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# On the average chord length in reactor physics

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## Abstract

By elaborating on the meaning of the average chord length in reactor physics, it is shown that the average chord length for a convex body in an isotropic flux is given by the body's volume divided by its average projection area. The relation is known in literature (Weinberg, A.M., Wigner, E.P., 1958. *The Physical Theory of Neutron Chain Reactors*. The University of Chicago Press, Chicago), but in view of a recent technical note on the average chord length by Sjöstrand [Ann Nucl Eng 29 (2002) 1607] it seems useful to explain the background of the simplicity of the average chord length for a convex body in an isotropic flux. For another angular flux distribution the average chord length cannot be expressed in such an elegant way, and has to be calculated for each body (and orientation) separately.

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## 1. Introduction

The first-flight escape probability, which is used in reactor physics to evaluate resonance integrals, can be related to the averaged chord length of the fuel lump. In a recent paper Sjöstrand (2002) warns against the uncritical use of the equation for the average chord length,  $R_{av}$ , of a convex body (Czuber, 1884):

$$R_{av} = \frac{4V}{S}, \quad (1)$$

where  $V$  is the volume of the body and  $S$  the outer surface. This is the expression that one finds if one assumes an isotropic flux distribution at the boundary, which gives for the angular current through the boundary surface area a cosine distribution

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with respect to the inward normal. However, when Sjöstrand remarks that the average chord length of a sphere in an isotropic flux equals  $r$ , the radius of the sphere, he misinterprets the assumption of an isotropic flux. In fact, he assumes an isotropic angular *current* through the surface area to calculate the average chord length, for which the number of particles per unit area of the body surface is indeed proportional only to the solid angle. It is analogous with a uniform isotropic surface source at the boundary surface area. However, this case is not very useful in reactor physics, because an isotropic angular current distribution is not realistic. Note that according to Eq. (1), the average chord length for a sphere in an isotropic flux is  $\frac{4r}{3}$ .

Elaborating on the technical note by Sjöstrand (2002), we obtained some useful insight into the average chord length presented in this technical note.

## 2. Two-dimensional case

Let us start with the simple case of a circle with radius  $r$ . Intuitively, one inclines to calculate the chord length by taking a point on the circle, randomly choosing another point on the circle, and calculating the distance between the two. Repeating this process and taking the average, one finds for the chord length  $l_{av} = \frac{4r}{\pi}$ . This procedure gives a chord length equivalent to that obtained assuming an isotropic angular (2D) current through the circle perimeter or a uniform isotropic (2D) line source on the circle perimeter.

Suppose now that we take this circle and draw equidistant parallel lines through it as is done in Fig. 1 for an arbitrary convex figure. Then the average chord length for a circle equals  $l_{av} = \frac{\pi r}{2}$ . The same result is obtained if one would weight the chords in our first example with the cosine of the angle between the chord and the normal.

Now let us continue to the general case of a convex two-dimensional figure, and assume that the number of chords is proportional to the cosine of the angle between the chord and the inward normal on the boundary, as would be the case for an isotropic neutron flux. In analogy with the derivation by Dirac (In fact, we have used Case et al. 1953 instead), the probability that a chord is between length  $l$  and  $l + dl$  is given by:

$$\phi(l)dl = \frac{\int \int_{l_s=l} \vec{m} \cdot \vec{n} d\theta ds}{\int \int \vec{m} \cdot \vec{n} d\theta ds}, \quad (2)$$

with  $\vec{m}$  the direction of the chord,  $\vec{n}$  the inner surface normal,  $\theta$  the polar angle between  $\vec{m}$  and an arbitrary plane with fixed orientation, and  $s$  the position on the boundary of the figure, and with the restriction that  $\vec{m} \cdot \vec{n} > 0$ . Since

$$\int \int \vec{m} \cdot \vec{n} d\theta ds = \int \int_{-\pi/2}^{\pi/2} \cos(\varphi) d\varphi ds = 2S, \quad (3)$$

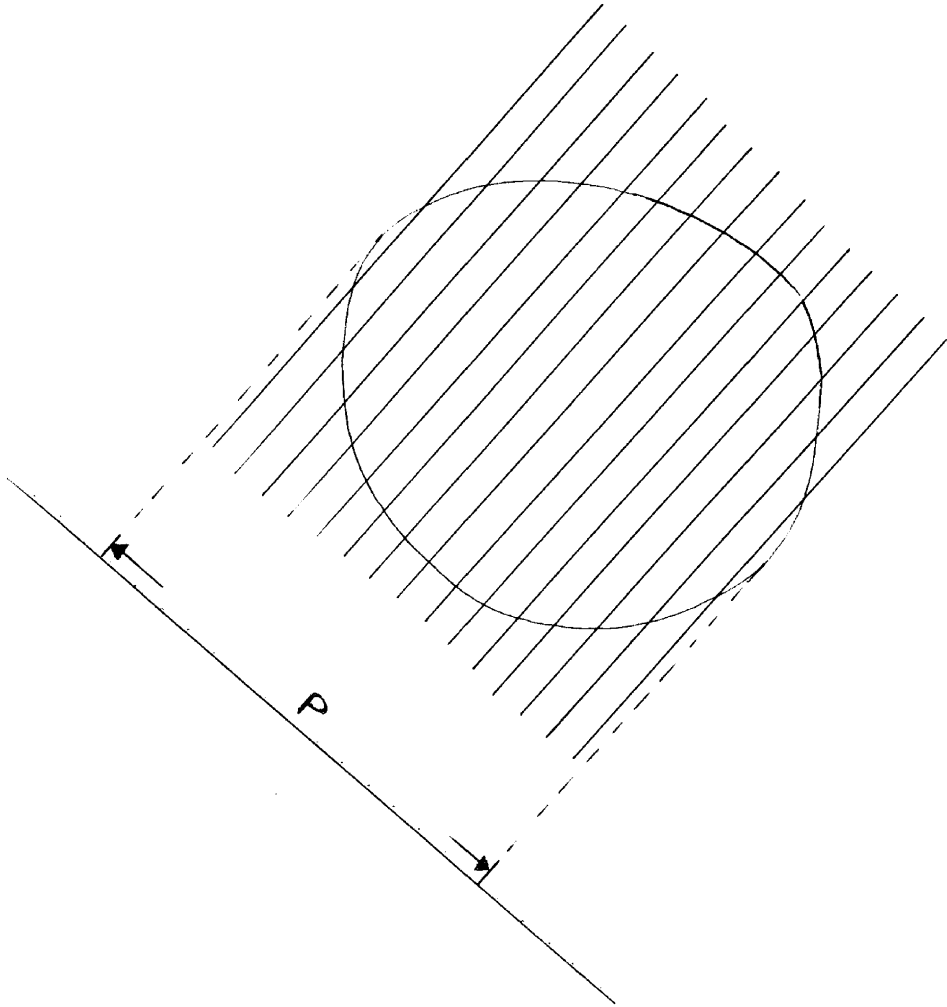


Fig. 1. Arbitrary convex figure with chords drawn for one angle, and projection  $p$  on the line normal to the chords.

with  $\varphi$  the angle between the chord and the normal ( $\vec{m} \cdot \vec{n} = \cos(\varphi)$ ), and  $S$  the perimeter of the figure, the average chord length is given by:

$$l_{av} = \frac{\int \int l(\theta, s) \vec{m} \cdot \vec{n} d\theta ds}{2S}. \quad (4)$$

And because:

$$\int l(\theta, s) \vec{m} \cdot \vec{n} ds = A, \quad (5)$$

with  $A$  the area of the figure, the average chord length becomes:

$$l_{av} = \frac{A\pi}{S}. \quad (6)$$

Another way of looking at it is by considering the average projection of the figure onto a line segment perpendicular to the direction of the chords as shown in Fig. 1. The average projection length is given by:

$$p_{av} = \frac{\int \int \vec{m} \cdot \vec{n} d\theta ds}{\int d\theta} = \frac{S}{\pi}. \quad (7)$$

The average chord length is thus given by:

$$l_{av} = \frac{A}{p_{av}}. \quad (8)$$

### 3. Three-dimensional case

The derivation for the general three-dimensional case for an isotropic flux is extensively treated in literature and is similar to the one above (e.g. Case et al., 1953). However, we would like to point at the relation between the average chord length and the average projection area of a convex body,  $P_{av}$  (Weinberg and Wigner, 1958). The latter can be obtained by averaging the projection area in the direction  $\Omega$  of the chords over all possible directions. With the restriction that  $\Omega \cdot \vec{n} > 0$  this gives (Cauchy, 1908):

$$P_{av} = \frac{\int \int \vec{\Omega} \cdot \vec{n} d\Omega ds}{\int d\Omega} = \frac{S}{4}, \quad (9)$$

with  $s$  the position on the surface,  $\vec{n}$  the normal, and  $S$  the total surface area, which means that in analogy with the two-dimensional case the average chord length can be given by:

$$R_{av} = \frac{V}{P_{av}}. \quad (10)$$

This is a useful equation in obtaining some insight into the meaning of Eq. (1). The background of the simplicity of Eq. (1) lies in the fact that *the average projection of a convex body onto a plane is equal to one quarter of its surface area*. This in analogy with the fact that *the average projection of a convex two-dimensional figure onto a line segment is equal to the perimeter divided by  $\pi$* . Although having extensively used Eq. (1) in the past, the authors were not aware of these relations.

For a non-isotropic angular flux distribution the average chord length cannot be expressed in such an elegant way, and has to be calculated for each body (and orientation) separately. An orientation-independent equation for the average chord length for a realistic case can only be found for an isotropic flux distribution or for a sphere. The case of an isotropic angular current distribution is unrealistic.

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