*, and the straight line passing through the pole and the center of curvature is the *principal axis*. Any other straight line passing through the center of curvature

is a *secondary* axis. The angle MCM' is the *aperture* of the mirror. Since the radii of spheres are normal surfaces, any ray of light coming to the mirror at any point on the axis will be reflected back upon itself, as if the mirror were a plane at that point of incidence.

448. Principal Focus of Concave Mirrors.—When a concave mirror is held across a sunbeam in such a way that it reflects the beam on a screen, the sun's image is seen and is most brilliant at a certain distance in front of the mirror. The image is then said to be at the *principal focus* and in the *principal focal plane*, and its distance from the pole of the mirror is the *principal focal length*. The sun is so large compared with terrestrial distances and is so remote that its rays may be regarded as practically parallel and its wave fronts plane. Parallel rays or plane waves then are brought to a real focus after reflection by a concave spherical mirror.

Wave Construction. Suppose a train of plane waves in a medium to move along the principal axis of a concave mirror (Fig. 255). The outer parts of the waves reach the mirror first and are reflected as to form a converging pencil with the focus F approximately between the vertex and the center of curvature.

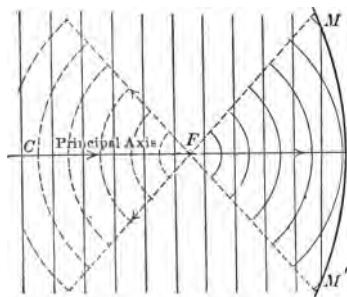


FIG. 255. — Wave Construction for the Principal Focus of a Concave Mirror.

The waves after reflecting converge to this principal focus and then spread out in a diverging manner.

Ray Construction. Let AMN (Fig. 256) be a concave spherical mirror with its center of curvature at C and its vertex at A .

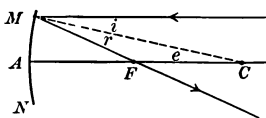


FIG. 256. — Ray Construction for the Principal Focus of a Concave Mirror.

along the principal axis CA is reflected and passes through F , a point midway between C and A . Let any other ray BM parallel to the principal axis strike the mirror. It will be reflected so as to make the angle of incidence i equal the angle of reflection r , and the reflected ray will pass very near to F , provided the aperture is small. *The principal focus of a concave spherical mirror lies then halfway between the center of curvature and the vertex.* Conversely, when a point source is placed at the principal focus, the pencil or cone of illumination that reaches the concave mirror near its principal axis will be reflected as a beam of parallel rays.

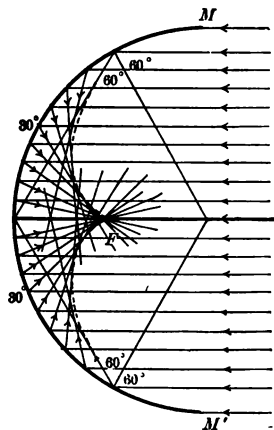


FIG. 257.—Spherical Aberration.

449. Spherical Aberration.—When a beam of light is reflected from a concave mirror (Fig. 257), its focus is not a point, but is spread along the axis between the principal focus and the vertex.* When more and more of the outer portions of the mirror are covered with non-reflectors of light, the focus becomes smaller and more distinct. Obviously all the parallel rays are not brought to the same focus when reflected from a concave mirror. The smaller the portion of the sphere or cylinder from which the mirror is taken, the more nearly do the rays come to a point focus. Thus, when the image of a lamp or candle is caught upon a screen after reflection from a concave mirror, the sharpness of outline may be increased by covering all but the central portion of the mirror, although thereby the image is made fainter because less light is used in projecting it. Experiments such as these, coupled with a geometric study, prove that parallel rays are not brought to one and the same focus by reflection from a spherical concave mirror, and that this is especially noticeable with broad beams incident upon mirrors with large apertures. This scattering of the focus is desig-

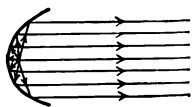


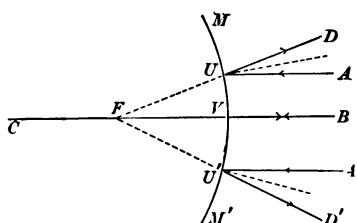
FIG. 258.—Parabolic Mirror.

* Lay a plain gold ring on a piece of paper, and note the reflections produced. Cut off portions of its reflecting surface by interposing strips of paper, and note how the distinctness of the focus is affected.

nated as *spherical aberration*, and the image made by the reflection of diffused light is called the *caustic by reflection*. The smaller the aperture of the mirror, the less is the confusion of focus, and with an aperture of less than 10° , the aberration is hardly noticeable. By gradually lessening the curvature of the mirror from the vertex outward, that is, by making the cross section of the mirror *parabolic* instead of circular, the aberration may be done away with entirely (Fig. 258).

450. Principal Focus of Convex Mirror.—When a sun-beam is reflected from a convex mirror, a broad pencil is observed, which, inasmuch as it diverges in front of the

mirror, must converge behind it. Its principal focus is therefore virtual.



Wave Construction. The construction is identical with that for a plane wave reflected from a circular surface (Fig. 187).

FIG. 259. — Ray Construction for the Principal Focus of a Convex Mirror.

Ray Construction. Let MM' (Fig. 259) represent a convex spherical mirror. The ray BV will be reflected upon itself, since it coincides in direction with the principal axis, and the focus will lie upon it or upon its prolongation behind the mirror. Let AU be any other ray parallel to the principal axis, and construct UD as the reflected ray. As UD and BV diverge in front of the mirror, they must meet behind the mirror, as at F . F is the principal focus and is located approximately halfway between V , the vertex, and C , the center of curvature.

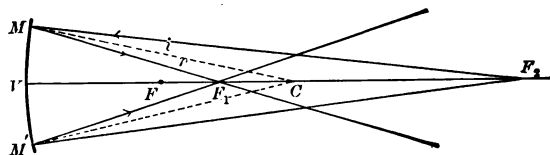


FIG. 260. — Conjugate Foci of a Concave Mirror.

451. Conjugate Foci.—A point-source F_1 (Fig. 260) on the principal axis of a concave mirror somewhere between

the principal focus and the center of curvature sends out diverging rays of light, which by reflection become a converging pencil with its focus F_2 on the principal axis on the farther side of the center of curvature. If, now, the point source is placed at the focus F_2 , its light will be brought to a focus at F_1 . These two foci are interchangeable. Such pairs of foci are named *conjugate foci*.

If F_1 is moved nearer and nearer to the principal focus, its conjugate focus is formed farther and farther away, until finally, when F_1 coincides with F , F_2 will be at an infinite distance from the mirror, or in other words, the rays after reflection will never meet; they will be parallel. The conjugate focus of the principal focus is therefore an infinite distance from it; that is, in reality, there is none. When the point source is at the center of curvature, the rays of light are normal to the mirror and are reflected upon themselves. The conjugate foci are consequently at the same point. With one focus F_1 (Fig. 261) between the vertex and the principal focus, the conjugate focus F_2 is behind the mirror and is virtual.

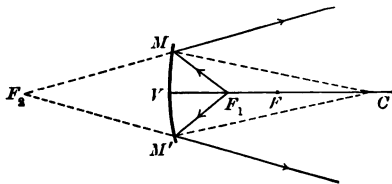


FIG. 261. — One conjugate focus is virtual.

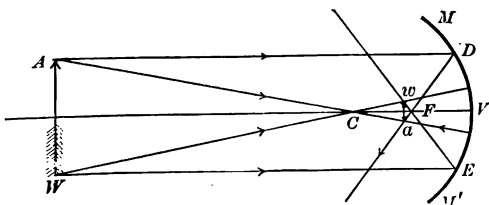


FIG. 262. — Image of a Point not on the Principal Axis of a Concave Mirror.

452. Foci not on the Principal Axis.—When the source of light is not on the principal axis, its conjugate focus is

found as follows. Let the point source be A (Fig. 262). Draw through it the secondary axis AC . The ray along this axis will be reflected upon itself. The ray AD parallel to the principal axis will be reflected so that it will pass through the principal focus F and intersect the other ray at a , which is, therefore, the conjugate focus of A .

453. Formation of Images by Concave Mirrors. — Geometrical constructions for the images of objects are made by constructing the foci of two or more of their points. Six cases present themselves:

I. *An Object at an Infinite Distance.* As the rays coming from an object at an infinite distance are parallel, they are reflected to the principal focus, provided they are parallel to the principal axis. The image is therefore reduced almost to a point (Fig. 256).

II. *A Small Object at the Principal Focus.* This is the reverse of Case I. The rays are parallel after reflection and give no image.

III. *An Object at the Center of Curvature.* The rays are reflected on themselves so that the image is real, inverted, of the same size as the object, and at the center of curvature.

IV. *An Object between the Principal Focus and the Center of Curvature.* The construction (Fig. 262) shows that the image is real, inverted, larger than the object, and beyond the center of curvature.

V. *An Object at a Finite Distance beyond the Center of Curvature.* This is the reverse of Case IV. The image is real, inverted,

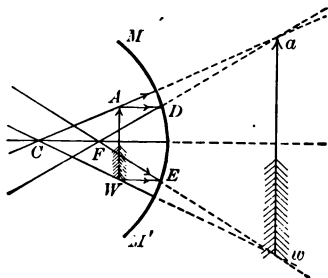


FIG. 263. — Image of an Object between the Principal Focus and the Concave Mirror.

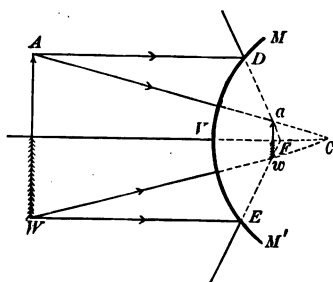


FIG. 264. — Image of an Object in a Convex Mirror.

smaller than the object, and between the principal focus and the center of curvature (Fig. 262).

VI. *An Object between the Principal Focus and the Mirror.* The construction (Fig. 263) shows the image to be virtual, erect, larger than the object, and located farther behind the mirror than the object is in front of it.

454. Formation of Images by Convex Mirror. — The construction (Fig. 264) shows the image to be virtual, erect, between the principal focus and the mirror, and smaller than the object.

REFRACTION AT CURVED SURFACES

455. Lenses. — Any transparent substance with at least one curved surface is called a *lens*. The surfaces of most lenses are spherical and their cross sections are bounded by arcs of circles (Fig. 265). The centers of the spheres

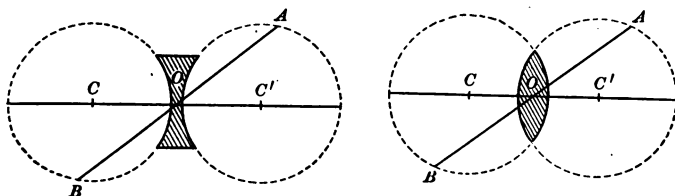


FIG. 265. — Lenses bounded by Spherical Surfaces.

C and C' , the surfaces of which bound a lens, are its *centers of curvature*, and the straight line connecting these centers

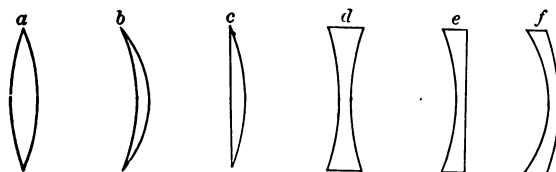


FIG. 266. — a , b , and c are converging, and d , e , and f , diverging lenses.

is the *axis*. The *optical center* O is a point such that any ray (*secondary axis*) that passes through it and the lens suffers no change in direction. Lenses may be divided into two classes, each comprising three typical forms (Fig. 266). Most of our discussions will be confined to biconvex and biconcave lenses. The effect of a convex lens on rays

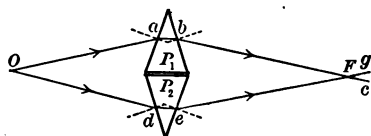


FIG. 267 a.—Converging Lenses and Prisms Compared.

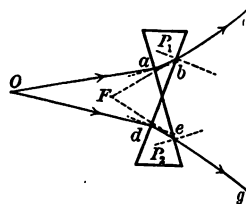


FIG. 267 b.—Diverging Lenses and Prisms Compared.

of light is like that of two prisms with their bases placed together (Fig. 267 a), and that of a concave lens is similar to that of two prisms joined at their apexes (Fig. 267 b).

456. Principal Foci of Lenses.—When a converging lens is held with its principal axis parallel to the direct rays of the sun, and a piece of paper placed at the focus, the concentration of the heat rays by means of this “burning glass” soon

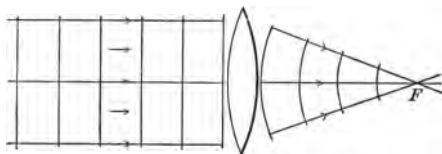


FIG. 268.—The focus of a convex lens is real.

raises the temperature of the paper to its kindling point. The distance between the paper screen and the optical center of the lens is the *principal focal length*. Parallel rays are brought by a converging lens to a real focus on

the side opposite to the object (Fig. 268); while they are brought to a virtual focus on the same side by a diverging lens (Fig. 269).

For biconvex and biconcave lenses of glass with refractive index equal to $\frac{3}{2}$, and with equal radii of curvature,

the principal focal length is approximately equal to the radius of curvature.

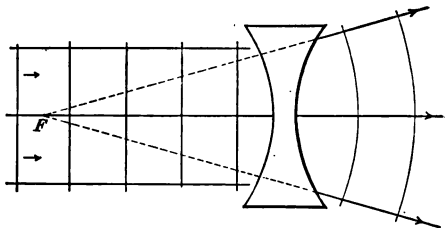


FIG. 269. — The focus of a concave lens is virtual.

457. Spherical Aberration of Lenses. — Spherical lenses like spherical mirrors do not bring all the rays parallel to the principal axis to a point focus. By the use of a screen (diaphragm) covering the marginal parts of the lens, the sharpness of focus is improved, but the image is fainter because so much light is shut off. By properly changing the curvature of lenses, it is possible to correct this aberration due to sphericity (§ 449).

458. Location of Images. — Let F and F' be the principal foci (Fig. 270) of a lens, and P a point source of light. The ray PL parallel to the principal axis will after refraction

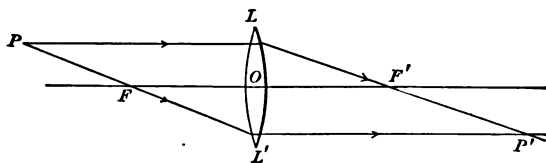


FIG. 270. — Location of an Image.

pass through F' , and the ray PL' , which passes through F , will after refraction be parallel to the axis. The image of the point will be located at the intersection P' of these

refracted rays. The image of an object may in like fashion be found by locating the positions of the images of the

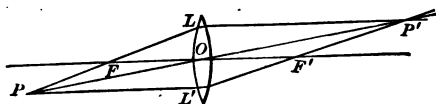


FIG. 271. — Location of an Image.

prominent points of an object. Another method of locating positions of images depends upon the fact

that secondary axes passing through the optical center of a lens do not have their directions changed. Thus the secondary axis POP' (Fig. 271) remains straight throughout its length.

459. Conjugate Foci. — Experiment shows that the following relation exists between the distances of an object and its image from the center of a lens, and its principal focal length:

$$\frac{1}{\text{object-distance } (v)} + \frac{1}{\text{image-distance } (u)} = \frac{1}{\text{focal-distance } (f)}$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}.$$

Image and object are located at the conjugate foci of the lens.

Experiment also shows that

$$\frac{\text{object-size}}{\text{image-size}} = \frac{\text{object-distance}}{\text{image-distance}}.$$

Similar relations are also true of curved mirrors.

EXERCISES

1. Explain why it is that a convex mirror can never yield an image.
2. What effect will be caused on the focal length of a lens by immersing it in water instead of air?

3. Why can the image formed by a concave lens never exceed in size the object?

4. A candle placed 25 cm. in front of a concave mirror has its image formed 50 cm. in front of the mirror. Find the radius of the mirror.

5. If the focal length of a concave mirror is 16 cm., at what distance from the mirror will be the image of an object that is 18 cm. from the mirror? Describe this image.

6. A simple convex lens of 20-cm. focal length is placed in one end of a box and is used to give an image of a small object which is 200 cm. distant from the lens. How long must the box be in order that the image may be in focus on the end of the box opposite the lens?

7. What are the characteristics of the image of an object set 20 cm. in front of a biconvex lens of 15-cm. focus?

8. When a converging lens of 10-cm. focus was coupled with a diverging lens, it was found that the two lenses together produced no change in the apparent size of objects viewed through them. What is the principal focal length of the diverging lens?

OPTICAL INSTRUMENTS

460. The Photographic Camera. — We have learned that by means of a pinhole camera (§ 417) an image of an object may be formed upon a screen, and that the size of the image depends upon the relative distances of screen and object from the pinhole. To get sharp definition, the pinhole must be small. Hence so little light is admitted that a long exposure is necessary in order to affect a photographic plate adequately. This disadvantage is surmounted by substituting a converg-

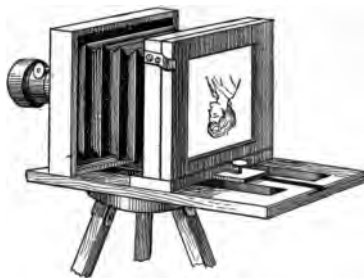


FIG. 272. — Camera.

ing lens for the pinhole, and placing the screen in a position that is the conjugate focus of the position occupied by the object (Fig. 272). The lens admits enough light into the box or *camera obscura* to bring about rapidly the chemical changes in the substances on the plate, which is afterward developed into the negative. In self-focusing cameras of the kodak type, the lens is so shaped as to bring objects at a distance greater than about six feet to approximately the same focus.

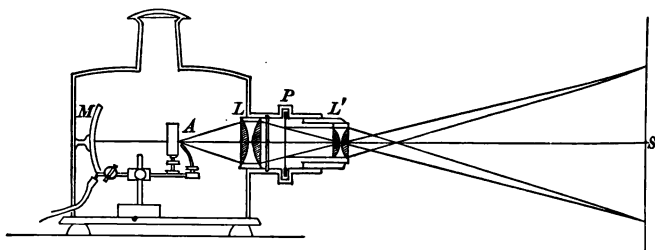


FIG. 273. — Projection Lantern.

461. The Magic or Projection Lantern. — The projection lantern may be regarded as a photographic camera in which the positions of image and object are interchanged. The strongly illuminated object P (Fig. 273) (usually a transparent slide) is placed a little beyond the principal focus of the lens L' so that a much enlarged and inverted image is produced on the screen S . The image is a much larger than the object as the distance of the screen from the lens is greater than the distance of the lens from the object. To illuminate the slide as brilliantly as possible, the light from a powerful source, such as that of a calcium light A or of an electric arc, is reflected from the mirror M and passed through a so-called *condensing lens* L .

462. The Eye.—The eye is essentially similar to a photographic camera, and projects upon its own retina images of outside objects

(Fig. 274). With the photographic camera the lens must be adjusted according to the

distances of the objects in order to get sharply defined images on the plate. The curvature of the crystalline lens can by the action of certain muscles be changed so as to make its focal length *accommodate* itself to objects at different distances. Unconsciously and almost instantaneously we change the curvature of the lens so as to form on the retina a well-defined image of an object a few inches or many miles away. The iris acts as a diaphragm which

not only corrects spherical aberration in a measure, but also regulates the amount of light admitted into the eye.

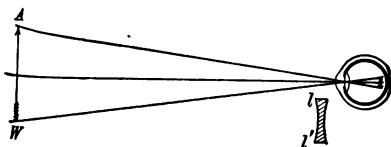


FIG. 274. — A Normal Eye.

FIG. 275. — A Myopic Eye.

strained can bring parallel rays to a focus on the retina. If the eyeball is too long from front to back, the rays come to a focus in front of the retina except when the illuminated object is very close to the eye (Fig. 275). Such an eye is *myopic* and the defect is *myopia* or *shortsightedness*. By making the

light pass through diverging lenses (in spectacles or eyeglasses) of proper focal length, the image may be made to reach the retina and the vision will be distinct. If, on the other hand, the axis of the eye-

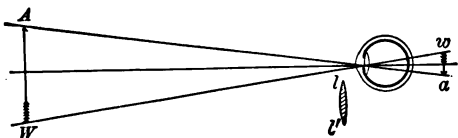


FIG. 276. — A Hypermetropic Eye.

463. Defects of Vision.—

The normal eye when not

ball is too short, rays will come to a focus beyond the retina, except when the object is far distant (Fig. 276). Such an eye is *hypermetropic* and the defect is *longsightedness* or *hypermetropia*. It is remedied by

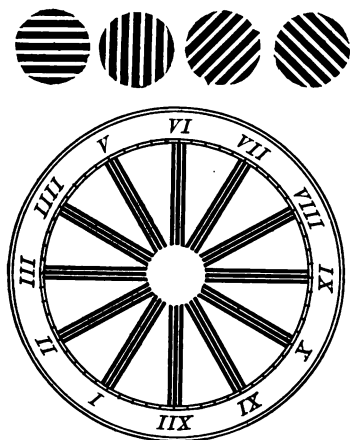


FIG. 277. — Test for Astigmatism.

using converging spectacle lenses of suitable focal length. Farsightedness may also be due to loss of the power of accommodation, the crystalline lens not adjusting itself to rays of considerable divergence, such as those coming from near objects. Such an eye is *presbyopic*, and *presbyopia* is corrected by converging spectacle lenses. *Astigmatism* is due to the inequalities of curvature of the cornea in different directions. To an astigmatic eye viewing the diagram (Fig. 277) the vertical lines may be in focus and distinctly seen, while the horizontal

ones will be out of focus, and consequently appear blurred, or the reverse may be the case. This defect is remedied by using spectacles with lenses ground cylindrically as well as spherically.

464. Limit of Distinct Vision.—A normal eye cannot easily accommodate itself so as to see distinctly objects that are less than 25 cm. = 10 in. from it. At such distances its refractive power is insufficient to bring the rays to a sharp focus; in consequence, the object seems blurred, and the eye is uncomfortably strained. Now it is the size of the image upon the retina which gives us our measure of the size of an object, and the size of the image depends upon the visual angle. While we have learned in viewing objects with the naked eye to make allowance for their distances, when we peer into the eyepiece of a microscope or telescope we are governed in our judgment of the size of the image solely by the angle that it subtends upon the

retina. The purpose of microscopes and telescopes is to increase the divergence of the rays sent into the eye.

465. The Simple Microscope or Magnifying Glass.—The simple microscope consists of a converging lens or system of lenses suitably mounted so that the eye may be brought close to one of its sides, and the object placed at a little less than the principal focal distance on the other side (Fig. 278). The image is then virtual, erect, and larger than the

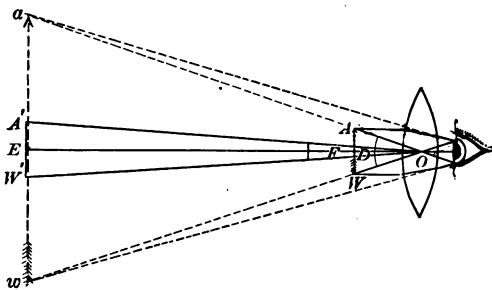


FIG. 278. — Simple Microscope.

object. The amount of magnification is the ratio of the size of the image to that of the object, which is equal to the ratio of the distance of the image to the distance of the object, both measured from the center of the lens. But as the image appears largest and most distinct when it is apparently 25 cm. from the eye, the object has to be placed at such a distance from the lens that its virtual image is formed 25 cm. from the eye. The *magnifying power* is defined to be the ratio of the angle subtended by the image (its angular size) to the angle subtended by the object, both being at the least distance of distinct vision. But this ratio is approximately equal to $aw : AW$, which, in turn, equals $EO : DO$. As it is almost invariably the case that the lens has a short focus (less than 5 cm.), the principal focal length f is so nearly equal to DO that it may be substituted for it without appreciable error.

Hence, as EO is equal to 25 cm., the ratio $\frac{25}{f}$ may be taken as an approximate measure of the magnifying power of a simple microscope.

466. The Compound Microscope and the Telescope.—Both these instruments have essentially the same construction, but differ in details because of the different services re-

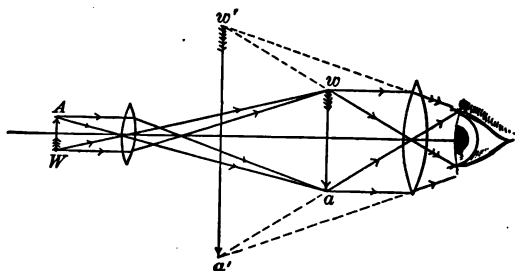


FIG. 279. — Compound Microscope.

quired of them. Each receives the light from the object by means of an *objective*; this is a converging lens or system of lenses that produces a real and inverted image

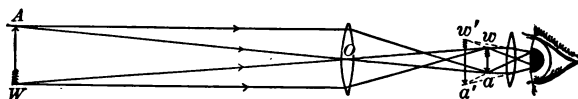


FIG. 280. — Astronomical Telescope.

within the tube. The image is then viewed through an *eyepiece* or *ocular*, which is a simple microscope placed at a little less than its focal distance from the real image, so that a virtual image is seen (Figs. 279, 280, and 281).

Telescopic objects are far enough away for the waves received by the objective to be plane-fronted, so that they therefore converge at the principal focus; and since

the objects cannot be illuminated by the observer, the objective must be made large so as to collect enough light to make the image distinct.

Microscopic objects, on the other hand, can be placed very close to the objective, so that the waves are spherical-fronted and are brought to the conjugate focus. As the objects may be strongly illuminated by means of a mirror and a condenser, the objective may be made small.

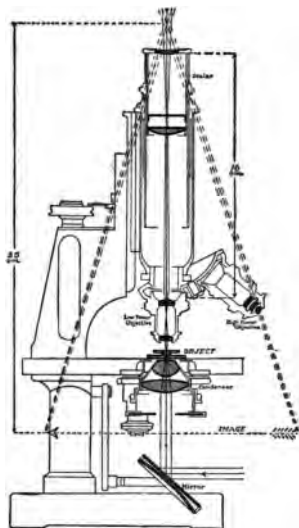


FIG. 281. — Construction of a Compound Microscope.

467. Magnifying Power of the Astronomical Telescope. — If the eye were placed 25 cm. from the real image formed by the objective, the angle subtended by the image would be as many times larger than the angle subtended by the object as the focal length f' of the objective is times greater than the distance of distinct vision. The objective therefore increases the visual angle of the object $\frac{f'}{25}$ times. The eyepiece, in turn, magnifies this image $\frac{25}{f}$ times, so that the combination

of lenses gives a total magnification equal to $\frac{f'}{25} \times \frac{25}{f} = \frac{f'}{f}$. *The magnifying power of an astronomical telescope is measured by the ratio of the focal length of the objective to the focal length of the eyepiece.*

A high magnifying power, then, is secured by using an objective of great focal length and an eyepiece of small focal length. The focal length of the objective of the Yerkes telescope is about 25 m., so that with an eyepiece of .5 cm. focal length, its magnification is 5000.

468. The Magnifying Power of the Compound Microscope. — The objective produces an image as many times larger than the object

as the distance between the image and the lens is greater than that between the object and the lens. As the distance between the lens and the object is approximately equal to its principal focal length f' , and the distance between the lens and the image is approximately equal to the length of the microscope tube L , and since the eyepiece magnifies the image $\frac{25}{f}$ times, the magnifying power of the combination of lenses is $\frac{L}{f'} \times \frac{25}{f}$. A high magnifying power is therefore obtained when the focal lengths of the lenses are as short as possible and the microscope tube is as long as possible. Thus, if a microscope is 16 cm. long, the focal length of the objective 5 cm. and that of the eyepiece 2 cm., its magnifying power is $25 \times 16 \div .5 \times 2 = 400$.

469. The Terrestrial Telescope or Spyglass. — While it is an unimportant matter that heavenly bodies viewed through an astronomical

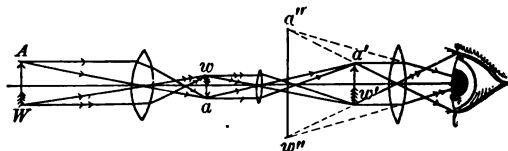


FIG. 282. — Terrestrial Telescope.

telescope should appear inverted, terrestrial objects viewed through a telescope must be erect. The terrestrial telescope has a lens between the objective and the eyepiece, by means of which the inverted image produced by the objective is made erect, thereby enabling the eye to see the object in its natural position (Fig. 282).

470. The Galilean Telescope and Opera Glass. — These instruments have objectives like the other forms of telescopes, but their eyepieces

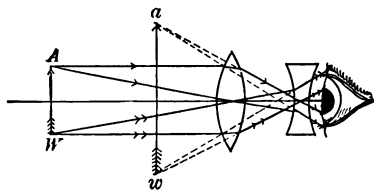


FIG. 283. — Galilean Telescope.

are concave lenses with a focal length equal to that of the observer's eye. As the diverging effect of the eyepiece counteracts the converging action of the lens of the eye, the objective, in effect, forms its image directly upon the retina (Fig. 283). The size of

the image produced by the objective is as many times larger than the size of the image produced by the naked eye as the focal length of the objective is times greater than the focal length of the eye. And since the focal length of the eyepiece is equal to that of the eye, the magnifying power of the Galilean telescope is equal to the ratio of the focal lengths of the objective and eyepiece. Objects viewed through this instrument appear erect for the reason that the image formed on the retina is inverted, just as is the case with the image formed by the unaided eye.

The opera glass consists of two Galilean telescopes mounted together so that both eyes may be used at once.

471. Stereoscopic Vision. — When an object is viewed with both eyes at once, the image formed on one retina differs slightly from that formed on the other retina, because the eyes are looking at the object from slightly different positions (Fig. 284). The union of these two dis-

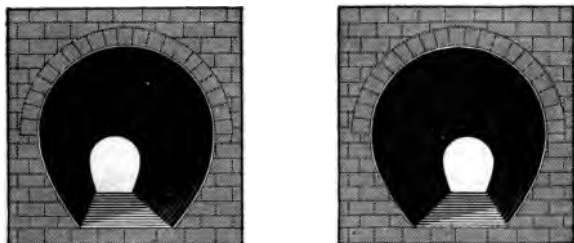


FIG. 284. — Stereoscopic Vision. Hold a postal card between the two pictures so that each eye can see only the picture on its side. Then adjust the eyes as if they were looking through the book at a point some distance behind it. The pictures will blend into a single view that has the appearance of greater depth.

similar views gives us the impression of relief; the object appears to have depth or solidity. If a picture of an object be presented to each eye as it appears from its point of view, and the eyes be so adjusted as to see both pictures at the same time, the appearance of relief will be reproduced.

472. The Stereoscope. — The stereoscope is an instrument that makes pictures appear in relief. Two pictures of the same object from slightly differing points of view are made. When they are both seen at the

same time through two prismatic lenses which superpose them on the retina as well as magnify them, they seem to form but one picture, having all the characteristics that the object would have if actually seen by both eyes (Fig. 285).

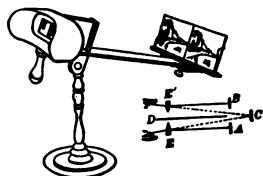


FIG. 285. — Stereoscope.

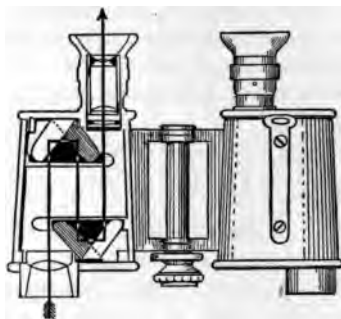


FIG. 286. — Zeiss Binocular.

473. Reflecting Prism Binoculars.— The *Zeiss binocular* is essentially a double astronomical telescope in which the light undergoes total internal reflection between two prisms (Fig. 286) in such a way that it traverses the length of the tube three times in passing from the objective to the eyepiece; the instrument is therefore shorter and more compact than the corresponding telescope would be. The reflections also make the image erect instead of inverted. Inasmuch as the distance between the objectives is greater than that between the eyes, the stereoscopic effect is more pronounced than in the case of the unaided eye or of the opera glass.

EXERCISES

1. In what relation does the magnifying power of a lens stand to its principal focal length?
2. What is the purpose of the bellows of a photographic camera? What kind of camera is not provided with bellows? Under what defects as to focus does such a form of camera labor?
3. What are the points of superiority of the opera glass over the spyglass?
4. Why are astronomical telescopes of high magnifying power made very long?

5. If spectacles fitted to a person having myopia are put on by a person enjoying normal eyesight, is the image formed in front of or behind the retina?

6. Presbyopic eyes are sometimes fitted with spectacles having double lenses. What advantage is thereby gained?

7. If a picture thrown upon a screen by a projection lantern is too large, in what direction must the lantern be moved in order to reduce the size of the view on the screen?

8. What is the focal length of a simple microscope that magnifies five diameters?

9. How would the magnifying power of a telescope be changed by doubling the focal length (*a*) of the objective? (*b*) of the eyepiece?

10. What is the magnifying power of a $\frac{1}{4}$ -in. lens used as a simple microscope?

11. Which will give the larger image in a projecting lantern at a fixed distance from the screen, a 6-in. or a 12-in. objective?

12. If a telescope has an eyepiece of 1-in. focal length and an objective of 20-ft. focal length, what is its magnifying power?

13. If a compound microscope has the regulation length of 160 mm., an eyepiece of 1-in. focal length and an objective of $\frac{1}{4}$ -in. focus, what is its magnifying power?

14. Taking the focal length of the eye as 2.5 cm. and the focal length of the objectives of an opera glass as 10 cm., calculate the magnifying power of the glass.

15. The telescope at Lick Observatory is nearly 60 ft. long. What is its magnifying power when used with an eyepiece of (*a*) 2-in. focal length? (*b*) $\frac{1}{2}$ -in. focal length?

CHAPTER XX

COLOR

COLOR AND WAVE LENGTH

474. Dispersion. — We have assumed in our treatment of reflection and refraction of light that the beams and pencils are homogeneous, *i.e.* that one ray is like any other. Such, however, is seldom the case, as is shown by the following experiment first performed by Newton in 1666. A beam of sunlight is admitted into a darkened room so as to strike a prism obliquely (Fig. 287). The refracted beam when projected upon a white screen is seen to consist of a number of colors. If this beam of many-colored light is incident upon a second prism of the same shape and material as the first and held in an inverted position, the refracted beam

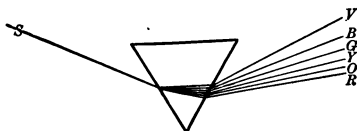


FIG. 287. — Analysis of White Light.



FIG. 288. — Synthesis of White Light.

makes a white spot on the screen, the colors having combined to produce white light (Fig. 288). Like results are obtained whenever any so-called white light source is employed. These experiments teach that white light is made up of many colors which may be separated by refraction, or rather by *dispersion*, — a special name given to refraction when a change of color occurs. The colored

band upon the screen is called the *spectrum*. The most prominent colors of the spectrum of the sun are red, orange, yellow, green, blue, and violet. The waves producing the sensation of violet are refracted most, and those giving rise to the sensation of red are refracted least. The violet wave length is 39×10^{-6} cm. and the red wave length is 76×10^{-6} cm., the latter being almost twice the former.

475. Newton's Rings. — Newton pressed the curved face of a plano-convex lens against a piece of plate glass, and observed concentric rings of variously colored light alternating with dark spaces, between the glass and the lens. These rings are due to the interference of the waves of light reflected from the upper side of the curved surface of the lens and from the upper surface of the glass plate. Newton's experiment, when modified by using two plates of glass clamped together at one end and separated slightly at the other (Fig. 289), and by employing monochromatic illumination, presents to view a series of dark bands alternating with a series of bright bands. These bands are all of equal breadth and are called *interference fringes*. The essential thing in experiments of this nature is a very thin wedge-shaped layer of some transparent substance, such as air, separating the two reflecting surfaces.

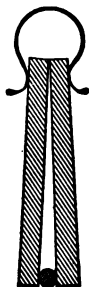


FIG. 289. —
Apparatus
for Pro-
ducing In-
terference
Fringes.

476. Interference Fringes. — How these rings and bands are produced is rendered clear by considering the course of a beam of monochromatic light incident at one of the surfaces of the wedge. Let *ANE* (Fig. 290) represent

the wedge of air. Part of the beam ab will be reflected from the surface AN , and part will pass through AN and be reflected from the surface EN . The waves of the

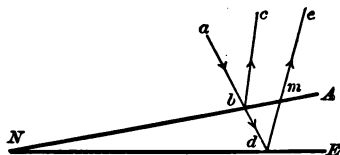


FIG. 290. — How Interference Fringes are Produced.

reflected beams bc and de may differ in phase, since the latter have traveled the greater distance bdm . If this distance is such that the waves along me are half a wave length behind those along bc , being in opposite

phase, they will combine to produce darkness, and a dark band will be formed. But when the distance bdm is such that the reflected rays are in like phase, they combine to produce a bright band. If the distance bdm is such that the waves along me lag any odd multiple of half wave lengths behind the waves along bc , *destructive interference* takes place and dark bands are produced, while if the lag is equal to any even multiple of half wave lengths, *constructive interference* occurs and bright bands are produced.

Thin and transparent films of any substance give rise to these interference phenomena. Thus traces of oil upon water often exhibit beautiful and changing colors, the particular color being due to the thickness of the oil film, and the changeableness to variations in the thickness of the film. Soap bubbles also show interference films; and the magnificent display of color of the plumage of some birds, of the wing cases of various insects, and of certain shells is due to interference.

477. Measurements of Wave Lengths by Interference Fringes.—

Dark bands (Fig. 291) are noticed in the positions marked d and bright bands in the positions marked b . Between any two consecutive bright or dark bands the distance from NA to $N'E$ increases by one half of a wave length. Suppose that the monochromatic light used is green and that the wedge of air is 60 mm. long and .01 mm. thick at its broad end. If 40 bright bands are observed, the bands are $\frac{60}{40} = 1.5$

mm. apart, and as 40 bands are formed by a divergence in the plates amounting to .01 mm., each band is formed by a divergence of $.01/40 = .00025$ mm.; this is the length of half a wave of green light. By illuminating the same plates with lights of other colors and counting the interference fringes, the wave lengths of these colors also may be determined.

478. Chromatic Aberration. — When a beam of sunlight falls upon a converging lens, and a screen is placed near its principal focus, the image is seen to be surrounded by a colored fringe. This phenomenon is known as *chromatic aberration*, and is due to the fact that the different waves constituting white light do not have the same index of refraction. Thus the focus of the more refrangible violet rays is nearer the lens than is the focus of the less refrangible red rays. When the screen is placed at the focus of the violet rays, the image is fringed with red, while when it is at the focus of the red rays, the image has violet borders (Fig.

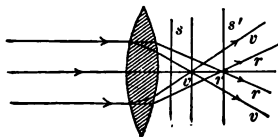


FIG. 292.—Chromatic Aberration.

292). The foci for the intermediate colors of the spectrum lie between those for red and for violet rays.

479. Achromatic Lenses. — Chromatic aberration is a serious drawback in optical instruments, and lessens their usefulness greatly unless means are at hand to counteract its effects. The remedy was discovered by Dolland in 1757, and is based upon the fact that there is no proportionality between the amount of dispersion and the amount of refraction of different substances. Thus, while flint

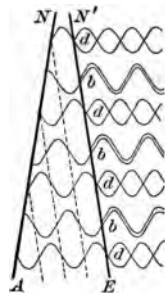


FIG. 291. — Wave Lengths Measured by Interference Fringes.

glass and crown glass have but slightly different refractive powers, the dispersive power of crown glass is only about

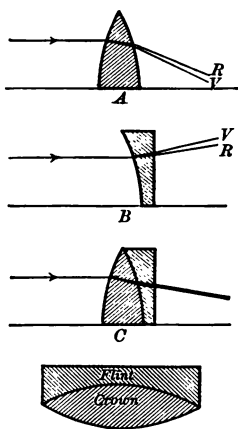


FIG. 293. — Principle of the Achromatic Lens.

half as great as that of flint glass. If, then, light after passing through a converging lens of crown glass is made to traverse a diverging lens made of flint glass, the chromatic aberration will be almost entirely suppressed, and yet enough refraction will take place to make the combination of lenses useful in optical instruments (Fig. 293). To reduce chromatic aberration to a minimum, various combinations of lenses have been devised (Fig. 281), such achromatic lenses forming the objectives of microscopes and telescopes.

480. Action of a Raindrop on Sunlight. — Let a section of a raindrop be represented by a circle (Fig. 294), and let SO be a line passing from the sun through the center of the drop, and ri a ray striking the drop so as to make the angle iOc equal to $59^\circ 23\frac{1}{2}'$. Some of the waves along ri will be reflected at i , but those that enter will undergo refraction and dispersion. At r some of the waves will pass out of the drop, while the others will be reflected to d where they will again be refracted and dispersed. The angle given was chosen because for that value alone the frequency of the emergent waves corresponds to the color red, so that to an observer viewing the drop from E — a position where dE makes an angle of about 42° with EA — the drop appears red. Waves meeting the drop at other points will be so refracted, dispersed, and reflected as

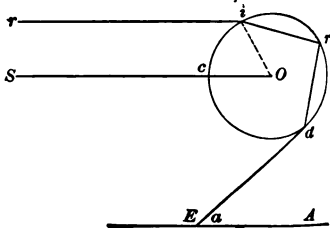


FIG. 294. — Formation of Rainbow.

to make the drop appear to an observer at different positions a little to the left of E to have the other colors of the spectrum. The violet rays make an angle of about 40° with EA , the other colors coming between 40° and 42° .

481. The Rainbow. — The rainbow is a solar spectrum with raindrops for refracting media and the retina of the eye for a screen. A rainbow is seen only when the sun is not more than about 40° above the horizon, when the observer has his back to the sun, and drops of rain are falling before him (Fig. 295). On a small scale, rainbows may be seen in the spray of a waterfall or of a lawn sprinkler. Sometimes a *secondary bow* is seen in addition to the inner *primary bow*, both bows having the same center. The primary bow is caused by two refractions and one reflection, while the secondary bow is due to two refractions and two reflections in the drops. The consequence of this second reflection is that the secondary bow is the dimmer and has the order of its colors reversed.

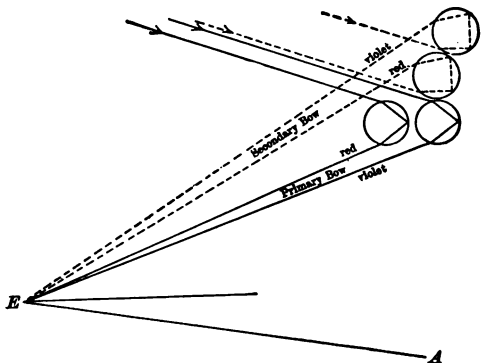


FIG. 295. — Primary and Secondary Rainbows.

SPECTRA

482. Pure Spectrum. — When the beam of light incident upon a prism is broad, the spectrum is made up of overlapping images of the different colors. To prevent this overlapping, the light is admitted by as narrow a slit as possible, and the rays are brought to a focus on the screen by means of

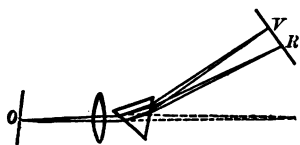


FIG. 296. — Production of a Pure Spectrum.

a converging lens, the prism being placed in its position of minimum deviation (Fig. 296). An image of the slit in each color is projected on the screen, and the overlapping is diminished.

483. The Spectroscope.—To produce pure spectra the *spectroscope* is employed. This instrument (Fig. 297) con-

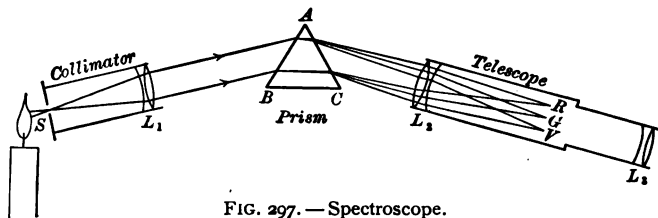


FIG. 297. — Spectroscope.

sists of three essentials, *collimator*, *prism*, and *telescope*, with accessory parts in the better instruments. The collimator is a tube with a slit of adjustable width at one end and a converging lens near the other end at its focal distance from the slit. The diverging rays of the narrow beam of light admitted by the slit are made parallel by this lens so that a beam of parallel rays falls upon the prism. The view telescope is an astronomical refractor and serves to magnify the spectrum.

484. Diffraction Grating.—A diffraction grating is made by scratching with a diamond point parallel lines that are very close together (5000 to 15000 to the inch) on a glass or metal plate. A grating may be used instead of a prism in a spectroscope, and the spectra thus produced by the interference of light waves are very pure.

485. Classes of Spectra.—When the lights from many *flames* or *white-hot solids* are examined by means of a spectroscope, the spectra are similar, showing all the prismatic colors. Such sources of light therefore contain

waves of all degrees of refrangibility, and are said to have *continuous spectra*.

When the light emitted by *incandescent vapors* or *gases* is examined by a spectroscope, the spectra consist of one or more bright lines of different colors standing out on a dark background. Such sources of light contain only certain waves of differing refrangibility; they are said to give *discontinuous* or *bright-line spectra*.

When light capable of giving a continuous spectrum is passed through various substances, some of the waves of certain lengths are absorbed by the medium. The light corresponding to these wave lengths is thereby quenched, with the result that dark lines or bands appear in the spectra, which are consequently called *dark-line* or *absorption spectra*.

486. Bright and Dark-line Spectra.—When the light due to the incandescent vapor of the metal sodium (produced by heating compounds of sodium, such as common salt or soda in a colorless Bunsen or alcohol flame) is examined spectroscopically, its spectrum is seen to consist of two bright yellow lines so close together as to appear as a single line except in the best instruments. If sodium is boiled and its vapor interposed between the slit and the yellow sodium light, the spectrum vanishes. The sodium vapor is at a comparatively low temperature and absorbs the yellow light sent out by the incandescent sodium vapor. Likewise, when a light yielding a continuous spectrum is made to pass through sodium vapor, a dark line appears in the yellow part of the spectrum. In general, it is found that a vapor at a low temperature absorbs the waves that it emits when incandescent. The dark lines in a spectrum indicate, therefore, that either the source of light does not

contain the waves corresponding to them or that been quenched by interposed substances.

487. Fraunhofer Lines.—The solar spectrum dark lines that coincide in position with the b

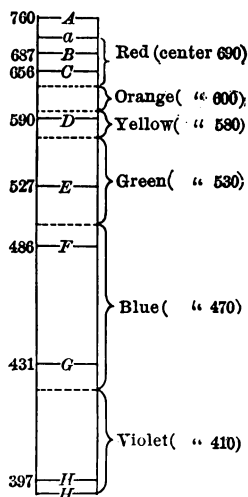


FIG. 298. — Fraunhofer Lines. The most prominent are named according to letters. The numbers are the wave lengths, the unit being the *micron*, a millionth of a millimeter.

488. Spectrum Analysis.—Every elementary substance give rise to a characteristic bright-line spectrum, and man show distinct absorption spectra. By examining the sp substances of unknown composition and comparing them with of known substances, it is possible to arrive at a definite to the composition of the substances under examination. 4

analysis is of extreme delicacy; a speck of salt so small as to be invisible even with a powerful microscope can yield a bright yellow line.

489. The Invisible Spectrum. — A delicate thermometer placed beyond the red part of the visible spectrum shows that heat is radiated from this *infra-red* part of the spectrum; and a photographic plate exposed beyond the violet part shows, after being developed, that it has been affected by the *ultra-violet rays*. These invisible parts of the spectrum are more extensive than the visible part. Thus the range of the visible spectrum is, so to speak, barely an octave, while the heat spectrum extends at least five octaves below the red, and the chemical spectrum at least two octaves above the violet.

COLOR BY TRANSMITTED AND REFLECTED LIGHT

490. Color by Transmitted Light. — Many substances permit ether waves of certain lengths to pass through them, but prevent the passage of others; certain colors are selected and absorbed by the medium, and it is to this *selective absorption* or *transmission* that the color of transparent substances is due. The color exhibited is that of the parts of the spectrum that are not absorbed.

When the light does not contain waves of such length as can pass through a given substance, as when sodium light falls upon blue glass, all of the light is absorbed. The color of a transparent substance therefore depends largely upon the color of the light illuminating it. When white light is all transmitted, the object is colorless; while if no light is transmitted, the object is black.

491. Selective Absorption for Dark Heat Waves. — Substances that transmit the long invisible waves are said to be *diathermanous*, while those that absorb them are *athermanous*. A transparent substance is not necessarily diathermanous. Thus, while sunshine passes freely through glass, the dark heat waves radiated from the objects warmed by the sun's direct rays can escape but in a small measure through the glass. Dry air is diathermanous, but the presence of water vapor in it makes it to a great degree athermanous. The waves of light from

the sun pass to the earth, where they are changed into heat which are effectually prevented from escaping from the atmosphere by the aqueous vapor that it holds.

492. Color of Opaque Bodies.—When a strip of paper placed on a black background is viewed through a prism, a spectrum is seen similar to that of sunlight, but fainter. A red strip similarly arranged and viewed gives a spectrum containing only the red parts, the other colors being very dim or totally wanting. Similarly, a blue strip gives a spectrum in which all the colors except blue are largely suppressed or wanting. When a strip of colored material is held in a solar spectrum, it appears black except the red parts of the spectrum; a blue strip appears blue only in the blue parts, appearing black elsewhere, and a piece of black paper appears black in all parts of the spectrum.

The color of an opaque substance must then be determined by both the light received and the light reflected. A substance is red when it absorbs all colors except red, and cannot appear red unless there are red waves in the light illuminating it. Substances cannot properly be said to have a color of their own, since they can show no color not present in the light shed upon them. The discrepancies experienced in matching colors by artificial light and changes in the shade of color of materials viewed by natural light and in artificial light are illustrations in point. Natural artificial lights do not have precisely the same composition as sunlight; hence the light reflected by a material is therefore its color, varies with the nature of the source of illumination.

493. Color by Reflected Light.—If a piece of colored glass or the surface of a colored liquid be used

mirror, it will be seen that the colors of the image are practically the same as those of the object. The color of the mirror has but little to do with the color of the object. Manifestly, then, it is not the light reflected at the surface that gives the colored substances their distinctive colors. If the light has to pass through some of the substance on its way to or from the surface reflecting it to the eye, although the distance traversed may be excessively minute, it suffices for selective absorption to come into play, and the color presented is due to selective absorption followed by reflection. The color may be modified because of the fact that some substances, notably the metals, exercise a *selective reflection* whereby certain colors are reflected more strongly than others.

494. Absorption and Reflection of Radiant Heat.—When dark heat waves strike a body, their reflection, absorption, and transmission depend mainly upon the nature of the surface of the body. The waves transmitted or reflected do not change the temperature of the body—only those that are absorbed are effective in that respect. When a body is at a higher temperature than its surroundings, we are concerned chiefly with its radiating power, while when it is at a lower temperature, our interest lies in its absorbing power. It has been found in general that the radiating and absorbing powers of the same body are equal. It is obvious that as more heat is reflected from a surface, less is absorbed by the substance. Hence good radiators are good absorbers and bad reflectors, while bad radiators are bad absorbers and good reflectors. Polished metals are the best reflectors, while lampblack is the poorest reflector known.

495. The Radiometer.—This instrument consists of a glass bulb almost exhausted of air. In it is a cross of light wire supported on a vertical axis and carrying vanes of mica, one face of each being covered with lampblack (Fig. 299). When exposed to sunshine or heat radiation from any adequate source, the blackened faces move away from the source, thus causing the cross to spin round.

The bulb is so nearly exhausted of air that the “free path,” *i.e.* the

distance a molecule of air remaining in the bulb can travel before colliding with another, is at least equal to the distance between the

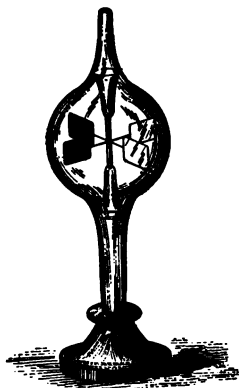


FIG. 299. — Radiometer.

vanes and the walls of the bulb. Now the blackened sides of the vanes absorb more heat than the other sides, and the gas molecules striking against these warmer faces rebound with a greater velocity than from the unblackened surfaces. This results in an unbalanced pressure being applied against the blackened faces, which is sufficient to overcome the slight friction of the bearings and to set the cross spinning.

496. Color of Pigments. — A solution of potassium dichromate allows only orange-red light to pass through it, together with a little green. A solution of copper sulphate, on the other hand, transmits blue light chiefly, as well as some green; the red, orange, and yellow being absorbed. When a beam of white light is sent through these solutions successively, the emergent light is green, as the other colors of the spectrum have been absorbed. Mixtures of the two solutions in the right proportions also appear green for the same reason. This is true of mixtures of other red and blue substances.

Similar effects are obtained by mixing the above substances when finely powdered. With suitable proportions the red and blue powders give a green mixture. The light illuminating the mixture is not all reflected until it has penetrated below the surface. Selective absorption then occurs, and the light reflected to the eye is green.

497. Fluorescence. — Solutions of certain substances, such as quinine and eosin, when exposed to the action of ether waves so short as to be invisible, are observed to emit a bluish light. The substances have the

property of absorbing the actinic waves, and of emitting light waves; they transform the energy received so as to make the invisible radiation visible. Many substances show more or less of this ability to change the length in ether waves; and as fluorspar was among the first substances observed to have this property in a marked degree, the name of *fluorescence* was assigned to the phenomenon.

EXERCISES

1. When a white object is viewed by reflection from a piece of red glass, two distinct images are seen, one white and one red. How is this explained?
2. What colors seem to be wanting in the light of a candle? gas flame? incandescent gas mantle flame? incandescent electric light? arc light? Cooper-Hewitt mercury arc light?
3. Why is it that any object seen through a piece of red glass looks either red or black?
4. What changes in the color of a red rose and a green leaf would be noticed if both were placed (1) in the red part of the spectrum? (2) in the green part?
5. Why is it that mother-of-pearl, which may present the most gorgeous colors, becomes white and dull like chalk when ground to a powder?
6. Why are the walls of a photographic dark room best painted black?
7. Knowing the wave lengths of various colors, how can you estimate the thickness of a soap-bubble film?
8. How does the glass of a greenhouse catch and imprison the sunshine?
9. Why is a rainbow never seen during the middle of the day?
10. How does the spectrum of moonlight compare with that of sunlight?
11. Why cannot two observers ever see exactly the same rainbow?
12. Which is hotter on the foot, a dusty shoe or a polished one — a patent leather or a tan?
13. Why is it hotter in the sunshine and cooler in the shade on the top of a mountain than in the valley?
14. Why is it that snow melts around the trunk of a tree more rapidly than it does on the ground a short distance away?
15. Which can be set on fire with a burning-glass the more quickly, black or white paper of the same texture and quality?
16. If the yellow and green are removed from white light, what will be the color of the residue?

CHAPTER XXI

STATIC ELECTRICITY

498. Electrification.—It is sometimes observed in cold weather that tiny sparks accompanied by a slight shock pass between the finger and a radiator or a chandelier, especially when the feet have been rubbed over the carpet; also, that a series of crackling sparks is obtained by rubbing a cat's back. We say that these phenomena are due to *electricity*. Furthermore, if a rubber comb or fountain-pen is rubbed on the coat sleeve and then brought near bits of paper, lint, or other light bodies, these move toward the pen or comb. We say that the rubber articles have been *charged with electricity*, or are *electrified by contact or friction*. The essential condition in getting these electrical manifestations is the putting of two different kinds of matter in close contact. When they are separated, each is found to be electrified. As the result of innumerable experiments it has been proved that any substance, when brought into intimate contact with a different substance, or even with the same substance in a different condition (rougher surface or higher temperature), becomes electrically charged, so that, when removed, it exercises this attractive force.

499. Conductors and Nonconductors.—While all substances may be electrified by contact, the presence of a charge cannot be detected upon very many, for the reason

that the charge passes from one body into others. Such substances as permit the easy escape of electric charges are called *conductors*, while those that refuse to part with their charges readily are called *nonconductors* (also *insulators* and *dielectrics*). In insulators the charge is localized, and remains on the portions of the body originally charged. In conductors the charge spreads over the whole body, and passes into other conductors that may be in contact with it. A conductor supported by means of an insulator is said to be insulated and is prevented from losing its charge.

500. Electrical Conductivities. — The metals are the best conductors. The best nonconductors are rubber, glass, silk, and wool. There are, however, no perfect conductors nor perfect insulators. The range of conductivity is enormous; silver, the best conductor, having a conducting power nearly 10^{20} times greater than that of gutta-percha, the best insulator. Moist air is a far better conductor than is dry air. Hence the dry air of winter is favorable for experiments in static electricity, while the moist air of summer discharges electrified bodies quickly. The human body and solutions of acid and saline substances are also good conductors.

501. Frictional Electricity. — When one substance touches another, there is actual contact between but few of their points, because of the roughness of the surfaces. By pressing them together, and especially by rubbing, the number of points of contact is increased, and the electric effects are correspondingly magnified. It is not the friction itself that generates the electricity; the rubbing is only a convenient means of rapidly multiplying the number of points of contact.

502. Electrification and Electricity. — The name *electricity* is derived from the Greek word for *amber* (*electron*), because amber when rubbed with wool was the first substance noticed to develop electrica¹

properties.* This name does not explain anything; it merely stands for a description of a class of phenomena. Electrification signifies the putting of a body into the condition that amber presents when it has been rubbed with wool. To do this requires work, and the work done is stored up in the bodies as a form of potential energy. While we are ignorant of the real nature of electricity, we know that to electrify a body we must do work upon it. Electrification is therefore energy, and electricity is the unknown something associated with electrified portions of matter.

503. Kinds of Electrification.—Typical electrical effects are obtained when silk and glass, or wool and hard rubber,

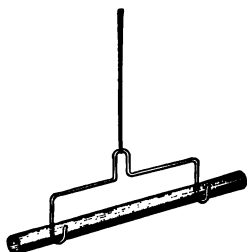


FIG. 300.—Electrified Rod
Free to Turn.

are brought together. Hence we shall confine ourselves to the study of the electrical behavior of these substances and regard it as typical. Rub a glass rod with silk. Both the glass and the silk are found to be electrified, since each attracts light bodies. Rub a vulcanite rod with flannel, and both substances become electrified. Suspend the electrified glass rod in a stirrup hanging from a support (Fig. 300), and bring near its rubbed portion the rubbed portion of a second electrified glass rod. The rods repel each other. Likewise two vulcanite rods charged by rubbing with flannel are found to repel each other. But when a glass rod is charged and brought near a charged and suspended vulcanite rod, there is mutual attraction. Again, if the flannel used in rubbing the vulcanite is brought near it when suspended, there is attraction. In short, when glass is rubbed with silk, and vulcanite with wool,

* This observation was first made by the Greek, Thales, about 600 B.C. Similar observations on other substances were not made until 1600 A.D., when Gilbert, Queen Elizabeth's physician, greatly extended the knowledge of electrification.

the mutual actions of the four substances may be classified as follows :

REPULSION BETWEEN		ATTRACTION BETWEEN	
+ Glass	and + Glass	+ Glass	and - Vulcanite
+ Wool	and + Wool	+ Glass	and - Silk
- Vulcanite	and - Vulcanite	- Vulcanite	and + Wool
- Silk	and - Silk	- Silk	and + Wool

Where oppositeness of effect is observed, it must be assumed to be due to oppositeness of kinds of electrification. To distinguish between these two kinds, the adjectives *positive* and *negative* are employed. *Any substance is said to have a positive (+) charge when it behaves like glass that has been rubbed with silk. And any substance has a negative (-) charge when it behaves like vulcanite previously rubbed with wool.* The generalization of such results as the above leads to the law: *Bodies bearing like electric charges repel each other; and those bearing unlike charges attract each other.*

504. Electroscopes. — Electroscopes are instruments for detecting in a body both the presence and the kind of electrification. They consist essentially of very light conductors mounted on insulated supports. The commonest forms are the *pith-ball* and the *gold-leaf electroscopes*.

Pith-ball Electroscope. A ball of pith (a wad of cotton or wool may be used instead) is suspended by a silk thread from a stand (Fig. 301). The pith is often covered with gold leaf to increase its conducting power. When a charged body is brought close to the ball, this moves toward it, and after receiving a part of the charge, flies away, being repelled because the charges of the body and the ball are the same. If any other charged body is brought near the

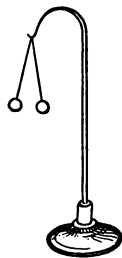


FIG. 301. — Pith-ball Electroscope.

ball, repulsion indicates that its charge is the same as that of the ball.

Gold-leaf Electroscpe. A metal rod passes through the insulated stopper of a vessel bearing upon its upper end a metal knob or plate and on its lower end two gold leaves (Fig. 302). If a charged body is touched to the knob, some of the charge is conducted to the leaves, which diverge because of their having like charges. As there is danger that a charged body may give up so great a charge to the leaves that they may be torn apart by the violence of the action, it is advisable to abstract a portion of the charge by means of a *proof plane* (Fig. 303), and then present this to the electro-



FIG. 302.—Gold-leaf Electroscpe.

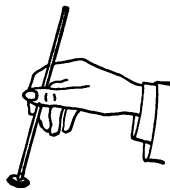


FIG. 303.—Proof Plane. It consists of a metallic disk having an insulating handle.

505. Equality of Charges. — Rub a neutral, that is, an unelectrified, glass rod with a neutral silk pad by twisting the pad around the rod but not moving it at all along the length of the rod. Keeping the silk wrapped about the rubbed part of the rod, bring them both together near an electroscpe. No attractions or repulsions are observed, the pair of substances acting like an unelectrified body. But after the glass and silk are separated, each is found to produce equal but opposite electrical effects. Hence the contact of the two substances must have produced equal amounts of opposite kinds of electrification.

506. Positive and Negative Charges. — It should be stated that glass does not always receive a positive charge, for if it is rubbed with fur it takes on a negative charge, the fur acquiring a positive charge. Only by experiment can we learn what kinds of electrification are produced

by the contact of substances. It has been found that any substance in the following brief list is electrified positively when rubbed with any substance standing below it in the list, and charged negatively when rubbed with any one above it.

1. Fur	5. Silk	9. Rosin
2. Flannel	6. Wood	10. Shellac
3. Glass	7. Metals	11. Sulphur
4. Cotton	8. Sealing wax	12. Gutta-percha

507. Quantity of Electrification. — The most noticeable effect of an electrically charged body is the motion it tends to produce in another body, and obviously the more electrification a body has, the greater is the amount of the motion. But motion is due to force, and to prevent the motion of charged bodies, a force at least equal to the amount of electrical attraction or repulsion must be applied. This force may be measured in any convenient unit, but in electrical matters it is customary to use the dyne. Quantities of electrification may then be measured by determining the number of dynes required to produce or prevent the same motion as may be produced in a body by electrifying it. *The unit quantity of electrification is defined to be that quantity which when present on each of two bodies one centimeter apart in air causes them to repel each other with a force equal to one dyne.*

EXERCISES

1. Why should a glass rod used in electrostatic experiments be kept warmer than the air of the room?

2. Two plates of metal insulated from each other are held horizontally a few centimeters apart, and pith balls placed upon the lower plate. The lower plate is connected with the earth, and the upper one to the pole of an electric machine. When the machine is worked, the pith balls rapidly pass back and forth between the plates. Account for this behavior.

3. A brass rod held in the bare hand and rubbed with a silk pad cannot be made to show any signs of being electrified; but if a rubber glove is put on the hand, the rod will be electrified when rubbed. Why is this?

4. How could you determine whether a rod of some unknown material was a good or bad conductor of electricity?

5. Why is it that in cold weather, if a dry sheet of paper be placed against the wall and rubbed with the dry hand, it will stick to the wall?

ELECTRICAL DISTRIBUTION

508. Distribution of Electrification. — If a spherical conductor be placed on an insulating support and given a charge of electrification, a proof plane applied to any part of its surface will take away the same amount of electrification, for, if the charge on the proof plane be imparted to an electroscope, similar effects will be produced. The charge is therefore uniformly distributed over the sphere. If the conductor has any shape other than spherical (Fig. 304), the proof plane is found to take away greater quan-

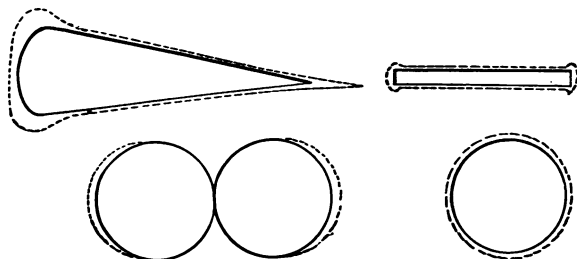


FIG. 304. — The distances between the dotted and full lines indicate the degree of electric density.

ties of electrification where the curvature of the surface is sharper. The electrification seems to have a greater *density*, *i.e.* there seems to be a greater charge per unit area where the radius of curvature is less. At any sharp points on conductors the electrification becomes, so to say, heaped up in such a way that the surrounding air particles become heavily charged and are quickly repelled by the points. Fresh particles rush in to take their places, and after being electrified are in turn repelled. The consequence is that a current of air — an “electric wind” — is produced (Fig. 305). The exposed ends of apparatus used

in experiments on static electricity are on this account made spherical, since, with pointed ends, the leakage of a charge is very rapid.

If a hollow conductor is charged, not the slightest sign of electrification can be detected within it; all of the charge collects on the outside surface. This is in accord with the fact that like charges repel each other. Hence a charge tends to escape from a body and collects on the outer surface of a conductor where it is ready to pass off whenever the opportunity presents itself.*

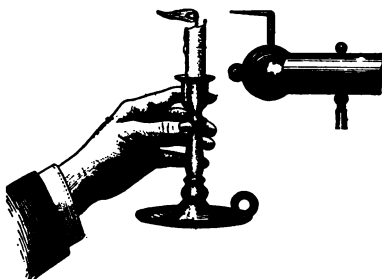


FIG. 305. — The air repelled from the charged pointed end deflects the flame.

EXERCISES

1. Which will hold the larger charge of electricity, a brass solid sphere or a silver hollow sphere, both of the same size?

2. If the insulated hollow cylinder (Fig. 306) is charged, the top pith balls diverge, but the inner pith balls do not move. Account for this.

3. How can an electroscope be protected from the influence of an electrical machine that is in operation near by?

4. Why do the leaves of a charged gold-leaf electroscope speedily collapse when the point of a pin or needle is held near the knob?

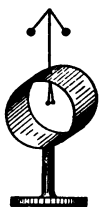


FIG. 306.

* Faraday proved this on a large scale as follows: He constructed a 12-foot-square cage lined with paper covered with tin foil and placed on insulating supports. He connected this cage with a powerful electrical machine, and going inside, found the conditions there to be as follows: "I went into the cube and lived in it, using electrometers and all other tests of electrical states; I could not detect the least influence upon them, though all the time the outside of the cube was powerfully charged, and large sparks and brushes were darting from every part of its outside surface."



FIG. 307.

5. A solid metallic sphere is suspended by an insulating thread and electrically charged. Two hemispherical cups with insulating handles are brought together over and in contact with the sphere and then removed (Fig. 307).

The shells are found to be electrified, but the sphere has totally lost its charge. Why is this?

6. A sheet of tin foil is so fastened to a glass rod that it can be rolled up on the rod (Fig. 308). The foil is unrolled, its lower end connected with a gold-leaf electroscope, and a feeble charge imparted to it. As the foil is rolled up, the leaves of the electroscope diverge more and more. Account for this behavior.

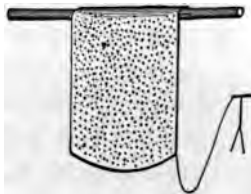


FIG. 308.

POTENTIAL AND ELECTROMOTIVE FORCE

509. Potential. — Electrical potential is analogous to temperature or to liquid level. Just as a body at a higher temperature gives out heat to a body at a lower temperature, and just as liquids tend to flow from higher levels to lower, so does electrification on a conductor at a higher potential tend to pass over to a conductor at a lower potential. Just as the greater the amount of heat imparted to a substance, the higher is the temperature, and just as the greater the amount of liquid put into a vessel, the higher is the level, so the greater the amount of electrification given to a body, the higher is the potential. Electrical potential then has reference to the relative conditions of conductors which determine the direction of the transfer of electrification; and of two similar bodies, the one that has the greater charge is at the higher potential.

510. Zero of Potential. — Just as a zero of temperature is fixed arbitrarily, so is a zero potential agreed upon by physicists. As the earth has the largest capacity (because of its size) of any body available to

man, and as no additions or subtractions of electrification that man is able to make can affect its potential appreciably, the potential of the earth is called zero. To bring conductors to zero potential, they are connected with the earth by touching them with the finger so that the body acts as a conductor to the earth, or by connecting them with wire to water or gas pipes; which are themselves conductors partly buried in the earth.

511. The Measure of Difference of Potential.—The difference in level between two tanks of water may be measured by the amount of work required to raise a unit weight of water from the lower to the higher tank. Thus, if ten foot pounds of work are needed to raise one pound of the liquid from the lower to the higher tank, their difference of level is ten feet. In analogous fashion the difference of potential between two charged bodies is measured by the amount of work expressed in ergs that has to be done to transfer the unit quantity of electrification from one body to another. Thus, if a body *A* is charged positively and a body *B* negatively, a certain number of ergs will be required to move a third body bearing a unit negative charge from *A* to *B*, for this body will be repelled by *B* and attracted by *A*; a force must therefore act through a distance against an opposing force, so that resistance is overcome and work thereby done. Physicists have agreed that the *unit difference of potential exists between two bodies when one erg of work is required to transfer the unit charge from the one body to the other*. Another unit, however, which is $\frac{1}{300}$ of the unit just defined, is the one in common use. This *practical unit* is called the *volt* in honor of Alessandro Volta, who was the first to make measurements on differences of potential.

512. Measurements of Potential Differences.—A gold-leaf electroscope is so constructed that its case can be connected with the earth

and thus kept at zero potential. A charged body is then connected with the electroscope, and the divergence of the leaves measured. The greater the divergence, the greater is the potential difference.

Estimates of potential difference may be made by means of measurements of the lengths of sparks passing in air between two bodies, for spark length is approximately proportional to potential difference. As a basis for this rough-and-ready method it is well to remember that it takes about 30,000 volts to make a spark one centimeter long.

513. Electromotive Force. — To cause a flow of electricity, that is, to cause an electric current, force — *electromotive force* — must act. Just as a difference of water levels calls forth a force — a pressure — so does a difference in the potentials of two bodies cause an electromotive force. The pressure of the water is due to the difference in levels, and the electromotive force is due to the difference of potentials. The analogy between the two may be illustrated by considering the action of a bicycle pump. When the piston is pressed down, a current of air issues from the nozzle, but if the nozzle be closed, the air will be compressed — will be subjected to a strain, and in consequence will force the piston back when the compressive stress is removed. The pressure on the piston corresponds to electromotive force, the current of air to electric current, and the strain to electric strain. Just as an air current is possible only when the nozzle is open, so is an electric current possible only when electromotive force acts upon some conductor; and just as a strain in the air is occasioned when pressure is applied with the nozzle closed, so is an electric strain called forth when electromotive force acts against some nonconductor. Furthermore, just as a sufficiently great fluid pressure may break the nozzle and thus cause an air current, so may a large enough electromotive force break down a nonconductor and compel it to become a conductor. When a charged body is brought near to

a conductor, the electromotive force acting upon the intervening dielectric (air or some other nonconductor) succeeds, if the air layer is thin enough or the electromotive force is great enough, in breaking down its insulating power, so that an electric current passes through it, accompanied by a flash of light.

514. Electric Field and Lines of Force. — When a charged body is brought near a gold-leaf electroscope, the leaves diverge while the body is still at some distance. This and similar phenomena indicate that the space round a charged body is in a peculiar condition. This space in which electrical manifestations are apparent is known as an *electric field*. An investigation of the field shows that a particle carrying a positive charge tends to move along a definite path called a *line of force*. The field is made up of these lines of force which start from positively electrified bodies and end on negatively electrified bodies. Their directions are such as would be taken by stretched, mutually repellent, elastic bands; that is to say, the lines of force tend to become shorter and at the same time to diverge from one another. The greater the number of lines passing through a given area of the field, the greater is the intensity shown by that portion of the field.

515. Electrostatic Action as related to Lines of Force. — Consider two bodies with equal and opposite charges, and suppose their lines of force to be mapped when the bodies are close together (Fig. 309). The lines of force are most numerous near the straight line joining the two bodies, and there are more of them between mn and pq than there are to the left of mn and to the right of pq combined. Since these lines of force are striving to shorten, being in a state of tension, their preponderance between mn and pq results in a tendency for the bodies to approach each other. We thus see how the conception of lines of force helps to explain the attraction of bodies bearing opposite charges. Con-

sider now the field of two electrified bodies having like charges and placed near each other (Fig. 310). In this case there are fewer lines

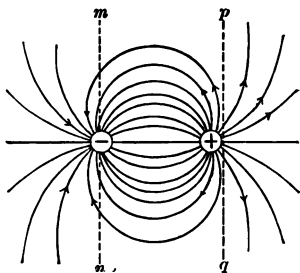


FIG. 309. — Lines of Force of Oppositely Charged Bodies.

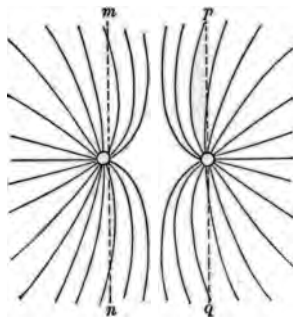


FIG. 310. — Lines of Force of Similarly Charged Bodies.

of force between mn and pq than on the outsides of these lines. Hence the preponderance of lines of force to the left of mn and to the right of pq results in a tendency for the bodies to move away from each other.

516. Influence of the Medium. — Bring an electrified rod just near enough to the knob of a gold-leaf electroscope to cause the least perceptible divergence of the leaves. Then, holding the rod steadily in that position, place between it and the electroscope a thick plate of glass or vulcanite, or a cake of wax or paraffin. The increased divergence of the leaves indicates that the electrical action is increased by substituting for a portion of the intervening air some other dielectric. *The force manifested by electrified bodies varies with the medium through which it acts.*

517. Coulomb's Law of Electrical Action. — We have frequently observed in earlier experiments that the force of attraction or repulsion varies with the amount of electrification as well as with the distance. The law of these variations was first found out in 1777 by Coulomb, who by

means of very delicate apparatus proved that the *attraction or repulsion between two charged bodies varies directly as the product of their charges and inversely as the square of the distance between them.*

Let e and e' stand for the amounts of electrification upon two bodies, d the distance between them, and D a quantity the value of which depends upon the nature of the medium between the two bodies. Then f , the force acting between the bodies, is

$$f \doteq \frac{1}{D} \frac{ee'}{d^2}.$$

EXERCISE

1. A ball charged with 250 units of positive electrification is placed at a distance of 10 cm. from a similar ball charged with 100 units of negative electrification. What is the direction and magnitude of the force acting between them? If the two balls are touched together and then again separated by a distance of 10 cm. what will be the direction and magnitude of the action?

ELECTROSTATIC INDUCTION

518. Electrostatic Induction.—When a charged conductor is brought into contact with another conductor, there is a distribution of the electrification over the surfaces of both conductors. We say that the second conductor has been electrified by contact. We shall now consider a method of electrifying a body in which actual contact is not necessary.

Let two insulated and neutral conductors m and n (Fig. 311) be placed in contact, and let a body charged with positive electrification be brought near to one conductor but not near enough for a spark to pass. The uncharged bodies are thus brought within the electric field of the charged body. If a pith-ball electroscope be brought

near to the conductors, it will be attracted at the ends *A* and *C* but hardly at all at *B*. This fact indicates that the conductors have become charged and that most of their charge is at their extremities. Keeping the charging body in the same position, and handling the farther conductor by an insulator, remove it to some distance from the nearer conductor, and then lay aside the charging body. Bring the conductor that was nearer the charging body to an electroscope charged negatively, and the repulsion observed proves that this conductor is charged negatively. In similar fashion, the farther body may be shown to bear a positive charge. Furthermore, if the two conductors be now

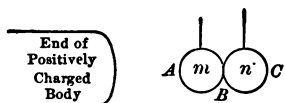


FIG. 311. — Induction.

brought in contact, they will be found to have lost their charges, thus proving that the amount of positive electrification was equal to the amount of negative electrification. The two bodies have by *induction* received equal and opposite charges. But how have these conductors been thus charged?

519. Two-fluid Theory. — In attempting to answer this question, let us make use of the following convenient but entirely artificial assumption concerning the nature of an electric charge. A neutral body is not devoid of electricity; on the contrary, it contains large quantities of the “electrical fluid.” This does not cause any electrical manifestations, however, because the quantity of positive electrical fluid equals the quantity of negative electrical fluid. When there is an excess of either fluid in a body, it shows a charge characteristic of that fluid. A body can then produce electrical effects only when there is inequality in the amounts of positive and negative electrification

borne by it. When one body exhibits a positive charge, some other body or bodies must have an equal negative charge. Each of these two fluids exerts a repellent action upon itself and an attractive action upon the other.

520. Induction Explained. — When a positively charged body is brought into the vicinity of an uncharged conductor, the $+E$ of the uncharged body is repelled to the side that is farthest from the positively charged body, and its $-E$ is attracted to the side nearest the charging body (Fig. 312). By breaking the conductor in two, or by constructing it in two parts that may be easily separated, the charges induced spread over the two parts, so that there is an excess of $-E$ on the nearer and an excess of $+E$ on the more remote part. These quantities of $+E$ and of $-E$ are equal.

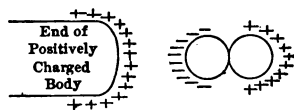


FIG. 312. — Induction Explained.

521. Induction precedes Attraction. — When any body is brought near a charged body, that is, is placed in its electric field, induction takes place, and there is a separation of the $+E$ and of the $-E$ of the body. As the charging body induces the opposite kind of electrification on the side of the neutral body nearest to it, and the same kind of electrification on the side farthest from it, the attraction exceeds the repulsion, and the bodies tend to approach each other.

522. Charging by Induction. — Let a charged body be brought near to a neutral conductor (Fig. 313). Then touch the conductor with the finger so that conduction may take place through the body into the earth. The repelled charge is free to escape to the earth; but the attracted charge is held or *bound* by the charge on the in-

* The abbreviations $+E$ and $-E$ will be used for positive electrification and for negative electrification, respectively.

ducing body. When the body being charged by induction is removed from the vicinity of the charging body, *i.e.* is removed from its electrical field, the induced charge spreads over the whole conductor in accordance with the laws of electrical distribution (Fig. 304). A body may therefore be charged without contact, *i.e.* through induction, by the following operations:

(1) Bring the body close to a charged body.
 (2) Remove the free charge by connecting with the earth or "grounding" for a moment.
 (3) Separate the two bodies.

The body thus charged by induction receives the kind of electrification which is opposite to that of the charging body.

523. Electrophorus. — A plate of vulcanite or of sealing wax supported on a metal base, and a disk of metal furnished with an insulating handle constitute an *electrophorus* (Fig. 314), a contrivance for obtaining in a convenient way electrification by induction. The electrophorus is operated in

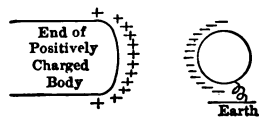


FIG. 313.—Charging by Induction.



FIG. 314.—Electrophorus.

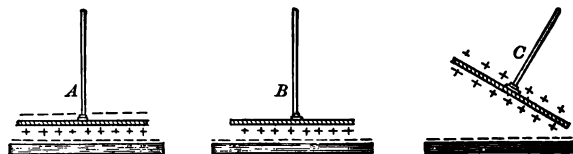


FIG. 315.—Action of Electrophorus.

the following way (Fig. 315). Charge the nonconducting plate with $-E$ by rubbing it with flannel. Then

place the metal disk upon it, and touch its upper surface with a finger. On removing the finger and then lifting the disk, this will be found to be charged with $+E$. As there is actual contact between a few points only of the disk and plate, the amount of electrification received by conduction is negligibly small in comparison with that which the disk gets by induction.

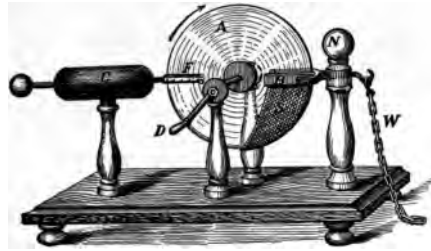


FIG. 316. — Friction Machine.

524. Electrical Machines. — The first electrical machines (Fig. 316) consisted of revolving cylinders or circular plates of sulphur or glass,

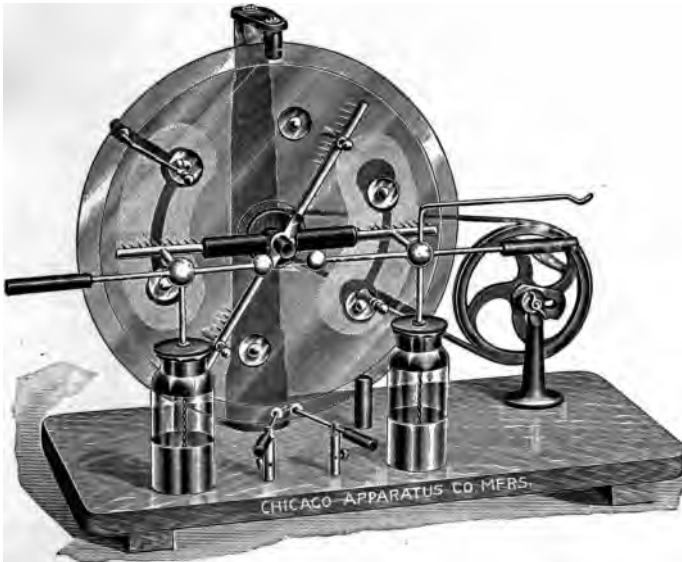


FIG. 317. — Toepler-Holtz Induction Machine.

which were made to rub against pads of silk or other suitable substance so as to become charged by contact. Later, much more powerful machines were invented (Fig. 317), their action depending upon induction and being similar to that of a series of revolving electrophori.

525. Conservation of Electricity.—How does the amount of an inducing charge compare with the amount of the induced charge? To

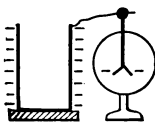
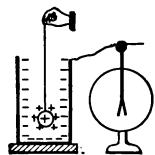
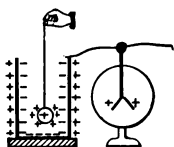


FIG. 318.—Faraday's Ice-pail Experiment.

answer this question, Faraday devised his so-called "ice-pail experiment." A metal pail is set on an insulator (Fig. 318) and connected by a wire to a gold-leaf electroscope. A metal ball suspended by an insulating thread is charged and lowered into the pail, without touching it, however. The pail becomes charged by induction and the leaves of the electroscope diverge. Upon touching the pail the free charge escapes, and the leaves collapse. If, after the removal of the finger, the ball is taken out of the pail, the leaves again diverge; for then the bound charge passes from the inside of the pail to the outside. The ball is now charged with one kind of electrification, and the pail and electroscope with the other. On touching the ball to the pail, the leaves fall together, showing that the two kinds of electrification have neutralized each other, leaving no excess of either. The general conclusion can therefore be drawn that *the induced charge always equals the inducing charge.*

One system of bodies cannot be charged with one kind of electrification without an equal and opposite charge being imparted to another system of bodies. *The sum total of positive and negative electricity in existence must equal zero.*

EXERCISES

1. Given a piece of flannel and a stick of sealing wax, in what two ways can you charge an insulated conductor with positive electricity?
2. What has induction to do with the attraction between an electrified body and light bodies, such as snips of paper, straw, pith?
3. How can an electric charge on a sphere be divided into three equal parts?

4. Given a positively charged insulated conductor, how can two other insulated conductors be charged, one negatively and one positively, without diminishing the charge on the first conductor?

ELECTRICAL CONDENSERS

526. Condensers. — A conducting plate B (Fig. 319) mounted on an insulating stand is connected by a wire with a gold-leaf electroscope and then given a negative charge sufficient to make the leaves diverge somewhat. When a similar plate A that is connected with the earth is brought nearer and nearer to the first plate, the leaves collapse more and more. The distribution of the electrification has changed, the electrical density on the leaves being less when A is near B than when it is remote. Now it is found that, when A and B are close to each other, many times the original charge has to be given to the electroscope before its leaves diverge as much as before. The amount of electrification that B can hold has therefore been much increased by putting it close to another grounded conductor. The electrification is, so to say, *condensed* on the plates. Such an arrangement of two conductors separated by a nonconductor forms a *condenser*.

527. Induction in Condensers. — When A (Fig. 319) is brought near to B , the negative electrification of B acts inductively across the dielectric (air, in this case) and induces positive electrification on the near side, and negative on the far side, the latter escaping to the earth. The positive electrification then induces some of the negative charge to pass from the electroscope to the near side of B . The positive charge on A and the negative charge on B are

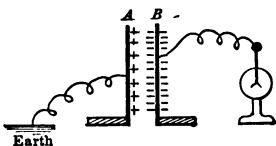


FIG. 319. — Action of Condenser.

mutually held or *bound* together, thus permitting additional electrification to be given to the plate connected with the electroscope. A condenser, then, can accumulate, by means of induction, large amounts of electrification upon comparatively small surfaces.

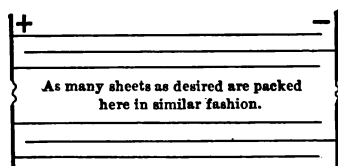


FIG. 320.—Compact Form of Condenser. The alternate sheets of tinfoil are connected together.

528. Capacity of Condensers.—

The amount of electrification which can be held by a condenser depends upon the nature of the dielectric, is directly proportional to the area of the conductors, and inversely proportional to the thickness of the dielectric.

Condensers of large capacity are commonly made of sheets of tin foil placed between sheets of mica or paraffined paper and connected as shown in Fig. 320.

529. The Leyden Jar.—One of the commonest and most convenient forms of condensers is the *Leyden jar* (Fig. 321). This consists of a glass jar covered, inside and outside, for about two thirds of its height with tin foil, and provided with a nonconducting stopper through which passes a brass rod tipped with a knob and carrying at its lower end a chain reaching to the inside coating at the bottom.

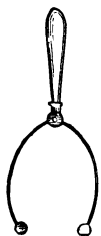


FIG. 322.—Discharger.

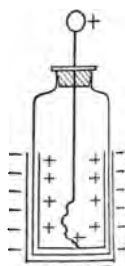


FIG. 321.—Leyden jar.

The *jar* is *charged* by holding it in the hand placed upon its outside coating so that the knob is in contact with the conductor of an electric machine. The charge thus given to the inner coating acts inductively through the glass upon the outside coating, the repelled electrification passing off through the body into the earth.

The *jar* is *discharged* by connecting its outer and inner coatings

with a wire or other conductor provided with an insulating handle (Fig. 322) to safeguard the hand from shock.

A *battery of Leyden jars* consists of two or more jars with their outer coatings connected by placing them upon the same conductor, and their inner coatings joined by means of a wire wound around their projecting rods.

530. Seat of the Charge in the Condenser. — A Leyden jar with movable coatings (Fig. 323) is put together, charged in the usual way, and placed upon an insulating plate. The inner conductor is then lifted out by means of a rod of insulating material, and the glass vessel raised from the

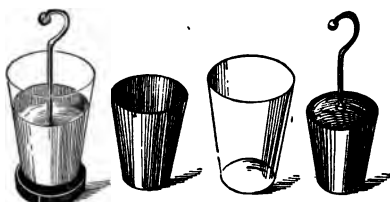


FIG. 323. — Dissectible Leyden Jar.

outer conductor by grasping it by the rim. On testing the three parts for electrification, but little, if any, is found on the conductors, most of it being present on the glass. The parts are again assembled, and when discharged, give nearly as big a spark as if the jar had not been dissected.

The proper interpretation of the experiment is that the charges of electricity are resident upon opposite surfaces of the glass, the coatings serving merely to distribute the original charge and to allow of its escape when they are connected. It is the dielectric, and not the conductors, which holds the charge. The dielectric is thereby strained, and if too great a charge be put into a condenser, the glass will break. Because of this disruptive effect the discharge of a condenser of any kind is frequently referred to as the *disruptive discharge*.

531. Oscillatory Character of the Disruptive Discharge. — A disruptive discharge takes place whenever the strain in a dielectric due to

the action of electromotive force becomes so great as to result in a breakdown of its insulating power; a discharge therefore signifies a release from electric strain. Now we have learned that when matter is released from strain, it is itself set vibrating or oscillating. Just as a sudden release from strain in elastic bodies gives rise to vibratory motions, so may release from electric strain result in oscillations. An electric spark takes from a thousandth to a ten millionth of a second to pass between two conductors, but during that time from ten to thirty separate discharges have occurred, each yielding its own spark. By correspondingly rapid successive exposures of a photographic plate, it is possible to obtain photographs of these separate sparks.

532. Electric Waves. — The general condition for the formation of waves is a sudden release from a strained condition. The disruptive discharge of a condenser or a static machine consists in a sudden release from electric strain, and as a consequence, it calls into being *electric* (also called *electromagnetic*) *waves*. These waves travel through the ether at the same speed as do the ethereal waves that give rise to light and to heat. Indeed, the only essential difference in all these ethereal waves is that of length. Electrical waves have been produced as short as a few millimeters and as long as several thousand miles — an enormous range. These electrical waves can be reflected, refracted, and diffracted, and can produce the phenomena of interference.

EXERCISES

1. Why can every charged body be regarded as one of the coatings of a condenser?

2. Why cannot a Leyden jar be charged at all heavily when it is supported on an insulating plate?

3. One person holding a charged Leyden jar in his hand by its outer coating touches its knob to the knob of a second uncharged jar held in the hand similarly by another person. What happens?

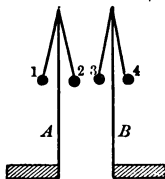


FIG. 324.

4. Two similar conducting plates are mounted on insulated stands and are provided with pith balls hung by conducting threads on opposite sides (Fig. 324). Plate *A* is charged positively and *B* is brought parallel and close to it, no spark, however, being allowed to pass between them. What changes will occur in the positions of pith balls 1, 2, 3, and 4? If *B* be grounded, what

changes in the pith balls will take place? If B is then disconnected from the earth, how will the pith balls be affected? What will the pith balls do when B is then removed far from A ?

ATMOSPHERIC ELECTRICITY

533. Lightning.—While scientific men as early as the sixteenth century were inclined to believe that lightning was nothing but an electric discharge, it was not until 1752 that Benjamin Franklin proved the identity of a lightning flash with an electric spark by means of his famous kite experiment. The kite was covered with silk so as not to be affected by rain, and the hempen cord used to fly it was insulated at one end by a silk ribbon, a key being connected with the string near the ribbon. A contemporary account of the experiment is as follows:

534. Franklin's Kite Experiment.—“Furnished with this apparatus, on the approach of a storm, he went out upon the commons near Philadelphia, accompanied by his son, to whom alone he communicated his intentions, well knowing the ridicule which would have attended the report of such an attempt should it prove to be unsuccessful. Having raised the kite, he placed himself under a shed, that the ribbon by which it was held might be kept dry, as it would become a conductor of electricity when wetted with rain, and so fail to afford that protection for which it was provided. A cloud, apparently charged with thunder, soon passed directly over the kite. He observed the hempen cord, but no bristling of its fibers was apparent, such as was wont to take place when it was electrified. He presented his knuckle to the key, but not the smallest spark was perceptible. The agony of his expectation and suspense can be adequately felt by those only who have entered into the spirit of such experimental researches. After the lapse of some time he saw that the fibers of the cord near the key bristled, and stood on end. He presented his knuckle to the key and received a strong, bright spark. It was lightning. The discovery was complete, and Franklin felt that he was immortal.”

535. Protection against Lightning. — Objects on the earth beneath a thundercloud become charged by induction, and when the difference in potential between cloud and earth becomes great enough to break down the insulating power of the intervening air, a lightning flash results. Steeples, towers, chimneys, trees, and the like are made of substances that conduct electricity better than air does. Hence lightning usually includes them in its path when they are present. Franklin conceived the idea of running a rod of some very good conducting material such as copper from deep in the ground to the projecting parts of buildings so that a very easy path would be offered to the passage of electricity from clouds to earth. While such lightning rods undoubtedly do diminish considerably the liability to lightning strokes, yet there are certain kinds of electric discharges from which the rods fail to protect buildings. Because of these occasional failures the use of lightning rods has fallen into some disrepute.

CHAPTER XXII

MAGNETISM

MAGNETIZATION

536. Natural Magnets. — A certain iron ore occurring in various parts of the earth has the property of attracting bits of iron brought near to it. When a specimen of the ore is rolled in iron filings and then lifted, tufts of the filings are seen to cling to two spots on opposite sides of it. And when a specimen is suspended by a cord so that the line joining the two spots is horizontal, it swings round until this line is in a north and south direction. The ancients knew of the attraction between this mineral and iron, and as the specimens that came under their observation were found in Magnesia in Asia Minor, they were called *magnets*. The name of *lodestone* (meaning *leading stone*) was later also given them because of their property of assuming a definite position when suspended. The spots of greatest attraction are called the *poles* of the magnet, and the line joining them is the *magnetic axis*. The pole that tends to point north is termed the *north-seeking, marked, N, or. + pole*, while the other is called the *south-seeking, unmarked, S, or - pole*.

537. Artificial Magnets. — When pieces of iron or steel are brought in contact with a lodestone, they also acquire the property of attracting iron; they become *magnetized* and form *artificial magnets*. When removed from the vicinity of the lodestone, some kinds of iron lose their

magnetic quality, while others retain it. In general, soft iron * acquires magnetism easily and loses it as readily, while hard iron and steel do not magnetize readily, but when once magnetized, retain the quality well. Permanent magnets are prepared from varieties of cast iron or steel which, through tempering or admixture of certain substances, have been made to acquire great *retentivity* or power of retaining magnetism. The magnetic quality of the lodestone does not seem to suffer loss from this process of magnetization, and any number of artificial magnets may be prepared from pieces of steel by contact with one natural magnet. Now that artificial magnets can be made by the action of an electric current[†], the lodestone method has fallen into disuse.

538. Shapes of Magnets. — Magnets are commonly given the shape of a straight rod or bar (*bar magnets*), or are bent into a U-shape, forming so-called *horseshoe magnets*. *Compound magnets* are built up of thin plates separately magnetized and fastened together with their similar poles facing in the same direction. A piece of soft iron placed across the opposite poles of two bar magnets laid side by side, or the two ends of a horseshoe magnet is called the *armature* or *keeper*; its purpose is to aid in preventing the loss of magnetic quality.

539. Conditions Affecting Permanency of Magnetic Quality. — A jar or shock to a permanent magnet often causes it to lose a portion of its magnetism, and long-continued jarring or tapping is sufficient to demagnetize it totally. When heated red hot, magnets lose all trace of magnetization, and do not regain it upon cooling.

* *Hard and Soft Iron.* A sample of iron is said to be *soft* when it can be easily filed, and *hard* when it is filed with difficulty. The mechanical properties of iron and steel depend both upon their chemical composition, *i.e.* upon the proportions of carbon and other substances that they may contain, and upon the processes of tempering and annealing undergone. Their magnetic properties also depend upon these conditions.

540. Magnetic Action. — Either pole is capable of attracting iron. This attraction is mutual; the iron attracts the magnet just as much as the magnet attracts the iron.

But what action have magnets upon one another? Bring the north-seeking pole of a bar magnet near the north-seeking pole of another magnet suspended horizontally. The suspended magnet swings round and presents its south-seeking pole to the north-seeking pole of the magnet brought near to it; that is, the north-seeking poles repel each other. In like fashion it may be shown that the north-seeking pole of the one magnet attracts the south-seeking pole of the other. *Like poles repel, and unlike poles attract each other.*

541. Magnetic Strength. — Magnets differ in their powers of attraction and repulsion. To compare their strengths, a unit of magnetic strength must be adopted. *A magnetic pole is defined to have unit strength when, if placed in air one centimeter from another like pole of equal strength, a force of one dyne is exerted between them.* A pole, then, that exerts a repelling force of m dynes upon a pole of unit strength, when they are one centimeter apart, has a strength equal to m dynes. Coulomb, in 1785, demonstrated that *the force acting between two poles is directly proportional to the product of their strengths and inversely proportional to the square of the distance between them.* The force f acting between a pole of strength m and a pole of strength m' placed d centimeters apart is therefore

$$f = \frac{mm'}{d^2}.$$

542. Magnetic Field. — The space surrounding a magnet is called a *magnetic field*, and particles of iron or other

magnets when brought into this field exhibit the characteristic attractions or repulsions. A magnetic field is mapped as follows:

I. A sheet of smooth paper is laid over a magnet and fine iron filings are sprinkled evenly upon the paper.

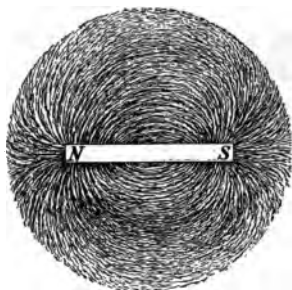


FIG. 325.—Field of Force of a Bar Magnet.

When the paper is gently tapped, the filings arrange themselves in curved lines (Fig. 325).

II. A small compass is placed on a sheet of paper near a magnet, and the direction taken by the needle marked (Fig. 326). By repeating this operation in various portions of the field characteristic curved lines are obtained, which are called *lines of*

force. Since a compass needle placed in a field of magnetic force turns until it is parallel with the lines of force at that point, the direction of these lines is that indicated by the compass. When the lines of force of a particular portion of a field are approximately parallel, that portion is said to present a uniform field of force.

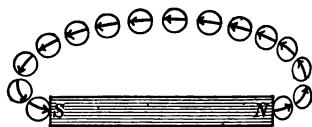


FIG. 326.—A Line of Force Plotted by a Compass.

While in the fields of permanent magnets the lines of force appear to end at the boundaries of the magnet, in reality they pass through the magnet so as to form closed loops. Every line of force must form a closed loop or circuit, part of which passes through air or other nonmagnetic substance, and may be mapped by the methods given above, while the other part passes through the magnetic substance itself.

Field of Force between Poles of Two Magnets. — s. 327 to 328 are shown the fields of like and of poles facing each other. The lines of force of the poles pass straight across, while those of the like werve aside. Now as unlike poles attract and like repel each other, the lines in these two fields do explain why these take place. For, regarding the lines as representing stretched elastic bands that tend to shorten as to repel one another, we perceive that when they pass straight across between the poles, they tend to draw these poles together, when they swerve aside between the poles, they tend to draw them apart.

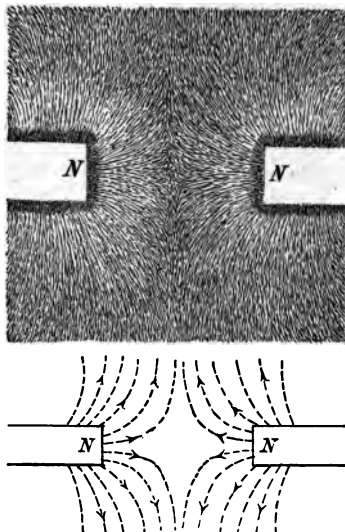


FIG. 327.— Magnetic Field between Like Poles.

Strength of Magnetic

At any point in a magnetic field there is a direction in a direction tangent to the line of force passing through that point. If several adjacent lines of force are traced throughout their entire length, it is found that sometimes they spread apart and sometimes they crowd together. Where the lines are closest together, as when they are near the poles, the magnetic force is greatest. We may take the number of lines of force passing through a given area perpendicular to their direction as means of indicating the strength or intensity of any

portion of a magnetic field. If one dyne of magnetic force is acting at a given part of a magnetic field, one line of

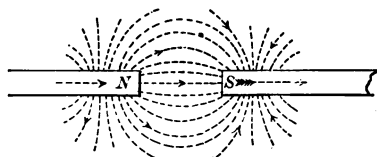
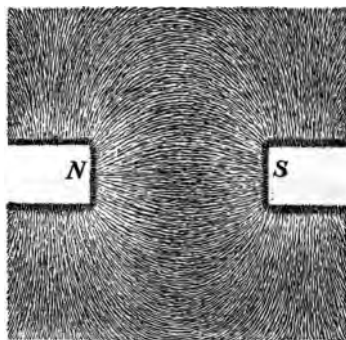


FIG. 328. — Magnetic Field between Unlike Poles.

force per square centimeter is taken to represent the magnetic intensity and that part of the field is said to have unit strength. One hundred lines of force per square centimeter represent a field in which 100 dynes are acting, and,

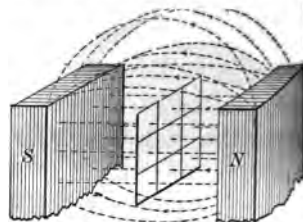


FIG. 329. — The number of lines of force per square centimeter is the measure of magnetic strength.

in general, m lines of force per square centimeter represent an intensity of m dynes (Fig. 329).

545. Magnetic Induction. — If one end of a soft iron rod is dipped in iron filings and a pole of a bar magnet brought near the other end (Fig. 330), the iron of the rod becomes temporarily magnetized, and a

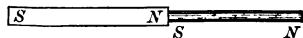


FIG. 330. — Induction of Magnetism.

tuft of filings will cling to it as long as the magnet is kept near the other end. The rod is said to be *magnetized by induction*. Bodies capable of magnetization develop poles and behave like magnets whenever brought into a mag-

netic field. A *N*-pole is induced next to the *S*-pole of the magnet producing the field, and a *S*-pole is induced farthest from the *S*-pole of the inducing magnet, and vice versa.

546. Permeability. — Suppose the unlike poles of two large bar magnets to be fixed near and opposite each other. The lines of force directly between them will be practically parallel (Fig. 331). But if a ring of soft iron is placed in such a uniform field, the lines of force are found to pass through the iron rather than through the air (Fig. 332). Iron therefore is said to be more

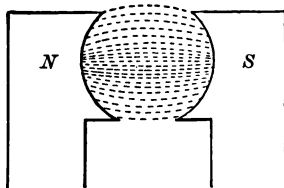


FIG. 331. — A Field of Force in Air.

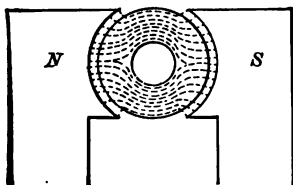


FIG. 332. — Lines of force pass more readily through iron than through air.

permeable than air for magnetic lines of force, and this property of gathering in and conducting lines of force is known as *permeability*. It is because of the high permeability of iron that the strength of magnets is conserved by their armatures, which prevent in a large measure the leakage of lines of force; and in all kinds of machines and apparatus in which magnetic force is used, iron is employed to prevent leakage and consequent dissipation of lines of force.

547. Theory of Magnetization. — If a magnet be broken in two, each part will develop poles and itself become a magnet (Fig. 333). Each of these magnets if again broken will yield two more magnets, and so on. This multiplication of magnets from successive divisions will evi-

dently end only when further division is impossible; and that is when the molecule is reached. Therefore the

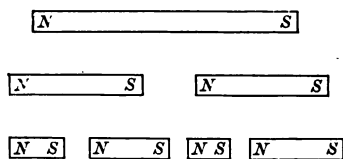


FIG. 333.—Effect of Breaking a Magnet.

molecules themselves must be magnets, and magnetization is a molecular phenomenon. A glass tube filled with steel filings, when moved repeatedly over a strong pole, soon becomes a magnet, the filings

being seen to arrange themselves in definite positions. Upon shaking up the filings so as to destroy this arrangement, the magnetic quality at once disappears. The magnetization must then depend upon the orderly arrangement of the filings.

To account for such phenomena as these we assume that in an unmagnetized piece of iron or steel the molecules have no uniform arrangement; the molecules individually may indeed be magnetized, but since on an average there are



FIG. 334.—Arrangement of Molecules in an Unmagnetized Body.

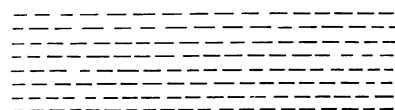


FIG. 335.—Arrangement of Molecules in a Magnetized Body.

as many *N*-poles facing in one direction as in any other, the body as a whole does not exhibit magnetic properties. But when the body is placed

in a magnetic field, the molecules are induced so to arrange themselves that their like poles turn and face in the same direction. Each of the molecular magnets contributes, then, its share of magnetic quality, and the body as a whole becomes magnetic. The body thereby takes on magnetic

properties which persist until the orderly arrangement is destroyed by heating or jarring (Figs. 334 and 335).

548. Saturation. — There is a limit to the amount of magnetization that a piece of iron or steel can receive, however strong the magnetizing force may be. This limit depends upon the particular kind of iron or steel used; in soft iron the limit is not reached until something like 20,000 lines of force per square centimeter are passing through it. When the magnetism of a body reaches this limit, it is said to be *saturated*. In a saturated magnet all the molecules are supposed to be lined up in parallel rows.

TERRESTRIAL MAGNETISM

549. The Compass. — Any magnet free to turn round a vertical axis may be used as a *compass* (Fig. 336). As is well known, the needle of a compass points in a northerly direction, and because of this constancy of position, compasses are used to guide men over trackless areas. It is to be noted that the compass as a whole does not tend to move bodily toward the north;



FIG. 336.—A Magnetic Needle or Compass.

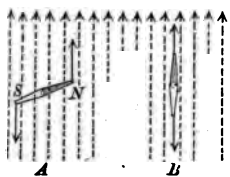


FIG. 337.—Action between a Compass and the Earth.

hence there can be no single resultant force acting upon it, for the effect of such a force would be motion in a straight line. The magnet simply turns; hence one of its poles must be attracted toward the north by a force having a magnitude equal to that of an opposite force urging the other pole toward the south. But two opposite, equal, and parallel forces constitute a mechanical couple (§ 128), and the

earth's action upon a magnet is therefore referred to as its "directive couple" (Fig. 337).

550. Magnetic Meridian and Variation.—The actual direction in which a compass needle points is called the *magnetic north*, and the vertical plane passing through the magnetic north is named the *magnetic meridian*. The magnetic meridian in many localities differs from the geographical meridian, the angle between the two being known as the *magnetic declination*. The declination is subject to daily and secular variation.

551. Inclination or Dip.—If a steel needle is balanced on a horizontal axis so as to be in neutral equilibrium, then magnetized and placed in the magnetic meridian, it is found to *incline* or *dip* so as to form an angle with the horizon (Fig. 338). At Chicago this angle is 73° . North of the equator the north-seeking pole dips downward, while south of the equator the south-seeking pole dips. At many places near the equator there is no dip, and an imaginary line drawn through places of zero inclination is called the *magnetic equator*.

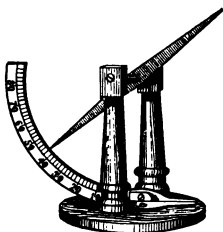


FIG. 338.—Dipping Needle.

552. The Earth a Magnet.—The behavior of the compass and of the dipping needle can be accounted for by regarding the earth as a huge magnet with its poles below the points on its surface where the dip is 90° . The magnetic poles do not coincide with the geographical poles (Fig. 339). As the declination and inclination in any locality have been found to vary slowly with the time, it follows

that the positions of the earth's magnetic poles are not fixed and invariable. They seem to swing irregularly from east to west, each swing requiring centuries for completion.

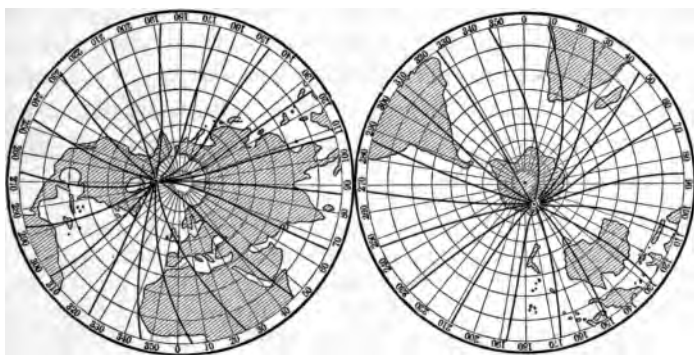


FIG. 339. — Northern Hemisphere. Southern Hemisphere.
The heavy lines represent magnetic meridians.

553. Inductive Action of the Earth. — Since the earth behaves like a gigantic magnet, lines of force must extend from pole to pole. While the earth's magnetic field is of small intensity compared with that of even feeble magnets, its presence may be readily detected not only from the behavior of compass and dipping needle, but also from its inductive action. Thus an unmagnetized steel rod held in the magnetic meridian so as to point in the same direction as the dipping needle is traversed longitudinally by the earth's lines of force. If, now, it is tapped sharply, it becomes magnetized, a north-seeking pole being produced at its lower end (in the northern hemisphere). By turning the rod over and tapping it, the positions of its magnetic poles are reversed. Because of the earth's inductive action most steel objects, such as columns, masts, and tools, become magnetized.

ELECTROMAGNETISM

554. Magnetic Field due to Electric Current.—The space in the vicinity of a lodestone or of certain pieces of steel that have acquired magnetic quality is in a peculiar condition, as is shown by the behavior of iron articles brought into this magnetic field. The question now arises: Can magnetic fields be produced by other means? This can best be answered after a consideration of the following experiments.

I. A strong, steady current of electricity, such as that furnished by an electric battery or a dynamo, is passed through a wire and a portion of the conductor dipped into iron filings. Filings cling to the wire in a tuft much as they would cling to a magnet.

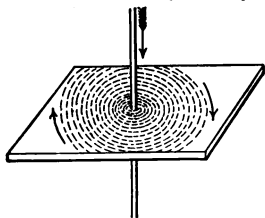


FIG. 340. — Lines of Force around a Current-carrying Wire.

II. A vertical wire is thrust through a piece of cardboard supported horizontally, over which iron filings are strewn.

When a strong, steady current is sent through the wire, the filings are seen to arrange themselves in concentric rings around the conductor (Fig. 340).

III. A loop of wire is suspended by a thread so that its two ends dip into mercury (a liquid conductor) contained in a vessel of insulating material shaped as shown in Fig. 341. A current of electricity sent into one mercury cup has to pass through the wire loop before it can escape through the other mercury cup. The loop is found to turn until its plane is at right angles to the lines of force of the earth's magnetic field, a position in which it incloses the largest possible number of the earth's lines of force. The

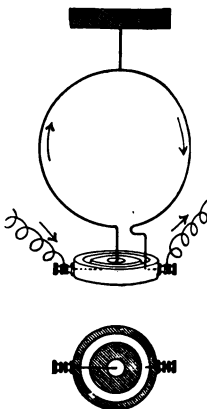


FIG. 341. — The central cavity and the annular groove are filled with mercury, the connections with the binding posts being shown in the lower figure.

loop behaves like a magnetized disk with poles located on opposite faces.

Furthermore, if a bar magnet be brought close to the current-bearing loop, attractions or repulsions will be observed such as would occur if the loop were a magnetized disk.

Evidently, then, a conductor through which an electric current is passing is surrounded by a magnetic field, and therefore is itself a magnet.

555. Solenoids.—By winding an insulated wire in a spiral, a series of loops is obtained (Fig. 342). As each loop behaves like a magnetized disk when an electric current is passed through the wire, the whole spiral is equivalent to a series of such magnetized disks with their *N*-poles facing in one direction and their *S*-poles facing in the other. The alternate *N*- and *S*-poles neu-

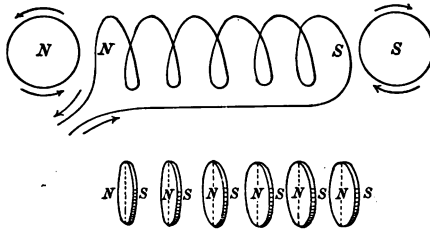


FIG. 342.—A solenoid behaves like a series of magnetized disks. The direction of the current determines its polarity.

tralize each other's action, but an unneutralized *N*-pole is left at one end and an unneutralized *S*-pole at the other end. Each of the loops, however, contributes lines of force so that the magnetic effects are proportionately increased. A current-carrying spiral or *helix* is called a *solenoid*. It has poles at its extremities, and behaves in all respects like a bar magnet.

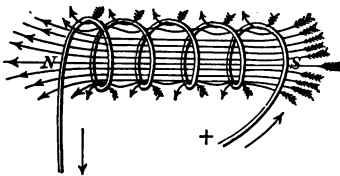


FIG. 343.—Lines of Force of a Solenoid.

556. Iron Cores.—The field of force of a solenoid is shown in Fig. 343. As is

seen, some of the lines of force appear at the sides of the spiral. But if the interior of the spiral be filled up with soft iron, so much easier is it for lines of force to pass through the iron than through air, that nearly all these side lines of force pass through the iron core and appear only at the ends of the solenoid. The number of lines of force at the ends is increased by the insertion of an iron core, and the strength of the field at the poles is correspondingly increased.

557. Electromagnets. — An electromagnet consists of an iron rod or a bundle of iron wires (Fig. 344) around which is wound an insulated wire. When a current is passed through the wire, the ends of the spiral become polarized. The greater the number of turns and the stronger the current, the stronger the electromagnet becomes. Hence it is possible to make electromagnets of a strength enormously transcending that of the most powerful permanent magnets. When the current ceases to flow, the coil at once loses its magnetic quality.

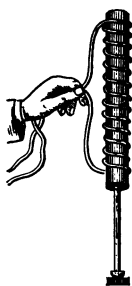


FIG. 344. — Straight Electromagnet.

The ease and promptness with which a magnetic field can be made to appear and disappear by means of an iron-cored coil of wire is of great importance in the action of various electrical devices. Ordinarily, electromagnets are made after the horseshoe pattern, advantage being thereby taken of the action exercised by both poles. The wire is wound (Fig. 345) in such a fashion that if the U-magnet were

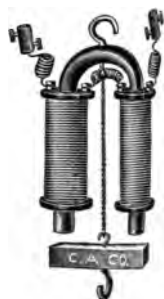


FIG. 345. — Horseshoe Electromagnet.

straightened out and the two spirals placed end to end, the direction of the winding would be the same on both.

EXERCISES

1. How can a mixture of iron and brass filings be separated?
2. Why will two long iron wires suspended by magnetic attraction near each other from the same pole of a bar magnet not hang parallel?
3. Given a bar magnet and a darning-needle, describe how the needle should be rubbed with the magnet in order to make it a magnet with its north-seeking pole at its point.
4. Why does not a magnet supported on a floating cork tend to move as a whole toward the north?
5. A nail is suspended by magnetic attraction near the positive pole of a bar magnet, and a similar second magnet slipped over the first until their unlike poles come over each other. Why does the nail fall off?
6. How can a compass be made out of an irregular lump of lodestone?
7. Indicate points of resemblance and of difference between (1) gravitation; (2) electrostatic action; (3) magnetic action.
8. Why is it that the lower end of a rod of soft iron pointing toward the north magnetic pole of the earth repels the north-seeking pole of a compass?
9. What will happen to a rod that has been magnetized by the earth's induction, if it is tapped sharply when held in an east-and-west position?
10. Why is it that steel tools that have just been tempered are not magnetized, while after use they often show magnetic behavior?
11. How would (a) a compass, (b) a dipping needle, behave if placed over one of the magnetic poles of the earth?
12. What is the strength of a magnetic pole that attracts with a force equal to 1 g. another pole which is 10 cm. away, and has a strength of 490 units?

CHAPTER XXIII

CURRENT ELECTRICITY

THE VOLTAIC CELL

558. Steady Currents.—When an electrically charged body is connected by means of a conductor with another body charged to a different potential, a transfer of electrification takes place between the bodies; a current of electricity passes. This current is, however, of momentary duration. Until about 1800 the only means of producing electricity were those already discussed (Chap. XXI), and because of the brief existence of the currents thereby produced, but little was learned about them. But near the beginning of the nineteenth century the discovery was made of a novel way of getting electric currents that could be made to flow steadily for some time. A new vista was thereupon opened up in electricity, and the study of *steady* or *continuous currents* was eagerly pursued. These steady currents are produced by means of *voltaic* or *galvanic cells*, as they are variously called after the names of the two men, Volta and Galvani, who were instrumental in their invention.

559. Action of Sulphuric Acid on Zinc and on Copper.—When a strip cut from a sheet of commercial zinc is placed in sulphuric acid diluted with water, hydrogen bubbles rise from the strip, which soon dissolves. This act of solution differs from that of salt in water, in that, instead of zinc, a

white substance, known as zinc sulphate or white vitriol, is obtained when the liquid is removed by evaporation. A chemical action has taken place in which zinc has been substituted for the hydrogen that was a constituent of the acid. Heat is evolved in this reaction, chemical energy being transformed into heat energy.

Now if a similar strip of zinc is wet with the acid, and then rubbed over with a little mercury, a mirror-like coating of mercury is spread over the strip, and the zinc is said to be *amalgamated*. Such a strip, when placed in the acid, is not acted upon appreciably, no hydrogen bubbles appearing on its surface. How the mercury protects the zinc will be discussed later.

When copper is placed in dilute sulphuric acid, no hydrogen bubbles appear, and the copper does not dissolve; no chemical action takes place.

560. The Simple Voltaic Cell. — When a strip of copper and a strip of amalgamated zinc are immersed in the same vessel of dilute sulphuric acid, no action occurs so long as the strips are not in contact. But let the strips touch each other, and at once bubbles arise, *not from the zinc, but from the copper*. After a little time the zinc will all dissolve, the mercury falling to the bottom of the vessel; but the copper will not be affected. Such a combination of zinc (the *active element*) and copper (the *passive element*) with sulphuric acid (the *exciting fluid*) constitutes a *simple voltaic cell*. Two or more cells joined together form a *battery*, although the words battery and cell are frequently used synonymously.

Let wires be soldered or clamped to the upper parts of the strips of copper and zinc immersed in the same vessel of dilute sulphuric acid. By pressing the extremities of

these wires (their *terminals*) together, the same results as before are obtained. Bring a compass needle near and parallel to the joined wires (Fig. 346), and it will be moved, thus proving the existence of a magnetic field about the wire. Connect the terminals by means of a very fine and short wire, and it will become hot.

Moving a compass needle or heating a wire implies the expenditure of energy. The wires connecting the strips,

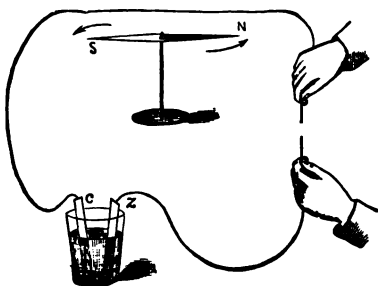


FIG. 346. — The electric current generates a magnetic field.

as well as the space surrounding them, have acquired the power of doing work. The source of this energy is the chemical energy of the reacting substances. We say that an electric current is traversing the wires, and that chemical energy is being transformed into electrical

energy. This electrical energy can be transformed in part into mechanical energy, as in moving a compass needle, and in part into heat energy, as in heating a wire.

561. Electromotive Force of Cell. — Whenever two different substances, be they solid, liquid, or gaseous, are in contact, a difference of potential is at once set up between them. Two different solids dipped into the same liquid conductor will as a rule be at different potentials, for the liquid has the same potential throughout, and the potentials of the solids differ from that of the liquid in varying degrees. If the solids conduct electricity and are joined by a wire, a current will traverse the wire because of the electromotive force (E. M. F.) due to the dif-

rence of potential.' The current will continue to pass as long as the difference of potential is maintained; and this occurs at the expense of chemical energy. It has been found that copper is at a potential about $+ .5$ volt higher than that of sulphuric acid, while zinc is at a potential about $- .5$ volt lower than that of the acid. Hence the potential difference (P. D.) between the two metals placed in the acid amounts to about $+ .5 - (- .5) = 1$ volt, and the E. M. F. urges the current from the copper to the zinc through the connecting wire.

562. E. M. F. and Steady Currents. — In order to keep up a steady flow of water, a constant difference of level producing a pressure must be maintained, and, analogously, in order to keep an electric current steady, a constant difference of potential producing an "electrical pressure" or E. M. F. must be preserved. Steady currents can be produced, then, only by generators that are able promptly and continuously to repair any breakdown in E. M. F. By rapidly working a static machine, very nearly steady currents are obtained, the E. M. F. being built up almost as soon as broken down.

563. Static Machine Currents compared with Voltaic Cell Currents. — A static machine may deliver a current that has the same amount of energy as the current from a voltaic battery, but the effects of the two currents are very different. Thus the leaves of an electroscope which are made to diverge widely by a static machine current are barely moved by a voltaic cell current. Again, the length of a spark between the terminals of a static machine is far greater than that which can pass between the terminals of a voltaic cell. We have learned that the degree of divergence of the leaves of an electroscope, as well as the length of a spark, is a measure of the E. M. F. We therefore conclude that the E. M. F. of the static machine is much larger than that of the voltaic cell.

But the current of a static machine, sent through a wire connecting its knobs, does not appreciably affect a compass needle nor warm a wire rapidly. As we shall soon see, these effects depend upon the quantity of electricity flowing through the wire. Hence we conclude that the quantity of electricity delivered by a voltaic cell greatly exceeds that of a static machine.

564. Classes of Conductors. — Many solids, especially the metals, conduct electricity without undergoing any deep-seated changes. Such conductors are said to belong to the *first class*. Very few pure (*i.e.* consisting of a single substance) liquids conduct electricity, excepting molten metals. But the solutions of certain substances, notably water solutions of substances such as acids, salts, and bases, are good conductors, and are known as *conductors of the second class*. These conductors undergo chemical changes when they carry electric currents.

565. The Dissociation Theory. — All acids, bases, and salts have the same general composition ; their molecules consist of two parts, each of which preserves a certain individuality in chemical actions. Now according to a theory which, since its proposal in 1887, has received an ever increasing acceptance, the molecules of acids, bases, and salts, when dissolved in water, break up into these two parts ; the water in dissolving them also dissociates their molecules. One part, the metallic or metal-like part, is always charged with positive electricity, while the other part, the nonmetallic part, is always charged with an equal amount of negative electricity. These *ions*, as the electrically charged part-molecules are called, lead a practically independent existence in the solution, but since the number of positive ions (*cations*) equals the number of negative ions (*anions*), the solution as a whole is electrically neutral. Sulphuric acid, for example, when it is mixed with water, dissociates into *hydrogen cations* and *sulphuryl anions* or *sulphions*.

566. Solution Tension. — When a plate of zinc (this is likewise qualitatively true of any other metal soluble in an acid) is placed in dilute sulphuric acid, a few zinc atoms leave the plate and pass into the acid in the form of positively charged ions. The zinc is said to have a tendency to pass into solution ; it has a *solution tension*. The separation of these positively charged ions leaves the zinc plate charged with an equal quantity of negative electricity, the difference of potential being about .5 volt. The case is similar to the one in which a static

charge is generated by placing glass and silk in contact, equal charges of positive and negative electrification appearing on the two substances, respectively. Only a few zinc ions form, however, before the zinc plate becomes so strongly charged with negative electricity that it pulls any positive ions, beyond a very small number, back to itself. A zinc plate in a diluted solution of sulphuric acid is therefore charged negatively and is surrounded by a layer of positively charged zinc ions.

567. Action of Voltaic Cell according to the Dissociation Theory.—The hydrogen ions already in the diluted acid are repelled by the zinc ions about the negative plate inasmuch as they both bear positive charges. If a copper plate is also in the solution, some of these hydrogen ions impart their charge to it, thereby changing into gaseous hydrogen which collects on the negative plate. But as soon as the potential of this plate rises about .5 volt above that of the solution, the charging process ceases. When the external circuit is closed, a current of electricity passes from copper to zinc in the external circuit, and the conditions for equilibrium are changed. As a consequence, more zinc ions pass into solution and drive more hydrogen ions out of solution. A procession of zinc ions driving a procession of hydrogen ions before them then passes through the solution. As these ions carry positive charges with them, positive electricity is conveyed from the zinc to the copper plate; in other words, a current of electricity traverses the solution from the positive to the negative plate.

Conductors of the second class, according to this theory, carry currents by a kind of convection; there is a movement of matter with the electricity. This is in sharp contrast with the method of conduction of conductors of the first class, where no such transference of matter is to be observed. Conductors of the second class are often called *electrolytes* (from the Greek *electro* and *lasis* = loosening); and the term *electrolytic conduction* is used in contrast with *metallic conduction*.

568. Before taking up the consideration of certain objectionable peculiarities of the simple cell and the means that have been devised to avoid them, let us pass in review some of the effects produced by steady currents. The most important of these are the magnetic, heating, and chemical effects, the preliminary study of which will enable us to better understand the action of cells.

MAGNETIC EFFECTS

569. Oersted's Experiment. — The Danish physicist Oersted, in 1819; while in the midst of a lecture, suddenly thought of testing whether or not a current-carrying wire would affect a compass needle. He brought the wire, held

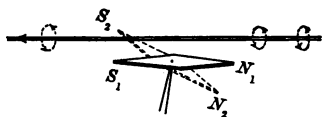


FIG. 347. — Magnetic Effect of a Current.

horizontally and parallel to the needle, down over it (Fig 347), and he and his audience saw the needle promptly swing

round and settle itself at an angle with the wire. When he reversed the direction of the current or placed the wire beneath the compass, the needle rotated in the opposite direction. Thus was made the discovery of *electromagnetism*, the relation between magnetism and electricity upon which rests the action of dynamos and motors as well as of a host of other electrical devices.

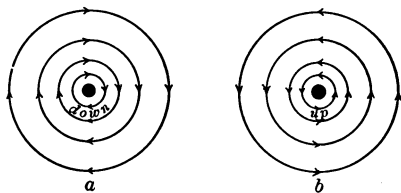


FIG. 348. — The diagrams show the cross section of a current-carrying wire normal to the plane of the paper. The arrows indicate the direction of the lines of force according as the current flows down or up.

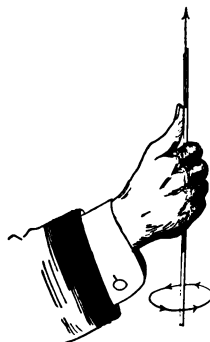


FIG. 349. — The Hand Rule.

570. Direction of Current and of Lines of Force. — The space around a current-carrying wire is filled with lines of force encircling the wire. The directions of the current

and of the lines of force (Fig. 348) are analogous to the turning and driving in of a corkscrew. If the wire be grasped by the right hand (Fig. 349) in such a way that the thumb points in the direction of the current, the fingers will point in the direction of the lines of force. The N-pole of a compass turns toward the west either when a current is flowing *north over* or *south under* the needle (Fig. 350). If the compass be placed at the center of a vertical loop of a current-carrying wire, the lines of force within the loop will have the same direction and the deflection will be increased.

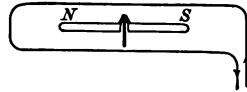


FIG. 350.—A loop around the needle increases the deflection.

571. Current Detectors or Galvanoscopes.—To find out whether or not a current is traversing a wire, all that is



FIG. 351.—Galvanoscope.

necessary is to try its effect upon a compass. To get as many lines of force as possible to act upon the

compass, the wire is wound into a coil and the compass needle placed at its center. Such an arrangement (Fig. 351) constitutes a *galvanoscope*, by the aid of which the presence as well as the direction of a current may be detected.

572. Measure of Current Strength.—The greater the current sent through a conductor, the greater will be the intensity of the magnetic field, so that current strengths may be measured by their magnetic effects. As Ampère was the first to recognize this fact, the unit of current strength

has been named the *ampere* in his honor. Let a current of known strength be passed through a galvanoscope provided with a scale, and the deviation of the needle noted. Suppose now that another current of different origin



FIG. 352. — Tangent Galvanometer, a Fixed Coil Instrument.

and unknown strength be passed through the same galvanoscope. If this second current makes the needle turn through an equal angle, its strength must be equal to the



FIG. 353. — D'Arsonval Galvanometer, a Movable Coil Instrument.



FIG. 354. — Lecture Table Galvanometer.

different known strengths, the galvanoscope becomes a *galvanometer*; the current detector is turned into a current measurer.

By fixing under the needle a scale divided into amperes instead of degrees of arc, the galvanometer becomes an *ampere-meter* or *ammeter*.

573. Types of Galvanometers. — We have thus far considered galvanometers in which the coil is fixed and the magnet movable (Figs. 351 and 352). But instruments are also made with the magnets fixed and the coil movable (Figs. 353 and 354), and for

most purposes the latter are superior to the former. The sensibility, *i.e.* the ability to measure feeble currents, of a galvanometer is increased by having a large number of turns in the coils, and by making the moving parts as light and frictionless as possible.

EXERCISES

1. What is the difference between an electroscope and a galvanoscope?
2. A compass is held over a wire through which a current is flowing from north to south. In what direction will the N-pole of the needle turn?
3. The S-pole of a compass is deflected toward the west when it is held underneath a current-carrying wire. In which direction is the current passing?
4. If a flat coil of wire lying upon a table is traversed by a current in the same direction as the hands of a watch move, what is the direction of the lines of force at the center of the coil?

HEATING EFFECTS

574. Heat produced by Electric Current.—When a current of electricity is sent through a conductor, heat is evolved all the time that the current passes; electrical energy is being transformed into heat energy. *With the same steady current the amount of heat evolved in the unit of time is found (1) to vary with the nature of the conductor; (2) to be directly proportional to its length; (3) to be inversely proportional to its cross-sectional area. And with the same conductor, the heat evolved in the time-unit is found to be directly proportional to the square of the current strength.*

575. Resistance.—By *resistance* is meant the opposition offered by a conductor to the passage of an electric current through it. *The amount of current which at a given E.M.F. can pass through a conductor is dependent upon its nature, and varies inversely as its length and directly as its size.*

The properties (nature and dimensions) of a conductor that determine its resistance are the same as those that determine the amount of heat evolved during the passage of the current. *Resistance (r) varies with the nature of the conductor (c), is directly proportional to its length (l), and is inversely proportional to its cross-sectional area (s):*

$$r = c \frac{l}{s}. \quad (1)$$

576. Specific Resistance or Resistivity.—The quantity c occurring in Equation (1) is a constant for any one substance under similar conditions. By making a substance of unit length and unit cross-sectional area, the resistance becomes equal to this constant, and in that case its numerical value is known as the *specific resistance* or *resistivity* of the particular substance. Silver is the best conductor of all known substances, and if its specific resistance be set equal to 100, the resistivities of a number of other substances will be represented by the numbers in the table below.

RESISTIVITIES OF SOME SUBSTANCES RELATIVE TO THAT OF SILVER

Silver	100	Soft iron	740
Copper	113	Nickel	785
Gold	125	Platinum	900
Aluminum	200	German silver	2040
Zinc	373	Mercury	6250

577. Resistance and Temperature. — The property of resistance, like most other properties of a substance, varies with a change of temperature. The resistance of metallic conductors is found to increase with a rise of temperature, while that of nonmetallic conductors, such as carbon, and of liquid conductors, such as solutions of acids, bases, and salts, is found to decrease with a rise in the temperature.

578. Unit of Resistance, the Ohm. — As resistance is a property of a conductor, to establish a unit of resistance of a certain substance of specified dimensions and under fixed

conditions must be chosen. The practical unit is called the *ohm*, and is defined to be *the resistance of a column of mercury at 0° C. with a length of 106.3 cm., and a cross-sectional area of one square millimeter.* The mass of such a column is 14.4521 g.

579. Resistance and Potential. — When water flows from a higher level to a lower, its pressure continually decreases all along the conducting pipes, and the difference of water levels, which determines the pressure at any point, becomes smaller and smaller (Fig. 355). Analogously, when an

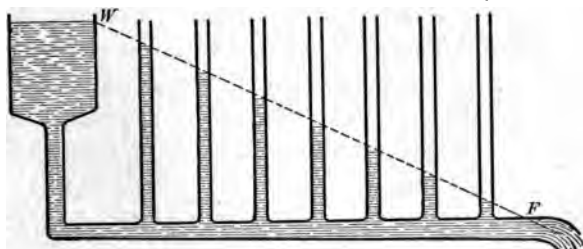


FIG. 355. — Water Analogy of Fall of Potential.

electric current is flowing from a higher potential to a lower, the potential becomes smaller and smaller. The resistance determines the amount of drop of potential. The P. D. between any two points in an electric circuit is directly proportional to the resistance between those points. Thus, if a circuit contains two conductors joined end to end in such a way that the same current traverses both, and if one of the conductors has twice the resistance of the other, the drop of potential will be twice as great between the ends of the larger resistance as between the ends of the smaller resistance. The total fall of potential is equal to the sum of the individual falls of potential.

580. Resistance and Current Strength. — The strength or intensity of a current is the amount of whatever is flowing (fluid or electricity) that passes the cross section of the conductor during the unit of time. While the speed may vary at different parts of the conductor, the amount of flow will remain constant. The water of a swiftly flowing brook, for instance, discharging into a large lake will create no perceptible flow in the lake as a whole. Yet there must be a movement, a drift, of the water of the lake, for if the lake's level remain the same, as much water must flow out as flows in. The amount of electricity flowing through one part of a circuit is the same as that through every other part, provided that all parts are joined *in series*, i.e. one after the other. The greater the resistance opposing the flow, the less is the intensity of the current. And the greater the difference of level or potential causing the flow, the greater is the current strength.

581. Ohm's Law. — Ohm, in 1827, was the first to discern clearly the relation between current strength, potential difference (equal numerically to electromotive force), and resistance; hence the enunciation of this relation is known as Ohm's law: *The strength of a current (C) is directly proportional to the E. M. F. (E) and inversely proportional to the resistance (R):*

$$C = \frac{E}{R}. \quad (2)$$

This law is of universal application in electricity. If the strength of current and the resistance of a circuit is known, the E. M. F. can be determined, for, clearing of fractions and transposing, we find that

$$E = CR. \quad (3)$$

the P. D. or E. M. F. and the current strength are known, the resistance can be determined, for

$$R = \frac{E}{C}. \quad (4)$$

If the E. M. F. is measured in volts, the current strength in amperes, and the resistance in ohms,

$$\text{Amperes} = \frac{\text{Volts}}{\text{Ohms}}; \quad (5)$$

$$\text{Volts} = \text{Amperes} \times \text{Ohms}; \quad (6)$$

$$\text{Ohms} = \frac{\text{Volts}}{\text{Amperes}}. \quad (7)$$

582. Measure of Electrical Energy.—When a stream of water falls through a vertical distance of 550 feet at the rate of one pound per second, one horse power is developed. Analogously, when one ampere of current flows from any point in a conductor to another where the difference of potential is one volt, one watt of power is developed. Amounts of electrical energy then can be measured in watts:

$$\text{Volts} \times \text{Amperes} = \text{Watts}. \quad (8)$$

$$\text{Volts} = \frac{\text{Watts}}{\text{Amperes}}. \quad (9)$$

$$\text{Amperes} = \frac{\text{Watts}}{\text{Volts}}. \quad (10)$$

583. Electrical and Heat Energy.—When the electrical energy in a certain part of a circuit is transformed into heat, the amount of heat energy developed must equal the amount of electrical energy expended. Now one calorie is equivalent to 4.19 joules (§ 316); or, conversely, one

joule is equivalent to .24 cal. But a watt is a joule per second. Hence

$$\text{Volts} \times \text{Amperes} = 4.19 \text{ cal. per second}; \quad (11)$$

or,

$$\text{Calories per second} = .24 \text{ Volts} \times \text{Amperes}. \quad (12)$$

The total number of calories H developed in t seconds will therefore be

$$H = \text{Volts} \times \text{Amperes} \times t \times .24. \quad (13)$$

But, by Ohm's law, volts = amperes $\times$ ohms, so that by substitution [Equation (6)]

$$H = \overline{\text{Amperes}}^2 \times \text{Ohms} \times t \times .24; \quad (14)$$

or, using C for amperes and r for ohms,

$$H = C^2 r t \times .24. \quad (15)$$

This equation shows that *the heat generated in a conductor is proportional to the time, to the resistance, and to the square of the current strength.* The relation is known as Joule's law.

584. Production of Current by Heating. — We have learned that electrical energy may be transformed into heat energy. But can heat

energy be transformed into electrical energy? This question is answered by the following experiment. Twist pieces of copper wire round the ends of an iron wire (Fig. 356), and connect the other ends of the copper wires to the terminals of a galvanometer. Heat the left-hand junction of the copper and iron wires with a Bunsen

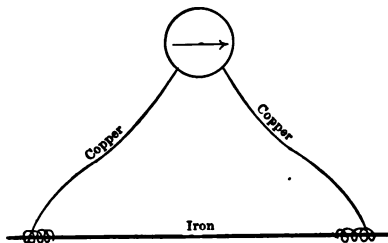


FIG. 356. — Thermo-electric Couple.

flame; the pointer of the galvanometer is deflected. Then heat the right-hand junction; the pointer moves in the opposite direction.

The conditions for such *thermo-electric currents* are a circuit consisting of two conductors joined together at two places, and the maintenance of different temperatures at these junctions. The direction of the current depends upon what metals are used. Even when two points of a conductor of the same metal are kept at the different temperatures, an E. M. F. (called the Thomson E. M. F.) is set up, which causes a feeble current to circulate in the conductor.

EXERCISES

1. If both the length and the diameter of a wire be doubled, what will be the effect on its resistance? $R \propto l$ $R \propto \frac{1}{d^2}$ $R \propto \frac{l}{d^2}$
2. A wire tapers down to half its largest diameter. When a current is sent through it, how do (1) the strengths, (2) the temperatures in the largest and smallest portions compare?
3. Calculate the resistance of a cylindrical column of mercury 50 cm. long and 1 cm. in diameter.
4. Conductance (c) is the reciprocal of resistance. * State Ohm's law using conductance instead of resistance.
5. What E. M. F. can maintain a current of 2 amperes through a resistance of 4 ohms?
6. If the resistance of the insulation between a trolley wire and the ground is 600,000 ohms, what current will leak through to the ground if the wire has a voltage of 600 volts?
7. The resistance of the human body from the hand to the foot or from one hand to the other is about 5000 ohms, and a current of 1 ampere passing through the body may prove fatal. Is the 110-volt lighting circuit dangerous to life? A 500-volt trolley circuit?
8. A current of 2 amperes is passed through a coil of wire having a resistance of 7 ohms, and immersed in 500 g. of water at 15°. What will be the temperature of the water in 15 min., if no heat is lost by radiation?
9. A 4-ampere current is passed for one minute through a 4-ohm coil placed in 200 g. of water. What will be the rise in the temperature of the water?

CHEMICAL EFFECTS

585. Electrolysis.—If carbon or platinum plates are made the terminals of a source of current of two or three volts, and are immersed in a solution of copper sulphate

copper will be deposited upon the plate at which the current leaves the solution, while at the other plate bubbles of oxygen will appear. In time, all of the copper will be removed from the solution, sulphuric acid taking its place. If the direction of the current be reversed, the copper will be deposited upon the other plate, which is then where the current passes from the solution. The name of *electrolysis* was given by Faraday to decompositions of conductors of the second class by means of electric currents. The solution he called the *electrolyte*, and the plates the *electrodes*. The plate that leads the current to the solution is the *anode*, and the plate by which the current leaves the solution, the *cathode*.* Every soluble acid, salt, or base can be electrolyzed, the products, however, varying according to the reactions that take place between the plates, the water, and the ions.

586. The Dissociation Theory applied to Electrolysis.—According to the dissociation theory, copper sulphate when dissolved dissociates into copper ions, charged positively, and sulphions, charged negatively. The copper ions, being attracted by the negative plate, move toward it, and are hence called cations (“down-goers”). They there lose their charges and become metallic copper. The negative sulphions are attracted by the positive plate, and hence move toward it; they are the anions (“up-goers”). The sulphions after giving up their charges act upon the water to form sulphuric acid. Under the influence of an E. M. F., there are two processions, moving in opposite directions, of ions carrying the electricity through the electrolyte—a procession of cations carrying positive electricity to the cathode, and a procession of anions carrying negative electricity to the anode.

587. Current Strength measured by Electrolysis.—Faraday, in 1834, demonstrated that, whatever the nature of the solution containing a certain element as cation or

* These words are derived from the Greek, *ode* = *path* or *way*, *an* = *up*, *cath* = *down*, *ion* = *goer*, and *lusis* = *loosening*.

anion, the mass of the element deposited is always the same for a steady current of given strength flowing for a given time. Thus a current of one ampere flowing for one hour through a solution of a silver salt, always causes a deposit of 4.025 g. of silver. Under the same conditions of current strength and of time 2.444 g. of gold, 1.181 g. of copper, and 1.203 g. of zinc are deposited. These masses are called the *electrochemical equivalents* of the respective metals. When once the electrochemical equivalent of an element has been found, the strength of a current may be measured by passing it through a solution of the element for a definite time and finding the mass of the deposit. As time and mass can be measured with extreme accuracy, this method of measuring current strength is employed in the most refined work.

588. Applications of Electrolysis. — Chemical decompositions brought about by electric currents are in constant use in certain important arts, some of which are the following:

I. *Refining of Metals.* Copper as it is obtained from the treatment of most of its ores is not pure. As its impurities impair some of its most valuable properties, notably its conductivity, it must be refined, and the electrolytic method is the one most used. A slab of the impure metal is used as the anode, and a thin sheet of pure copper as the cathode, the electrolyte being a solution of copper sulphate. When a current from a dynamo is sent through the solution, the copper in the anode dissolves as fast as it is deposited on the cathode, the solution carrying the copper in the ionic state from the one plate to the other without its strength being changed.

II. *Electrotyping.* A mold of a page of type is made of wax. As wax is a nonconductor, this mold is coated with a very thin layer of finely powdered graphite, which is a conductor. It is then made the cathode of a "bath" consisting of a solution of copper sulphate, the anode being a thick plate of pure copper. When the current has car

a deposit of a layer of the metal upon the mold about twice as thick as a sheet of this paper, the wax is removed, and a backing of type metal is cast into the copper film to make it stiff and rigid.

III. *Electroplating.* The silver-plating process is typical, gold, nickel, brass, and copper plating differing from it only in certain details. The articles are thoroughly cleaned and are then suspended from a rod connected with the negative terminal of a source of current, thus forming the cathode. The anode is a plate of silver and the electrolyte, a mixture of the solutions of silver and potassium cyanides. The current must be passed slowly, or the plating will not adhere well. After the articles are removed from the bath, they are rubbed with a smooth iron (burnisher) to give them polish and luster.

EXERCISES

1. How do you account for the fact that during the electrolysis of copper sulphate in solution, the blue color of the solution at the anode soon fades out?

2. How can galvanometers be made into ammeters by means of the chemical method of measuring currents?

3. What classes of chemical compounds are always electrolytes when dissolved in water?

4. What economy is there in electrotyping books? Why are daily newspapers seldom, if ever, electrotyped?

5. Two zinc plates are connected with a source of current and then placed in a zinc sulphate solution. How can the direction of the current be ascertained by the changes in weight that the two plates undergo?

6. A current when passed through a solution of silver nitrate for 20 minutes caused a deposition of .447 g. of silver. What was the strength of the current? How much copper would such a current deposit?

PRIMARY CELLS

589. Primary Cells or Batteries. — Any two conductors, one of which is acted upon by a liquid more than the other, may form a cell, but of the hundreds of such combinations that have been devised, only a half dozen or so have proved of commercial value. Zinc is universally

chosen for the positive plate, and is usually amalgamated to lessen the local action. The negative plate is commonly copper; but, in case that metal is acted upon by the exciting liquid, carbon is used. To distinguish the modifications of the simple voltaic cell from another type of cells, the former are called *primary*, and the latter *secondary* cells.

590. Local Action. — Pure zinc is insoluble in most exciting liquids, but the common commercial grade used in cells dissolves readily. The reason for this difference in behavior is to be found in the fact that the cheap grade contains impurities, especially iron and carbon. When such zinc is put in an acid, a multitude of tiny voltaic cells is formed, the negative plates of which are the particles of iron and carbon present in the zinc. The currents from these local cells have to complete their short circuits within the zinc plate and never get to the external circuit. Their energy is all transformed into heat.

591. Remedy for Local Action. — Iron as well as carbon is insoluble in mercury. When zinc (which is soluble in mercury) is amalgamated, the coating of the amalgam covers the iron and carbon, and protects them from the acid. As fast as the zinc on the outside of the amalgam ionizes and passes into solution, fresh quantities of zinc dissolve in the mercury, diffuse through the amalgam, and pass as ions into the solution.

592. Polarization. — Very soon after the circuit of a simple voltaic cell is closed, its E. M. F. falls off and the current is much reduced in strength; the cell is said to become *polarized*. The reason for this *polarization* is that hydrogen bubbles stick to the copper plate. Gaseous

hydrogen being a nonconductor, its presence on the negative plate acts like so much varnish or other insulating material. It diminishes the area of the plate in actual contact with the acid, thereby increasing the resistance. But the hydrogen does even more than this. It has a solution tension like that of zinc and tends to create an E. M. F. that opposes the E. M. F. of the simple cell.

593. Remedies for Polarization. — To remedy this defect due to polarization, some means must be devised to get rid of the hydrogen bubbles. Although they may be brushed or blown off, present-day commercial cells are provided with some substance that can combine chemically with the hydrogen to produce water or some other compound, the presence of which does not interfere with the working of the cell.

594. Depolarizers. — A chemical depolarizer may consist of a substance containing oxygen that is readily given up to the hydrogen to form water, or of a solution of a salt from which the metal can be displaced by hydrogen. The first depolarizers may be solutions of such substances as chromic acid or certain of its compounds; or they may be such solids as insoluble manganese dioxide and copper oxide. The second kind of depolarizer is usually a solution of copper sulphate.

595. E. M. F. and Size of Plates. — As difference of potential depends solely upon the nature of the substances brought in contact, the E. M. F. can be dependent only upon the nature of the plates and of the exciting liquid. Increasing the size of the plates affects the E. M. F. no more than increasing the size of a kettle affects the boiling point of the liquid within it. But just as a larger kettle can hold a larger amount of a boiling liquid, so can a larger cell furnish a larger quantity of electricity but still with the same E. M. F.

596. Internal Resistance. — The resistance offered by the electrolyte to the passage of a current of electricity is influenced by the following conditions.

(1) *The nature of the electrolyte.* The conductivity of different electrolytes varies greatly. Pure water has an enormous resistance, but the addition of acids, bases, and salts decreases it very much.

(2) *Chemical changes during action.* The exciting fluid acts upon the depolarizer in some instances, as well as upon the zinc, giving rise to products that may increase or decrease the resistance.

(3) *Area of the plates.* The larger the plates, the greater is the cross section of the column of liquid between them, and consequently the less is the resistance.

(4) *Distance between the plates.* The closer the plates are together, the shorter is the column of liquid between them, and hence the less the resistance.

SOME IMPORTANT CELLS

597. The Bichromate or Chromic Acid Cell is so called because its depolarizer is the bichromate of sodium or potassium, or chromic acid, dissolved in dilute sulphuric acid. The usual arrangement is to have two carbon plates one on each side of the zinc plate, as thereby the area of the electrolytic conductor is doubled and the internal resistance halved. This cell shows an E. M. F. of 2.2 volts, and its internal resistance ranges from .2 to .5 ohm. Since the depolarizer acts upon the zinc even when the circuit is open, it is necessary to remove the zinc from the liquid when the current is not being used. This is conveniently done by attaching the zinc plate to a sliding rod provided with a clamp by which it can be raised or lowered (Fig. 357). A special form of battery, called a *plunge battery*, is made, in which all the zinc plates are attached to a frame that can be lifted up or down by means of a windlass. While bichromate cells furnish a strong current for a short time, the materials are used up rapidly, and the E. M. F. fluctuates considerably.

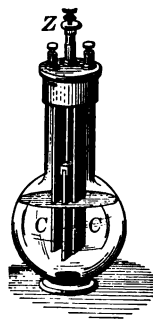


FIG. 357. — Bichromate Cell.

598. The Daniell Cell.—In this cell the zinc plate dips into a solution of the salt named zinc sulphate, and the copper plate into a solution of the salt called copper sulphate, the two solutions being prevented from undue mixing by keeping the first in a porous cup (Fig. 358). Polarization does not take place because the hydrogen

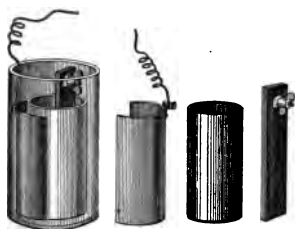


FIG. 358. — Daniell Cell.

ions that appear at the copper plate give up their charges to the copper ions (due to the presence of the copper sulphate, which dissociates into copper ions and sulphions) and combine with the sulphions to form sulphuric acid. As soon as the copper ions give up their charges to the copper plate they are changed into metallic copper which is deposited on the plate. A Daniell cell maintains a constant E. M. F. of 1.08 volts, and its internal resistance varies from 1 to 5 ohms. When the cell is left on open circuit, the copper sulphate gradually diffuses through the porous cup, reacts with the zinc, and covers it with a coating of copper and of copper oxide, that impairs the action of the cell. To prevent this, the circuit should always be kept closed, with enough resistance inserted in the external circuit to keep a current of small intensity always flowing. As the copper ions move away from the porous cup, there is then no danger of their diffusing through it.

599. The Gravity Cell.—As porous cups crumble and fall apart after being in use for some time, it is an advantage to employ some other means of keeping the solutions separated. Now a saturated solution of copper sulphate is so much heavier than a dilute solution of zinc sulphate that the latter will float upon the former. The construction of the gravity cell is based upon this fact (Fig. 359). The copper plate is placed in the bottom of the jar and is provided with an insulated wire extending to the top. Copper sulphate crystals are packed around the plate, and the jar is then about half filled



FIG. 359. — Gravity Cell.

with a saturated solution of copper sulphate. The zinc plate is shaped like a bird's foot (whence the popular name "crow-foot battery") and is hung from the edge of the jar. A weak solution of zinc sulphate or of sulphuric acid is carefully floated upon the copper sulphate solution. As long as the cell is kept on closed circuit, the two solutions mix but slightly, a sharply defined plane separating them.

600. The Sal-ammoniac or Leclanché Cell (Fig. 360).—Zinc dissolves in a solution of the salt, ammonium chloride (sal ammoniac), when in circuit with a plate of carbon, hydrogen being set free. If manganese dioxide is placed in a porous cup with the carbon, or better, if a mixture of powdered carbon and manganese dioxide is molded into an approved shape and then baked hard, the manganese dioxide combines with the hydrogen, thus depolarizing the cell. But the dioxide, being in the solid state, reacts but slowly with the hydrogen, so that if too large a current is taken from the cell, polarization



FIG. 360.—Leclanché Cell.

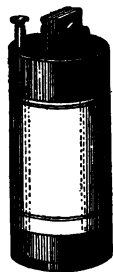


FIG. 361.—
Dry Cell.

will nevertheless occur; the depolarizing action cannot keep pace with the action furnishing the current. But if the cell be allowed to stand on an open circuit for some time, it will recover the E. M. F. it had at first. Such a cell has an E. M. F. of about 1.4 volts and an internal resistance of from .5 to 2 ohms. The advantage of the sal-ammoniac cell lies in its freedom from local action, so that it can be left on open circuit for a long time with but slight deterioration. On this account it is adapted to intermittent service, such as that of ringing door bells.

601. The Dry Cell (Fig. 361).—The dry cell consists of a zinc can in which is a carbon plate embedded in a mixture of sal ammoniac, zinc chloride, manganese dioxide, plaster of Paris, flour, and water. The reactions taking place in it are the same as those that actuate a Leclanché cell, of which it is a modification. As the jar is sealed, the cell can be used in any position, there being no danger of its pasty constituents

running out. Because of the convenience of dry cells they are extensively used for intermittent circuits. The E. M. F. is about 1.4 volts and the internal resistance in the neighborhood of .1 ohm.



FIG. 362. — Edison-Lalande Cell.

602. The Edison-Lalande (also called **Lalande-Chaperon**, and **Gladstone-Lalande**) Cell. — The exciting liquid is a solution of caustic soda in which zinc is soluble under the action of a current, and the depolarizer is copper oxide compressed into a cake, which also serves as the negative plate (Fig. 362). As a strong base such as caustic soda attacks glass somewhat, porcelain or iron jars must be used. The E. M. F. is .8 volt, and as the plates can be made large and set close together, the internal resistance may be made very small. Since the depolarizer acts quickly, very large and constant currents are furnished by these cells.

EXERCISES

1. How will the speed of solution of zinc in a Daniell cell be affected by short-circuiting (*i.e.* making the external resistance very small) the cell?
2. A certain dry cell when short-circuited gives a current of 20 amperes under a pressure of 1.36 volts. What is its internal resistance?
3. If a cell has an E. M. F. of 2 volts and an internal resistance of .4 ohm, what current can it furnish when the external resistance is negligibly small? What is its rate of expenditure of energy in watts?
4. If a Daniell cell has an E. M. F. of 1.08 volts and sends a current of .27 ampere through an external resistance of 2 ohms, what is the internal resistance of the cell?

603. Reversible Cells. — Specially treated platinum wires are sealed into the closed ends of two graduated tubes in such a way that they extend nearly to their open ends (Fig. 363). These tubes are filled with very weak sulphuric acid, and, after closing their mouths with the thumb, they are inverted in a vessel also containing acidulated water. When the external ends of the wire are

connected with a source of steady current of about three volts, gases appear upon the electrodes, which rise and displace the liquid. The *electrolysis of water* is effected, the liquid being decomposed into two volumes of hydrogen and one of oxygen.

According to the dissociation theory, the action that takes place is as follows. The positive hydrogen ions move to the negative plate, there lose their charges, and appear as hydrogen gas; the negative sulphions move to the positive plate, lose their charges, and then by reacting with the water present, form more sulphuric acid, at the same time setting oxygen gas free. After the evolution of these gases has continued for some time, let a very delicate galvanometer be substituted for the source of current. The galvanometer needle is deflected in such a way as to show that a current is traversing the circuit in a direction opposite to that originally sent through the electrolyte.

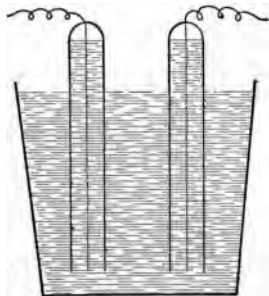


FIG. 363.—A Gas Battery.

As the current passes, the volumes of the two gases diminish, and after the gases are used up, the current stops. Such an arrangement constitutes a *gas battery*, in which the positive plate (platinum wire surrounded by hydrogen) is at a higher potential than the negative plate (platinum wire surrounded by oxygen). Its action is in nowise different, if instead of hydrogen and oxygen prepared by the electrolysis of water, these gases are prepared for use in the tubes by any other method.

The gas battery is an example of a *reversible cell*. The electrical energy sent through the acidulated water changes into chemical energy, the two gases, hydrogen and oxygen, possessing more energy when separated than when combined to form water. But when the gases reunite, their chemical energy is transformed back into electrical energy. The gases may be stored indefinitely above the liquid without any action taking place, but as soon as the circuit is closed, a current is generated.

Another example of a reversible cell is furnished by the Daniell cell. When this cell acts as an electrical generator, its current passes within

the liquids from zinc to copper, zinc being dissolved and copper deposited. But when a current with an E.M.F. greater than that of the Daniell cell is sent into the solution from the copper to the zinc, copper is dissolved and zinc deposited.

604. Secondary Cells.— Many combinations of two conductors and an electrolyte form reversible cells. But the

chief combination that has proved of commercial value consists of dilute sulphuric acid in which are immersed two lead plates, one plain and the other covered with lead peroxide. This cell is variously termed a *secondary cell*, a *storage cell*, or an *accumulator* (Fig. 364).

When two lead plates are made the terminals of a steady current of more than 2.5 volts and are immersed in dilute sulphuric acid, bubbles of hydrogen appear at the negative

plate (cathode), while the positive plate (anode) becomes covered with a dark brown deposit of lead peroxide, this compound resulting from the union of the oxygen with the lead. If, after this action has continued for some time, a galvanometer is substituted for the charging source of current, a current in the opposite direction traverses the cell.

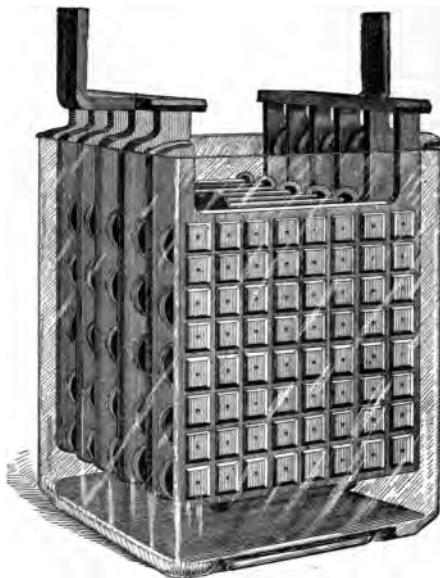


FIG. 364. — One Form of Storage Cell.

the peroxide gradually disappears, the dark brown color of its plate changing to the grayish color of lead.

605. Commercial Forms. — By repeated chargings and dischargings, the cathode becomes spongy in texture and the anode acquires a thick, porous crust of the peroxide. The action of the cell is then better than it was at first. To obviate this costly and time-robbing process of "forming" the plates, they are perforated with holes or cast in special forms (grids). The holes or interstices are packed with other oxides of lead (litharge or red lead), and a charging current is passed after they are placed in sulphuric acid. The hydrogen at the cathode combines with the oxygen in the oxides, leaving the lead in a spongy condition, while the oxygen at the anode changes the oxide into the peroxide. This charged cell can then be used to send a current through an external circuit, the direction of which in the electrolyte is from the lead grid to the peroxide grid.

606. Uses of Secondary Batteries. — Secondary cells were at first called storage cells or accumulators, because it seemed as if the electricity were stored up or accumulated in them. We now know that what they do is to transform electrical energy into chemical energy during the charging process, and chemical energy into electrical energy during the discharging process. At many large lighting plants part of the current from the dynamos is run into storage batteries during the time of day when there are comparatively few lights being used. Should an accident happen to the dynamos, the storage battery is switched on to the circuit, while the dynamos are being put into running condition again. Storage batteries are also used for running launches and carriages, being charged at fixed stations and discharged into the motors when the boat or vehicle is running.

EXERCISES

1. If the terminals of a battery are dipped into acidulated water, how can the direction of the current be ascertained from the relative abundance of bubbles of gas which arise from the respective terminals?
2. In what respect is the name *storage battery* often appropriately applied to secondary batteries? What is it that is really stored?
3. Which can be made to give the larger current and why — a Daniell cell or a storage cell of the same size?

4. The best forms of storage batteries furnish a current the total energy of which is about three fourths as great as that of the charging current. What are their efficiencies?

CONNECTIONS OF CELLS AND CIRCUITS

607. Combinations of Cells.—While a single cell can furnish only a limited amount of current of a certain E. M. F., combinations of cells can be devised by which larger E. M. F.'s and larger currents can be obtained. Three arrangements are practicable, each of which has its particular advantages.

FIG. 365.—Cells Connected in Parallel.

When all the positive poles are connected and likewise all the negative poles, the cells are said to be joined *in parallel* (or *in multiple*) (Fig. 365). When the positive pole of one cell is joined to the negative pole of the second, the positive pole of the second to the negative pole of the third, and so on, the cells are connected *in series* (or *in tandem*) (Fig. 366). When some of the cells are in series and some in parallel, they are said to be in *parallel-series* or *multiple-series* connection (Fig. 367).



FIG. 367.—Cells Connected in Parallel-Series.

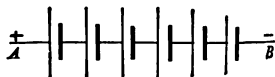


FIG. 366.—Cells Connected in Series.

608. Series Connection.—Suppose that we have a number of pumps each of which can raise water only 20 feet. How can we fill a tank 60 feet from the ground? Evidently the only arrangement would be to employ two intermediary tanks, one 20 and the other 40 feet from the source. One pump could fill the lowest tank, a second could raise the water from the lowest to the middle tank, and a third could raise the water from the middle to the highest tank Fig. 368).

Each pump increases then the hydrostatic pressure by 20 feet of water.

In analogous fashion, each cell connected in series raises the electrical pressure by an amount equal to its E. M. F. The E. M. F. furnished by a battery of cells in series is equal to the sum of the E. M. F.'s of each cell. Just as all the water lifted to the highest tank has to pass through each pump, so the whole current furnished by cells connected in series has to pass through each cell. The internal resistance of the battery is therefore equal to the resistance of a single cell multiplied by the number of cells. As the E. M. F. and the internal resistance are increased in like fashion, their ratio remains the same, provided the external resistance can be neglected. The current strength of a short-circuited battery of any number of cells in series is consequently not larger than the current strength of a single cell. But if the external resistance is increased more and more, the current furnished by the battery is increased more and more above that which a single cell can furnish in circuit with the same external resistance. Applying Ohm's law and denoting the number of cells by n , the external resistance by R_e , and the internal resistance by R_i , we have for cells in series

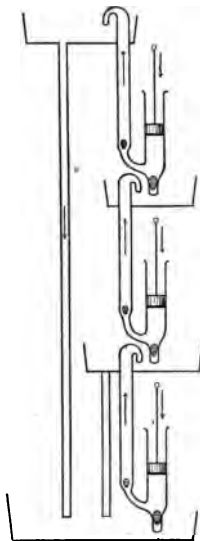


FIG. 368.—Water Analogy of Cells Connected in Series.

$$C = \frac{nE}{R_e + nR_i}. \quad (16)$$

When R_e is negligibly small in comparison with R_i ,

$$C = \frac{nE}{nR_i} = \frac{E}{R_i}; \quad (17)$$

but when R_i becomes negligibly small in comparison with R_e ,

$$C = \frac{nE}{R_e}. \quad (18)$$

In the latter case, the current furnished by n cells is practically n times that furnished by a single cell. *Series connections are therefore to be used when the external resistance is relatively large.*

609. Parallel Connection. — Suppose now that the pumps are to be used to supply a horizontal water main 20 feet high. If the main is shut off as soon as it is filled with water, the result will be a back pressure that will stop all the pumps (Fig. 369). If the main is wide open, all the pumps may be needed to keep it filled with water, each pump, however, having to raise water only 20 feet.

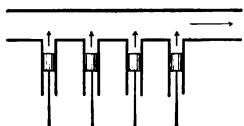


FIG. 369. — Water Analogy of Cells Connected in Parallel.

✓ A battery of cells in parallel has the same E. M. F. as a single cell. Just as each pump furnishes its quota of water to the main, so does each cell deliver its current to the external circuit. The whole of the electrical current does not pass through each of the cells any more than all the water in the main passes through each of the pumps. The battery acts like a single cell having plates of an area equal to the sum of the areas of all the plates of the cells composing the battery. Hence, if there are n cells, the internal resistance of the battery will be $\frac{1}{n}$ that of a single cell. Applying Ohm's law, we have

$$C = \frac{E}{R_e + \frac{R_i}{n}} \quad (19)$$

If on short circuit (R_e is then negligibly small), the current will be n times as great as that of a short-circuited single cell. If R_e is very large, the current will be but slightly greater than that furnished by a single cell. *Parallel connections are therefore to be used when the external resistance is relatively small.*

610. Parallel-series Connection. — By connecting cells, part in series and part in parallel, batteries can be made to meet the conditions imposed by any applications of current. Thus, if n cells be arranged in series, and p sets of the series-connected cells be connected in parallel, the E. M. F. of the battery will be nE , and the internal resistance will be $\frac{n}{p}R_i$. Hence

$$C = \frac{nE}{R + \frac{n}{p}R_i} \quad (20)$$

The general rule to be followed in connecting cells is: *Make the internal resistance as nearly equal as possible to the external resistance. The current strength will then be at its maximum with a given total number of cells.*

611. Series Circuits. — When all of a current flows consecutively through all the conductors composing its circuit, such as the wires connecting the various instruments, their coils of wire, and the electrolytes, the parts of the circuit are said to be *connected in series*. The total resistance of a series circuit is equal to the sum of all the resistances, whatever their nature, included in it.

612. Divided Circuits or Shunts. — When more than one path is offered a current of water, it will divide and a portion will pass through each of the paths, or *shunts*, as they are called. The drop of pressure between *A* and *B* (Fig. 370) is the same through all the shunts, and the amount of current that passes from *A* to *B* is equal to the sum of the amounts that pass through the shunts. Analogous statements are true of an electric current. The fall of potential in all the conductors *joined in parallel* is the same, and the total current is equal to the sum of all the partial currents (Fig. 371).

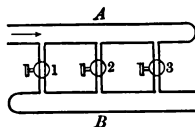


FIG. 370. — Water Analogy of Shunted Circuits.

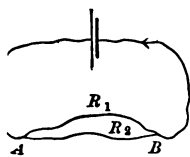


FIG. 371. — Shunted Circuits.

If the cock of shunt 1 (Fig. 370) is partially closed, less water will pass through it, and more current through the other shunts. The relative strengths of the shunted currents can be changed by turning the cocks more or less. If the same pressure is maintained between *A* and *B*, the amount of current through each

branch is inversely proportional to the amount of opposition to flow exerted by a cock.

Analogously, the amount of electricity flowing through a shunted circuit is inversely proportional to the resistance of the shunts; or, since resistance and conductance are reciprocals, it is directly proportional to the conductances of the shunts.

613. Ohm's Law applied to Shunts.—Let C denote the current strength in the undivided portion of a circuit, and C_1 and C_2 the current strengths in the two branches of it (Fig. 372), R , the total resistance between A and B , R_1 and R_2 , the resistances of the two branches, and D , the fall of potential between A and B . Then



FIG. 372.—Shunt Resistance.

$$C = C_1 + C_2, \quad (21)$$

and

$$C = \frac{D}{R}; \quad C_1 = \frac{D}{R_1}; \quad C_2 = \frac{D}{R_2}, \quad (22)$$

so that

$$D = CR = C_1 R_1 = C_2 R_2; \quad (23)$$

whence

$$\frac{C_1}{C_2} = \frac{R_2}{R_1}; \quad (24)$$

the strengths of the shunted currents vary inversely as the resistances of the shunts.

Furthermore, by substituting in Equation (21) the values given in Equation (22), and by cancelling the D 's,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}; \quad (25)$$

whence

$$R = \frac{R_1 R_2}{R_1 + R_2}; \quad (26)$$

the combined resistance of two shunts is obtained by dividing the product by the sum of their resistances.

EXERCISES

1. Under what conditions will a small cell give practically the same amount of current as a large cell of the same kind?

2. What kind of cells can be used to charge a storage battery made up of cells connected in parallel? How should they be connected?

3. Diagram an electric circuit including four cells, a galvanoscope, and a resistance box, marking the positive and negative poles of each cell, and arranging the cells to the best advantage, when (1) the external resistance is small, when (2) it is large.

4. Given six cells of 2 volts and 1.5 ohms each, should they be connected in series or parallel to give the maximum current?

5. Find the battery resistance of six cells connected (a) in parallel, (b) in series, the internal resistance of each cell being .2 ohm.

6. Calculate the E. M. F. of six cells connected (a) in series, (b) in parallel, the E. M. F. of a single cell being 1.4 volts.

7. What arrangement of 20 cells each having an E. M. F. of 1 volt and an internal resistance of 2 ohms will give the most current through an external resistance of 10 ohms?

8. By means of a diagram show how the 12 cells (2 volts each) of a storage battery can be connected so as to give 4 volts; 6 volts; 24 volts.

9. What is the joint resistance of two incandescent lamps in parallel, each lamp having a resistance of 250 ohms?

10. A current is made to pass through two wires in parallel, one having a resistance of 3 ohms, and the other a resistance of 4 ohms. What is the joint resistance of the two wires?

ELECTRICAL MEASUREMENTS

614. In the study of steady currents three things may be measured: the P. D. due to its E. M. F., both expressed in the same unit, volts; the strength, expressed in amperes; and the resistance of the circuit, expressed in ohms. By applying Ohm's law, the value of any one of these quantities may be computed from the values of the other two. When the number of amperes flowing between two points

of a conductor at a known difference of potential is ascertained, the amount of energy in watts is given by the product of the volts and amperes.

Many methods have been devised for measuring these electrical quantities, only a few of which can be described here.

615. Measurement of Current Strength. — The most direct and accurate method is to pass the current through an appropriate electrolyte for a definite time, and to find the mass or the volume of the products of the electrolysis, from which the strength in amperes can be readily calculated. A more convenient method is to use a galvanometer the readings of which are amperes or may be readily reduced to amperes.

Since the current strength is the same throughout the entire circuit, it is immaterial at what point the current measurer is inserted, provided that all the current to be measured passes through it. The resistance of the ammeter itself should be as low as possible so that its introduction into a circuit will not appreciably cut down the strength of the current.

616. Measurement of E. M. F. — The simplest method of measuring E. M. F. is by means of a voltmeter (Fig. 373),



FIG. 373.— Voltmeter.

— a galvanometer the coils of which consist of many turns of very fine wire so that its resistance is great. How such a galvanometer can be used to measure E. M. F. directly is readily explained by means of a water analogy. If a weight be hung from a cord wrapped around the axle of a water wheel fitting snugly in

the lower horizontal pipe (Fig. 374), the water will be forced from the left-hand tank into the right-hand tank when the wheel is turned by the descending weight. If the cock S in the upper pipe be closed, the wheel will stop turning as soon as the back pressure of the water due to the difference in levels attains a certain value. The state of affairs is then analogous to that of an open electric circuit where one terminal is kept at a higher P. D. than the other because of the action of a source of E. M. F., such as a voltaic cell (analogous to the water wheel). But when the cock is opened, water will flow through it, the difference in levels will decrease, and the water wheel will begin to turn again. But if the cock be opened

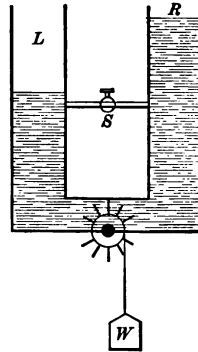


FIG. 374. — Water Analogy of Voltmeter.

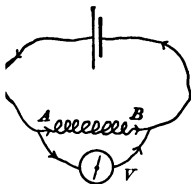


FIG. 375. — A voltmeter is always placed in a shunt.

such a little way that the wheel can pump water from L to R not faster than it can flow back, the difference of levels will not change. The greater the weight, the greater will be the difference of levels, and the faster the flow through the cock. Hence the amount of flow through the cock may be taken as a measure of the difference in the pressure that the pump can maintain. Analogously, if a shunt containing a galvanometer be connected at any two points of a circuit, the amount of current, as measured by the deflection of the galvanometer's needle, which traverses the shunt, may be taken as a measure of the P. D. between the two points, provided that not enough current is shunted to prevent the source of E. M. F. from

maintaining a constant current in the main part of the circuit (Fig. 375).

A voltmeter then is a sensitive galvanometer having a resistance so great that only a very small fraction of the main current can flow through it, and having a scale graduated to read in volts. To find the P.D. between two points of a circuit, the circuit is closed, and the two terminals of a voltmeter are connected in at these points. The reading of the voltmeter thus placed on a shunt gives the P.D. in volts.

617. Measurement of Resistance. — *Fall of Potential or Ammeter-voltmeter Method.* When the amperes and volts of a current are known, the ohms can be calculated by means of Ohm's law. An ammeter is inserted in series with the circuit, and the terminals of a voltmeter are joined on either side of the resistance to be measured, so as to be in a shunt. If the resistance of the voltmeter is at least several hundred times that of the resistance being measured, the reading of the voltmeter divided by that of the ammeter gives the resistance. If the resistance of the main circuit is large, the proportion of the current passing through the voltmeter becomes so great as to require correction; but, in that case, it is better to use another method.

618. Resistance Box. — The other methods of resistance measurement require the use of known resistances for purposes of comparison. These known standards of resistance consist usually of spools wound with wires of different lengths and sizes, inclosed in a box to protect them (Fig. 376). When such a



FIG. 376. — Resistance Box.

resistance box is placed in a circuit and all the plugs are firmly inserted in the blocks, the current passes from binding post to binding post through the large blocks and plugs, the resistance of which is so slight as to be negligible. By taking out plugs, the current is made to pass through the coils, the terminals of which are connected with the blocks on either side of the plugs (Fig. 377).

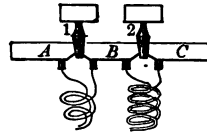


FIG. 377. — Construction of a Resistance Box.

619. Resistance by Substitution Method. — The resistance to be measured is connected in series with a galvanometer and a source of E. M. F., such as a voltaic cell or battery. The position of the pointer is then noted and a resistance box substituted for the unknown resistance. Plugs are taken out until the known resistance thereby introduced into the circuit regulates the amount of current so as to produce a deflection equal to that of the unknown resistance. The resistance to be measured is then equal to the known resistance inserted.

620. Resistance by the Wheatstone Bridge Method. — A water analogy helps to a clear understanding of this rather involved method. A water pipe supplied from a source S at a constant level divides into

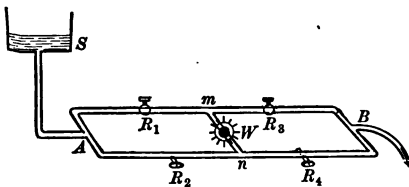


FIG. 378. — Water Analogy of Wheatstone Bridge.

two branches which later unite (Fig. 378). The two branches or shunts are provided with two valves or cocks each. Another pipe mn , in which is placed a frictionless water wheel W , forms a bridge between the two branches. Now, if the

cocks be so adjusted that the carrying capacity of the two branches is equal, no water will flow through the bridge, and the wheel will not turn. The drop in pressure between the points A and B

will then be the same in both branches, and there will be no difference in pressure at the points m and n . The quantity of water flowing from A to m will equal that flowing from m to B , as will also be the case in the quantities of water flowing from A to n and from n to B . Now let one of the cocks, say R_1 , be partly closed. The resistance to flow through this pipe will be thereby increased, so that the carrying capacity of pipe Am will be decreased. Hence the pressure at m will be less than at n , the consequence being that water will flow through the bridge from n to m and turn the wheel in a certain direction. In like fashion, if, after the cocks are again adjusted so as to allow no flow of water through the bridge, cock R_3 be partly closed, a current will flow from m to n and turn the wheel in the opposite direction. Like effects can be obtained by manipulating the other cocks. The condition for no flow in the bridging pipe is met when the ratio of the carrying capacity of Am to the carrying capacity of An is the same as the ratio of the carrying capacity of mB to the carrying capacity of nB .

Replace now the reservoir of water by a voltaic battery, the pipes by wires, the cocks by resistance boxes or their equivalents, and the

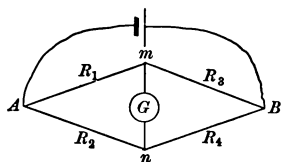


FIG. 379. — Diagram of Wheatstone Bridge.

wheel by a galvanometer; the apparatus for measuring resistance by the bridge method is then at hand. When the resistances are so adjusted that no current passes through the galvanometer, m and n (Fig. 379) are at the same potential. The P.D. between A and m then equals the P.D. between A and n . Hence, letting C_1 and C_3 and

R_1 and R_3 represent the current strengths and resistances in these respective branches, we have, by Ohm's law,

$$C_1 R_1 = C_3 R_3. \quad (27)$$

The P.D. between m and B equals the P.D. between n and B , so

$$\text{that} \quad C_2 R_2 = C_4 R_4. \quad (28)$$

By division,

$$\frac{C_1 R_1}{C_2 R_2} = \frac{C_3 R_3}{C_4 R_4}. \quad (29)$$

And since the current in Am must be the same as in mB , and the current in An the same as in nB ,

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}. \quad (30)$$

By means of this relationship, when any three of the resistances are known, the fourth may be found by calculation.

EXERCISES

1. Why is an ammeter always placed in series and a voltmeter in a shunt to a circuit?
2. Two cells in series give .2 ampere through an external resistance of 10 ohms and .3 ampere through an external resistance of 5 ohms. Find the E. M. F. and the internal resistance of each cell.
3. If one branch of a divided circuit has 10 times the resistance of the other, what is the ratio of the currents traversing them, and what fraction of the whole current is carried by the greater resistance?
4. How many storage cells must be used to keep a 50-volt lamp glowing? How must they be joined?
5. Given 12 cells each having an E. M. F. of 1.4 volts and an internal resistance of 1 ohm. What current passes when they are connected with 1.5 ohms external resistance (1) in series? (2) in parallel? (3) in three groups of four cells, the cells of each group being in series, and the groups in parallel?
6. What connection gives the strongest current from six cells of 2 volts and .2 ohm, with 1 ohm external resistance?
7. A current with a strength of 3 amperes passes through a coil of wire of 5 ohms' resistance immersed in 500 g. of water. How much will the temperature of the water change in 10 min.?
8. An incandescent lamp with a resistance of 250 ohms, through which a current of .45 ampere flows, is immersed for 10 min. in water. If the temperature of the water is thereby made to rise from 18° to 54.4° , find the mass of the water.

CHAPTER XXIV

INDUCED CURRENTS

ELECTROMAGNETIC INDUCTION

621. Currents produced by Magnets. — We have learned how to make magnets by means of electric currents, and the question arises: Can magnets produce electric currents? An affirmative answer was first given in 1831 by Faraday, who showed that, given a magnetic field and a conductor, a current is generated in the conductor whenever there is a change in the number of lines of force intercepted or "cut" by the conductor. Such currents are said to be due to *electromagnetic induction*, and are called *induced currents*. Let us pass in review some experiments designed to illustrate this method of generating electric currents.

622. Experiments in Induction. — I. Connect the terminals of a coil of insulated wire with those of a delicate galvanometer (Fig. 380).

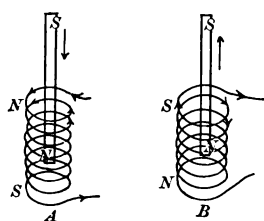


FIG. 380. — Induction of Currents.

Quickly thrust into the coil the *N*-pole of a bar magnet. The galvanometer needle will swing in a certain direction, thereby indicating that a momentary current has passed through the wire. After it swings back into its original position, jerk out the magnet. The pointer swings in the opposite direction; a brief current must therefore have moved through the coil in the opposite direction. If the magnet be rotated within the coil parallel to the

lines of force in such a way that the number of lines of force intercepted remains constant, no current will be obtained. These in-

duced currents last only while the magnet is moving, that is, while the number of lines of force passing through the coil is changing. When the number of intercepted lines is increasing, as when the magnet is entering the coil, the induced current flows in a direction opposite to its direction when the number of intercepted lines is decreasing. By moving the magnet to and fro in the coil across the lines of force, an *alternating current* is set up in the wire, *i.e.* the current flows now in one and now in the other direction.

II. Thrust the *S*-pole of the magnet into the coil. The induced current turns the needle in the opposite direction to that observed when the *N*-pole is inserted. Withdraw the magnet, and the galvanometer pointer swings in the same direction as when the *N*-pole is inserted.

III. Vary the conditions of the foregoing experiments by holding the magnet still and moving the coil over its poles. When the coil is put over the *N*-pole of the magnet, an induced current is generated in a direction opposite to that generated when it is taken off; and the currents induced by similar movements of the coil over the *S*-pole have directions opposite to those produced by the *N*-pole. Whether the coil or the magnet be moved, the results are the same.

IV. Another variation of these fundamental experiments is the substitution of electromagnets for the permanent magnets. The relative movements of the electromagnet and the coil connected with the galvanometer give results similar to those obtained with permanent magnets.

V. Finally, without any movements of the coil or the magnet, induced currents may be generated. Place an open-circuited electromagnet within a coil connected with a galvanometer. Then close the circuit, thereby generating a field of force within the coil. The motion of the pointer proves that an induced current is generated. Break the circuit of the electromagnet, thereby destroying the field, and the pointer turns in the opposite direction.

Further variations of such experiments, such as moving the coil (or magnet) nearer or farther from the magnet (or coil), making or breaking the circuit of the electromagnet placed by the side of the galvanometer coil, etc., all point to the same conclusion, namely: *Any change in the numb*

of lines of magnetic force passing through a closed circuit gives rise to an induced current.

623. Lenz's Law. — Let a voltaic cell be inserted in the galvanometer-and-coil circuit, and suppose the direction of the current to be such as to develop an *N*-pole at the upper end of the coil. Note which way the galvanometer pointer turns, remove the cell from the circuit, and bring the *N*-pole of a bar magnet near to the upper end of the coil (Fig. 381). The pointer will move in the same direction as when the cell was included in the circuit. Obviously, then, the induced current produces an *N*-pole at the upper end of

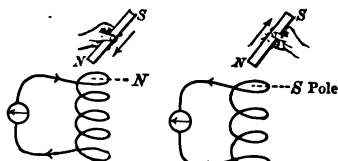


FIG. 381. — Lenz's Law.

the coil. The *N*-poles of a bar magnet and of a coil (solenoid) repel each other, and work must be done to overcome their mutual repulsion, which opposes their motion toward each other. When the mag-

net is moved away, the direction of the induced current is reversed, and an *S*-pole is developed at the coil's upper end. To overcome the attraction between these unlike poles, work has to be done. In general, then, energy must be expended to generate an induced current, and the direction of the induced current is such as to oppose the relative movements of coil and magnet. This was first recognized in 1834, by Lenz, who enunciated the law: *When a conductor or a magnet moves the one with respect to the other, an induced current is produced, the direction of which is such as to call forth a force that opposes the motion.*

624. Eddy Currents. — When a block of metal is moved in a magnetic field, induced currents are set up in it; and as they have no special path, such as a wire provides, in which their flow is confined,

they move irregularly about like eddies. The resistance of the block is small and the energy of these eddy currents is rapidly transformed into heat. When a metallic pendulum bob is made to execute torsional or transverse vibrations over an open-circuited powerful electromagnet, these vibrations will be regular until the circuit is closed, when the bob will promptly come to rest. As the bob oscillates in the magnetic field, eddy currents are induced in it, and it becomes a magnet because of their presence. The poles of the pendulum magnet are so placed that their interactions with the poles of the electromagnet stops the motion. Eddy currents are frequently called *Foucault currents*, after their discoverer.

Whenever a current is induced in a coil of insulated wire wrapped around an iron core, a not inconsiderable amount of the energy of the inducing current is transformed into heat due to the Foucault currents, this heat energy appearing at the expense of the electrical energy. When small wires or thin plates (laminæ) of iron insulated from one another are substituted for the solid core, it is found that the eddy currents are much decreased, with a consequent diminution in the amount of heating. Thus, if the pendulum bob be made up of disks clamped together, its oscillations in the electromagnetic field are not damped so quickly. In most practical applications of induced currents the heating of the iron cores is detrimental; hence the cores are constructed of wires or plates insulated by a coating of varnish or other nonconducting material.

625. Induced Electromotive Force. — An induced E. M. F. is generated whenever a conductor cuts lines of force, even if the conductor does not form a definite closed circuit and only eddy currents are produced. Just as the terminals of a voltaic cell on open circuit are at different potentials, so are the ends of any open circuit moving across a field of force also at different poten-

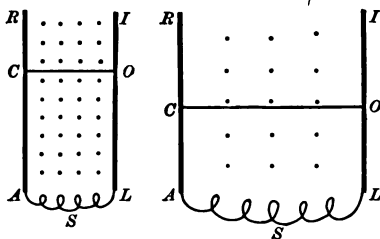


FIG. 382. — Induced Electromotive Force.

tials. In both cases the closing of a circuit permits a current to flow. The value of an induced E. M. F. depends upon the number of lines of force that the conductor passes through in a given time. In Fig. 382 the dots represent the ends of lines of force having directions normal to the plane of the paper. The straight conductor CO slides along the conducting rails RA and IL , the circuit being completed through the wire S . When CO is slid along the rails, the number of lines it passes through depends upon its velocity v , its length l , and the number of lines of force per unit of area (intensity or density of field) B . Then

Induced E. M. F. varies as vlB .

The unit difference of potential is induced in a conductor one centimeter long when it moves at the rate of one centimeter per second across a field having an intensity of one line per square centimeter, *i.e.* when it cuts one line a second. One hundred million of such units are equal to one volt. Hence the number of volts E induced in a moving conductor is given by the equation

$$E = vlB \times 10^{-8}. \quad (1)$$

When the conductor is wound into a coil, its length is proportional to the number of turns. Hence an *induced E. M. F. is proportional to the number of turns of a coil.*

TRANSFORMERS AND INDUCTION COILS

626. The Transformer. — Two coils of insulated wire are wound around a ring of soft iron (Fig. 383). When a current is sent through one of the coils, the *primary*, P , a momentary current flows through the other, the *secondary coil*, S ; and when the inducing current is stopped in the *primary coil*, a reverse induced current passes through the

secondary. The name *transformer* is given to this apparatus because, by varying the number of turns of primary and secondary windings, it is possible to change or transform the value of the induced E. M. F. Thus, if there are n times as many turns in the secondary as in the primary coil, the induced E. M. F. is n times as great as the inducing E. M. F., while if there are n times as many turns in the primary as in the secondary coil, the inducing E. M. F. is n times as great as the induced E. M. F. When the induced E. M. F. is greater than the inducing E. M. F. the name *step-up transformer* is applied, while in the reverse case, the name *step-down transformer* is adopted.

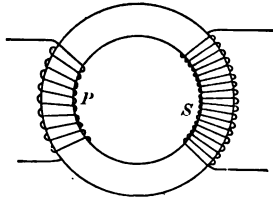


FIG. 383. — Ring Transformer.

When a definite amount of electrical energy is delivered to a step-up transformer, if the induced E. M. F. is thereby increased n -fold, the strength of the induced current is diminished n -fold, for electrical energy is equal to the product of the E. M. F. and the strength of the current, and any increase in the one factor must be accompanied by an equal decrease in the other. Furthermore, it takes additional energy to run a transformer, just as in the case of any other machine, because there must be some transformation of electrical energy into other forms, notably heat, accompanying its action.

627. Directions of Currents in Primary and Secondary Coils. — When the circuit is closed in the primary coil of a transformer, the coil and core constitute an electromagnet. Now Lenz's law affirms that the current induced in the secondary coil must have a direction such as to form an electromagnet, the poles of which are opposite to those produced in the primary-coil electromagnet. The induced current must therefore flow around the core in a direction

opposite to that of the inducing current in the primary. The closing of the primary circuit accordingly induces a current of opposite direction in the secondary circuit. But when the primary circuit is opened, its magnetic field disappears, and, by Lenz's law, the induced current must be in such a direction as to oppose the demagnetization. Hence, in that case, the direction of the primary current is the same as that of the secondary current. *At make, then, the currents in primary and secondary coils have opposite directions, while at break they have like directions.*

628. Self-induction ; Extra Current. — When a current is started by closing a circuit, it takes time (very little, however) for it to traverse the entire circuit. As the van of the current passes along, it builds up a progressing magnetic field, and the lines of force, as fast as they are generated, thread across the conductor. But wheresoever lines of force cut across a conductor, an induced E. M. F. is produced. Hence, on closing a circuit, an induced current is set up in the conductor itself which is carrying the inducing current. This phenomenon is known as *self-induction*, and the self-induced current is called the *extra current*. When the circuit is opened, the magnetic field rapidly dies out so that fewer and fewer lines of force thread through the conductor. Hence an extra current appears also when the circuit is broken. At the make of the circuit the induced current is found to have a direction opposed to that of the inducing current, while at the break both inducing and induced currents have the same direction. At make, then, the total value of the current is the difference between inducing and induced currents, while, at break, it is their sum. Accordingly, the electrical effects at breaking a circuit are much more pronounced than at its making, as is illustrated by the fact that a spark may appear at the break and not at the make.

629. Spark Coil. — When the circuit is made of a long wire coiled up compactly, the extra current developed is proportionately increased. The spark at break is thereby made much larger than would be the case if no such "spark coil" were used.

630. The Induction Coil. The induction coil is a step-up transformer provided with an automatic device for making

and breaking the circuit. Its usual form (Fig. 384) consists of a core made of soft iron wires, a primary coil of a hundred or so turns in two or three layers of coarse insulated wire, an automatic make-and-break connected with the primary coil and a battery, and a secondary coil encircling the primary and made up of many thousands of turns of fine copper insulated wire. Usually a condenser is placed in the base of the instrument. When a current is sent through the primary coil, a current of very high E. M. F. is induced in the secondary coil.

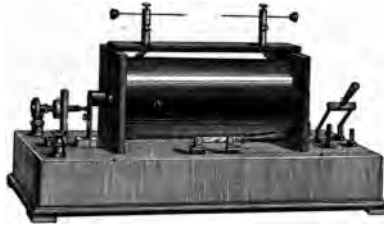


FIG. 384. — Induction Coil.

631. Action of Induction Coil. — The “make-and-break” (Fig. 385) consists of a soft iron hammer b facing one end of the core

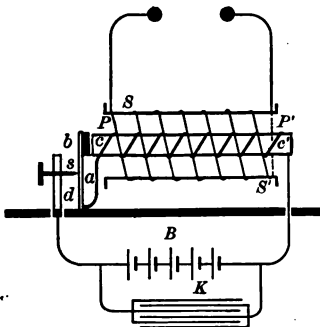


FIG. 385. — Diagram of an Induction Coil.

cc' with a spring brass handle a used to press the hammer against the point of an adjusting screw s . When the current from the battery B is started in the primary coil PP' , the core becomes magnetized and draws the hammer away from the adjusting screw. At the same time an induced current appears in the secondary coil SS' . The circuit is thereupon opened and a reverse induced current appears in the secondary. The core at once loses its power of attracting the hammer so that this springs back against the screw and again closes the circuit.

The above series of events thereupon repeats itself. By moving the secondary terminals close together, the E. M. F., induced both at make and at break, is large enough to cause a series of sparks to strike across the air gap. By separating the terminals, however, the E. M. F. in-

duced at make is, because of the action of the extra current, not great enough to pierce the air gap, so that only discharges due to the E. M. F. at break can appear. By moving the terminals farther apart still, the E. M. F. even at break cannot force a current across the gap.

632. Use of the Condenser.—It is found that the length of the spark between the secondary terminals may be increased by placing a condenser in a shunt around the make and break. When the primary circuit is broken, a spark tends to pass from the hammer to the contact screw because of the additive effect of the extra current, and as this spark or arc can conduct electricity in some degree, the duration of the break is prolonged, and the induced E. M. F. therefore lessened. But when a condenser is put into the circuit, the extra current which would pass through the arc is diverted into the condenser; and as it takes time to charge the condenser, the hammer is afforded an opportunity of getting far enough away from the contact screw to prevent the passing of a spark. A sudden break and a correspondingly greater E.M.F. is thereby insured.

633. Fleming's Right-hand Rule.—The direction of the induced E.M.F. depends upon the direction of the lines of

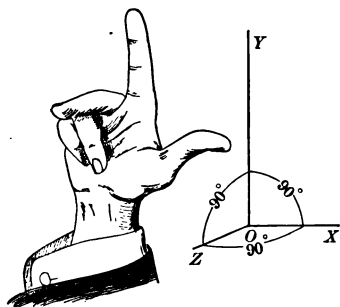


FIG. 386. — Fleming's Right-hand Rule.

force and of the motion of the conductor cutting the lines of force. To help in keeping track of the relationships of these directions, Fleming has proposed the following rule: *Point the forefinger of the right hand in the direction of the lines of force and the thumb in the direction of the motion of the conductor; then the middle finger, held at right*

angles to both forefinger and thumb, will point out the direction of the induced E. M. F. (Fig. 386). As this rule is frequently applied in the study of dynamos, it is often

called the *dynamo rule*; it also goes by the name of the *three-finger rule*.

Imagine yourself to be holding a straight conductor horizontally in front of you and standing at the *N*-pole of a magnet so as to face in the direction of the lines of force. If you let the conductor fall across the lines of force, will the induced current set up in it flow toward your right or left?

THE DYNAMO AND THE MOTOR

634. The Principle of the Dynamo. — Dynamo-electric machines, abbreviated to dynamos, transform mechanical energy into electrical energy. The word is derived from the Greek word for force (*dunamis*), for the dynamo utilizes the force of a man, or a heat engine, or a water wheel, or a windmill, to produce electric currents by means of mechanical work. All dynamos are so constructed that a coil is moved (rotated) in a magnetic field produced either by a permanent magnet or by an electromagnet. Permanent magnets exclusively were used in the earliest dynamos, whence arose the name *magneto-electric machine*, abbreviated into *magneto*; but electromagnets have taken their place, except in automobile magnetos and in the hand magnetos used in working "medical coils" and telephone call bells.

635. Action of the Dynamo. — *N* and *S* (Fig. 387) represent the poles of a magnet, and the parallel lines between them represent the lines of force. A straight conductor at right angles to the plane of the paper is supposed to move in the circular path indicated. The horizontal line is divided into equal parts at *a, b, c, . . . n*, which represent intervals of 30° in the rotation of the conductor. When the conductor passes through the posi-

tion 0° , it is moving parallel with the lines of force and no E. M. F. is induced. The value of the induced E. M. F. in this position is therefore zero, and it marks the starting point of the curve. While the conductor is moving to the position 30° , it cuts across the lines of force between 0° and 30° , and an E. M. F. is induced, the value of which is represented by the line extending from b to 30° on the curve. When the conductor passes through 90° , it is moving at right angles to the lines of force so that it cuts

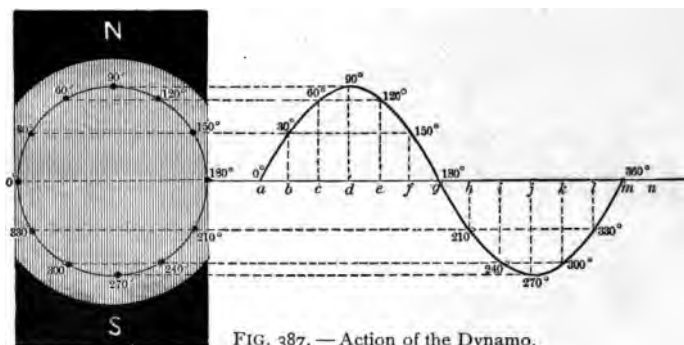


FIG. 387. — Action of the Dynamo.

the maximum number of them, and the maximum E. M. F. is induced. The curve therefore attains its highest point at 90° . During its passage from 90° to 180° the conductor intercepts an ever decreasing number of lines of force, so that the induced E. M. F. passes from its maximum value to zero again. Continuing its motion, the conductor will cut an ever increasing number of lines in the *opposite direction*, and the induced E. M. F. will also be in the opposite direction. To represent this, the curve is continued below the horizontal line. At 270° the maximum number of lines of force is cut, and again the E. M. F. has a maximum value but now in the opposite direction. A

decreasing number of lines of force is cut as the conductor moves around, and the induced E. M. F. decreases again to zero. The same series of changes or *cycle* will occur during each succeeding revolution; *the induced E. M. F. will pass from zero to a maximum, decrease to zero again, reverse its direction, increase to a maximum, and decrease to zero again.*

Suppose now that a loop be substituted for the single conductor, and suppose that one side of the loop passes through 0° while the other passes through 180° . When the loop is rotated, an E. M. F. is induced in one direction in one of its sides and in the opposite direction in the other. Hence a current would be started down through the plane of the paper in one branch and up through the paper in the other branch; in other words, it would start to circulate around the loop, and in case the loop formed a closed circuit, an induced current would pass around it. The induced E. M. F.'s act in the same direction in the loop, and their sum is the resultant E. M. F. Similar statements apply if a coil of any number of turns be substituted for the loop.

636. Simple Alternating Current Generator. — When a loop of wire rotates between the poles of a magnet, the direction of the induced E. M. F. changes every half-revolution. The current which may be produced is therefore alternating in direction. To keep the ends of the loop from twisting as it rotates, they are connected with two *collecting or slip rings*, *c* and *d* (Fig. 388). Rubbing lightly upon these rings are two stationary *brushes*, *e* and *f*, made of metal or carbon, which conduct the induced alternating current to the external circuit *HG*.

If the loop is set at right angles to the lines of force

and then rotated in the direction shown by the arrow, during the first quarter-turn there is an *increase* in the number of lines cut by *A* from *right to left*, and during the second quarter-turn, a *decrease* in the number of lines cut from *left to right*. Hence the direction of the induced E. M. F. remains the same during the first half-revolution, which, by Fleming's rule, is that shown by the arrowheads.

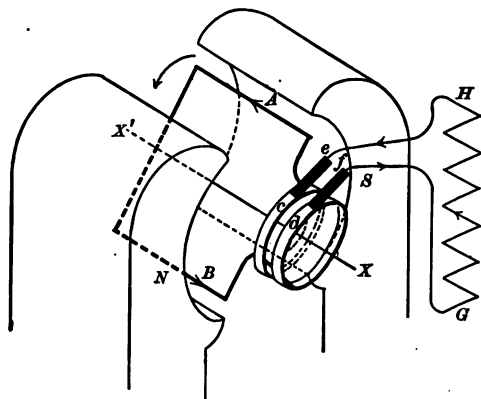


FIG. 388. — Simple Alternator.

An induced current will flow out from ring *f* through the external circuit *HG* back to ring *e*.

During the third quarter-turn, there is an *increase* in the number of lines cut by *A* from *left to right*. Consequently the induced E. M. F. reverses its direction. And during the fourth quarter-turn, there is a *decrease* in the number of lines cut from *right to left*. No change in direction takes place in the second half-revolution, during which the induced current has been passing from ring *e* through the external circuit back to ring *f*. The direction of the induced E. M. F. in part *B* of the loop is just the opposite of that in part *A*, as already explained,

the result being that the direction of the induced current through the loop remains the same during each half-revolution but alternates between the half-revolutions.

637. The Commutator. — If, instead of two collecting rings, a single ring split into halves insulated from each other is used (Fig. 389), the ends of the loop connected

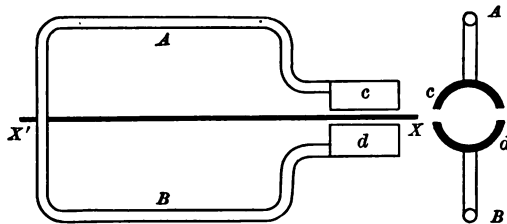


FIG. 389. — The left-hand diagram is a side view and the right-hand diagram an end view of the commutator.

with the two half-rings will during rotation send a current first into one half-ring and then into the other. Such an arrangement is called a *split-ring commutator (changer)*,

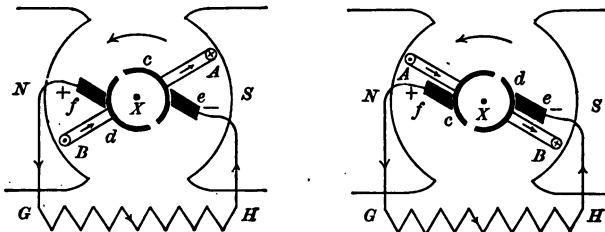


FIG. 390. — Direct Current Dynamo with a Single Loop Armature.

and its purpose is to change the alternating current into a direct current, *i.e.* one that has always the same direction. For this purpose the brushes (Fig. 390) are placed on opposite sides of the commutator, so as to rest on th

insulating strips between the half-rings when no E. M. F. is being induced. The brushes then lose contact with one segment of the ring and make contact with the other just when, from the rotation of the loop, the induced E. M. F. changes its direction. The current passing into the external circuit is thereby made to leave always by the same brush.

The loop (Fig. 390) is rotating in the direction indicated by the arrow, and the direction of the induced E. M. F. is shown by the signs $\odot$ (point of an arrow, meaning that the direction of the current is toward the spectator) and $\oplus$ (tail of arrow, meaning that the direction of the current is away from the spectator). Inspection of the diagrams will show how the brushes shift from one commutator segment to the other at the instant when the direction of the induced E. M. F. changes, so that the direction of the induced current in the external circuit GH always remains the same.

638. Direct Current Induced E. M. F. — The curve of induced E. M. F. for a direct current is shown in Fig. 391. It differs from

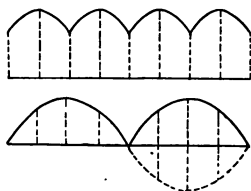


FIG. 391. — Curves for Direct Current Induced E. M. F.

that for an alternating current in that there are no lower portions, the upper portions being, instead, repeated. While the direction of the current is always the same, its E. M. F. rises to a maximum and falls to zero twice during a revolution, thereby producing a *pulsating current*. If, instead of one loop, there are two loops in planes at right angles to each other, the E. M. F. is at a maximum in one when it is zero in

the other. Hence the resulting E. M. F. will be represented by a curve which is compounded of the two curves representing the E. M. F.'s induced in the two loops (Fig. 392). By using a number of loops fixed at different angles to one another the curve is straightened out still more and the current becomes practically steady.

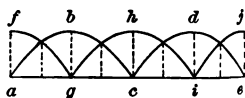


FIG. 392. — Curve for Direct Current Induced E. M. F. with a Double Loop Armature.

639. The Principle of the Motor. — Let RA and IL represent two conducting rails (Fig. 393), on which slides the conductor CO , and let the dots between them represent the ends of lines of force having directions normal to the surface of the paper. If a current flow through the conductor, it will slide up or down the rails according to the direction of the current and the direction of the magnetic field. The relationships between the three directions involved is given by *Fleming's left-hand rule*, also called the *motor rule*: *Point the forefinger of the left hand in the direction of the lines of force, and the middle finger, held at right angles to the forefinger, in the direction of the current sent through the conductor; then the thumb, held at right angles to both fingers, will indicate the direction in which the conductor tends to move.* Applying this rule (Fig. 393), we find that if the direction of the lines of force be upward through the paper and the direction of the current in the sliding conductor to the right, it will move toward the observer.

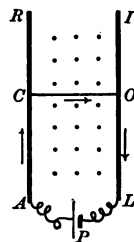


FIG. 393.—The Motor Principle.

Apply the motor rule and show that if the direction either of the lines of force or of the current be reversed, the direction of the sliding conductor will be reversed, but if the directions of the lines of force and of the current *both* be reversed, the direction of the motion will not change.

640. Action of the Motor. — If the loop (Fig. 388) be connected with a source of steady current in the external circuit HG , it will then behave like a magnetized disk with poles on its opposite faces. Like poles of the loop and of the magnet will repel each other, while unlike poles will attract each other. The loop will therefore turn until the unlike poles are as close together as possible and the

like poles as far apart as possible. But this can happen only when the plane of the loop is perpendicular to the field of force. The loop tends, therefore, to place itself so as to include the maximum number of lines of force. By the use of a commutator (Fig. 390) the current can be reversed in the loop. The poles of the loop will consequently be reversed also, after which the loop will turn halfway around. Another reversal of the supplied current then occurs, so that the loop continues to rotate. An electric current, therefore, may make a body move, electrical energy being converted into mechanical energy.

641. Dynamo and Motor Interchangeable.— There is no essential difference in the construction of a motor and of a dynamo. A dynamo can be run as a motor and a motor as a dynamo. In the dynamo, mechanical energy is expended in turning the loop (armature) within a magnetic field, and an electrical current is obtained; in the motor, electrical energy is spent in sending a current through the loop, and mechanical energy is obtained.

X-RAYS AND RADIOACTIVITY

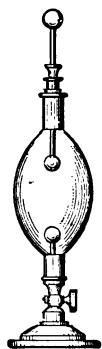


FIG. 394. —
Electrical
Egg.

642. Electric Discharges in Rarefied Gases.— The terminals of the secondary of an induction coil are connected with two wires sliding gas-tight through the ends of a glass tube (Fig. 394) which is joined to an air pump. If the air in the tubes is at the same pressure as the outside air, the terminals must be brought close together before a spark will pass between them when the coil is in action. But as the air is pumped out, the terminals can be moved farther and farther apart without breaking the spark, showing that a spark can pass the more readily through a gas the lower its pressure becomes.

: exhaustion proceeds, the white flash at first ob- spreads out into a luminous band of hazy purple

This fades when the exhaustion becomes very and a dark space appears round the negative il or cathode, which at last fills most of the tube.

s of the
hen glow
yellowish
ence.

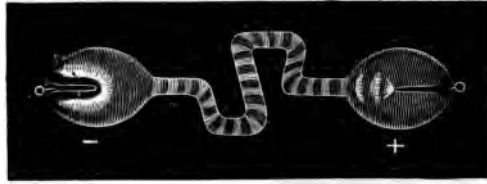
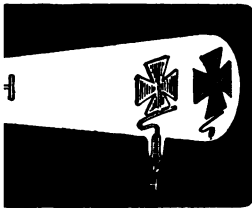


FIG. 395. — A Geissler Tube.

Geissler
— These
which are

ven quite fantastic shapes (Fig. 395), have two electrodes of sealed in their ends, and are exhausted of most of the gas they i. By filling the tubes before exhaustion with different gases, these remain after exhaustion and give rise to different colors

when the spark is passed. By making the tubes of uranium glass, which is highly fluorescent, or by surrounding them with jacketing tubes containing solutions of various substances, beautiful color effects are obtained.



—Shadow cast by Cathode Rays.

644. Cathode Rays. — The fluorescence observed when dis- of great E. M. F. from induction coils or static machines are rough highly exhausted tubes is said to be due to the action of zys, which stream the cathode and on the glass. rays move in lines normal to ode, for the fluo- is seen in a perpendicular

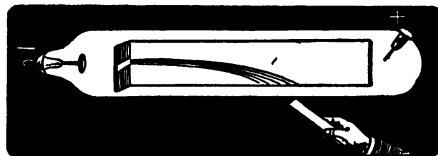


FIG. 397. — Effect of a Magnet upon Cathode Rays.

to the surface of the cathode, and the shadow cast by a metallic screen has well-defined borders (Fig. 396). They are deflected by a magnetic

field, for the presence of a magnet causes the positions of maximum fluorescence on the glass to shift somewhat (Fig. 397). They also produce heating effects, for if the cathode is made concave and platinum foil is placed at its focus, the latter becomes red-hot (Fig. 398).



FIG. 398. — Heating Effects of Cathode Rays.

645. Nature of Cathode Rays. — The behavior of cathode rays is best explained by assuming that they are charged particles moving with great speeds. Since these particles are repelled from the negative terminal, they must bear negative charges; and they move in straight lines. Since a moving charge is surrounded by a magnetic field, the streams of cathode-ray particles must be themselves mag-

netic, and hence they are affected by outside magnetic fields. Having both mass and speed, they must have energy, which can be transformed into heat.

646. X-Rays. — Wherever cathode rays impinge upon ordinary matter, as the walls of a vacuum tube, they give rise to another type of rays, called *x* or *Röntgen rays*. To concentrate the *x*-rays, the cathode is made concave and a piece of platinum placed at its focus (Fig. 399). This is fixed obliquely so that the *x*-rays coming from it may pass through the side of the tube.

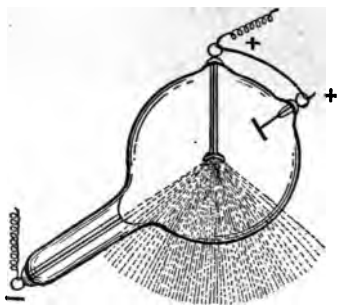


FIG. 399. — X-ray Tube.

647. Nature of X-Rays. — *X*-rays resemble cathode rays in that they move in straight lines, cannot be reflected or refracted, and produce fluorescence; but they differ in that they are not affected by a magnet and can pass through many substances impervious to the cathode rays. Röntgen, who discovered them in 1895, named them *x*-rays because their nature was

unknown, and they are still unexplained rays. It is assumed that the bombardment of surfaces by cathode rays must take place irregularly so that the checking of the motions of the electrically charged cathode-ray particles gives rise to a series of irregular jars or impulses, which in some fashion or other cause the *x*-ray effects.

648. X-Ray Pictures. — *X*-rays affect a photographic plate so that when it is developed, it will be more or less darkened according to the intensity of the rays reaching it. The picture obtained is merely a record of the shadows cast by the objects exposed to the rays. The denser a substance the more effectually it stops the passage of the rays. As flesh is much less dense than bone, the shadows of bones are darker than those of flesh (Fig. 400).

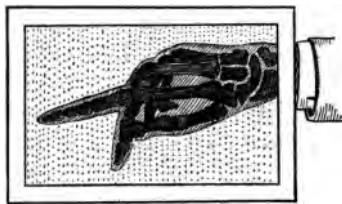


FIG. 400. — A "Shadowgraph."

649. The Fluoroscope. — *X*-ray shadow pictures of concealed objects may be seen directly by the aid of a *fluoroscope*. This consists of a box (Fig. 401), the large end of which is covered with a fluorescing substance, such as barium platinocyanide. The other end fits tightly around the eyes. Any object held between the screen and an *x*-ray tube casts a shadow which so affects the fluorescent coating as to be rendered visible.

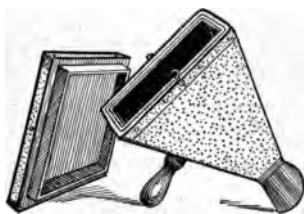


FIG. 401. — Fluoroscope.

650. Radioactivity. — Becquerel in 1896 wrapped a photographic plate in black paper, laid a coin upon the paper, and placed over the coin a mineral containing the metal, uranium. After keeping the whole for several days in a dark room, he found on developing the plate that a picture like that due to *x*-rays had been made. The mineral had emitted radiation of some sort which produced effects like those of *x*-rays.

Investigation revealed the fact that other substances also yield these *uranium* or *Becquerel rays*, and the mineral pitchblende was found to contain minute proportions of a compound of a new element which possesses this peculiar radioactive power to an extent nearly a milli-

fold greater than uranium. This new element was named *radium*. Radium also has the wonderful property of generating heat continuously so that its temperature is always above that of its surroundings.

651. Electrons. — Radioactive substances and cathode rays produce like effects; hence it is inferred that they are probably due to the same cause. These effects are generally ascribed to the action of very minute particles emitted by radioactive substances as well as from the cathode of a vacuum tube. These *electrons*, as they are called, have been found (1) to have mass, each electron having about $\frac{1}{1836}$ as much mass as an atom of hydrogen; (2) to move with great speed (many thousands of miles a second); (3) to carry electrical charges; (4) to have the same size and nature, no matter what their source; (5) to generate *x*-rays.

652. The Electron Theory of Matter. ~~This theory can here be sketched only in barest outline. It assumes that the atom of any substance has a nucleus charged with positive electricity. Associated with this nucleus is a large number of negatively charged electrons. As the atom in its normal condition is electrically neutral, the sum of the negative charges of the electrons must equal the positive charge of the nucleus. The electrons enjoy a certain freedom so that they can, to a limited extent, pass from one atom to another. An atom that has lost some of its electrons must therefore be charged positively, while one that has gained electrons must be charged negatively. The structure of an atom then would be similar to that of a solar system, the sun being analogous to the positive nucleus, and the planets, satellites, comets, and so on, analogous to the electrons.~~

653. Ionization. — We have learned that in conductors of the second class there is an actual transference of definite amounts of matter — the *ions*. Now the only way electricity can pass through a highly rarefied gas or a vacuum seems to be by a process similar to that of transfer by ions or electrons. Facts in support of this view are that a gold-leaf electroscope, even when as perfectly insulated as possible, is rapidly discharged by the proximity of an electron-producing substance or an *x*-ray machine. Under the radioactive influence the atoms of the gases are made to lose or gain electrons so that they become positively or negatively charged. These charged atoms are then attracted or re-

pelled, as the case may be, by the charge of the electroscope, which is thereby more or less rapidly discharged.

EXERCISES

1. What effect would be produced on a magnetic needle suspended by a thread, if a wire carrying a current were placed alongside the needle ?

2. Give two reasons for making the secondary of an induction coil of very fine wire.

3. How does Lenz's law account for the fact that the direction of an induced current produced by the movements of the *N*-pole of a magnet is the reverse of that of the induced current produced by similar movements of an *S*-pole ?

4. When one terminal of a strong battery is connected with the tang of a file, and the other terminal drawn over the teeth of the file, a series of minute sparks is obtained. But by including in the circuit an electromagnet, the size and brilliancy of the sparks are much increased. Why is this ?

5. How might a d'Arsonval galvanometer be modified so as to make it an electric motor ?

6. Ten amperes flow through a motor having a resistance of 1.5 ohms. What is the P. D. at the terminals of the motor ?

7. A certain dynamo delivers a current of 80 amperes at 250 volts. If its efficiency is 94 %, what horse power is required to drive it ?

8. What horse power is delivered by a dynamo of 80% efficiency if it is run by a water wheel with an efficiency of 70%, which in turn is driven by a stream of water discharging $60,000 \frac{\text{kg.}}{\text{min.}}$ from a height of 7.61 m.?

CHAPTER XXV

COMMERCIAL APPLICATIONS OF ELECTRICITY

654. Transference and Transformation of Electricity.—Electrical energy is characterized by the ease and rapidity with which it may be transferred from place to place. All that is needed is a conductor so insulated and connected that there is not a great loss of energy during the transfer. And its transference is possible even without special conductors, the ether itself acting as the medium, as in wireless telegraphy.

Electrical energy can also be readily transformed into other forms of energy. It may be generated in one place and conducted to another, where it may be turned into heat, light, or power. It is because of this readiness of transference and ease of transformation that the commercial applications have become so numerous and important.

The chief uses to which electrical energy is put are: (1) The transmission of messages; (2) the production of heat and light; and (3) the generation of power.

TRANSMISSION OF MESSAGES

655. The Electric Bell.—*Single-stroke Pattern.* A soft iron armature is fastened in front of the poles of a U-shaped electromagnet by a strip of brass supported on the base of the instrument. When the circuit is closed through the magnets, the armature is attracted and the attached clapper made to strike the bell; and when the circuit is opened, the armature springs back to its original position. Every time the circuit is closed the bell gives a single, brief ring.

Vibrating Pattern. To convert the single-stroke bell into the more common trembling or vibrating bell, an automatic circuit breaker similar

to that used on an induction coil is added (Fig. 402). Such a bell keeps on ringing as long as the circuit is closed. (Fig. 403.)

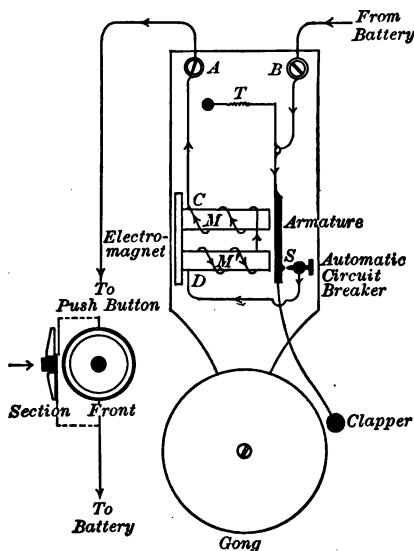


FIG. 402. — Electric Bell.

656. Buzzer.

— By omitting the bell and clapper, the buzzer results, the vibration of the armature making sufficient noise to attract the attention of a person near by.

657. Transmission of Messages by Means of Electric Bells.

— An electric bell can be used to transmit messages by the adoption of a code of signals. In the case of a doorbell one long ring means that some one is at the door, and two short rings mean the postman, while in hotel rooms one ring means ice water, two rings, bell boy, etc. Words may be transmitted by adopting an alphabet such as that devised by Morse for telegraphic use, in which certain combinations of long and short rings stand for certain letters. For example, if a dot (•) stands for a short ring and a dash (—) for a long ring, and if longer intervals of time intervene between the dots and dashes composing them, the sentence *Open the door* may be transmitted by the following rings :

•• •••• • — — •••• • —•• •• •• ••••
O P E N T H E D O O —

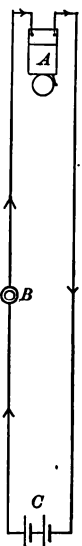


FIG. 403. — Bell Circuit. A, bell; B, push button; C, battery.

658. The Telegraph.—The Morse telegraph works on a plan similar to that of an electric bell circuit. The essential parts of a telegraphic circuit are the *key*, the *line*, and the *sounder*.

659. The Morse Key.—A lever has a contact point *P* on the lower side and a button on the upper side (Fig. 404).

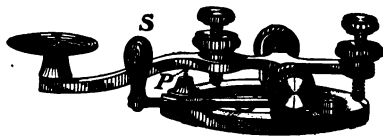


FIG. 404. — Morse Key.

When the lever is pushed down, the contact point touches another contact point on the base of the instrument, and closes the circuit. A

spring forces the lever up when released, the circuit being thereby broken. By means of a switch 'S' the circuit can be kept closed or open at the pleasure of the operator.

660. The Sounder.—The sounder (Fig. 405) is a U-shaped electromagnet in series with the line, having a soft iron armature pivoted across its poles, so as to play up and down in a notch. A spring forces the armature up against the upper part of the notch when the circuit is open. When a current is made to pass through the coils of the magnet, the armature is pulled down and strikes with a loud click against the lower part of the notch. When the current stops, the armature springs back and makes another click by striking against the upper part of the notch.

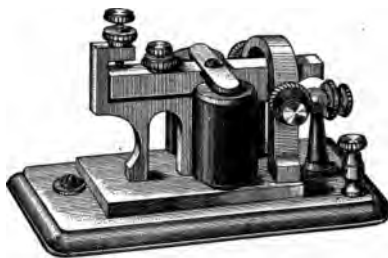


FIG. 405. — Sounder.

According as the interval of time between these two clicks is long or short, the signal is considered a dash or a dot.

661. The Line. — Galvanized iron wire, usually strung on poles, carries the current between sounder and key, the return circuit being through the ground. The wires are grounded by including in the circuit two large plates buried deep in the ground, where the earth is always moist.* A current (derived from gravity batteries or from dynamos) is constantly flowing through the circuit, the system being kept on a closed circuit.

662. Action of Telegraph. — Suppose that the operator at the station *A* (Fig. 406) wishes to send signals to the

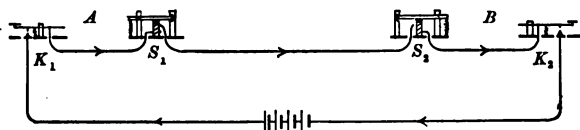


FIG. 406. — A Short Line Telegraph System.

operator at the station *B*. *A* releases his switch and operates his key so as to send through the sounder at *B* the signal previously agreed upon, which informs *B* that *A* wishes to communicate with him. *B* thereupon closes his switch, thus placing the circuit under the control of *A*. The sounders at both stations then click out *A*'s message

* As difficulty is sometimes experienced in understanding how the ground can act as part of the circuit and return the current to its starting place, the following analogy may be helpful. Suppose a water pipe to be placed across a pond and water to be pumped up from the pond on one side and delivered through the pipe on the other side. This will lower the level of the water in the pond on the pumping side and will raise it on the other side. Thus there will be a movement of the water in the pond toward the pumping side. While the water that is delivered will not be the same that is pumped up, yet there will be water supplied so as to keep up the flow.

to *B*. When *B* replies, *A* short-circuits his key so that the line is under the control of *B*, who by manipulating his key causes both sounders to click again.

663. The Relay. — When the distance between stations is considerable, the great resistance of the line renders the



FIG. 407. — Relay.

current so feeble that it cannot make the sounders click loudly enough to be heard clearly. To overcome this difficulty, either very strong currents must be used or

the delicacy of the receiving instruments much increased. The latter method is generally used. An electromagnet (Fig. 407), having very many turns of fine wire around its cores, and with an armature so light and so nicely balanced as

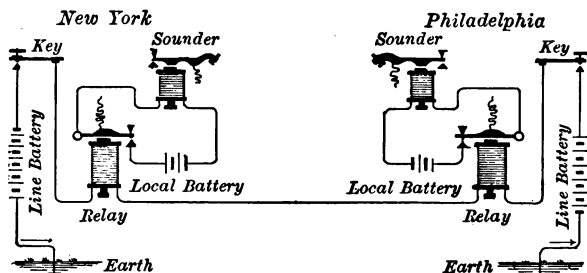


FIG. 408. — Arrangement of Instruments at Each Station of a Long Distance Telegraph System.

to move readily, even when the current is very weak, is placed in series in the line. When the circuit is opened or closed, the armature of this *relay* moves correspondingly. In its movements it acts as a key to open or close another circuit. In this *local circuit* are a battery and a sounder. When the

When the key opens or closes the local circuit, the sounders click at the stations (Fig. 408).

4. Wireless Telegraph.—Wireless or space telegraphy is a system of transmitting signals between stations without use of connecting wires. At each station is installed a *transmitting apparatus* by means of which electrical waves produced by disruptive electrical discharges are sent out into the ether, and a *receiving apparatus* which is affected by the waves.

5. The Transmitter.—The transmitting apparatus (Fig. 409) consists of an induction coil *I* between the

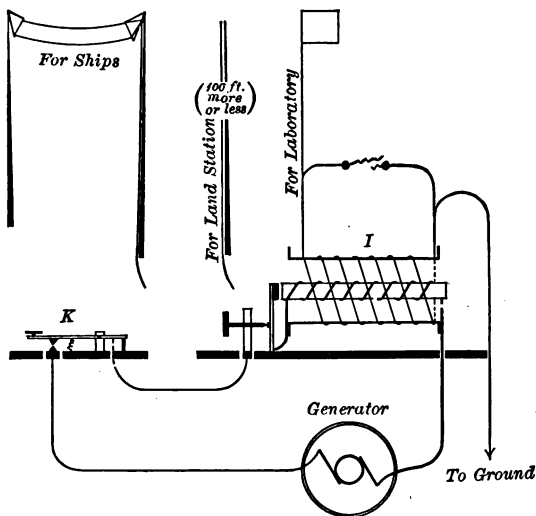


FIG. 409. — Transmitting Apparatus of Wireless Telegraph.

by means of which oscillatory discharges may be produced by closing the circuit of the primary coil by a Morse key. In order to increase the distance through which the

electrical waves may be made effective, a long vertical wire (the *aerial wire* or the *antenna*) is connected with one knob of the induction coil while the other knob is grounded.

666. The Receiver. — The receiving apparatus consists of a *coherer* and a *decoherer* in circuit with a *battery* and *relay*, which operates a *battery* and *sounder circuit*.

667. The Coherer. — The middle of a glass tube *C* (Fig. 410) of about half a millimeter bore is filled with fine filings

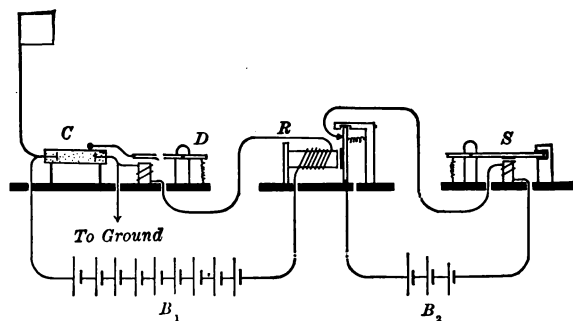


FIG. 410. — Receiving Apparatus of Wireless Telegraph.

of silver and nickel, on either side of which are inserted conductors, one connected with an aerial wire similar to that of the transmitter, and the other with a grounded wire — all placed in circuit with a battery and a relay *R*. The filings ordinarily conduct electricity so poorly that not enough current can pass through the circuit to work the relay. But when the waves sent out by the transmitter reach the filings, they cause them to cohere, *i.e.* to cling together, thereby increasing their conductivity. Sufficient current can then flow through them and the rest of the cir-

cuit to move the armature of the relay. The relay closes the circuit through the sounder *S* and a click is produced, corresponding to the action of the transmitter key.

668. The Decoherer. — The metal filings continue to cohere until the tube is tapped so as to jar them apart and thus break the relay circuit. To accomplish this tapping automatically, a buzzer *D* is included in the circuit, the clapper being set against the glass tube. When a current is made to pass through the circuit, the clapper vibrates, and tapping against the glass tube causes the filings to decohere. As often as the electrical waves make the filings cohere, the clapper makes them decohere, so that signals may follow one another rapidly.

669. The Telephone. — While in telegraphic systems a code of signals has to be used, in telephonic systems the original sounds are reproduced. Although wireless telephony has not yet reached the commercial stage, systems requiring a line have attained a high degree of perfection. In the case of short lines the receiver and transmitter may be precisely similar instruments, and the return circuit may be through the ground. But long lines require a complete metallic circuit and a special form of transmitter.

670. The Receiver. — Within the case of the receiver is a bar magnet *A* (Fig. 411) with its *N*-pole close to a soft iron diaphragm *E*. Over this pole is slipped a spool *B* wound with many turns of fine insulated wire, the ends of which pass back along the magnet to the bind-

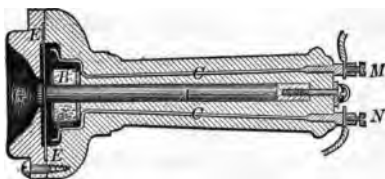


FIG. 411. — Telephone Receiver.

ing screws M and N , thus making connection with the line. When such a receiver is used as a transmitter, sound waves impinging against the diaphragm make it vibrate. Its slight

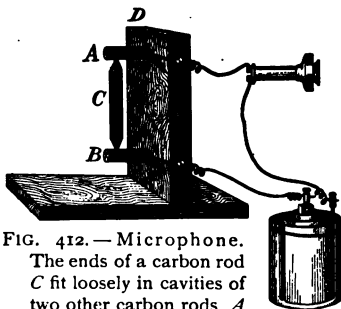


FIG. 412. — Microphone. The ends of a carbon rod C fit loosely in cavities of two other carbon rods, A and B , fastened to a sounding board D . The rods are in series with a telephone and a battery. Any vibration impressed upon the instrument causes changes in pressure between the rods, thereby altering the resistance. The telephone then emits sounds similar to and usually louder than the original.

to-and-fro movements alter the strength of the magnetic field of force inside the coil. The currents thereby induced are transmitted by the line to the receiver at the other end of the line. Here the induced currents cause similar changes in the strength of the field and make the diaphragm imitate the movements of the transmitter. Its vibrations set up in the air sound waves that closely resemble the original waves.

671. The Transmitter.—The action of modern transmitters depends upon the fact discovered by Hughes in 1878 that, when conductors are placed in loose contact, any jarring or disturbance of the conductors changes the extent of the contact and thereby changes the resistance of the circuit; the harder they are pressed together the less is the resistance, and the greater the current. Such a contact is called a *microphonic contact* (Fig. 412). The terminals of a local battery are connected, one with the diaphragm made of thin sheet carbon or iron, and the other with the solid

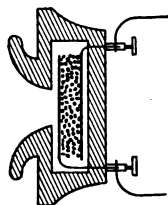


FIG. 413. — Solid-back Transmitter.

back of the transmitter, the chamber between being filled loosely with small grains of carbon (Fig. 413). Sound waves make the diaphragm vibrate, and the vibrations cause variations in the pressure of the many contact points of the granular carbon. The consequent variations of the resistance call forth corresponding fluctuations in the strength of the current. The primary of a step-up induction coil is placed in the battery circuit, the secondary being

in circuit with the line and the receiver (Fig. 414). The induced currents due to the changes in the strength of the primary current pass into the line and act upon the receiver at its remote end in such a way as to reproduce the sounds imparted to the transmitter.

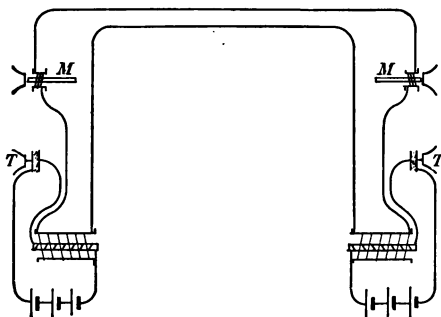


FIG. 414. — Arrangement of Instruments in a Telephone Circuit. T, T are transmitters; M, M, receivers.

PRODUCTION OF HEAT AND LIGHT

672. Electrical and Heat Energy. — Whenever a current of electricity encounters resistance, electrical energy is converted into heat energy. By variations in the strength of the current and of the resistance, corresponding variations in heat intensity are obtained. The convenience from most points of view of heating devices that derive their energy from electrical sources is unquestioned, and whenever the greater cost is not prohibitive, electric heating appliances will be chosen in preference to others.

673. Electrical Stoves.—An electrical stove consists essentially of a coil or coils of wire of considerable resistance contained in a suitable holder (Fig. 415).

674. Electric Welding.—When metals, especially iron and steel, are heated to a high temperature and pressed firmly together, they

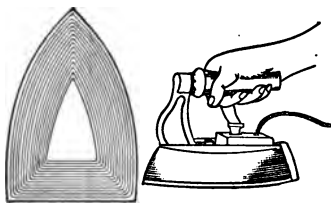


FIG. 415.—Electric Flatiron.

cohere, the joints proving to be as strong as the rest of the metal. This process of joining metals is called *welding* or *brazing*. Welding by means of an electric current is accomplished as follows. The two pieces of metal are made the terminals of a powerful current of low voltage usually supplied by an

alternator. When the pieces are placed in contact, enough heat is soon generated to raise them to the welding temperature. The circuit is then broken, and the two pieces pressed together with enormous force usually exerted by a hydraulic press.

675. Gas-engine Ignition.—To set on fire the explosive mixture of air and vapor drawn into the cylinder of an internal combustion engine, an electric spark is usually employed. To make the spark have sufficient heat, in the “make-and-break” igniter, a spark coil is inserted in the circuit, the extra current at break increasing the size of the spark. In the *jump-spark system*, which has an advantage over the *make-and-break system* in that it requires no moving parts within the cylinder, the battery current is closed by means of a “timer” through the primary of an induction coil (“vibrating-spark coil”). The secondary coil is in circuit with a “spark plug” screwed into the cylinder, across the terminals of which the igniting sparks pass.

676. Electric Furnaces.—The heat of an electric furnace is derived from the resistance offered by the heated materials to the passage of the current. By employing powerful currents, enormous heating effects may be obtained, the commercial application of which has resulted in the discovery and introduction into use of many new and valuable substances.

677. The Incandescent Lamp.—A current of electricity sent through a substance may heat it to such a degree

that it gives out light, a part of the heat energy produced by the current appearing as light energy. The *incandescent lamp* (Fig. 416) consists of a thread or filament of specially prepared carbon inclosed in a glass bulb from which the air has been exhausted. Carbon is used because it has a high resistance and melting point, and the air is removed, as otherwise the carbon would burn up. The ends of the filament are connected to two platinum wires fused into a glass tube projecting within the bulb. One wire is joined to the brass plug *g* and the other end to the brass casing *d*, the intervening space being filled with a non-conducting cement. When the lamp is screwed into its socket, electrical contact is made between the brass casing *d* and the socket, as well as between the plug *g* and a brass tongue *h* insulated from the other metallic parts of the socket and connected with one of the terminals of the source of current.

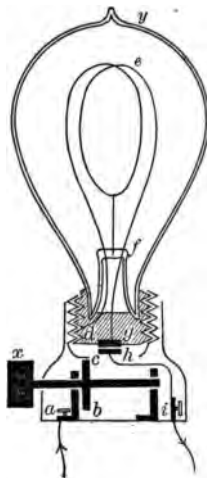


FIG. 416. — Incandescent Lamp. When the current is turned on, the circuit is as follows: *a, b, c, d, e, f, g, h, i.*

A 16 candle-power lamp used on a 110-volt circuit requires about .5 ampere, so that the energy consumed is about 55 watts or 3.5 watts per candle power.

678. The Tungsten Lamp.—Certain rare metals, such as osmium, tantalum, and tungsten have very high melting points, and fine wires of these metals may carry current enough to bring them to incandescence before their melting point is reached. Tungsten filaments have been successfully substituted for carbon filaments and show high efficiencies, the tungsten lamp requiring only about one watt per candle power.

679. The Nernst Lamp.—This lamp (Fig. 417) has a filament composed of the oxides of certain rare metals, being similar in composition to gas-light mantles. As the filament is incombustible, it does not have to be inclosed in an exhausted vessel. The substances composing the filament ("glower") conduct electricity only when heated to incandescence. Hence, to start the lamp, the filament must be heated by an alcohol flame or by a current of electricity passing through

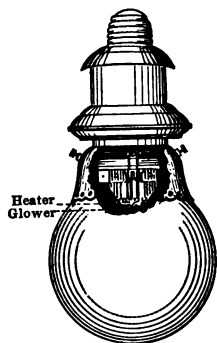


FIG. 417.—Nernst Lamp.

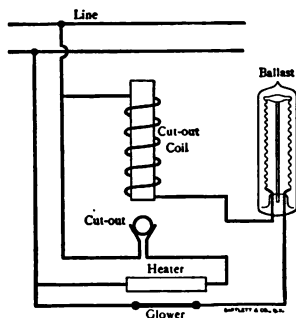


FIG. 418.—Diagram of parts of Nernst Lamp.

an encircling coil of platinum wire ("heater"), which is automatically cut out by an electromagnet of the circuit as soon as the filament reaches incandescence (Fig. 418). A resistance ("ballast") serving to steady the action of the lamp is placed in series with the glower. A Nernst lamp requires a little less than two watts' per candle power.

680. The Arc Lamp.—When the ends of two carbon rods forming the terminals of a strong current are brought into contact, the great resistance offered causes the ends to become very hot. If the carbons be separated a few millimeters, the current will still flow, being conducted by the incandescent carbon vapor that is produced. Carbon distills more rapidly from the positive to the negative rod than in the opposite direction, so that the former becomes hollowed out, forming the *crater*, while the latter assumes

a cone shape (Fig. 419). In the air the carbons burn at the rate of about an inch an hour, the positive pole wasting away about twice as fast as the negative. By inclosing the arc in a tight glass globe, it is protected from the action of the air, and the burning of the carbons is reduced to about an eighth of what it would be were they not so shielded. As the arc attains the highest temperature artificially procurable, — about 3800° F., — its light is the most powerful of all sources, being comparable to that of the sun.

An ordinary arc lamp gives about 1000 candle power and takes 10 amperes at 50 volts. When the carbons are made with a core consisting of certain highly infusible oxides, the arc is lengthened and assumes the appearance of a flame, and this *flaming arc* gives several times the light of an ordinary arc, with the same consumption of energy. Inclosed arc lamps are sometimes constructed so as to work under a pressure of 110 volts, and they can in that case be put upon an incandescent lighting circuit.



FIG. 419.—Electric Arc.

681. Hand and Automatic Feed Lamps. — As the carbons gradually burn up and as the distance between them should remain constant in order to give the best results, it is necessary to push them together as fast as they waste away. While this feeding may be done by hand in the case of lamps under constant supervision, such as those used in searchlights and in stereopticons, some kind of automatic feed is required for street lamps, one form of which is the following: Two solenoids C_1 and C_2 (Fig. 420), one of many turns and of very high resistance, and the other of a few turns and of low resistance, are provided with movable cores n and n' of soft iron. When a current is passed through these coils, their cores are drawn up by the magnetic action

of the solenoids. The high-resistance coil C_1 forms a shunt, while the low-resistance coil C_2 is in series with the carbons. Suppose

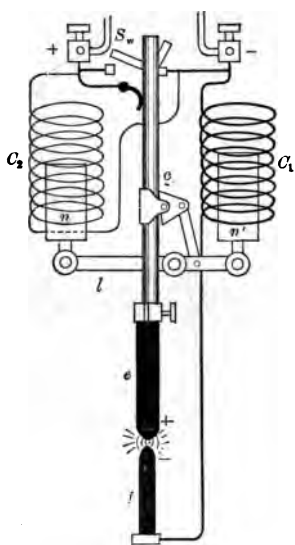


FIG. 420. — Automatic Arc Lamp.

the carbons to be separated when the current is turned on. The current passing through the shunt coil causes its core to rise and to draw up its end of the lever L . The clutch at c is there-upon loosened, and the upper carbon falls down upon the lower. Most of the current therefore leaves the shunt coil to pass through the carbons and the series coil, as their resistance is comparatively small. The series core rises, draws up its end of the lever and causes the clutch to grasp the upper carbon. The carbons are pulled apart, and an arc is struck between them. If the arc becomes too long, its resistance increases, and the current passes into the shunt so that the upper carbon is again let down. So nicely is the mechanism adjusted that a practically uniform length of arc is maintained.

682. The Cooper-Hewitt Lamp.—This lamp consists of a long glass tube from which the air has been exhausted. Platinum “leading-in” wires are sealed in the ends of the vacuum tube to which are attached the electrodes, the positive one being an iron plate and the negative one a small quantity of mercury. The vapor of mercury conducts the current and forms a long arc that emits a very brilliant and well-diffused light, which, as it lacks the red rays, is of a bluish cast. The lamp takes a direct current of 3.5 amperes at 50 to 150 volts, and its efficiency is very high, requiring but .3 watt per candle power. Where the peculiar color of the light is not objectionable, the lamp is being used extensively.

683. Price of Electricity.—The price of electricity is based upon the amount of electrical energy consumed in a certain unit of time.

A *watt-hour*, for instance, since a watt is equal to a *volt-ampere*, is the amount of electrical energy delivered in one hour by a current of one ampere having a potential difference of one volt. A thousand times greater unit, the *kilowatt-hour*, is the one universally used in buying and selling electricity. Incandescent lamps were formerly labeled according to their candle power, but it is now the custom to rate and label them in watts. The common 50-watt lamp kept glowing for 20 hours would use one kilowatt-hour of electric power, and, if the price were 10 cents per kilowatt-hour, the cost of using the light would be half a cent an hour.

EXERCISES

1. What great advantage has electrical energy over other forms of energy?
2. Point out in what particulars a telephone transmitter may be regarded as a dynamo, and a receiver as a motor?
3. What effect would it have on the light of a Cooper-Hewitt lamp to enclose it in red glass?
4. Why is the Cooper-Hewitt lamp especially well adapted to photographic work?
5. What will it cost per hour to run a 16-candle-power lamp supplied with current at 110 volts, if its resistance is 220 ohms and electricity costs 10¢ per kilowatt-hour?
6. Make a diagram of the proper connections for ten 110-volt lamps in the ceiling of a trolley car operated on a 550-volt circuit.
7. In welding rails a current of 30,000 amperes at 6 volts is commonly used. If the current is applied for two minutes, what is the total amount of heat generated?
8. How many calories are produced in five hours by an arc light with a resistance of 5 ohms and a P. D. of 50 volts?
9. Two electric bells with coils having a resistance of 4 ohms each are connected in parallel. If the resistance of the line is 1 ohm and that of the battery is 3 ohms, what is the total resistance of the circuit?
10. If a 110-volt tungsten lamp requires a current strength of .75 ampere and gives 70 candle power, what is its resistance, and how many watts does it require?
11. How does the cost per candle of the lamp in Exercise 5 compare with that of the lamp in Exercise 10?

GENERATION OF POWER

684. Parts of a Dynamo.—The principal parts of a direct current dynamo are the *field magnets*, the *armature*, the *commutator*, and the *brushes*. The field magnets are electromagnets the current for which is derived either from a separate source, as in *separately excited dynamos*, or from the dynamo itself, as in *self-excited dynamos*. The armature rotates between the magnetic poles, and consists of coils of wire wound about a soft iron core built up of thin sheets. The ends of the coils are joined to the segments of the commutator, on which press the brushes that conduct the current to and from the external circuit. The induced E. M. F. is increased by having many coils, as each coil adds its E. M. F. to that of the others. A large number of coils also renders the current steady.*

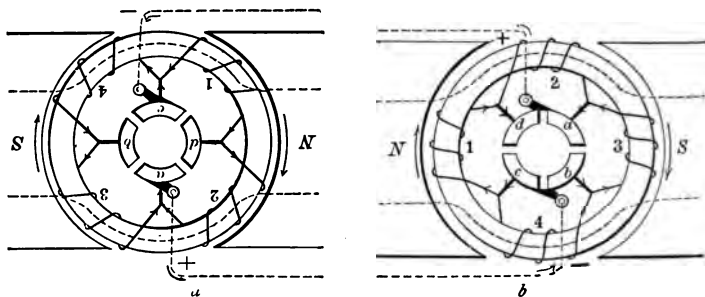


FIG. 421. — Diagram of Gramme-ring.

685. The Ring-armature D. C. Dynamo.†—A coil of wire wound about an iron ring constitutes an armature with a *ring winding*, often referred to as a *Gramme-*

* Alternating current dynamos (*alternators*) have the same parts as direct current dynamos, except that collecting rings are substituted for the commutator.

† The abbreviations D. C. for Direct Current and A. C. for Alternating Current are in common use with electrical engineers.

ring, after the name of its inventor, Gramme. A four-coil ring winding with a four-part commutator is represented in Fig. 421. One end of coil 1 is joined to the commutator segment *c*, and the other end to segment *d*. The other coils are similarly joined to their adjacent segments. The brushes bear on diametrically opposite segments. The lines of force pass only through the iron of the ring, as indicated by the dotted lines, and do not penetrate within the ring. When the armature is made to rotate, only such lines of force are cut by the portions of the coils as are on the outside of the ring, those portions within the ring serving merely to connect the outside conductors in series, and not themselves generating E. M. F. As each coil is made up of three turns, the induced E. M. F. is three times what it would be if there were but one turn.

The coils 1 and 3 (Fig. 421, *a*) are just entering the magnetic field, and coils 2 and 4 are just leaving it. The induced current passes from the external circuit through the lower brush to commutator segment *c*, where it divides and passes in series through coils 1 and 2 on the left and coils 4 and 3 on the right. These two currents then unite and pass through segment *a* and the upper brush into the external circuit. Since the two pairs of coils are in series, the total E. M. F. induced will be the sum of the induced E. M. F.'s of the two coils.

In Fig. 421, *b*, the armature has rotated an eighth of a turn (45°) farther than in Fig. 421, *a*, the upper brush now making contact between segments *a* and *d*, and the lower brush between segments *b* and *c*. Coils 1 and 3 are now cutting the maximum number of lines of force, and hence developing the maximum E. M. F. Coils 2 and 4 are moving parallel to the lines of force, and are therefore producing no E. M. F.; they are said to be in the *neutral position*. The current from the external circuit passes from the lower brush to the upper through coils 1 and 3 in parallel, no current passing through coils 2 and 4; for segments *a* and *d* are connected through the upper brush, and segments *b* and *c* through the lower brush, and as these connections have a low resistance, coils 2 and 4 are short-circuited.

During the next quarter turn, coils 1 and 3 will be approaching the

neutral position, while coils 2 and 4 will be developing E. M. F. in the opposite direction; but through the shifting of the commutator segments, the brushes will continue to receive the current in the same direction as before.

While there is a fluctuation of E. M. F. during the rotation of the armature, the addition of more coils reduces it, the E. M. F. being practically constant when thirty or more coils are used. In machines built with more than one pair of poles the number of circuits increases proportionately, and consequently the number of pairs of brushes, although, indeed, there may be but a single pair if all the positive terminals are connected together as well as all the negative terminals.

686. The Drum-armature D. C. Dynamo. — Drum windings differ from ring windings in that all the wires are on the outside of the core, thus forming a cage surrounding the core. The only parts of drum windings which do not cut lines of force are at the ends of the core, where the wires (*end-conductors*) pass across so as to be practically parallel to these lines (Fig. 422). To make all these end-conductors of the same form and length, the brushes are usually placed opposite the poles instead

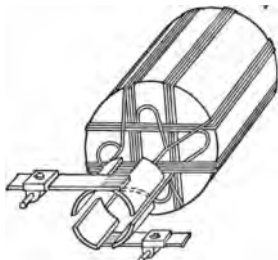


FIG. 422. — Drum Armature.

of between them, as in ring-wound dynamos. The action of drum-wound dynamos is similar to that of ring-wound machines. The parts of the wires on one side of the line joining the brushes have induced E. M. F.'s in one direction, while the parts on the other side have E. M. F.'s induced in the reversed direction. The commutators collect the currents and deliver them through the brushes to the external circuit.

687. The Alternator. — In many large alternating current machines the field magnets are made to rotate around a stationary armature, while

in others the armature turns between the magnets. The rotating part of an alternator of either type is called the *rotor*, and the stationary part the *stator*. A diagram of a common form of alternator is given in Fig. 423, the rotor being an eight-coil armature, and the stator an eight-pole electromagnet which is separately excited by a small D. C. dynamo (not shown in the figure). The winding of the electromagnet is such that the adjacent poles have opposite polarity.

The alternate coils of the armature are wound in opposite directions and are connected in series, the terminals passing to the collecting rings. In the position shown coils 1, 3, 5, and 7 are in front of *N*-poles, while coils 2, 4, 6, and 8 are in front of *S*-poles. As the windings of the adjacent coils are in opposite directions, the currents induced all have the same direction and therefore unite and pass through the collecting rings and brushes to the external circuit.

But when the armature has rotated an eighth of a turn, the odd-numbered coils are in front of *S*-poles, and the even-numbered in front of *N*-poles, and the direction of the current is reversed. The current will therefore alternate in direction eight times per revolution of the rotor, so that there will be four cycles in the reversals of the induced currents, and with 900 R. P. M. (revolutions per minute) there will be 60 cycles per second.

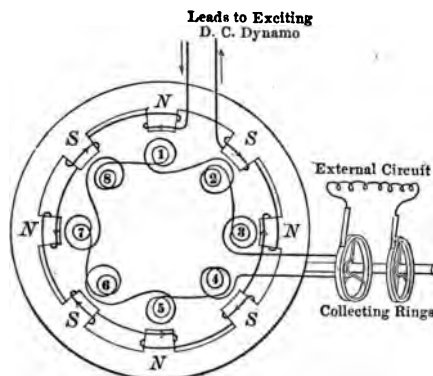


FIG. 423. — Diagram of Alternator.

688. Commercial Transformers. — The ring-core transformer (Fig. 383) is not economical for the reason that the magnetic circuit is not confined closely enough to the core, many lines of force streaming out into the air. To avoid this leakage, the primary and secondary coils are placed very close together, and the shape of the core is chosen

as to direct as many lines as possible through the coils. To prevent eddy currents and the consequent conversion of electrical energy into heat, the core is built up of thin sheets of the softest iron (Fig. 424). Properly constructed transformers have an efficiency as high as 98 %, the electrical energy changing into heat energy.



FIG. 424. — Commercial Transformer. *P, P*, primary coil; *S, S*, secondary coil.

689. Purpose of Transformers.

— Transformers are employed to change the E. M. F. of alternating currents. As it is more economical to transmit high voltage currents than those of low voltage, the current from an alternator is passed through a step-up transformer (unless, indeed, the alternator itself can furnish a current of great enough E. M. F.) to the main line. Step-down transformers are then inserted in the main line wherever current is used,

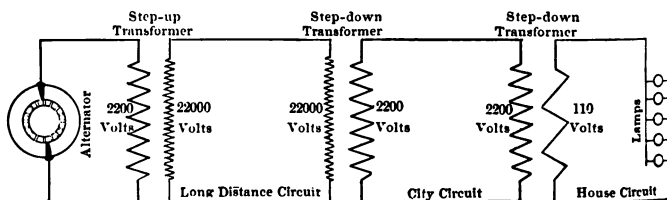


FIG. 425. — Long Distance Power and Lighting Circuit.

and through them the voltage is lowered to the value desired. Thus the E. M. F. of the current of an incandescent lighting circuit is usually 1100 to 2200 volts, while the lamps require but about 110 volts. Hence a transformer

is used to lower the voltage, and as currents having an E. M. F. of more than 1000 volts are too dangerous to be run directly into residences, the transformers are placed outside on the poles supporting the main line (Fig. 425).

If an alternator generates a current of 50 amperes at 2200 volts, the power developed is 110 kilowatts. A current of 50 amperes would require for transmission a very large conductor of small resistance, otherwise much of the electrical energy would be dissipated as heat. If the current be transformed into 5 amperes and 22,000 volts, the number of kilowatts will be the same, but a smaller conductor may be used. Where electrical power is to be transmitted long distances, an alternating current with high voltage and low amperage is used. A large amount of electrical power can be transmitted over a long distance with but slight heat loss by a small current and correspondingly small conductor, if the voltage be very high.

690. Self-excitation Field. — When a self-excited dynamo is run for the first time, no current is developed for the reason that there is no magnetism in the field magnets. Hence at the factory the field is excited by running a current from another dynamo through the coils of the electromagnets. Some of the magnetism is retained by the iron core for long periods of time. So, when the dynamo is started again, this *residual magnetism* suffices to generate a feeble current which, flowing through the field magnets, strengthens them so that still stronger currents are induced in the armature. In this fashion the strength of the field is rapidly built up to its maximum value.

691. Constant Potential and Constant Current Dynamos.
— The power in a dynamo's external circuit is equal to the product of the difference of potential E at the brushes by the current strength C , *i.e.* it is equal to $E \times C$. The amount of work that is done on the circuit is called its *load*, and is proportional to $E \times C$. The magnitude of EC is increased by increasing either E or C , and the load on a dynamo is increased when it is required to supply a larger current of the same voltage or a higher voltage of the same current

strength. As E is measured in volts and C in amperes, their product is watts. So long as the product of E and C remains constant, the same amount of energy is being delivered by the dynamo.

Now for incandescent lighting and power distribution, a current of constant voltage is required. Hence *constant potential dynamos* must be so constructed that the same number of lines of force is cut in the unit of time. E then will be constant, and C , as well as EC , will change in order required to meet the demands upon the external circuit. In arc lighting a constant current rather than a constant E. M. F. is required. Hence the design of a dynamo for an arc-lighting system must be such as to keep C constant, while E may vary according to the number of lamps, although too great variations of E are not permissible.

692. Classes of D. C. Dynamos. — Direct current dynamos are divided into three classes according to the manner in which the external circuit and the field coils are interconnected.

Series Dynamos. When the current from the brushes passes *in series* through the external circuit and the field magnets, the dynamo is said to be *series-wound* (Fig. 426). As the field coils carry all the current, they are made of a large size of wire, but

because of the large current they carry, only a comparatively few turns are needed to give the field the required strength. When the resistance of the external or main circuit is increased, the current strength is proportionately decreased, and the strength of the magnetic field decreases for the reason that less current passes through its coils. Hence fluctuations in the main circuit or load call forth fluctuations in E. M. F. Series dynamos can therefore be advantageously used only with circuits that have a nearly constant load, as is the case with arc-light circuits.

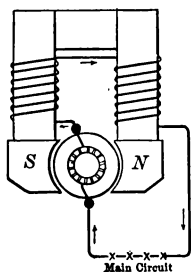


FIG. 426.—Series Dynamo.

Shunt Dynamos. In *shunt-wound* machines the induced current is offered two paths, one through the external circuit and one through the field coils. The field windings consist of many turns of fine wire, so that only a small part of the total current is shunted through them (Fig. 427). The greater the external resistance, the less will be the proportion of the total current which passes through the field coils, and hence the less the E. M. F.; as the load decreases, the E. M. F. increases.

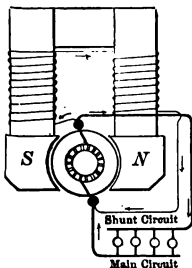


FIG. 427. — Shunt Dynamo.

Compound Dynamos. The field magnets are wound with two wires, one fine, forming a shunt with many turns, the other thick, having only a few turns (Fig. 428). The shunt and series coils are so proportioned as to make the E. M. F. practically constant for all loads.

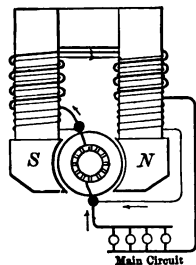


FIG. 428. — Compound Dynamo.

693. Wiring of External Circuits of Dynamos. — Both alternating and direct currents may be used, the arrangement of the circuits being essentially the same.

Arc-light System. A series-wound dynamo is commonly chosen, and the lamps are connected in series. As in most cases each lamp takes about 10 amperes at 50 volts, the power furnished each lamp amounts to 500 watts. If, then, there are 50 lamps on the circuit, the dynamo must be capable of furnishing 25,000 watts.

Incandescent-light System. A compound-wound dynamo capable of maintaining a P. D. of from 110 to 115 volts between the main wires

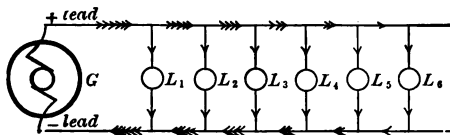


FIG. 429. — Wiring of Incandescent Lamp Circuit.

or *leads* is used, and the lamps are connected in parallel across the leads (Fig. 429). The *three-wire system* is applied where economy in the quantity of copper wire is practiced. In this system two 110-volt generators are connected in series, so that there is a P. D. of 220 volts between their outer leads

(Fig. 430). The middle or neutral wire acts as a common conductor for both the other leads, the 110-volt lamps being connected between this neutral wire and the outer leads. With equal numbers of lamps glowing on either side of the neutral wire the current will flow through the pairs of lamps in series so that they will take half the current at

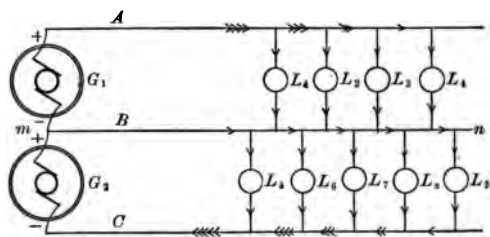


FIG. 430. — Three-wire System of Wiring.

twice the voltage that they would bear if arranged on a two-wire system. Then the *line loss*, i.e. the amount of energy consumed in overcoming the resistance of the conductors, not counting the lamps or motors on the circuit,

will be only one fourth as great (cf. Joule's law, p. 388). If the same number of lamps be glowing on each circuit, no current at all will flow through the neutral wire. When, however, a lamp is cut out of the lower circuit (negative side), the current can then no longer return by the lower (negative) wire, and hence has to return by the neutral wire. And if one of the lamps is cut out of the upper circuit, the extra lamp left over on the lower circuit receives its current through the neutral wire. A lighting circuit is so laid out that as far as possible there will be nearly equal demands for current from both circuits. Any excess or deficit will be taken care of by the neutral wire. Since the positive and negative wires carry only half the current at twice the voltage that a two-wire system using the same amount of power carries, their weight may be reduced to one fourth: and since the length of the neutral wire is half the combined lengths of the positive and negative wires, its weight is only half their weight. Hence the total weight of the conductors of a three-wire system is $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$ that required for a two-wire system, a saving of 62.5% in the cost of the wire required for a two-wire system.

694. Dynamo as Motor. — Any dynamo may without essential modification act as a motor. If a current be sent into the dynamo armature, this will spin around, and its motion may be applied to run machinery. That a motor is nothing but a dynamo working under a reversal of con-

ditions, the following considerations will show. Let a current enter the positive brush of a ring-wound dynamo (Fig. 421, *a*). It will divide, one half passing through the coils on one side of the brushes, and the other half passing through the coils on the other side. The reunited current then passes out through the negative brush. Each half of the ring armature becomes an electromagnet, having an *S*-pole above and an *N*-pole below (apply the right-hand rule, p. 422). The attractions and repulsions of these poles with regard to the poles of the field magnets causes the armature to rotate in the opposite direction to that indicated by the large arrows (apply the left-hand rule, p. 429). As long as the current is supplied at the brushes, rotation will be kept up, and the stronger the current, the greater will be the force acting to produce rotation.

The behavior of drum-wound armatures is essentially similar. The current entering one of the brushes will divide so as to flow always in one direction through the conductors on one side, and in the opposite direction through the conductors on the other side, the commutator shifting contacts so as to keep up the same direction of current.

695. Counter E. M. F. — The armature of a motor when rotating cuts the lines of force of the field. An E. M. F. is therefore induced in the armature windings, the machine acting both as motor and dynamo at once. The direction of this induced E. M. F., due to the dynamo action, is opposite to that of the current running the motor, as may be proved by applying Lenz's law. (Note the opposite directions of rotation of the Gramme machine in Fig. 421 when it is run as a dynamo and as a motor.) The greater the speed of the armature, the greater is the rate of cutting the lines of force in the field, and consequently the greater this *counter* or *back* E. M. F. (C. E. M. F.). When a current is passed into the armature and a force applied to prevent its turning, the magnitude of the current is by Ohm's law equal to the ratio of its E. M. F. to the resistance of the windings, this ratio being always small. But on releasing the armature, the counter E. M. F. opposes the applied E. M. F., so that the motor, if not required to do outside work, may have but a small current in its armature even when its speed is high. The current actually passing through the armature is due to the difference between the E. M. F. of the current applied and the counter E. M. F., so that the faster a motor spins, the less is the current taken from the mains.

696. Motor Equation and Starting Resistance. — The drop of potential in a motor armature is equal to the difference of the applied E. M. F. and the C. E. M. F., the former being always larger than the latter. Applying Ohm's law, we have then for the intensity of the current in the armature with a resistance of r :

$$C = \frac{\text{E. M. F.} - \text{C. E. M. F.}}{r}.$$

This *motor equation* shows that if the C. E. M. F. is at a minimum, the armature current will be at a maximum. When a motor is started, the C. E. M. F. has a small value, so that a large current passes through the armature windings. As this large current might overheat the armature windings and thereby destroy their insulation, it is necessary to throw in series a larger resistance when a motor is starting, and this resistance can be gradually reduced as the motor comes up to speed.

697. Applications of Motors. — Wherever current is at hand and power is demanded, the motor proves its worth. A mere list of all the applications of the motor would occupy many pages. As trolley cars

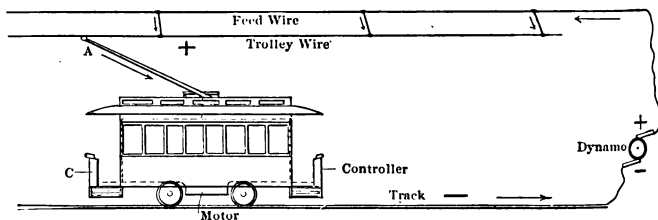


FIG. 431. — Electric Road System.

are perhaps the most familiar of motor-run objects, something may be said of their operation. Cars are nearly always run by D. C. series motors geared on the axles (Fig. 431). The current is supplied by compound dynamos installed at the power house, maintaining a P.D. of from 500 to 600 volts between the trolley wire and the track, or between the third rail and the track in elevated road cars. The motor-man applies the power by means of a controller, turning the handle so as to throw resistance in or out according to the speed of the car.

EXERCISES

1. What effects of alternating currents are the same as those of direct currents?

2. Why is a compound-wound dynamo best adapted to incandescent lighting systems?

3. Observe a motorman starting a trolley car. He turns the controller round by degrees. What effect would it have on the motor if he should turn the controller all the way around at once?

4. What effect does it have upon the direction of rotation of the armature of a motor to: (1) reverse the direction of the lines of force? (2) reverse the direction of the armature current? (3) reverse the direction of both (apply the motor rule)?

5. How do the directions of rotation of the armature of series-wound generators compare: (1) when run as a dynamo? (2) when run as a generator?

6. Why is it that the armature of a dynamo that requires a powerful engine to run it when its full load is on can be rotated by hand when the load is off?

7. What is the ratio of the number of windings on the primary and secondary coils of a transformer stepping 2200 volts down to 110 volts?

8. The reading on the voltmeter of an electric-lighting plant is 110 and on the ammeter is 200. What power is being taken from the dynamo?

9. An ammeter connected in series with a fan motor gives a reading of 7 amperes when the motor is started, but only .4 ampere after the motor is running at full speed. Account for this.

10. Seventy-five lamps are glowing when connected between the positive and neutral wires of a three-wire system, and 89 lamps are likewise glowing when connected between the negative and the neutral wires. If each lamp requires .5 ampere, what is the direction and amount of flow through each of the wires?

11. The resistance of the wire connecting a dynamo to a number of lamps is .05 ohm. The dynamo can deliver 200 amperes. (1) What is the drop in voltage between the dynamo and the lamps? (2) How many watts are expended in sending the current through the wire?

INDEX

- Abscissa, 59.
 Absolute scale, 175.
 Absolute zero, 175.
 Absorption, 101.
 Absorption spectra, 327.
 Acceleration, 63.
 Achromatic lenses, 323.
 Activity, 122.
 Adhesion, 95.
 Aërodynamics, 20.
 Aëroplane, 79.
 Aërostatics, 20.
 Air pump, 51.
 Air thermometer, 174.
 Alarm thermometer, 170.
 Alternator, 425, 454.
 Altitude and boiling point, 192.
 Ammeter, 382.
 Ampere, 382.
 Amplitude, 213.
 Analysis of sounds, 266.
 Aneroid, 41.
 Angle of deviation, 290; of incidence, 290; of reflection, 283; of refraction, 290.
 Anions, 378.
 Antinode, 218.
 Aperture of mirror, 200.
 Arc lamp, 448.
 Arc-light wiring, 459.
 Archimedes' principle, 31.
 Armature, 360, 452.
 Artesian wells, 28.
 Aspirator, 55.
 Astigmatism, 312.
 Astronomical telescope, 314.
 Atmospheric electricity, 357.
 Atmospheric pressure, 40.
 Atmospheric refraction, 258.
 Atom, 3.
 Atomizer, 55.
 Back electromotive force, 461.
 Back pressure, 206.
 Ballistic curves, 109.
 Balloons, 44.
 Barometer, 41.
 Beam balance, 83.
 Beam of light, 273.
 Beats, 250.
 Becquerel rays, 433.
 Bellows, 53.
 Bells, 268.
 Bending stress, 17.
 Bichromate cell, 395.
 Boat sailing, 78.
 Boiling, 189.
 Boyle's law, 42.
 Breast wheel, 147.
 Bright-line spectra, 327.
 Bugle, 254.
 Bunsen's photometer, 281.
 Buoyancy, 30.
 Buzzer, 437.
 Caisson, 56.
 Caloric, 179.
 Camera obscura, 310.
 Capillarity, 100.
 Cathode rays, 431.
 Cations, 378.
 Caustic by reflection, 302.

- Cells, connections of, 402.
 Center of curvature of mirror, 299.
 Center of gravity, 85; of mass, 85;
 of oscillation, 117; of percussion,
 118.
 Centigrade scale, 166.
 Centrifugal force, 113.
 Centrifugal pumps, 149.
 Centripetal force, 113.
 Charles's law, 173.
 Chromatic aberration, 323.
 Chromosphere, 328.
 Circle of reference, 213.
 Coherer, 442.
 Cohesion, 95.
 Cold storage plant, 194.
 Collecting rings, 425.
 Collimator, 326.
 Collision, 71.
 Color, 329.
 Commutator, 427.
 Compass, 367.
 Compensated balance wheel, 171.
 Composite bar, 170.
 Composition of forces, 77.
 Condenser, 193; of inductive coil,
 422.
 Condensers, capacity of, 354; electri-
 cal, 353; induction in, 353.
 Conduction of heat, 157.
 Conductors, classes of, 378; electric,
 335.
 Cone of illumination, 273.
 Conjugate foci, of lenses, 308; of mir-
 rors, 302.
 Conservation of energy, 125.
 Convection of heat, 157.
 Cooper-Hewitt lamp, 450.
 Cornet, 254.
 Corpuscle, 3.
 Corpuscular theory of light, 274.
 Coulomb's law, 346.
 Couple, 83.
 Critical angle, 296.
 Cubical expansion, 169.
 Current strength, measurement of, 408.
 Cymbals, 269.
 Dalton's law, 189.
 Daniell cell, 396.
 D'Arsonval galvanometer, 382.
 Davy lamp, 159.
 Dead points, 205.
 Decoherer, 443.
 Density, 12; electric, 340; and spe-
 cific gravity, 32.
 Depolarizers, 394.
 Dew point, 191.
 Diatonic scale, 243.
 Dielectrics, 335.
 Differential pulley, 141.
 Diffraction of light, 275; of sound,
 236; of waves, 223.
 Diffraction grating, 326.
 Diffused light, 284.
 Diffusion of gases, 91; of liquids, 93.
 Dipping needle, 368.
 Discord, 262.
 Dispersion of light, 320.
 Displacement, 31.
 Disruptive discharge, 355.
 Dissociation theory, 378.
 Distillation, 192.
 Divided circuits, 405.
 Diving bell, 55; suit, 56.
 Doppler's principle, 242.
 Drums, 269.
 Dry cell, 397.
 Ductility, 97.
 Dynamics, 20.
 Dynamo, 452; compound, 459;
 drum-armature, 454; principle of,
 423.
 Dynamo and motor, 460.
 Dynamos, classes of, 458.
 Dyne, 69.
 Echoes, 235; multiple, 235.
 Eddy currents, 416.
 Edison-Lalande cell, 398.

- Efficiency, 134; of steam engine, 205.
 Effort, 80, 133.
 Elasticity, 15, 96.
 Electric bell, 437; furnace, 446;
 wind, 340.
 Electrical conductors, distribution,
 340; machines, 351; stove, 446.
 Electricity, conservation of, 352; price
 of, 450.
 Electrification, 334; unit of, 339.
 Electrochemical equivalents, 391.
 Electrolysis, 389; of water, 399.
 Electrolytic conduction, 379.
 Electromagnet, 372.
 Electromagnetic induction, 414.
 Electromagnetism, 370.
 Electromotive force, 344; measure-
 ment of, 408.
 Electron, 3.
 Electrons, 434.
 Electrophorus, 350.
 Electroplating, 392.
 Electroscopes, 337.
 Electrotyping, 391.
 Element, 3.
 Emission theory of light, 274.
 Energy, 3, 124; dissipation of, 128;
 electric, 387; of stress, 126.
 Equilibrant, 80.
 Equilibrium, 87.
 Ether, 2, 270.
 Evaporation, 191.
 Expansion of gases, 173; of liquids, 174.
 Extension, 10.
 Extra current, 420.
 Eye, 311.
 Factor of safety, 97.
 Fahrenheit scale, 166.
 Fahrenheit's hydrometer, 35.
 Falling bodies, 103.
 Fire syringe, 202.
 Fleming's right-hand rule, 422.
 Floating bodies, 31.
 Flue pipes, 255.
 Fluid, 18.
 Fluorescence, 332.
 Fluoroscope, 433.
 Foot pound, 122.
 Force, 13; counter electromotive,
 461; driving, 133.
 Force pump, 50.
 Forced vibrations, 225.
 Foucault currents, 417.
 Four-cycle gas engine, 209.
 Franklin's experiment, 190; kite, 357.
 Fraunhofer lines, 328.
 Freezing point, 183; of solutions, 188.
 Friction, 129; in fluids, 131; on in-
 clined plane, 144; sliding, 130.
 Frictional electricity, 335.
 Fusing point, 183.
 Galilean telescope, 316.
 Galvanometer, 382.
 Galvanoscopes, 381.
 Gas, 17.
 Gas battery, 399; engine, 208.
 Gas-engine ignition, 446.
 Gas equation, 176.
 Gear wheels, 151.
 Geissler tubes, 431.
 Gongs, 269.
 Gram-calorie, 201.
 Gramme ring, 452.
 Gravitation, 110.
 Gravity cell, 3.
 Gravity wheel, 147.
 Hardness, 97.
 Harmony, 262.
 Heat, a form of energy, 153; nature
 of, 521; sources of, 154.
 Heat capacity, 178.
 Heat of fusion, 184; of solution, 187;
 of vaporization, 194.
 Heating in vacuo, 190.
 Hero's engine, 203.
 Hooke's law, 16.
 Horse power, 123; hour, 205.

- Hot-air furnace, 160.
 Hot-water heater, 161.
 Hydraulic crane, 151; press, 23; ram, 148.
 Hydrodynamics, 20.
 Hydrogen thermometer, 175.
 Hydrometers, 34.
 Hydrostatic paradox, 27.
 Hydrostatics, 20.
 Hypermetropia, 312.
 Hypothesis, 5.
- Ice calorimeter, 186; machines, 194.
 Illuminated body, 274.
 Illumination, 279.
 Images, 285; in curved mirrors, 303; by lenses, 307; multiple, 287.
 Impact, 70.
 Incandescent lamp, 447.
 Incandescent-light wiring, 459.
 Inclination, magnetic, 368.
 Inclined plane, 77; as machine, 143.
 Index of refraction, 291.
 Induced electromotive force, 417.
 Induced currents, 414.
 Induction coil, 420.
 Induction, electrostatic, 347; magnetic, 364.
 Inertia, 66.
 Insulator, 335.
 Interference, constructive, 322; destructive, 322; of sound, 249; of waves, 225; of light, 275.
 Interference fringes, 321.
 Internal reflection, 295.
 Ionization, 434.
 Ions, 378.
- JackscREW, 150.
 Joule's law, 388.
- Kettledrums, 269.
 Kilogram-calorie, 201.
 Kilogramm-meter, 122.
 Kilowatt, 123; -hour, 451.
- Kinetic energy, 126; theory, 90; theory and heat, 154.
- Lactometer, 35.
 Latent heat, 184.
 Law of boiling, 192; of conservation of matter, 4; of conservation of energy, 4; of gravitation, 110; of machines, 135; of motion, 65; of nature, 4.
 Laws of vibrations of strings, 254.
 Leclanché cell, 397.
 Lenses, 305; condensing, 310; principal foci of, 306; secondary axis of, 306.
 Lenz's law, 416.
 Lever, 81; as machine, 136.
 Levers, classes of, 137.
 Leyden jar, 354.
 Light propagation, 273.
 Lightning, 357.
 Limiting angle of friction, 145.
 Limits of audibility, 242.
 Linear expansion, 168.
 Line loss, 460.
 Line of direction, 87.
 Lines of force, electric, 345; magnetic, 362.
 Liquid, 18.
 Load, 133; of dynamo, 457.
 Local action in cells, 393.
 Longitudinal vibrations, 211.
 Longsightedness, 312.
 Loudness, 238.
 Luminous body, 274.
- Machine, 133.
 Magic lantern, 310.
 Magnetic field, 961; strength of, 363.
 Magnetic meridian, 368.
 Magnets, 359.
 Magnifying glass, 313.
 Magnifying power, 313.
 Major triads, 262.
 Malleability, 97.

- Manometers, 45.
 Manometric capsule, 267.
 Matter, 2; properties of, 10; states of, 17.
 Mechanical advantage, 134.
 Mechanical equivalent of heat, 199.
 Mechanics, 20.
 Melody, 262.
 Melting point, 183; and pressure, 187.
 Metallic thermometer, 171.
 Metronome, 118.
 Microphone, 444.
 Microscope, compound, 314; simple, 313.
 Mirrors, 284; curved, 299; parabolic, 301; plane, 284; principal axis of, 299; principal focus of concave, 300; principal focus of convex, 302; secondary axis of, 300; spherical, 299.
 Molecular forces, 90.
 Molecule, 3.
 Moment of force, 82.
 Momentum, 67.
 Monochord, 253.
 Morse key, 438.
 Motion, 58; curvilinear, 113; reflected, 71; simple harmonic, 212.
 Motor and dynamo, 460.
 Motor, principle of, 429.
 Musical glasses, 269.
 Musical intervals, 243; notation, 244; scales, 243.
 Myopia, 311.

 Nernst lamp, 448.
 Newton's laws of motion, 66.
 Newton's rings, 321.
 Nicholson's hydrometer, 36.
 Node, 217.
 Nonconductors, electric, 334.

 Ocular, 314.
 Ohm, 384.
 Ohm's law, 386.
 Opera glass, 316.

 Optical center, 306; density, 290.
 Ordinate, 59.
 Osmose of gases, 92.
 Osmotic pressure, 95.
 Overshot wheel, 147.
 Overtones of strings, 258; of wind instruments, 261.

 Parallel forces, 80.
 Parallelogram of velocities, 75.
 Pascal's law, 21.
 Pelton wheel, 147.
 Pencil of light, 273.
 Pendulum, 115; compensated, 170; conical, 212; of clock, 118.
 Penumbra, 276.
 Period of vibration, 213.
 Permeability, 365.
 Perpetual motion, 136.
 Phase, 213.
 Phenomenon, 1.
 Photographic camera, 309.
 Photometry, 280.
 Photosphere, 328.
 Physics, defined, 2; methods of, 4; subject matter of, 1.
 Pinhole camera, 278.
 Pitch, 239.
 Pneumatic appliances, 47.
 Polarization, 393.
 Potential, 342; energy, 126; zero of, 342.
 Power, 122.
 Presbyopia, 312.
 Pressure on fluids, 20.
 Primary cells, 392.
 Projectiles, 108.
 Projection lantern, 310.
 Proportion, 6.
 Pulleys, 138.
 Pumps, 49.
 Pure intonation, 264.
 Pyrometers, 167.

 Quality of sound, 253, 257.

- Radiant energy, 270.
 Radiation of heat, 157.
 Radioactivity, 433.
 Radiometer, 331.
 Radium, 434.
 Rainbow, 325.
 Range finding, 109.
 Ray of light, 273.
 Reaction, 69.
 Reaumur scale, 166.
 Reciprocating engine, 203.
 Recoil, 70.
 Reed pipes, 255.
 Refining of metals, 391.
 Reflecting prism binocular, 318.
 Reflection of light, 283; of sound, 234; of waves, 221.
 Refraction of light, 289; in prisms, 295; through a plate, 294.
 Refraction of sound, 236; of waves, 223.
 Relay, 440.
 Residual magnetism, 457.
 Resistance, 80; electric, 383; internal, 395; of a machine, 133; measurement of, 410; and temperature, 384, 391.
 Resistance box, 410.
 Resolution of velocities, 75.
 Resonance, 246.
 Resonators, 248.
 Resultant, 80.
 Retardation, 64.
 Reverberation, 235.
 Reversible cells, 398.
 Ring-armature dynamo, 452.
 Ring winding, 452.
 Rotor, 454.

 Safety valve, 204.
 Sal-ammoniac cell, 397.
 Savart's wheel, 239.
 Screw, 146.
 Secondary cells, 400.
 Seebeck's siren, 239.
 Selective absorption, 329.
 Self-excitation field, 457.
 Self-excited dynamo, 452.
 Self-induction, 420.
 Series dynamo, 458.
 Shadowgraphs, 433.
 Shadows, 276.
 Sheave, 138.
 Shortsightedness, 311.
 Shunt dynamo, 459.
 Sine curve, 219.
 Siphons, 47.
 Snell's law, 291.
 Solenoid, 371.
 Solid, 18.
 Solution, 93.
 Solution tension, 378.
 Sonometer, 253.
 Sound media, 229.
 Sound and vibrations, 227.
 Sounder, 438.
 Spark coil, 420.
 Specific gravity, 32.
 Specific heat, 178.
 Specific resistance, 384.
 Spectra, 327; continuous, 327; dark-line, 327; discontinuous, 327.
 Spectroscope, 326.
 Spectrum, 321; analysis, 328.
 Speed, 60; of light, 276; of sound, 230; ratio, 134.
 Spherical aberration of lenses, 307; of mirror, 301.
 Spherical waves, 220.
 Spyglass, 316.
 Standard pitch, 244.
 Statics, 20.
 Stator, 454.
 Steady currents, 374.
 Steam calorimeter, 196.
 Steam engine, 203; compound, 207.
 Steelyard, 83.
 Stereoscope, 318.
 Stereoscopic vision, 317.
 Still, 193.
 Storage cell, 400.

- Strain, 15.
- Stress, 15; compressive, 17.
- Sublimation, 188.
- Surface expansion, 169; tension, 199; viscosity, 100.
- Sympathetic vibrations, 224.
- Synthesis of sounds, 268.

- Tangent galvanometer, 382.
- Tangential wheel, 147.
- Telegraph, 438.
- Telephone, 443.
- Telescope, 314.
- Temperature, 156; measurement of, 163.
- Tempered scale, 264.
- Tenacity, 96.
- Tensile strength, 96; stress, 17.
- Terrestrial magnetism, 367.
- Terrestrial telescope, 316.
- Thermo-electric couple, 388.
- Thermometer, 163.
- Thermometric scales, 166.
- Thermopile, 167.
- Theory, 5.
- Thomson E. M. F., 389.
- Three-wire system, 460.
- Torricelli's experiment, 39.
- Torsional vibrations, 211.
- Total reflection, 296.
- Trade winds, 161.
- Transformer, 418.
- Transformers, commercial, 455.
- Transverse vibrations, 211.
- Triangle of velocities, 75.
- Trombone, 255.
- Tubular boiler, 204.
- Tungsten lamp, 447.
- Turbine steam engine, 207.
- Turbine wheel, 148.
- Two-cycle gas engine, 209.
- Tympani, 269.

- Umbra, 276.
- Undershot wheel, 147.

- Uniform acceleration, 64.
- Units, metric, 11.
- Uranium rays, 433.

- Vapor pressure, 188.
- Vaporization, 188.
- Variation, 6.
- Vector, 76.
- Velocity ratio, 134.
- Ventilation, 161.
- Vibration of plates, 268; of strings, 252.
- Vision, 312; defects of, 311.
- Visual angle, 279.
- Volt, 343.
- Voltaic cell, 375.
- Voltmeter, 408.

- Water equivalent, 180; power, 146; waves, 215; works, 28.
- Watt, 123.
- Watt-hour, 451.
- Wave-length measurements, 322.
- Wave motion, 211; speed, 218; trough, 215.
- Waves, circular, 220; plane, 220; stationary, 216.
- Waves and energy, 226.
- Waves in cords, 217.
- Wedge, 145.
- Weight, 14, 111.
- Welding, electric, 446.
- Wheatstone bridge, 411.
- Wheel and axle, 137.
- Whispering galleries, 235.
- Wind, electric, 341.
- Wind instruments, 254.
- Windlass, 138; geared, 151.
- Wireless telegraph, 441.
- Work, 121.
- Worm condenser, 193.
- Worm wheel, 150.

- X-rays, 432.

- Zeiss binocular, 318.
- Zeppelin airship, 45.









