Physics I
The Textbook

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Chapter 1

Foundations

Week 1

0.1 · What Physics Is

By the end of this section you can: describe what physics studies; explain why measurement sits at the center of it; and describe how this book is organized to help you learn.

Physics is the study of how the world moves and why. It is the most basic of the sciences, because it asks the simplest questions. What is motion? What is force? What is energy? Every other science sits on top of the answers. A chemist studying a reaction, a biologist studying a beating heart, and an engineer designing a bridge are all leaning on the rules you will learn this year.

Here is the promise of the course, and it is a big one. A small handful of ideas, written as short equations, describe an enormous range of the world. The same rule that predicts how far a dropped ball falls also predicts how a car skids to a stop, how a rocket climbs, and how a planet circles the sun. You will not memorize a hundred disconnected facts. You will learn a few powerful ideas and practice using them in new situations. That is what it means to think like a physicist.

I spent six years as an engineer building instruments in a supersonic wind tunnel, and the thing I want you to take from this chapter is the thing that job taught me. **The measurement is the physics.** A beautiful theory means nothing until you can measure the world and check it. Before we study motion, forces, or energy, we spend a week on the humble skills that make everything after it trustworthy: units, measurement, significant figures, graphs, and a clear way to work a problem. This is the toolbox. Take the time to sharpen it now.

A tool of the trade: a Swiss mechanical stopwatch. Look closely at the dial — it is divided into 100 parts, not 60, so each tick is a hundredth of a minute. Physics begins with careful measurement, and careful measurement begins with knowing exactly what your instrument's marks mean.

How this book works

Every chapter follows the same rhythm, so you always know where you are. Each section opens with a short list of what you should be able to do. Worked examples show the thinking, not just the answer, and many of them star Sparky, our lab robot, who makes the same mistakes you might. Watch for three kinds of boxes: a blue box adds a helpful connection, a red box warns you about a common trap, and a green box pauses to consider how the order of creation points beyond itself. Every section ends with a short "Check yourself" you can open to test your understanding before moving on.

Check yourself: 0.1

1. In one sentence, what does physics study?
   How the world moves and why, described by a small set of basic rules about motion, force, and energy.
2. Why does this book spend a week on measurement before studying motion?
   Because a theory is only trustworthy once you can measure the world and check it. Measurement is the foundation the rest of the course is built on.

0.2 · Units and the SI System

By the end of this section you can: name the SI base units for length, mass, and time; attach correct units to every measurement; and convert between units with the factor-label method.

A number by itself is almost useless in physics. If a friend says a hallway is "12" long, you have to ask, twelve what? Twelve feet is a small room. Twelve meters is a large classroom. Twelve miles is a road trip. The number is only half of the answer. The **unit** is the other half, and it is not optional. In this class, a number written without a unit is treated as wrong, because it is.

Scientists everywhere use one shared system so their measurements mean the same thing in every country. It is the **SI system** (from the French *Système International*), often called the metric system. Three base units carry most of physics:

Quantity	SI base unit	Symbol	A feel for its size
length	meter	m	a little longer than a yard; a doorknob is about 1 m high
mass	kilogram	kg	about the mass of a full 1-liter water bottle
time	second	s	about one relaxed heartbeat

Every other unit you meet this year is built from these three. Speed is meters per second (m/s). Force, as you will see in Chapter 4, is measured in newtons, and one newton is just $1\ \text{kg} \cdot \text{m/s}^2$ in disguise. These built-from-base units are called **derived units**. Because everything traces back to meters, kilograms, and seconds, physics problems have a hidden safety check: if you carry units through your work and the final units come out wrong, you know you made a mistake before you ever check the number.

Metric prefixes

The metric system scales by tens, which makes big and small numbers easy to name. A prefix in front of a unit multiplies it by a power of ten. These five cover most of what you need:

Prefix	Symbol	Means	Example
kilo-	k	$\times$ 1000	1 km = 1000 m
centi-	c	$\times$ 0.01	1 cm = 0.01 m
milli-	m	$\times$ 0.001	1 mm = 0.001 m
micro-	μ	$\times$ 0.000001	1 μs = 0.000001 s
nano-	n	$\times$ 0.000000001	1 nm = 0.000000001 m

Converting units the safe way

The reliable way to change units is the **factor-label method**, also called dimensional analysis. The idea is simple. Multiplying by a fraction equal to 1 never changes the true size of a quantity, only the units it wears. A conversion factor like $\frac{1000\ \text{m}}{1\ \text{km}}$ equals 1, because the top and bottom are the same length. Set up the fraction so the unit you want to cancel sits on the opposite level, and it divides out.

Sparky rolls at 5.0 meters per second. How fast is that in kilometers per hour? Change meters to kilometers, then seconds to hours, one factor at a time so each unit cancels:

$$5.0\ \tfrac{\text{m}}{\text{s}} \times \frac{1\ \text{km}}{1000\ \text{m}} \times \frac{3600\ \text{s}}{1\ \text{h}} = \boxed{18\ \text{km/h}}$$

Check the units: meters cancel meters, seconds cancel seconds, and what is left is km/h, exactly what we wanted. Check the size: 18 km/h is a brisk walk, which feels right for a rolling robot. When the units land right and the size makes sense, you can trust the number.

How many centimeters are in 3.5 m? Use the factor that cancels meters and leaves centimeters (100 cm = 1 m):

$$3.5\ \text{m} \times \frac{100\ \text{cm}}{1\ \text{m}} = \boxed{350\ \text{cm}}$$

Going to a smaller unit gives a bigger number, which makes sense: it takes more centimeters to cover the same length.

How many seconds are in 2.0 hours?

$$2.0\ \text{h} \times \frac{3600\ \text{s}}{1\ \text{h}} = \boxed{7200\ \text{s}}$$

Time conversions trip people up because minutes and hours are not powers of ten. Keep the factors (60 s/min, 3600 s/h) handy.

A car drives 72 km/h. Convert this to m/s.

$$72\ \tfrac{\text{km}}{\text{h}} \times \frac{1000\ \text{m}}{1\ \text{km}} \times \frac{1\ \text{h}}{3600\ \text{s}} = \boxed{20\ \text{m/s}}$$

Or just divide by 3.6. Since physics works in m/s, get comfortable turning any speed into these units.

Misconception check: a conversion factor must equal 1, with the same real amount on top and bottom (1000 m and 1 km are the same distance). Flipping a factor upside down is the most common error. If your answer comes out wildly too big or too small, you probably multiplied where you should have divided. Write the units every step and let them police you.

Check yourself: 0.2

1. Why is a number without a unit treated as wrong in physics?
   The unit carries half the meaning. "12" could be meters, miles, or seconds; without the unit the measurement says nothing definite.

2. Convert 90 km/h to m/s.
   90 km/h × (1000 m / 1 km) × (1 h / 3600 s) = 25 m/s.
3. How many millimeters are in 2.5 m?
   2.5 m × (1000 mm / 1 m) = 2500 mm.

0.3 · Measurement, Precision, and Significant Figures

By the end of this section you can: explain the difference between accuracy and precision; record a measurement to the right number of digits; and apply the significant-figure rules to a calculation.

No measurement is perfect. Every ruler, scale, and stopwatch has a limit, and an honest measurement admits that limit instead of hiding it. Two words name the two ways a measurement can be good. **Accuracy** is how close your value is to the true value. **Precision** is how tightly repeated measurements agree with each other. They are not the same. A cheap scale that always reads 2.0 kg high is precise but not accurate. A dart player who scatters darts all around the bullseye is neither. The goal is both.

When you read an instrument, record every digit you know for certain, plus one last digit that you estimate between the smallest marks. Those trustworthy digits are the **significant figures**, and they are a promise about how well you actually know the number. Writing "3.0 m" and writing "3.00 m" are different claims. The second one says you measured ten times more finely. Do not write digits you did not earn.

Counting significant figures

- Every nonzero digit counts. 452 has three.
- Zeros between nonzero digits count. 4005 has four.
- Leading zeros never count; they only place the decimal. 0.007 has one.
- Trailing zeros after a decimal point count. 2.50 has three, and that third digit is a real claim of precision.

Carrying significant figures through math

Your answer cannot be more precise than the least precise measurement you started with. You cannot manufacture certainty by multiplying. Two rules cover almost everything:

- **Multiplying or dividing:** the answer keeps as many significant figures as the measurement with the fewest.
- **Adding or subtracting:** the answer keeps as many decimal places as the measurement with the fewest decimal places.

Sparky measures a desktop as 1.2 m long and 0.75 m wide. What is its area? Area is length times width: $1.2\ \text{m} \times 0.75\ \text{m} = \boxed{0.90\ \text{m}^2}$. The calculator says 0.9, but we write **0.90 m²**. Here is the reasoning. The measurement 1.2 m has only two significant figures, so the answer may keep only two. Reporting 0.9 would throw away precision we earned; reporting 0.9000 would claim precision we never had. Two significant figures, no more and no less, is the honest answer.

How many significant figures are in each: 452, 4005, 0.007, and 100.0?
452 has **three** (all nonzero digits count). 4005 has **four** (zeros trapped between nonzero digits count). 0.007 has **one** (leading zeros only place the decimal). 100.0 has **four** (the trailing zero after the decimal is a real claim of precision).
The decimal point is the key: it tells you whether trailing zeros are meaningful or just placeholders.

Compute 4.56×1.4 to the correct number of significant figures. The calculator gives 6.384. The factor 1.4 has only two significant figures, so the answer may keep only two:

$$4.56 \times 1.4 = \boxed{6.4}$$

The least precise measurement sets the limit. You cannot end up knowing the product better than you knew one of the things you multiplied.

Add $12.11\ \text{m} + 1.2\ \text{m}$ to the correct precision. The exact sum is 13.31, but addition follows the *decimal-places* rule, not the sig-fig rule. The value 1.2 has only one decimal place, so the answer keeps one:

$$12.11 + 1.2 = \boxed{13.3\ \text{m}}$$

Notice the difference from multiplying: here we count decimal places, because you can only add digits that both measurements actually reached.

Misconception check: significant figures are not a picky rule invented to cost you points. They are how a scientist states honestly how well something is known. A bridge engineer who rounds carelessly and a lab that reports fake precision are both lying with numbers, just in opposite directions. Report exactly what you measured, no more and no less.

Check yourself: 0.3

1. What is the difference between accuracy and precision?
   Accuracy is closeness to the true value. Precision is how tightly repeated measurements agree with each other.
2. How many significant figures are in 0.03060?
   Four: the 3, 0, 6, and the trailing 0 after the decimal. The two leading zeros do not count.

3. Compute 3.14 m × 2.0 m to the correct number of significant figures.
   6.3 m^2 (two significant figures, limited by the 2.0).

0.4 · Scientific Notation and Estimation

By the end of this section you can: write very large and very small numbers in scientific notation; and make a quick order-of-magnitude estimate to sanity-check an answer.

Physics runs from the tiny to the enormous. The mass of an electron is a decimal point followed by thirty zeros and then a 9. The distance to the sun is about 150,000,000,000 meters. Writing those out is clumsy and easy to get wrong. **Scientific notation** tames them by splitting a number into a digit part between 1 and 10 times a power of ten.

The distance to the sun becomes 1.5×10^{11} m. The rule is quick: the exponent counts how many places the decimal moves. A positive exponent means a large number, so the decimal moves right. A negative exponent means a small number, so the decimal moves left. So 3×10^5 is 300,000, and 3×10^{-4} is 0.0003. Scientific notation also makes significant figures honest, because you write only the digits you actually know.

Estimation, the physicist's superpower

Before you trust any answer, ask whether it is even in the right ballpark. An **order-of-magnitude estimate** is a rough calculation using round numbers, aimed only at the nearest power of ten. It will not give you the exact answer. It will tell you instantly if your exact answer is nonsense. If you calculate that a thrown baseball travels at 4000 m/s, estimation should set off an alarm, because that is faster than a rifle bullet. A good physicist checks the size of an answer as reflexively as checking a mirror before changing lanes.

Roughly how many seconds are in a school year of 180 days? Round and multiply powers of ten: about 180 days × 24 h × 3600 s $\approx 2 \times 10^1 \times 2 \times 10^1 \times 4 \times 10^3 \approx \boxed{1.6 \times 10^7 \text{ s}}$. So a school year is roughly ten million seconds. The exact figure is about 15.6 million, so our estimate landed in the right ballpark. That is all an estimate promises, and it is enough to catch a wrong answer.

Write 93,000,000 in scientific notation. Move the decimal left until one nonzero digit remains in front of it, and count the moves: 9.3, having moved 7 places.

$$93{,}000{,}000 = \boxed{9.3 \times 10^7}$$

A large number gets a positive exponent. The exponent is just how many places the decimal traveled.

Write 2.5×10^{-3} as an ordinary number. A negative exponent means a small number, so the decimal moves left 3 places:

$$2.5 \times 10^{-3} = \boxed{0.0025}$$

Reading the sign of the exponent tells you instantly whether the number is large (positive) or tiny (negative).

Compute $(2 \times 10^3) \times (3 \times 10^4)$. Multiply the digit parts and add the exponents:

$$(2 \times 3) \times 10^{3+4} = \boxed{6 \times 10^7}$$

This is why scientific notation is so handy: huge multiplications become one small multiply plus a little addition.

Check yourself: 0.4

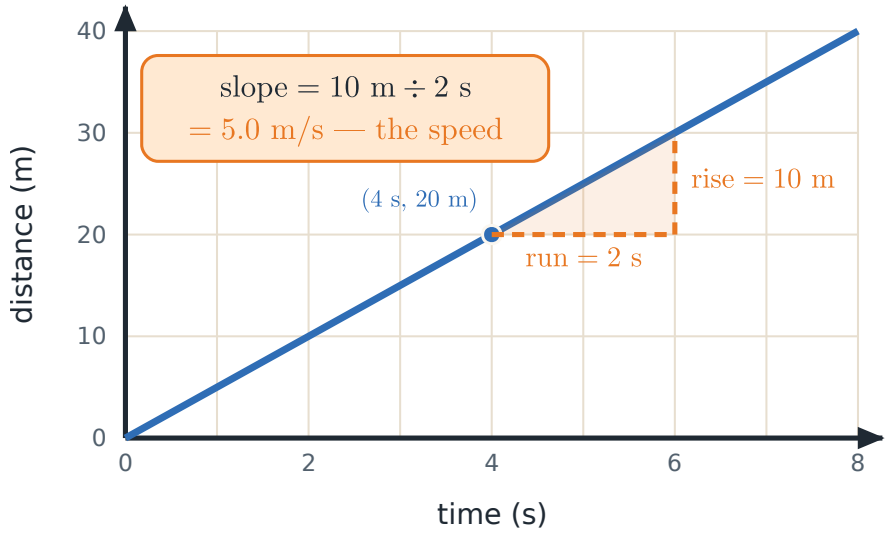
1. Write 0.00047 in scientific notation.
   4.7×10^{-4}.
2. Write 6.2×10 as an ordinary number.
   6,200,000.
3. Why is estimation worth doing even when you have a calculator?
   It catches nonsense answers. If the exact result is far from the estimate, you know a mistake slipped in.

0.5 · Reading the Story in a Graph

By the end of this section you can: identify the variables on a graph's axes; interpret the slope of a line; and explain what a straight-line graph tells you about two quantities.

Physics is a science of relationships. When one thing changes, what happens to another? A graph is the clearest way to see a relationship at a glance, and this year you will read graphs constantly, especially in the next chapter on motion. So it is worth learning to read them well now.

Two features carry most of the meaning. The **slope** is the steepness of the line, found by dividing the rise (the change up the vertical axis) by the run (the change across the horizontal axis). Slope answers "how fast is the vertical quantity changing as the horizontal one changes?" A steep line means rapid change; a flat line means no change at all. The second feature is shape. A straight line means the two quantities are **proportional**, changing at a steady rate. A curve means the rate itself is changing, which is a story all its own.



Slope tells the story. *On a distance-versus-time graph, the slope (rise over run) is the speed. A steeper line means a faster motion. You will use this idea all through Chapter 1.*

Axes carry units too. Always read what each axis measures and in what unit before you interpret a graph. The slope of a distance (m) versus time (s) graph has units of m/s, which is a speed. The units of the slope tell you what the slope physically means. That is a theme you will see again and again.

On a distance-versus-time graph, a line runs straight from the origin to the point (4.0 s, 20 m). Find the slope and say what it means.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{20\ \text{m}}{4.0\ \text{s}} = \boxed{5.0\ \text{m/s}}$$

The units of the slope are m/s, so the slope *is* a speed: this object moves at a steady 5.0 m/s.

A graph of temperature (°C) versus time (min) is a straight line that rises 6.0°C over 3.0 min. Find the slope and interpret it.

$$\text{slope} = \frac{6.0\ ^\circ\text{C}}{3.0\ \text{min}} = \boxed{2.0\ ^\circ\text{C/min}}$$

The slope's units, °C per minute, tell you it is a heating rate: the temperature climbs 2.0°C every minute. Reading the units of a slope always tells you what it physically means.

Check yourself: 0.5

1. How do you calculate the slope of a line?

Divide the rise (change on the vertical axis) by the run (change on the horizontal axis).

2. On a distance-versus-time graph, what does a steeper line mean?
   A faster speed, because more distance is covered in the same time.
3. What does a straight-line graph tell you about the two quantities?
   They are proportional: one changes at a steady rate as the other changes.

0.6 · A Problem-Solving Framework

By the end of this section you can: use a consistent method to work any physics problem; and explain why showing your work matters more than the final number.

Most students who struggle in physics do not struggle with the math. They struggle because they jump straight to hunting for an equation before they understand the situation. The cure is a habit. Use the same steps on every problem until they become automatic. We call it the **GUESS** method, and it is less about guessing than about being organized.

1. **G: Given.** List every number the problem tells you, with its unit and symbol.
2. **U: Unknown.** Write down exactly what you are solving for, with its symbol and unit.
3. **: Sketch.** Draw the situation before you touch an equation. GUESS is spelled with five letters but worked in six steps: the Sketch is **the silent S**. You do not hear it in the name, but I had better see it on your paper. Once we reach forces in Chapter 4, this sketch grows up into a free-body diagram.
4. **E: Equation.** Choose a relationship that connects your givens to your unknown.
5. **S: Substitute.** Put the numbers in, units and all.
6. **S: Solve.** Do the arithmetic, then check the units and the size of the answer.

Notice that nearly everything happens before you touch the arithmetic. That is the point. The thinking is the physics; the arithmetic is just bookkeeping. It is also why you show your work in this class. A correct final number with no work earns little credit, because it does not show that you understood anything. A clear solution with a small arithmetic slip earns most of the credit, because the reasoning, which is the part that matters, is right there on the page.

Sparky travels 150 m in 30 s at a steady speed. What is his speed? Given: distance d = 150 m, time t = 30 s. **Unknown:** speed s = ? in m/s. **Sketch:** a straight line with Sparky at one end, 150 m marked off, an arrow showing his direction. **Equation:** speed is distance over time, $s = \frac{d}{t}$. **Substitute:** $s = \frac{150\ \text{m}}{30\ \text{s}}$. **Solve:** $s = \boxed{5.0\ \text{m/s}}$. Check: the units are m/s, which is a speed, and 5 m/s is a reasonable rolling pace. The method looks like overkill on an easy problem, and that is exactly when to practice it. By the time the problems get hard, the habit will already be yours.

Faith & Science , "He determines the number of the stars; he gives to all of them their names" (Psalm 147:4). Physics rests on a quiet assumption that we rarely stop to notice. The universe is orderly. The same laws hold yesterday and today, in this classroom and across the galaxy, or none of our measuring and predicting would work at all. That dependability is not something science proves; it is something science leans on before it starts. For a believer, the order we measure is not a lucky accident but the fingerprint of a faithful Creator, the One who numbers the stars and holds them in their courses. Studying that order carefully, and measuring it honestly, is one good way to pay attention to it.

Practice Problems

Work these on your own, then open the solutions to check yourself. Show the setup, carry units, and mind your significant figures.

1. Convert 250 cm to meters.
2. How many seconds are in 45 minutes?
3. A runner moves at 36 km/h. Convert this to m/s.
4. How many significant figures are in each: 3080, 0.0025, and 6.500?
5. Compute $5.00\ \text{m} \times 2.0\ \text{m}$ to the correct number of significant figures.
6. Add $7.25\ \text{s} + 0.3\ \text{s}$ to the correct precision.
7. Write 320,000 in scientific notation.
8. Write 4.1×10^{-2} as an ordinary number.
9. On a distance-versus-time graph, a straight line runs from the origin to (12 s, 60 m). Find the slope and state what it represents.

Solutions to the Practice Problems

1. $250\ \text{cm} \times \dfrac{1\ \text{m}}{100\ \text{cm}} = 2.5\ \text{m}$.
2. $45\ \text{min} \times \dfrac{60\ \text{s}}{1\ \text{min}} = 2700\ \text{s}$.
3. $36 \div 3.6 = 10\ \text{m/s}$ (or $36 \times \frac{1000}{3600}$).
4. 3080 has **three** (trailing zero with no decimal does not count); 0.0025 has **two**; 6.500 has **four**.
5. $5.00 \times 2.0 = 10$, but limited by 2.0's two sig figs: **10.** m^2 (two significant figures).
6. Addition uses decimal places: 0.3 has one, so $7.25 + 0.3 = 7.6$ s.
7. $320{,}000 = 3.2 \times 10^5$.
8. $4.1 \times 10^{-2} = 0.041$.
9. $\text{slope} = \dfrac{60\ \text{m}}{12\ \text{s}} = 5.0\ \text{m/s}$; the slope is a speed.

Challenge Problems

The hard ones. Each of these takes more than one step and more than one idea; some borrow tools from earlier chapters. Work them in GUESS order (Given, Unknown, Sketch, Equation, Substitute, Solve) before opening the solutions.

1. **Sunlight Is Old News.** Light travels at 3.0×10^8 m/s. The Sun is 1.5×10^{11} m from Earth. When you look at the Sun, how old is the light you are seeing? Answer in minutes.

Scan to watch: Sunlight Is Old News — how old is the light hitting your eye?

2. **A Lifetime of Heartbeats.** A typical heart beats about 72 times per minute. Estimate how many times a heart beats in a 75-year lifetime. Give the answer in scientific notation.

Scan to watch: A Lifetime of Heartbeats (worked example)

3. **The Whole Story in One Graph.** A distance-versus-time graph shows three straight segments: from (0, 0) to (20 s, 150 m), then flat until 35 s, then rising to (50 s, 300 m). Find the speed during each segment and the average speed for the whole trip. Why is the average not just the average of the three speeds?

Scan to watch: The Whole Story in One Graph

4. **One Field, Two Rules.** A student measures a rectangular field as 105.6 m long and 38 m wide. Find (a) the perimeter and (b) the area, each to the correct precision. Then explain why the area must be written in scientific notation to be honest.

Scan to watch: One Field, Two Rules (sig figs)

Solutions to the Challenge Problems

1. **Sunlight Is Old News.** Rearrange $s = d/t$ for time: $t = d/v = (1.5\times10^{11})/(3.0\times10^{8}) = 500$ s. Convert: $500 \times \frac{1\text{ min}}{60\text{ s}} \approx$ **8.3 minutes**. Divide the powers of ten first and the arithmetic is easy; that is the whole point of scientific notation.

2. **A Lifetime of Heartbeats.** Chain the conversions: $72\ \frac{\text{beats}}{\text{min}} \times 60\ \frac{\text{min}}{\text{h}} \times 24\ \frac{\text{h}}{\text{day}} \times 365\ \frac{\text{days}}{\text{yr}} \times 75\text{ yr} \approx 2.8\times10^{9}$ **beats**, about three billion. Estimation is just unit conversion with courage.

3. **The Whole Story in One Graph.** Slopes: $150/20 = 7.5$ m/s, then 0, then $150/15 = 10$ m/s. Average speed is TOTAL distance over TOTAL time: $300/50 =$ **6.0 m/s**. Averaging the three speeds gives 5.8 m/s, which is wrong because the segments last different amounts of time.

4. **One Field, Two Rules.** Perimeter (addition rule, match the least precise decimal place): $105.6 + 105.6 + 38 + 38 = 287.2 \rightarrow$ **287 m**, because 38 m is only known to the ones place. Area (multiplication rule, match the fewest sig figs): $105.6 \times 38 = 4012.8 \rightarrow 4.0\times10^{3}$ **m**^{2}, two figures. Writing 4000 is ambiguous; 4.0×10^{3} says exactly what we know. Same field, two operations, two rules.

Chapter 0 Summary

- Physics studies how the world moves and why, using a small set of powerful ideas. Measurement is the foundation that makes those ideas trustworthy.
- Every measurement needs a unit. The SI base units are the meter (length), kilogram (mass), and second (time), and all other units are derived from them. Convert with the factor-label method and let units check your work.
- Accuracy is closeness to the true value; precision is agreement among repeated measurements. Significant figures report how well a value is known: an answer is no more precise than the least precise measurement it came from.
- Scientific notation writes very large and very small numbers as a digit part times a power of ten. Order-of-magnitude estimation catches nonsense answers before you trust them.
- A graph shows a relationship at a glance. Slope is rise over run, and its units tell you what it means; a straight line means the two quantities are proportional.
- Work every problem the same way: Given, Unknown, Equation, Substitute, Solve. Showing clear reasoning matters more than the final number.

Key terms

unit
the standard amount that gives a measurement its meaning (meter, second, kilogram)

SI system
the shared international (metric) system of units used in science

derived unit
a unit built from the base units, such as m/s for speed or kg $\cdot$ m/s^2 for force

factor-label method
converting units by multiplying by fractions equal to 1 so unwanted units cancel

accuracy
how close a measurement is to the true value

precision
how closely repeated measurements agree with one another

significant figures
the digits in a measurement that are actually known, reporting how well it is known

scientific notation
writing a number as a digit part between 1 and 10 times a power of ten

order-of-magnitude estimate
a rough calculation aimed at the nearest power of ten, used to sanity-check an answer

slope
the steepness of a line, rise divided by run; its units reveal its physical meaning

proportional
related by a steady rate, shown by a straight-line graph through the origin

Chapter 2

1-D Kinematics

PHYS1.PS2.1–2.2

1.1 · Position, Distance, and Displacement

By the end of this section you can: describe motion relative to a reference point; tell the difference between a scalar and a vector; and calculate distance and displacement for a trip.

Kinematics is the study of motion by itself, without yet asking what causes it. Before we can describe motion, we need a starting line. Motion is always measured relative to a **reference point**, a fixed spot we agree to call zero. Your **position** is where you are compared to that point, and we usually call it x. Choose a positive direction, and a position on the other side is negative. None of this is the "real" zero of the universe; it is a choice we make to keep the bookkeeping clear.

Now imagine you walk 3 m east, then 3 m back west to where you started. How far did you travel, and where did you end up? Those are two different questions, and physics gives them two different names. **Distance** is the total length of the path you walked, which is 6 m. **Displacement** is the straight-line change in position from start to finish, written Δx, which here is 0 m, because you ended where you began. Distance only grows. Displacement can be positive, negative, or zero, because it has direction.

Scalars and vectors. This is your first meeting with a distinction that runs through all of physics. A **scalar** is a quantity with size only, like distance or time. A **vector** is a quantity with size *and* direction, like displacement. Distance is a scalar; displacement is a vector. Chapter 2 is devoted to vectors, but the idea starts here.

Misconception check: distance and displacement are not the same word for the same thing. On a round trip they can be wildly different: a runner who laps a 400 m track ends with a distance of 400 m but a displacement of 0 m. When a problem says "how far," read carefully whether it wants the path length (distance) or the change in position (displacement).

Sparky rolls 12 m east, then 5 m west. Find the distance he traveled and his displacement. Distance is the whole path, so add the legs: $d = 12+5 = 17$ m. Displacement is the net change in position. Taking east as positive:

$$\Delta x = (+12) + (-5) = \boxed{+7 \text{ m (east)}}$$

He walked 17 m of pavement but ended up only 7 m from where he started. The two numbers answer two different questions.

Sparky starts at position $x_i = -3$ m and moves in a straight line to $x_f = +9$ m. What is his displacement? Displacement is always final position minus initial position:

$$\Delta x = x_f - x_i = (+9) - (-3) = \boxed{+12 \text{ m}}$$

Subtracting a negative adds, which is why crossing the zero point covers the full 12 m. He moved 12 m in the positive direction.

A jogger runs 150 m to a mailbox and then straight back to the start. Find the distance and the displacement. The path is out and back, so $d = 150 + 150 = 300$ m. But the finish is the start, so the change in position is:

$$\Delta x = \boxed{0 \text{ m}}$$

Distance can never shrink, but displacement can be zero, positive, or negative. A completed round trip is the cleanest case of zero displacement.

Check yourself: 1.1

1. What is the difference between a scalar and a vector?
   A scalar has size only (distance, time). A vector has size and direction (displacement).
2. You walk 5 m north, then 2 m south. Find your distance and your displacement.
   Distance = 7 m (total path). Displacement = 3 m north (change in position).
3. Can displacement ever be zero while distance is not? Give an example.
   Yes. Any round trip: you run a full lap, so distance is the lap length but displacement is zero.

1.2 · Speed and Velocity

By the end of this section you can: distinguish speed from velocity; compute average speed and average velocity; and explain the difference between average and instantaneous values.

Just as distance and displacement are a scalar-and-vector pair, so are speed and velocity. **Speed** is how fast you move, a scalar, found by dividing distance d by time t. **Velocity** is speed with a

direction, a vector, found by dividing displacement Δx by time:

$$s = \frac{d}{t} \qquad\qquad v = \frac{\Delta x}{t}$$

On a round trip the average speed is some positive number, but the average velocity is zero, because the displacement is zero. Velocity carries the sign of the direction you actually got somewhere. We also separate two time frames. **Average velocity** looks at the whole trip; **instantaneous velocity** is how fast you are going at one single moment, the number a speedometer shows. A car trip might average 80 km/h even though the speedometer swung between 0 and 120 the whole way.

Sparky rolls 40 m east in 8.0 s, then 10 m west in 2.0 s. Find his average speed and average velocity for the whole trip. Total distance $d = 40 + 10 = 50$ m; total time $t = 8.0 + 2.0 = 10$ s.

$$s = \frac{d}{t} = \frac{50}{10} = \boxed{5.0 \text{ m/s}}$$

Taking east as positive, total displacement $\Delta x = (+40) + (-10) = +30$ m:

$$v = \frac{\Delta x}{t} = \frac{30}{10} = 3.0 \text{ m/s (east)}$$

The two answers differ. Speed counts every meter walked; velocity counts only the net progress east.

A car's speed is 90 km/h. Express it in meters per second. There are 1000 m in a kilometer and 3600 s in an hour, so:

$$90 \ \tfrac{\text{km}}{\text{h}} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ h}}{3600 \text{ s}} = \boxed{25 \text{ m/s}}$$

A quick shortcut: divide any km/h value by 3.6 to get m/s. In this course we almost always work in m/s, so this conversion shows up constantly.

A family drives 210 km in 3.0 hours. What is their average speed?

$$s = \frac{d}{t} = \frac{210 \text{ km}}{3.0 \text{ h}} = \boxed{70 \text{ km/h}}$$

This is an average: they surely sped up and slowed down along the way, but over the whole trip the distance and time work out to 70 km/h.

How long does it take Sparky to travel 300 m at a steady 6.0 m/s? Rearrange $s = d/t$ to solve for time, $t = d/s$:

$$t = \frac{d}{s} = \frac{300 \text{ m}}{6.0 \text{ m/s}} = \boxed{50 \text{ s}}$$

The same simple relationship gives distance, speed, or time depending on which two you know. Cover the one you want and the formula tells you what to do.

Check yourself: 1.2

1. What is the difference between speed and velocity?
   Speed is a scalar, $s = d/t$. Velocity is a vector, $v = \Delta x/t$, so it carries direction.
2. How can average velocity be zero when average speed is not?
   On a round trip the displacement is zero, so velocity is zero, but distance is not zero, so speed is positive.
3. What does a speedometer show, average or instantaneous speed?
   Instantaneous speed, the value at that single moment.

1.3 · Acceleration

By the end of this section you can: define acceleration as the rate of change of velocity; compute it with correct units and sign; and explain what a negative acceleration does and does not mean.

Velocity tells you how position changes. **Acceleration** tells you how velocity changes. It is the rate at which velocity changes over time, where v_i is the initial velocity and v_f is the final velocity:

$$a = \frac{\Delta v}{t} = \frac{v_f - v_i}{t}$$

Because velocity is measured in m/s and time in s, acceleration comes out in m/s^2. Read it out loud: an acceleration of 3 m/s^2 means the velocity changes by 3 m/s during every second that passes. Acceleration is a vector, so its sign matters. A negative acceleration does *not* automatically mean slowing down; it means the acceleration points in the negative direction.

Misconception check: "negative acceleration" is not a synonym for "slowing down." The rule that always works: if acceleration and velocity have the *same* sign, the object speeds up; if they have *opposite* signs, it slows down. Check the two signs, not just one.

A car moving east at 20 m/s brakes to 5 m/s in 3.0 s. Find its acceleration. Given: $v_i = 20$ m/s, $v_f = 5$ m/s, $t = 3.0$ s, east positive.

$$a = \frac{v_f - v_i}{t} = \frac{5 - 20}{3.0} = \boxed{-5.0 \text{ m/s}^2}$$

The acceleration is 5.0 m/s^2 pointing west. Velocity points east and acceleration points west, opposite signs, so the car slows down. That matches the story of braking.

A sprinter reaches 24 m/s from rest in 6.0 s. Find the acceleration. Given: $v_i = 0$, $v_f = 24$ m/s, $t = 6.0$ s.

$$a = \frac{v_f - v_i}{t} = \frac{24 - 0}{6.0} = \boxed{4.0 \text{ m/s}^2}$$

Velocity and acceleration are both positive, so the sprinter speeds up, gaining 4.0 m/s every second.

A car slows from 30 m/s to 12 m/s while accelerating at -3.0 m/s^2. How long does that take? Rearrange $a = (v_f - v_i)/t$ to solve for time, $t = (v_f - v_i)/a$:

$$t = \frac{12 - 30}{-3.0} = \frac{-18}{-3.0} = \boxed{6.0 \text{ s}}$$

Both the top and bottom are negative, so the time comes out positive, as time always must. The negatives simply confirmed the car was slowing.

An object moves at -15 m/s with an acceleration of $+3.0$ m/s^2. Is it speeding up or slowing down, and what is its velocity after 2.0 s? Velocity is negative and acceleration is positive, opposite signs, so the object is *slowing down*. Find the new velocity:

$$v_f = v_i + at = -15 + (3.0)(2.0) = \boxed{-9 \text{ m/s}}$$

It is still moving in the negative direction, but at 9 m/s instead of 15, so its speed dropped. The signs told the story before the arithmetic did.

Check yourself: 1.3

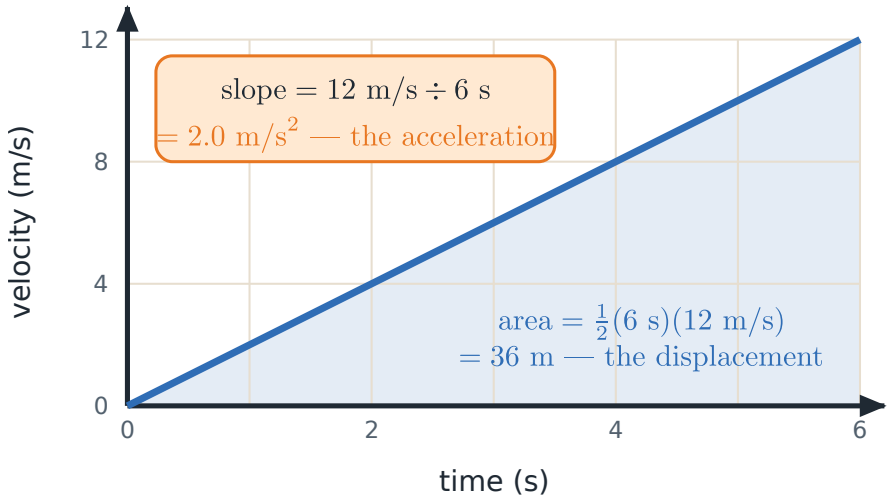
1. What does an acceleration of 4 m/s^2 mean in plain words?
   The velocity changes by 4 m/s during each second that passes.
2. An object has a negative velocity and a negative acceleration. Is it speeding up or slowing down?
   Speeding up, because velocity and acceleration point the same way (both negative).
3. A jogger goes from 2 m/s to 6 m/s in 2.0 s. Find the acceleration.
   $a = (6 - 2)/2 = 2$ m/s^2.

1.4 · Reading Motion Graphs

By the end of this section you can: interpret position-time and velocity-time graphs; connect slope to velocity and acceleration; and connect the area under a velocity-time graph to displacement.

On a **position-time graph**, the slope is the velocity, $v = \dfrac{\Delta x}{t}$, because slope is the change in position over the change in time. A flat line means the object is standing still; a steep line means it is moving fast; a curved line means the velocity is changing.

On a **velocity-time graph**, two features carry meaning. The slope is now the acceleration, since it is the change in velocity over the change in time. And the *area* between the line and the time axis is the displacement. A velocity of 5 m/s held for 4 s covers 20 m, which is exactly the area of the rectangle under the line.



Two questions for a velocity-time graph: the slope is the acceleration, and the shaded area between the line and the time axis is the displacement.

On a position-time graph, an object moves along a straight line from $x = 2$ m at $t = 1$ s to $x = 14$ m at $t = 4$ s. Find its velocity. The slope of a position-time graph is the velocity, rise over run:

$$v = \frac{\Delta x}{t} = \frac{14 - 2}{4 - 1} = \frac{12}{3} = \boxed{4.0 \text{ m/s}}$$

Because the line is straight, the velocity is steady at 4.0 m/s the whole way.

A velocity-time graph shows a constant 8.0 m/s for 5.0 s. Find the displacement. Displacement is the area between the line and the time axis. A constant velocity makes a rectangle:

$$\Delta x = \text{area} = (8.0)(5.0) = \boxed{40 \text{ m}}$$

This is just $v = \Delta x/t$ turned around, but seeing it as an area is what lets us handle graphs that are not flat.

On a velocity-time graph, velocity rises in a straight line from 0 to 12 m/s over 6.0 s. Find the acceleration and the displacement. The slope gives the acceleration:

$$a = \frac{\Delta v}{t} = \frac{12 - 0}{6.0} = \boxed{2.0 \text{ m/s}^2}$$

The area under the sloped line is a triangle, $\frac{1}{2}$ (base)(height):

$$\Delta x = \tfrac{1}{2}(6.0)(12) = 36 \text{ m}$$

One velocity-time graph answered two questions at once: the slope for acceleration, the area for displacement.

Check yourself: 1.4

1. On a position-time graph, what does the slope represent?
   The velocity.
2. On a velocity-time graph, what do the slope and the area represent?
   The slope is the acceleration; the area between the line and the time axis is the displacement.
3. A position-time graph is a flat horizontal line. What is the object doing?
   Standing still: its position is not changing, so its velocity is zero.

1.5 · The Kinematic Equations

By the end of this section you can: list the four kinematic equations for constant acceleration; choose the right one for a given problem; and solve for an unknown.

When acceleration is constant, and in this course it usually is, four equations connect the five quantities of motion: distance d, initial velocity v_i, final velocity v_f, acceleration a, and time t. Learn to recognize which one to reach for, and most of this unit becomes straightforward.

Equation	Leaves out	Use it when
$v_f = v_i + at$	d	you do not know or need the distance
$d = v_i t + \frac{1}{2}at^2$	v_f	you do not know the final velocity
$v_f^2 = v_i^2 + 2ad$	t	you do not know or need the time
$d = \frac{1}{2}(v_i + v_f)\,t$	a	you do not know the acceleration

Read the problem, list which of the five quantities you are given and which you want, then pick the equation that contains your knowns and your unknown and leaves out the one quantity you neither have nor need. The "Leaves out" column is your map.

Sparky starts from rest and accelerates at 2.0 m/s² for 6.0 s. How far does he travel? Given: $v_i = 0$, $a = 2.0$ m/s^2, $t = 6.0$ s. **Unknown:** d. We are not told the final velocity, so choose the equation that leaves out v_f.

$$d = v_i t + \tfrac{1}{2}at^2 = (0)(6.0) + \tfrac{1}{2}(2.0)(6.0)^2 = \boxed{36 \text{ m}}$$

Units check: $(\text{m/s}^2)(\text{s}^2)$ gives meters. The answer is a distance, as it should be.

A cart starts from rest and accelerates at 3.0 m/s² over a distance of 54 m. Find its final velocity. Given: $v_i = 0$, $a = 3.0$ m/s^2, $d = 54$ m. We have no time and want v_f, so use the equation that leaves out t:

$$v_f^2 = v_i^2 + 2ad = 0 + 2(3.0)(54) = 324$$

$$v_f = \sqrt{324} = \boxed{18 \text{ m/s}}$$

Don't forget the square root at the end. Forgetting it is the single most common slip on this equation.

A car traveling 25 m/s brakes at −5.0 m/s². How far does it travel before stopping? Given: $v_i = 25$ m/s, $v_f = 0$, $a = -5.0$ m/s^2. No time is given, so use the same time-free equation, solved for d:

$$d = \frac{v_f^2 - v_i^2}{2a} = \frac{0 - 25^2}{2(-5.0)} = \frac{-625}{-10} = \boxed{62.5 \text{ m}}$$

That is over 200 feet, and stopping distance grows with the *square* of speed, which is the physics behind every "slow down" sign.

An object speeds up steadily from 4.0 m/s to 20 m/s while covering 48 m. How long did it take? Given: $v_i = 4.0$, $v_f = 20$, $d = 48$ m. We are not told the acceleration, so use the equation that leaves out a:

$$d = \tfrac{1}{2}(v_i + v_f)\,t \;\Rightarrow\; t = \frac{2d}{v_i + v_f} = \frac{2(48)}{4.0 + 20} = \frac{96}{24} = \boxed{4.0 \text{ s}}$$

Choosing the equation that skips the quantity you neither have nor need turned a messy problem into one clean step.

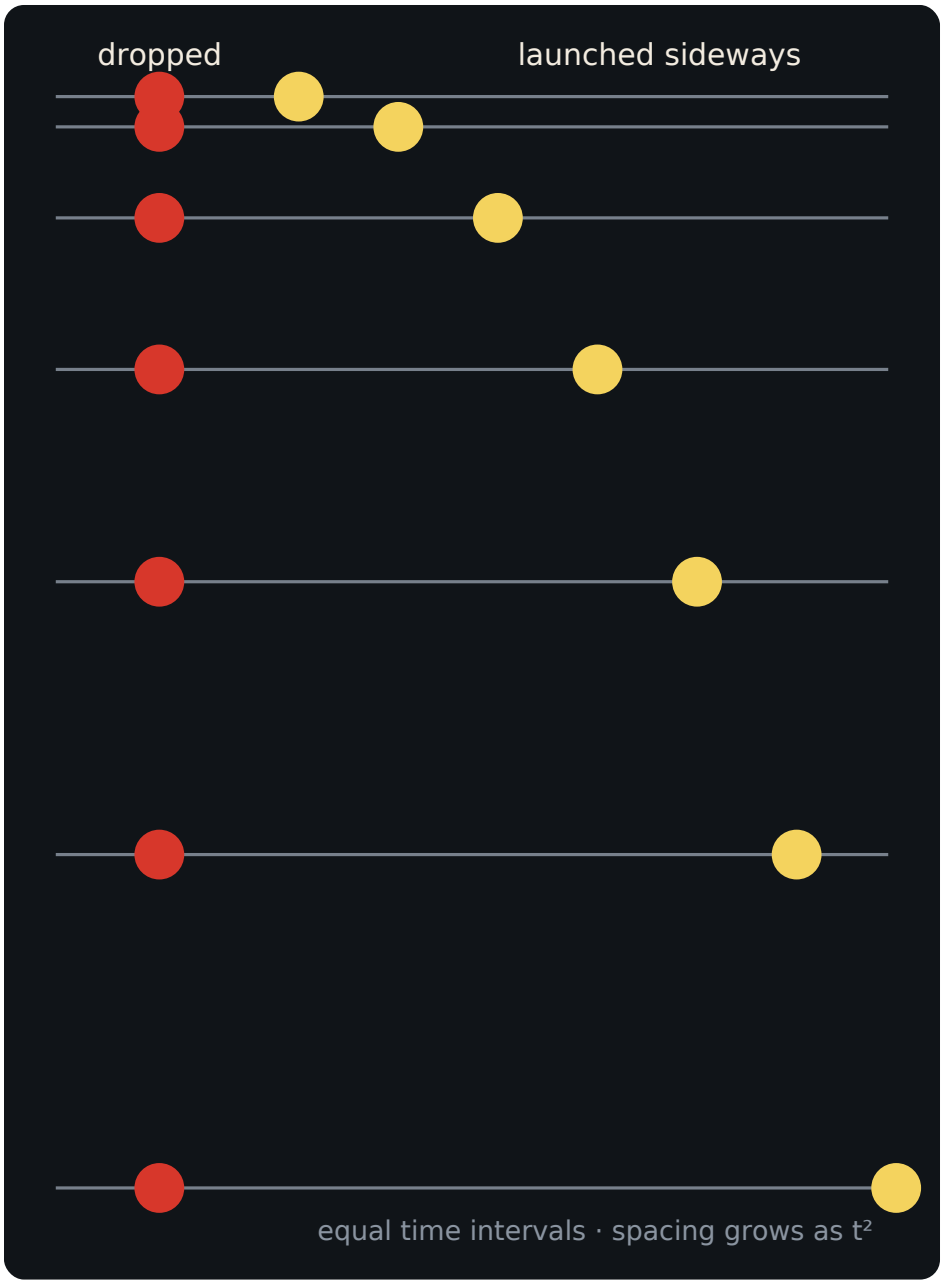
Check yourself: 1.5

1. Which equation would you use if you know v_i, a, and t, and want v_f?
   $v_f = v_i + at$ (it leaves out distance, which you neither have nor need).
2. A car starts at 10 m/s and accelerates at 3.0 m/s^2 for 4.0 s. Find its final velocity.
   $v_f = 10 + (3.0)(4.0) = 22$ m/s.
3. Why does choosing the equation that "leaves out" one quantity work?
   It matches the four knowns you have to the unknown you want, skipping the single quantity you neither have nor need.

1.6 · Free Fall

By the end of this section you can: describe free fall as motion under gravity alone; use $g = -9.8$ m/s^2; and apply the kinematic equations to falling and rising objects.

Drop a rock and a crumpled piece of paper at the same time and they land together, once air resistance is out of the picture. An object moving under the pull of gravity alone, with air resistance ignored, is in **free fall**, and near Earth's surface every free-falling object has the same downward acceleration, g. Taking up as the positive direction, $g = -9.8$ m/s^2: the negative sign says gravity points down. A feather and a hammer dropped on the airless Moon land at the same instant, which the Apollo 15 astronauts actually tested on camera.



Free fall, drawn strobe-style with exact physics. Both balls are shown at equal time intervals, and the vertical gaps grow as the square of the time: that is what constant acceleration looks like. Bonus: the yellow ball was launched sideways at the same instant the red one was dropped, and they fall step for step. Hold that thought for Chapter 3.

Free fall is not a new topic; it is the kinematic equations with the acceleration already known. Set $a = g = -9.8\ \text{m/s}^2$ and keep your sign convention consistent through the whole problem. A ball thrown straight up is still in free fall the entire time, even at the very top, where its velocity is momentarily zero but its acceleration is still $-9.8\ \text{m/s}^2$, already turning the motion around.

A ball is dropped from rest off a bridge and falls for 3.0 s before hitting the water. How fast is it going at impact, and how high is the bridge? Take up as positive, $g = -9.8$ m/s^2. Given: $v_i = 0$, $a = g = -9.8$ m/s^2, $t = 3.0$ s. Velocity at impact:

$$v_f = v_i + gt = 0 + (-9.8)(3.0) = \boxed{-29 \text{ m/s}}$$

Displacement (negative means below the start):

$$d = v_i t + \tfrac{1}{2} g t^2 = 0 + \tfrac{1}{2}(-9.8)(3.0)^2 = -44 \text{ m}$$

The ball hits the water moving 29 m/s downward, and the bridge is 44 m above the water. The minus signs simply record that "down" is the negative direction we chose. Check the sizes: 29 m/s is about 65 mph, and 44 m is roughly a 14-story building. Both are reasonable.

Sparky is thrown straight up at 19.6 m/s. How long until he reaches the highest point? At the very top his velocity is momentarily zero. With $v_i = +19.6$, $v_f = 0$, $a = g = -9.8$:

$$t = \frac{v_f - v_i}{g} = \frac{0 - 19.6}{-9.8} = \boxed{2.0 \text{ s}}$$

Even at the top, where he is briefly not moving, gravity is still pulling at -9.8 m/s^2, already turning him back around.

For the same throw ($v_i = 19.6$ m/s upward), how high does Sparky go? Use the time-free equation with $v_f = 0$ at the top:

$$v_f^2 = v_i^2 + 2gd \;\Rightarrow\; 0 = (19.6)^2 + 2(-9.8)\, d$$

$$d = \frac{-(19.6)^2}{2(-9.8)} = \frac{-384.16}{-19.6} = \boxed{19.6 \text{ m}}$$

He rises 19.6 m. Notice we never needed the time to get the height; the right equation skipped it.

A stone is dropped from rest off a 44.1 m cliff. How long until it lands? ($g = -9.8$ m/s^2) With $v_i = 0$, displacement is -44.1 m (down). Use $d = v_i t + \frac{1}{2} g t^2$:

$$-44.1 = 0 + \tfrac{1}{2}(-9.8)\, t^2 \;\Rightarrow\; t^2 = \frac{-44.1}{-4.9} = 9.0$$

$$t = \sqrt{9.0} = \boxed{3.0 \text{ s}}$$

Keeping "down" negative on both the displacement and g let the signs cancel cleanly, leaving a positive time.

Faith & Science "Which of you by being anxious can add a single hour to his span of life?" (Matthew 6:27). There is something steadying about free fall. Every object, the heavy and the light, falls under the very same law, and none of them can hurry it or slow it by wanting to. Jesus points at the same kind of trust when he tells his listeners to stop striving anxiously, because the God who orders the falling of a sparrow also orders their days. The universe is dependable, and a dependable world is one you can both study and rest in.

Practice Problems

Work these on your own, then open the solutions to check yourself. Take up as positive and use $g = -9.8$ m/s^2 where needed.

1. You walk 60 m north, then 25 m south. Find your distance and your displacement.
2. Convert 120 km/h into meters per second.
3. A motorcycle accelerates from rest to 30 m/s in 5.0 s. Find its acceleration.
4. A car moving at 8.0 m/s accelerates at 2.5 m/s^2 for 4.0 s. Find its final velocity.
5. A runner starting at 5.0 m/s accelerates at 2.0 m/s^2 for 6.0 s. How far does she travel?
6. A cyclist moving at 18 m/s brakes at -4.0 m/s^2. How far does she travel before stopping?
7. A rock is dropped from rest and falls for 2.5 s. Find its velocity at impact and the height it fell.
8. A ball is thrown straight up at 24.5 m/s. Find the time to reach the top and the maximum height.
9. On a velocity-time graph, velocity rises in a straight line from 0 to 10 m/s over 4.0 s. Find the displacement.
10. On a position-time graph, an object moves in a straight line from $x = 5$ m at $t = 0$ to $x = 20$ m at $t = 5.0$ s. Find its velocity.

Solutions to the Practice Problems

1. **Distance** is the whole path: $60 + 25 = 85$ m. **Displacement** (north positive): $\Delta x = (+60) + (-25) = +35$ m, i.e. 35 m north.
2. $120 \div 3.6 = 33$ m/s (more precisely 33.3 m/s). Or $120 \times \frac{1000}{3600} = 33$ m/s.
3. $a = \dfrac{v_f - v_i}{t} = \dfrac{30 - 0}{5.0} = 6.0$ m/s^2.
4. $v_f = v_i + at = 8.0 + (2.5)(4.0) = 18$ m/s.
5. $d = v_i t + \frac{1}{2}at^2 = (5.0)(6.0) + \frac{1}{2}(2.0)(6.0)^2 = 30 + 36 = 66$ m.
6. No time given, so use $v_f^2 = v_i^2 + 2ad$: $d = \dfrac{0 - 18^2}{2(-4.0)} = \dfrac{-324}{-8} = 40.5$ m.
7. $v_f = gt = (-9.8)(2.5) = -24.5$ m/s (24.5 m/s downward). Height: $d = \frac{1}{2}gt^2 = \frac{1}{2}(-9.8)(2.5)^2 = -30.6$ m, so it fell about 30.6 m.

8. At the top $v_f = 0$. Time: $t = \dfrac{0 - 24.5}{-9.8} = 2.5$ s. Max height: $d = \dfrac{-(24.5)^2}{2(-9.8)} = 30.6$ m.

9. Displacement is the area, a triangle: $\Delta x = \frac{1}{2}(4.0)(10) = 20$ m.

10. Slope of a position-time graph is velocity: $v = \dfrac{20 - 5}{5.0 - 0} = 3.0$ m/s.

Challenge Problems

The hard ones. Each of these takes more than one step and more than one idea; some borrow tools from earlier chapters. Work them in GUESS order (Given, Unknown, Sketch, Equation, Substitute, Solve) before opening the solutions.

1. **Gas, Then Brakes.** A car accelerates from rest at 3.0 m/s^2 for 5.0 s, then brakes at -5.0 m/s^2 until stopped. Find the total distance traveled and the total time on the road.

Scan to watch: Gas, Then Brakes (worked example)

2. **Off the Roof.** A ball is thrown straight up at 15 m/s from the edge of a 20 m tall roof and falls all the way to the ground. Find its speed when it lands and the total time in the air. (Up positive, $g = -9.8$ m/s^2)

Scan to watch: Off the Roof (free fall worked example)

3. **The Speed Trap.** Sparky rolls past a parked police car at a steady 6.0 m/s. The instant he passes, the police car accelerates from rest at 2.0 m/s^2. How long until it catches him, how far has it traveled, and how fast is it moving at that moment?

Scan to watch: The Speed Trap (worked example)

4. **The Trip in a Graph.** A velocity-time graph shows a bus that speeds up from rest to 8.0 m/s in 4.0 s, holds 8.0 m/s for 6.0 s, then brakes to rest in 2.0 s. Find the acceleration in each segment, the total displacement, and the average velocity.

Scan to watch: The Trip in a Graph

Solutions to the Challenge Problems

1. **Gas, Then Brakes.** Phase 1: $v_f = 0 + (3.0)(5.0) = 15$ m/s; $d_1 = \frac{1}{2}(3.0)(5.0)^2 = 37.5$ m. Phase 2 starts at 15 m/s: $t_2 = 15/5.0 = 3.0$ s; $d_2 = \frac{0-225}{2(-5.0)} = 22.5$ m. Totals: **60 m and 8.0 s**. Solve each phase separately; the final velocity of one is the initial velocity of the next.
2. **Off the Roof.** Avoid the quadratic: velocity first. $v_f^2 = 15^2 + 2(-9.8)(-20) = 617$, so $v_f \approx$ **25 m/s downward**. Then $t = (v_f - v_i)/g = (-24.8 - 15)/(-9.8) \approx$ **4.1 s**. The velocity equation has no time in it, so use it first; time then falls out of $a = \Delta v/t$.
3. **The Speed Trap.** Set positions equal: $6.0t = \frac{1}{2}(2.0)t^2 = t^2$, so $t =$ **6.0 s**, $d = 6.0(6.0) =$ **36 m**, $v = (2.0)(6.0) =$ **12 m/s**. The police car crosses at exactly twice Sparky's speed; starting from rest, its average had to match his.
4. **The Trip in a Graph.** Slopes: $a_1 = 8.0/4.0 = 2.0$ m/s^2, $a_2 = 0$, $a_3 = -8.0/2.0 = -4.0$ m/s^2. Areas: $\frac{1}{2}(4)(8) + (6)(8) + \frac{1}{2}(2)(8) = 16 + 48 + 8 =$ **72 m**; average velocity $72/12 =$ **6.0 m/s**. Slope tells the acceleration story; area tells the distance story.

Chapter 1 Summary

- Motion is measured from a reference point. Distance (a scalar) is total path length; displacement Δx (a vector) is the change in position and carries a sign.
- Speed is $s = d/t$ (scalar); velocity is $v = \Delta x/t$ (vector). Average values describe a whole trip; instantaneous values describe a single moment.
- Acceleration is the rate of change of velocity, $a = (v_f - v_i)/t$, in m/s^2. An object speeds up when acceleration and velocity share a sign and slows down when their signs are opposite.
- On a position-time graph the slope is velocity. On a velocity-time graph the slope is acceleration and the area is displacement.
- For constant acceleration, four kinematic equations connect d, v_i, v_f, a, and t. Choose the one that contains your knowns and unknown and leaves out the rest.
- Free fall is motion under gravity alone, with $a = g = -9.8$ m/s^2 (up positive). A thrown object is in free fall the whole time, even at the top of its path.

Key terms

reference point
the fixed spot chosen as zero, from which position is measured

position (x)
where an object is relative to the reference point, with a sign for direction

distance (d)
the total length of the path traveled; a scalar

displacement (Δx)
the straight-line change in position from start to finish; a vector

scalar
a quantity with size only (distance, speed, time)

vector
a quantity with size and direction (displacement, velocity, acceleration)

speed (s)
distance divided by time, $s = d/t$; a scalar

velocity (v)
displacement divided by time, $v = \Delta x/t$; a vector

acceleration (a)
the rate of change of velocity, $a = (v_f - v_i)/t$, in m/s^2; a vector

free fall
motion under gravity alone, with $g = -9.8\ \text{m/s}^2$ (up positive)

Chapter 3

Vectors & 2-D Motion

PHYS1.PS2.4

2.1 · Picturing a Vector

By the end of this section you can: represent a vector as an arrow; state its two parts; and explain why direction changes everything.

In Chapter 1 you met the idea that some quantities carry a direction. A **vector** has both a size, called its **magnitude**, and a direction. Velocity, displacement, acceleration, and force are all vectors. We draw a vector as an arrow: the length of the arrow shows the magnitude, and the way it points shows the direction. This simple picture is powerful, because it lets us add directions instead of just numbers.

Why does direction matter so much? Because two motions of the same size can combine to give very different results depending on how they point. Walk 3 m east then 4 m east and you are 7 m from home. Walk 3 m east then 4 m north and you are only 5 m from home, off at an angle. Same two numbers, different directions, different answer. Handling that correctly is what this chapter teaches.

Two forces act on a sled: 8.0 N east and 5.0 N east. Find the net force. Then find it if the 5.0 N instead points west. Along one line, vectors just add with signs. Taking east as positive, both east: $8.0 + 5.0 = 13$ N east. With the 5.0 N pointing west (negative): $8.0 + (-5.0) = \boxed{3.0 \text{ N east}}$. Direction is doing real arithmetic here. The same two sizes give 13 N or 3 N depending only on how the second one points.

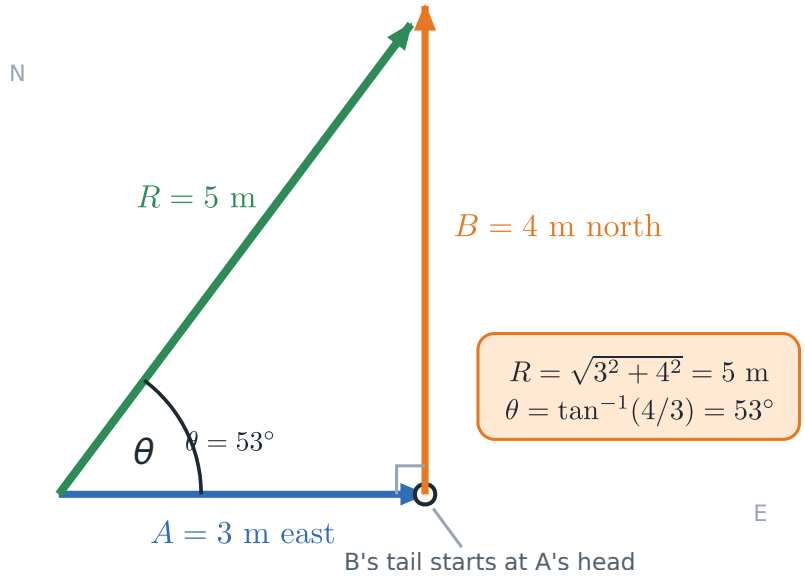
A velocity vector is drawn 4 cm long pointing due west, using a scale of 1 cm = 5 m/s. State its magnitude and direction. Length gives magnitude: $4 \text{ cm} \times 5 \text{ m/s per cm} = \boxed{20 \text{ m/s}}$. The arrow points west, so the velocity is **20 m/s west**. A vector is never complete until you have stated both parts.

2.2 · Adding Vectors Head to Tail

By the end of this section you can: add two vectors graphically; identify the resultant; and use the Pythagorean theorem for perpendicular vectors.

To add vectors, we line them up **head to tail**: draw the first arrow, then start the second arrow at the tip of the first. The single arrow drawn from the very start to the very end is their sum, called the **resultant**. It represents the one motion that would have the same result as taking both in turn. The order does not matter; you get the same resultant either way.

When the two vectors are perpendicular, the resultant is the hypotenuse of a right triangle, so its magnitude comes straight from the Pythagorean theorem. Our "3 east then 4 north" walk makes a 3-4-5 triangle, so the resultant is 5 m, pointing northeast at an angle you can find with trigonometry.



Head to tail. *The second arrow starts where the first ends. The resultant runs from the very start to the very end. Perpendicular vectors make a right triangle, so 3 and 4 give 5.*

Misconception check: you cannot just add the magnitudes of vectors unless they point the same way. 3 m and 4 m combine to 7 m only if both point in the same direction. At right angles they give 5 m, and at other angles something in between. The direction is part of the arithmetic, not decoration on top of it.

A boat travels 6.0 m east, then 8.0 m north. How far is it from where it started, and in what direction? The legs are perpendicular, so the resultant is the hypotenuse:

$$R = \sqrt{6.0^2 + 8.0^2} = \sqrt{100} = \boxed{10 \text{ m}}$$

Direction, measured north of east: $\theta = \tan^{-1}\left(\frac{8.0}{6.0}\right) = 53°$. So it ends up 10 m from the start, 53° north of east.

A hiker walks 12 m east and 5.0 m south. Find the magnitude and direction of the displacement.

$$R = \sqrt{12^2 + 5.0^2} = \sqrt{169} = \boxed{13 \text{ m}}$$

Angle south of east: $\theta = \tan^{-1}\left(\frac{5.0}{12}\right) = 23°$. Recognizing the 5-12-13 triangle is a handy shortcut, but the Pythagorean theorem always works.

Two ropes pull on a ring: 7.0 N north and 24 N east. Find the single resultant force.

$$R = \sqrt{7.0^2 + 24^2} = \sqrt{625} = \boxed{25 \text{ N}}$$

Angle north of east: $\theta = \tan^{-1}\left(\frac{7.0}{24}\right) = 16°$. One 25 N pull at 16° would do exactly what the two ropes do together.

Check yourself: 2.2

1. What is the resultant of two vectors?
   The single vector, drawn from the start of the first to the end of the last, that has the same effect as adding them.
2. A boat goes 6 m east and 8 m north. How far is it from the start?
   $(6^2 + 8^2) = 100 = 10$ m.
3. Why can't you add 5 N and 5 N to always get 10 N?
   Only if they point the same way. At right angles they give about 7 N; pointing opposite they give 0 N.

2.3 · Vector Components

By the end of this section you can: break a vector into perpendicular components; and use sine and cosine to find them.

Adding arrows by drawing works, but it is slow and imprecise. The professional method runs the idea backward. Instead of combining two vectors into one, we take any single vector and split it into two perpendicular pieces, one along the x-axis and one along the y-axis. These pieces are the vector's **components**. Every diagonal vector is really an east-west part and a north-south part traveling together, and once we separate them, each axis becomes a simple one-dimensional problem like Chapter 1.

Right-triangle trigonometry gives the components. For a vector of magnitude V at angle θ measured from the x-axis:

$$V_x = V\cos\theta \quad \text{and} \quad V_y = V\sin\theta$$

A helpful memory aid: the component next to the angle (adjacent) uses cosine, and the component across from the angle (opposite) uses sine. Draw the triangle every time until it becomes second nature.

Sparky fires a dart at 20 m/s, aimed 30° above the horizontal. Find the horizontal and vertical components of its velocity. Given: V = 20 m/s, $\theta = 30°$. **Unknown:** V_x and V_y. Horizontal: $V_x = 20\cos 30° = 20(0.866) = 17$ m/s. Vertical: $V_y = 20\sin 30° = 20(0.500) = \boxed{10 \text{ m/s}}$. Check: both components are smaller than the full 20 m/s, as they must be, and the more horizontal aim (30°) gives a larger horizontal part than vertical part, which matches the picture. These two numbers are exactly what Chapter 3 needs to handle a projectile.

A rope pulls a crate with 100 N of force directed 25° above the horizontal. Find the horizontal and vertical components.

$$F_x = 100\cos 25° = \boxed{91 \text{ N}} \qquad F_y = 100\sin 25° = \boxed{42 \text{ N}}$$

Only the 91 N horizontal part slides the crate along the floor; the 42 N vertical part tries to lift it. Splitting the pull tells you what each part actually does.

A ball moves at 30 m/s directed 40° above the horizontal. Find its horizontal and vertical velocity components.

$$v_x = 30\cos 40° = \boxed{23 \text{ m/s}} \qquad v_y = 30\sin 40° = \boxed{19 \text{ m/s}}$$

These are the two independent motions the ball carries at once, exactly what Chapter 3 will treat separately.

Check yourself: 2.3

1. Why do we break a vector into components?
   So each axis becomes an independent one-dimensional problem, which is far easier than working diagonally.
2. A force of 50 N acts at 60° above horizontal. Find its horizontal and vertical components.
   F_x = 50 cos 60° = 25 N; F_y = 50 sin 60° = 43 N.
3. Which trig function gives the component next to (adjacent to) the angle?
   Cosine.

2.4 · Adding Vectors by Components

By the end of this section you can: add several vectors by summing their components; and rebuild the resultant's magnitude and direction.

Components turn vector addition into simple arithmetic, and this is the method you will use for the rest of the course. In this class we call it the **ROXY method**, named for the four columns

of its organizer: **R** (each vector's magnitude), θ (theta, its angle), and the **X** and **Y** components you compute from them. It is the very same procedure a spreadsheet or a vector calculator runs internally, written out so you can do it by hand and see every step. Because it is standardized, you will reuse this exact table for forces, momentum, and every other vector in this book.

The method flows one direction and then back. Start with each vector as an **R** and a θ. Convert it into components with $X = R\cos\theta$ and $Y = R\sin\theta$. Add up all the X's and all the Y's. Then convert those two totals back into a single resultant magnitude and angle:

$$R = \sqrt{X^2 + Y^2} \quad \text{and} \quad \theta = \tan^{-1}\left(\frac{Y}{X}\right)$$

The organizer is just a table: one row per vector holding its R and θ, then its X and Y, with a totals row at the bottom. Fill it in, sum the X and Y columns, and convert that last row back to a resultant:

Vector	R	θ	X = R cos θ	Y = R sin θ
A	R_A	θ_A	$R_A \cos\theta_A$	$R_A \sin\theta_A$
B	R_B	θ_B	$R_B \cos\theta_B$	$R_B \sin\theta_B$
Sum	—	—	Σ **X**	Σ **Y**

Then convert the totals back: $R = \sqrt{(\Sigma X)^2 + (\Sigma Y)^2}$ and $\theta = \tan^{-1}\left(\frac{\Sigma Y}{\Sigma X}\right)$.

The power of the method is that it works for any number of vectors at any angles. Three forces pulling on a ring, five legs of a hiking trip, it does not matter. Add a row for each one, convert every R and θ into X and Y, sum the two component columns, and convert back once at the end.

One motion, two independent stories. The deep idea hiding in components is that the x-motion and the y-motion do not affect each other. What happens east-west is a completely separate story from what happens north-south. That independence is the secret that makes projectile motion, the whole of Chapter 3, click into place.

Add $A = 5.0$ m pointing east (0°) and $B = 5.0$ m pointing north (90°) with a ROXY table.
VectorRθX = R cos θY = R sin θ A5.00°5.00 B5.090°05.0 **Sum——5.05.0**

$$R = \sqrt{5.0^2 + 5.0^2} = \boxed{7.1\text{ m},} \qquad \theta = \tan^{-1}\left(\frac{5.0}{5.0}\right) = \boxed{45^\circ}$$

Equal east and north parts give a 45° resultant, exactly as the picture demands.

A hiker walks $A = 10$ m at 30° and then $B = 8.0$ m at 120° (both measured from east). Find the total displacement. Fill the organizer, one row per leg:
VectorRθX = R cos θY = R sin θ A10 m30°+8.66+5.00 B8.0 m120°−4.00+6.93 **Sum——+4.66+11.93**

$$R = \sqrt{4.66^2 + 11.93^2} = \boxed{12.8\text{ m},} \qquad \theta = \tan^{-1}\left(\frac{11.93}{4.66}\right) = \boxed{69^\circ}$$

The 120° leg has a *negative* x-component, because it points up and to the left. Letting the signs come out of cosine and sine automatically is the whole advantage of the method.

A ROXY table has been summed to $\Sigma X = -30$ and $\Sigma Y = -40$. Find the resultant's magnitude and its direction as a standard-position angle (measured counterclockwise from east). The magnitude is safe, because squaring erases the signs:

$$R = \sqrt{(-30)^2 + (-40)^2} = \sqrt{2500} = 50$$

Now the angle, and here is the trap. Punch it in blindly and the calculator says:

$$\tan^{-1}\left(\tfrac{-40}{-30}\right) = \tan^{-1}(1.33) = 53^\circ$$

But 53° points up and to the right, into the **northeast** — and our vector has *both* components negative, so it must point down and to the left, into the **southwest**. The calculator is wrong by a half-turn. This is not a glitch: tan repeats every 180°, so $\tan^{-1}$ can only ever answer in the right half of the plane (quadrants I and IV). It literally cannot see quadrants II and III. You have to supply the missing information yourself, from the signs in your table:

$$\text{if } \Sigma X < 0, \text{ add } 180^\circ \quad \Rightarrow \quad \theta = 53^\circ + 180^\circ = \boxed{233^\circ}$$

So the resultant is **50 at 233°**, pointing southwest, exactly as a quick sketch of $(-30, -40)$ demands. The rule is short: compute $\tan^{-1}(\Sigma Y/\Sigma X)$, then **if your summed X is negative, add 180°**. That one check covers both quadrant II and quadrant III. Always sketch the summed components first, so you know which direction the answer *should* point and can catch the calculator when it lies.

Check yourself: 2.4

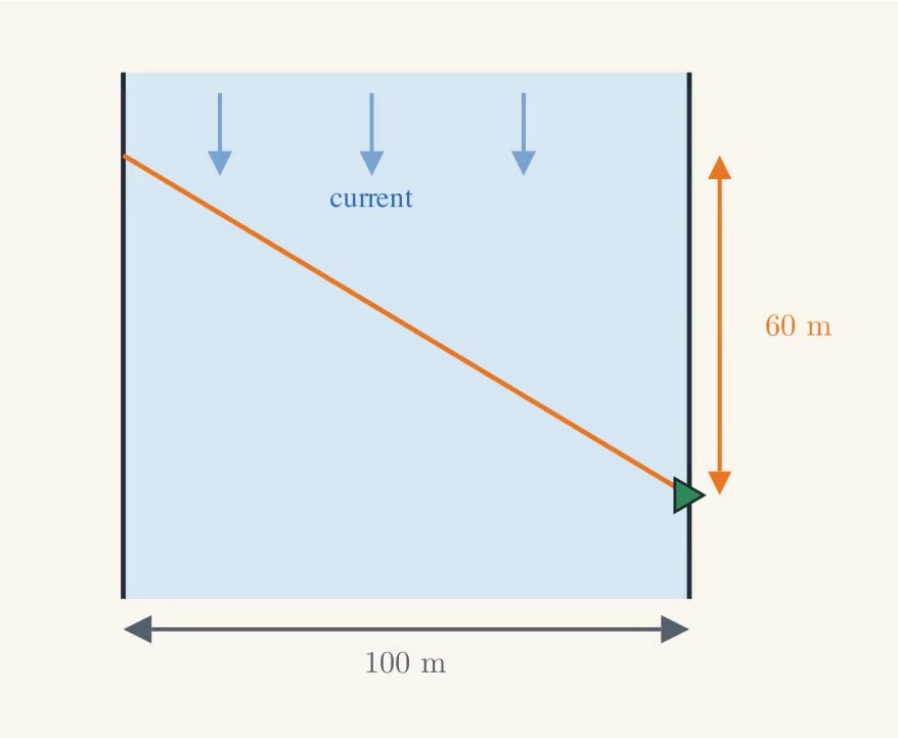
1. What do the letters in the ROXY method stand for, and what is the flow?
   R (magnitude), θ (angle), X, Y. Convert each vector's R and θ into X = R cos θ and Y = R sin θ, sum all the X's and all the Y's, then convert those totals back into a resultant R and θ.
2. A resultant has $R_x = 3$ m and R_y = 4 m. Find its magnitude.
   $R = (3^2 + 4^2) = 5$ m.
3. Why is the component method better than drawing arrows to scale?
   It is exact and works for any number of vectors at any angles, without careful drawing.

2.5 · Relative Velocity

By the end of this section you can: explain that velocity depends on the observer; and add velocities to find motion relative to the ground.

Walk forward on a moving bus and you stroll at a gentle pace relative to the seats, but to someone standing on the sidewalk you fly by at the bus's speed plus your own. Both are true. Velocity is **relative**: it depends on who is watching. To combine motions, you add the velocity vectors. Your velocity relative to the ground is your velocity relative to the bus plus the bus's velocity relative to the ground.

The classic case is a boat crossing a river. The boat aims straight across, but the current sweeps it downstream, so its true path relative to the shore is the diagonal resultant of the two velocities. Because those two velocities are perpendicular, you find the true speed with the Pythagorean theorem, exactly like Section 2.2. A plane flying in a crosswind is the very same problem wearing different clothes.



The river crossing, exactly as this chapter works it. The boat aims straight across at 5.0 m/s while the 3.0 m/s current carries it downstream. The orange line is the true path over the ground: 100 m across, 60 m of drift, and the crossing time never noticed the current. This frame comes from the Challenge 2.2 video, embedded later in this chapter.

Sparky pilots a boat straight across a river at 4.0 m/s. The current flows downstream at 3.0 m/s. What is the boat's speed and direction relative to the shore? **Given:** across velocity 4.0 m/s, downstream velocity 3.0 m/s (perpendicular). **Unknown:** resultant speed and angle. Speed: $R = \sqrt{4.0^2 + 3.0^2} = \sqrt{25} = 5.0$ m/s. Angle downstream from straight-across: $\theta = \tan^{-1}\left(\frac{3.0}{4.0}\right) = \boxed{37^\circ}$. So the boat actually moves 5.0 m/s along a path angled 37° downstream. Aiming straight across does not mean going straight across, because the river gets a vote too.

A plane flies north at 200 km/h. A 50 km/h wind blows from the west (toward the east). Find the plane's speed and direction relative to the ground. The two velocities are perpendicular, so combine them like Section 2.2:

$$R = \sqrt{200^2 + 50^2} = \boxed{206 \text{ km/h},} \qquad \theta = \tan^{-1}\left(\tfrac{50}{200}\right) = \boxed{14^\circ}$$

The plane travels 206 km/h along a path 14° east of north. The wind both speeds it up and blows it off its heading, which is why pilots aim slightly into the wind.

Sparky again points his boat straight across a river at 5.0 m/s, but now the current runs 12 m/s downstream. Find his speed and direction relative to the shore.

$$R = \sqrt{5.0^2 + 12^2} = \boxed{13 \text{ m/s},} \qquad \theta = \tan^{-1}\left(\tfrac{12}{5.0}\right) = \boxed{67^\circ}$$

With the current more than twice his own speed, the boat is swept far downstream: 67° off from straight across. The current really does get the louder vote here.

Sparky is tired of landing downstream. He wants to reach the point directly across from where he starts. His boat moves 5.0 m/s relative to the water, and the current runs 3.0 m/s downstream. Which way must he aim the boat, how fast does he actually cross, and how long does a 100 m river take? Given: boat 5.0 m/s (relative to the water), current 3.0 m/s downstream, river 100 m wide. **Unknown:** the heading angle upstream, the across-the-river speed, and the crossing time. This is the earlier problem run backward. Instead of aiming straight and asking where the river pushes him, Sparky aims upstream by an angle θ so that his upstream component exactly cancels the current:

$$5.0 \sin\theta = 3.0 \quad \Rightarrow \quad \theta = \sin^{-1}\left(\tfrac{3.0}{5.0}\right) = \boxed{37^\circ \text{ upstream of straight across}}$$

Whatever heading is left over points at the far bank, and that component is his true crossing speed:

$$v_{\text{across}} = 5.0 \cos 37^\circ = 4.0 \text{ m/s}, \qquad t = \frac{100 \text{ m}}{4.0 \text{ m/s}} = 25 \text{ s}$$

The current is the same 3.0 m/s as in Worked Example 2.5, but the question is inverted: not "where will the river push me?" but "how do I aim so it cannot push me off target?" Sparky trades some forward speed (4.0 m/s instead of his full 5.0) for a straight path. That trade, aiming into the flow, is exactly what a pilot does in a crosswind and what you do when you angle upstream to reach the dock across the river.

Faith & Science , "The heart of man plans his way, but the LORD establishes his steps" (Proverbs 16:9). Relative velocity is a quiet reminder that we never move through the world alone. The boat points where the pilot aims, but the current is moving too, and the true path is the sum of both. So it often is with a life. We choose a heading and pull hard, and still a larger current carries us in ways we did not chart. The physics does not make us passive; the pilot's aim genuinely matters to where the boat lands. It simply says the final path is bigger than our aim alone, which is a truth worth holding with humility.

Practice Problems

Work these on your own, then open the solutions to check yourself. Measure angles from the east (positive x) direction unless told otherwise.

1. Two forces act along a line: 12 N east and 7.0 N west. Find the net force.
2. A drone flies 8.0 m east, then 6.0 m north. Find the magnitude and direction of its displacement.
3. A force of 24 N acts at 60° above the horizontal. Find its horizontal and vertical components.
4. A velocity of 20 m/s is directed 35° above the horizontal. Find its components.
5. Add two displacements with a ROXY table: 40 m east (0°) and 30 m north (90°). Give the resultant's magnitude and direction.
6. A swimmer heads straight across a river at 1.5 m/s while the current carries her downstream at 2.0 m/s. Find her speed and direction relative to the bank.
7. A plane flies north at 150 km/h through a 40 km/h wind blowing toward the east. Find its ground speed and direction.
8. A plane heads due north at 120 km/h. A 30 km/h wind blows at 330° in standard position (that is, 30° south of east). The wind is not perpendicular to the heading, so use a ROXY table: find the plane's ground speed and direction.

Solutions to the Practice Problems

1. East positive: $12 + (-7.0) = 5.0$ N east.
2. $R = \sqrt{8.0^2 + 6.0^2} = 10$ m; $\theta = \tan^{-1}(6.0/8.0) = 37°$ north of east.
3. $F_x = 24\cos 60° = 12$ N; $F_y = 24\sin 60° = 21$ N.
4. $v_x = 20\cos 35° = 16$ m/s; $v_y = 20\sin 35° = 11$ m/s.
5. $R_x = 40,\ R_y = 30$; $R = \sqrt{40^2 + 30^2} = 50$ m; $\theta = \tan^{-1}(30/40) = 37°$ north of east.
6. $R = \sqrt{1.5^2 + 2.0^2} = 2.5$ m/s; $\theta = \tan^{-1}(2.0/1.5) = 53°$ downstream from straight across.
7. $R = \sqrt{150^2 + 40^2} = 155$ km/h; $\theta = \tan^{-1}(40/150) = 15°$ east of north.
8. ROXY: plane $(0,\ 120)$; wind $(30\cos 330°,\ 30\sin 330°) = (+26.0,\ -15.0)$. Sum $\Sigma X = 26.0$, $\Sigma Y = 105$. $R = \sqrt{26.0^2 + 105^2} = 108$ km/h; $\theta = \tan^{-1}(26.0/105) = 14°$ east of north. Here $\Sigma X > 0$, so no quadrant correction is needed. The ground speed is a touch under the 120 km/h airspeed because the wind carries a small southward part.

Challenge Problems

The hard ones. Each of these takes more than one step and more than one idea; some borrow tools from earlier chapters. Work them in GUESS order (Given, Unknown, Sketch, Equation, Substitute, Solve) before opening the solutions.

1. **The Treasure Hunt.** A treasure map says: walk 60 m east, then 80 m north, then 100 m at 216.9° (counterclockwise from east). Where do you end up relative to the start? And what single straight walk takes you back home?

Scan to watch: The Treasure Hunt (vector addition)

2. **The Full River Crossing.** Sparky points his boat straight across a river at 5.0 m/s. The current flows downstream at 3.0 m/s, and the river is 100 m wide. Find (a) how long the crossing takes, (b) how far downstream he lands, and (c) his speed and direction relative to the shore.

Scan to watch: The Full River Crossing

3. **Blown Off Course.** A plane flies with its nose pointed north at 200 km/h while a steady 50 km/h wind blows toward the east. After 2.5 hours, how far is the plane from its start, and how far east of its intended due-north course has it drifted?

Scan to watch: Blown Off Course (relative velocity)

4. **A Quartering Wind.** A plane holds its nose due north — that is, at 90° measured from the +x (east) axis — at 200 km/h through the air. A 50 km/h wind blows at 315° in standard position (that is, 45° south of east, toward the southeast), so it carries equal eastward and southward parts. Because the wind no longer meets the heading at a right angle, the Pythagorean shortcut fails you. Build a ROXY component table and find (a) the plane's speed and direction relative to the ground, and (b) how far east of its intended due-north course it drifts in 2.0 hours. Then explain why the ground speed comes out slower than the plane's 200 km/h airspeed.

Scan to watch: A Quartering Wind (non-perpendicular vectors)

5. **The Calculator's Blind Spot.** A search-and-rescue drone leaves base and flies three legs, each an angle in standard position (counterclockwise from east): 60 m at 210°, then 50 m at 240°, then 40 m at 150°. (a) How far is the drone from base, and in what direction (as a standard-position angle)? (b) What single heading would fly it straight back to base? Watch the quadrant carefully — your calculator will try to trick you on this one.

Scan to watch: The Calculator's Blind Spot (arctan quadrant trap)

Solutions to the Challenge Problems

1. **The Treasure Hunt.** ROXY table: $A = (60, 0)$, $B = (0, 80)$, $C = (100\cos 216.9°, 100\sin 216.9°) = (-80, -60)$. Sum: $R_x = -20$, $R_y = +20$, so $R = \sqrt{20^2 + 20^2} =$ **28 m at 135° (northwest)**. The walk home is the opposite vector: **28 m at 315°**.

2. **The Full River Crossing.** The motions are independent: only the boat's 5.0 m/s carries him across, so (a) $t = 100/5.0 =$ **20 s**. The current owns the drift: (b) $(3.0)(20) =$ **60 m downstream**. (c) $R = \sqrt{5.0^2 + 3.0^2} =$ **5.8 m/s at 31° downstream of straight across**. The crossing time does not depend on the current at all; that independence returns in Chapter 3 wearing a projectile costume.

3. **Blown Off Course.** Ground velocity: $R = \sqrt{200^2 + 50^2} = 206$ km/h at $\tan^{-1}(50/200) = 14°$ east of north. Displacement is velocity times time: total $(206)(2.5) \approx$ **520 km**; eastward drift $(50)(2.5) =$ **about 130 km**. A crosswind a quarter of the plane's speed pushes it 130 km off course in one flight; this is why pilots aim upwind on purpose.

4. **A Quartering Wind.** A wind at 315° points 45° below the east axis, so the right-triangle trick will not work; resolve everything into components (east = $+x$, north = $+y$). ROXY table: the plane is (0, 200); the wind is $(50\cos 315°,\ 50\sin 315°) = (+35.4,\ -35.4)$. Sum the columns: $R_x = 35.4$, $R_y = 200 - 35.4 = 164.6$. (a) $R = \sqrt{35.4^2 + 164.6^2} \approx$ **168 km/h** at $\tan^{-1}(35.4/164.6) \approx$ **12° east of north**. (b) Only the wind carries the plane sideways, so the eastward drift is (35.4 km/h)(2.0 h) $\approx$ **71 km**. The ground speed (168) is *slower* than the 200 km/h airspeed because this wind has a southward component that fights the plane's northward motion, the mirror image of "Blown Off Course," where a pure crosswind left the northward speed untouched and only added to it. A blind Pythagorean guess would have given $\sqrt{200^2 + 50^2} = 206$ km/h, which is not just wrong but wrong in the wrong direction; the oblique angle is why you must go to components.

5. **The Calculator's Blind Spot.** ROXY table, every angle from east: leg 1 $(60\cos 210°,\ 60\sin 210°) = (-52.0,\ -30.0)$; leg 2 $(50\cos 240°,\ 50\sin 240°) = (-25.0,\ -43.3)$; leg 3 $(40\cos 150°,\ 40\sin 150°) = (-34.6,\ +20.0)$. Sum the columns: $\Sigma X = -111.6$, $\Sigma Y = -53.3$. Magnitude $R = \sqrt{111.6^2 + 53.3^2} \approx$ **124 m**. Now the trap: $\tan^{-1}(-53.3/-111.6) = \tan^{-1}(0.478) = 25.5°$, but *both* sums are negative, so the drone is in the third quadrant, southwest of base. Since $\Sigma X < 0$, add 180°: the true bearing is $25.5° + 180° =$ **206°**. (b) Straight home is the exact reverse, $206° - 180° =$ **26°**. Here is the sting: the calculator's uncorrected 25.5° is *exactly* the heading home, the opposite of where the drone actually is. Trust it blindly and you search in precisely the wrong direction.

Chapter 2 Summary

- A vector has magnitude and direction and is drawn as an arrow. Direction is part of the arithmetic, so two equal magnitudes can add to many different results.
- Add vectors head to tail; the resultant runs from the first tail to the last head. Perpendicular vectors give a right triangle, so the Pythagorean theorem finds the resultant.
- Any vector splits into perpendicular components: $V_x = V \cos\theta$ and V_y = V sin θ. Components turn a diagonal problem into two simple one-dimensional problems.
- Add vectors with the ROXY method (R, θ, X, Y): convert each vector's R and θ into X = R cos θ and Y = R sin θ, sum the X and Y columns, then convert back with R = $(X^2 + Y^2)$ and $\theta = \tan^{-1}(Y / X)$. This standardized organizer works for any number of vectors and is reused throughout the book.
- Velocity is relative to the observer. Combine motions by adding velocity vectors; a boat in a current or a plane in wind is a perpendicular-vector problem.

Key terms

vector
a quantity with magnitude and direction, drawn as an arrow

magnitude
the size of a vector, shown by the arrow's length

head to tail
the method of adding vectors by starting each where the previous one ended

resultant
the single vector equal to the sum of two or more vectors

components
the perpendicular x and y pieces a vector splits into

ROXY method
a standardized organizer (R, θ, X, Y) for adding vectors: convert each R and θ to X = R cos θ and Y = R sin θ, sum the columns, and convert back to a resultant R and θ

relative velocity
velocity measured with respect to a particular observer, found by adding velocity vectors

Chapter 4

Projectile Motion

PHYS1.PS2.11–2.12

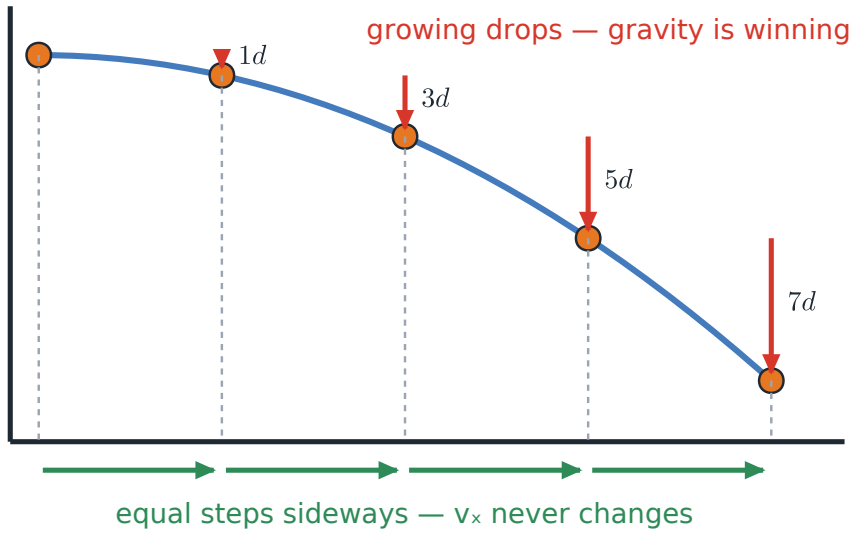
This unit is fully interactive. Alongside this reading, your unit hub has projectable lecture slides, a live Projectile Explorer simulation, self-checking practice, and printable guided notes. Read here first, then go play with the numbers.

3.1 · Two Motions at Once

By the end of this section you can: define a projectile; and state the independence of horizontal and vertical motion.

A **projectile** is any object thrown or launched that then moves under gravity alone. A kicked football, a thrown baseball, a stream of water from a hose, and a stunt car off a ramp are all projectiles once they leave the hand or the ground. Their curved path looks complicated, but it hides a beautifully simple secret that you already built in Chapter 2.

The secret is the **independence of motion**. A projectile's horizontal motion and vertical motion do not affect each other at all. Sideways, nothing pushes or pulls the object once it is flying, so it keeps a *constant horizontal velocity*. Vertically, gravity pulls down the whole time, so the object accelerates downward at $g = -9.8\ \text{m/s}^2$, exactly like the free fall of Chapter 1. Two separate stories, running at the same time, sharing only the clock.



The parabola decoded. *In equal time steps the horizontal gaps stay the same (constant velocity), while the vertical drops grow (gravity accelerates). Two independent motions make the curve.*

3.2 · Horizontal Launch

By the end of this section you can: analyze an object launched horizontally; and explain why it lands at the same time as one simply dropped.

Start with the simplest case: an object launched straight sideways off a table or cliff, with no upward or downward aim. Its vertical motion starts from zero vertical velocity, so vertically it is just a dropped object. Its horizontal motion is a steady cruise at the launch speed. The two run in parallel until it lands.

This leads to the most famous demonstration in the unit. Fire one ball horizontally off a table and drop a second ball straight down at the very same instant. They hit the floor *together*. It feels wrong, because the fired ball is clearly going somewhere, but the vertical stories are identical: both start with zero vertical velocity and both fall under the same g. The horizontal speed changes where the ball lands, not when. Gravity does not care that the ball is also moving sideways.

Misconception check: a projectile's horizontal velocity does not slow down on its own during flight (we ignore air resistance until Section 3.5). Students often expect the sideways motion to "run out." It does not. The only acceleration is downward. Keep horizontal speed constant in your equations and let gravity act only on the vertical.

Sparky rolls off a 1.25 m high table at 3.0 m/s. How long is he in the air, and how far from the table does he land? Take down as positive, g = 9.8 m/s². Vertical (a dropped-object problem): $v_{iy} = 0$, height = 1.25 m. From $y = \frac{1}{2}gt^2$: $1.25 = \frac{1}{2}(9.8)t^2$, so $t^2 = 0.255$ and $t = 0.50$ s. **Horizontal (constant velocity):** $x = v_x t = (3.0)(0.50) = \boxed{1.5 \text{ m}}$. He is airborne 0.50 s and lands 1.5 m out. Notice the two halves never mixed: time came from the vertical drop, then that same time drove the horizontal distance. The clock is the only thing the two motions share.

A ball is launched horizontally at 25 m/s off a 19.6 m cliff. Find the time in the air, the range, and how fast it is moving when it hits. (Take down as positive, g = 9.8 m/s².) Vertical (dropped-object): $19.6 = \frac{1}{2}(9.8)t^2 \Rightarrow t^2 = 4.0 \Rightarrow t = 2.0$ s. **Horizontal:** $x = v_x t = (25)(2.0) = 50$ m. **Impact speed:** the vertical velocity at landing is $v_y = gt = (9.8)(2.0) = 19.6$ m/s. Combine with the steady horizontal 25 m/s like a vector:

$$v = \sqrt{25^2 + 19.6^2} = \boxed{32 \text{ m/s}}$$

It lands 50 m out, moving 32 m/s along a downward slant. The horizontal part never changed; only the vertical part grew.

A rock is thrown horizontally at 12 m/s from a 78.4 m cliff. How long is it in the air, and how far does it land from the base? Vertical: $78.4 = \frac{1}{2}(9.8)t^2 \Rightarrow t^2 = 16 \Rightarrow t = 4.0$ s. **Horizontal:** $x = (12)(4.0) = \boxed{48 \text{ m}}$. Doubling the launch height did not double the range, because time grows with the *square root* of the height. The vertical drop always sets the clock.

Check yourself: 3.2

1. Two balls leave a table at the same instant, one fired sideways and one dropped. Which lands first?
   They land together: both have zero initial vertical velocity and fall under the same gravity, so horizontal speed does not change the fall time.
2. What is a horizontally launched projectile's vertical initial velocity?
   Zero, so vertically it behaves exactly like a dropped object.
3. What quantity links the horizontal and vertical parts of the problem?
   Time. The flight time is found from the vertical motion, then used in the horizontal motion.

3.3 · Launched at an Angle

By the end of this section you can: break an angled launch into components; and use the symmetry of the path to find flight time and height.

Now aim the launch upward at an angle, like a kicked field goal. The trick is the one from Chapter 2: split the launch velocity into components. The horizontal component $v_x = v\cos\theta$ stays constant the whole flight. The vertical component $v_y = v\sin\theta$ behaves like a ball thrown straight up, slowing as it rises, stopping for an instant at the top, then speeding up on the way down. Once split, each axis is a problem you already know.

The rising-and-falling vertical motion is beautifully **symmetric**. The time up equals the time down. The launch speed equals the landing speed. The object passes any given height at the same speed going up as coming down. This symmetry is a shortcut: find the time to the top, double it for the total flight time, and much of the problem falls into place.

A bouncing ball, caught by a strobe flashing 25 times per second. Every arc is a parabola, and look closely at the tops: the images crowd together there, because a projectile moves slowest at the peak of its flight. Same flash rate everywhere; the ball itself slows down. This one photograph is most of Chapter 3.

Sparky fires a dart at 20 m/s, 30° above horizontal, over level ground. How long is it in the air? (From Chapter 2, $\mathbf{v_x = 17}$ m/s and $\mathbf{v_y = 10}$ m/s. Take up as positive, $\mathbf{g = -9.8\ m/s^2}$.) Time to the top (where v_y momentarily equals 0), from $v_f = v_i + at$: $0 = 10 + (-9.8)t$, so $t = 1.02$ s. **Total flight time** by symmetry: $2 \times 1.02 = \boxed{2.0\text{ s}}$. The dart is airborne about 2.0 seconds. From here the horizontal range is one short step away, which is the next section.

A ball is kicked at 25 m/s, 53° above level ground. Find its total flight time and its maximum height. ($\mathbf{g = -9.8\ m/s^2}$) Split the launch: $v_x = 25\cos 53° = 15$ m/s and $v_y = 25\sin 53° = 20$ m/s. **Time to the top** ($v_y = 0$): $t = \dfrac{0 - 20}{-9.8} = 2.04$ s. **Total time** by symmetry: $2 \times 2.04 = 4.1$ s. **Max height** from $v_f^2 = v_i^2 + 2gd$: $d = \dfrac{-(20)^2}{2(-9.8)} = \boxed{20\text{ m}}$. The steeper 53° kick spends over four seconds aloft and climbs about 20 m.

A projectile is launched at 30 m/s, 40° above the horizontal. Find its total flight time and maximum height. $v_y = 30 \sin 40° = 19.3$ m/s. Time to top: $19.3/9.8 = 1.97$ s; total $= 3.9$ s. Max height: $d = \dfrac{-(19.3)^2}{2(-9.8)} = \boxed{19 \text{ m}}$. A slightly flatter, faster launch gives nearly the same height but a shorter hang time than the 53° kick.

Check yourself: 3.3

1. What happens to the horizontal component of velocity during an angled flight?
   It stays constant, because nothing accelerates the object horizontally.
2. At the very top of the arc, what is the vertical velocity? The horizontal velocity?
   Vertical velocity is zero; horizontal velocity is unchanged from launch.
3. How does symmetry help find total flight time?
   Time up equals time down, so total time is twice the time to the top.

3.4 · Range, Height, and Launch Angle

By the end of this section you can: find a projectile's range; and explain how launch angle affects range and height.

The **range** is the horizontal distance a projectile covers before landing. Because horizontal velocity is constant, range is just the steady horizontal speed multiplied by the total flight time: $R = v_x\, t$. For Sparky's dart, that is 17 m/s × 2.0 s = 34 m. Every angled-launch problem eventually comes down to this one multiplication, once you have the flight time from the vertical motion.

Launch angle trades height for distance. A steep launch sends the object high but not far. A shallow launch keeps it low but stretched out. On level ground the two effects balance at **45°**, which gives the maximum range for a given launch speed. Angles that add up to 90°, such as 30° and 60°, give the same range as each other, one arcing high and slow, the other flat and fast. Athletes feel this in their bones: a long jumper launches near 45°, while a line-drive throw stays low for speed.

Try it yourself. Open the Projectile Explorer and hold the launch speed fixed while you sweep the angle from 15° to 75°. Watch the range peak at 45°, and watch 30° and 60° land in the same spot. Seeing it beats memorizing it.

Finish the dart from Example 3.3: launched at 20 m/s, 30°, with $v_x = 17$ m/s and a total flight time of 2.04 s. Find its range.

$$R = v_x\, t = (17)(2.04) = \boxed{35 \text{ m}}$$

Range is always the steady horizontal speed times the total time aloft. The vertical motion gave the time; the horizontal motion spends it.

Use the kick from Example 3.3b ($v_x = 15$ m/s, total time 4.08 s). Find its range.

$$R = (15)(4.08) = \boxed{61 \text{ m}}$$

A long, high kick like this can clear more than half a football field.

Show that a launch of 30 m/s gives the same range at 30° as at 60°. At 30°: $v_x = 30\cos 30° = 26$ m/s, $v_y = 15$ m/s, total time $= 2(15)/9.8 = 3.06$ s, so $R = 26 \times 3.06 = 80$ m. **At 60°:** $v_x = 15$ m/s, $v_y = 26$ m/s, total time $= 2(26)/9.8 = 5.30$ s, so $R = 15 \times 5.30 = \boxed{80 \text{ m}}$. Same range, 80 m, by two different paths: the 30° shot is flat and fast, the 60° shot is high and slow. Complementary angles always tie.

Check yourself: 3.4

1. How do you find the range of a projectile?
   Multiply the constant horizontal velocity by the total time in the air: $R = v_x \cdot t$.
2. What launch angle gives the maximum range on level ground?
   45°.
3. Which reaches a greater height for the same launch speed, 30° or 60°?
   60°, the steeper launch. They share the same range, but 60° goes higher and slower.

3.5 · When Air Fights Back: Terminal Velocity

By the end of this section you can: explain how air resistance changes real projectile motion; and describe terminal velocity.

Everything so far ignored the air, which is a fair simplification for a dense, slow object over a short flight. But air pushes back, and for light objects or long falls it changes the story. **Air resistance** is a force that opposes motion through air, and it grows stronger the faster you go. That growing-with-speed behavior leads to a tidy result for a falling object.

When you first start falling, you are slow, so air resistance is small and gravity easily wins, speeding you up. But as you speed up, the upward air resistance grows, until it finally equals your weight. Now the forces balance, the acceleration drops to zero, and you fall at a steady speed called **terminal velocity**. A skydiver in a belly-flat spread reaches about 55 m/s, roughly 120 mph. A feather, with lots of air resistance for its tiny weight, reaches terminal velocity almost instantly at a gentle drift. This is exactly why the rock and the paper in Chapter 1 fell together only once we removed the air.

Faith & Science , "But they who wait for the LORD shall renew their strength; they shall mount up with wings like eagles" (Isaiah 40:31). There is a lesson in how a projectile stays aloft only for a while before gravity brings it home, and in how even a falling body finds a steady, bearable speed rather than an endless rush. The world is built with limits that hold us. An eagle does not defeat gravity; it works with the air, and it soars. The verse promises a strength that is given, not manufactured, to those who wait on God. Physics cannot preach that, but it can show a world where rising and resting are both built in, and where even the fall has a mercy in it.

Practice Problems

Work these on your own, then open the solutions to check yourself. Keep horizontal and vertical motion separate, and let time be the bridge between them. Use $g = 9.8$ m/s^2.

1. A ball rolls off a 44.1 m high roof at 10 m/s horizontally. Find the time in the air and the range.
2. A marble is launched horizontally at 6.0 m/s off a 1.8 m table. How long is it airborne and how far does it land from the table's edge?
3. A ball is launched at 40 m/s, 30° above level ground. Find its total flight time.
4. Using the same launch (40 m/s at 30°), find the range.
5. For that launch, find the maximum height.
6. What launch angle gives the greatest range on level ground, and what do we call two angles that give equal ranges?
7. Explain in one or two sentences why a horizontally fired bullet and a dropped bullet hit level ground at the same time.

Solutions to the Practice Problems

1. Vertical: $44.1 = \frac{1}{2}(9.8)t^2 \Rightarrow t^2 = 9.0 \Rightarrow t = 3.0$ s. Range: $x = (10)(3.0) = 30$ m.
2. $1.8 = \frac{1}{2}(9.8)t^2 \Rightarrow t^2 = 0.367 \Rightarrow t = 0.61$ s; $x = (6.0)(0.61) = 3.6$ m.
3. $v_y = 40 \sin 30° = 20$ m/s; time to top $= 20/9.8 = 2.04$ s; total $= 4.1$ s.
4. $v_x = 40 \cos 30° = 34.6$ m/s; $R = 34.6 \times 4.08 = 141$ m.
5. $d = \dfrac{-(20)^2}{2(-9.8)} = 20$ m.
6. 45° gives the maximum range; two angles that add to 90° (complementary angles, like 30° and 60°) give equal ranges.
7. Both start with zero vertical velocity and fall under the same gravity, and the horizontal motion is independent, so the sideways speed changes only *where* the bullet lands, not *when*.

Challenge Problems

The hard ones. Each of these takes more than one step and more than one idea; some borrow tools from earlier chapters. Work them in GUESS order (Given, Unknown, Sketch, Equation, Substitute, Solve) before opening the solutions.

1. **The Stunt Driver's Minimum.** A stunt car drives horizontally off a 30 m cliff and must clear a 24 m wide moat at the base. What is the minimum launch speed? (Down positive, $g = 9.8$ m/s^2)

Scan to watch: The Stunt Driver's Minimum (projectiles)

2. **Clearing the Fence.** A ball is kicked at 20 m/s, 53° above level ground, toward a 3.0 m tall fence 35 m away. Does it clear the fence, and by how much?

Scan to watch: Clearing the Fence (worked example)

3. **Off the Cliff at an Angle.** Sparky is launched at 20 m/s, 30° above horizontal, from the top of a 35 m cliff. Find the total time in the air, how far from the base he lands, and his speed at impact.

Scan to watch: Off the Cliff at an Angle

Solutions to the Challenge Problems

1. **The Stunt Driver's Minimum.** The fall sets the clock: $30 = \frac{1}{2}(9.8)t^2$ gives $t = 2.47$ s, no matter how fast the car goes. The horizontal motion must cover 24 m in that time: $v = 24/2.47 \approx$ **9.7 m/s**. The usual problem hands you the speed and asks for the range; this one runs the same physics backward.

2. **Clearing the Fence.** Components: $v_x = 20\cos 53° = 12.0$, $v_y = 20\sin 53° = 16.0$ m/s. "At the fence" is a time: $t = 35/12.0 = 2.91$ s. Height there: $y = (16.0)(2.91) - 4.9(2.91)^2 = 5.0$ m. It **clears the 3.0 m fence by about 2.0 m**. The horizontal motion tells you WHEN; the vertical motion tells you HOW HIGH.

3. **Off the Cliff at an Angle.** $v_x = 17.3$, $v_y = +10$ m/s. Velocity first (no quadratic): $v_{yf}^2 = 10^2 + 2(-9.8)(-35) = 786$, so $v_{yf} = -28.0$ m/s. Then $t = (-28.0 - 10)/(-9.8) = \mathbf{3.9\ s}$, range $= (17.3)(3.9) \approx \mathbf{67\ m}$, impact speed $= \sqrt{17.3^2 + 28.0^2} \approx \mathbf{33\ m/s}$. Components from Chapter 2, free-fall bookkeeping from Chapter 1, independence from Chapter 3: the full toolkit.

Chapter 3 Summary

- A projectile moves under gravity alone after launch. Its horizontal and vertical motions are independent, sharing only the flight time.
- Horizontal motion is constant velocity; vertical motion is free fall with a = g. A horizontally fired object lands at the same time as one simply dropped.
- For an angled launch, split the velocity into components. The horizontal component stays constant; the vertical component rises and falls symmetrically, so time up equals time down.
- Range is horizontal velocity times flight time, $R = v_x \cdot t$. On level ground 45° gives the maximum range, and complementary angles (like 30° and 60°) give equal ranges.
- Air resistance opposes motion and grows with speed. A falling object reaches terminal velocity when air resistance balances weight and the acceleration becomes zero.

Key terms

projectile
an object moving under gravity alone after being thrown or launched

independence of motion
the principle that horizontal and vertical motions do not affect each other

symmetric (path)
the up-and-down halves of a projectile's flight mirror each other in time and speed

range
the horizontal distance a projectile travels before landing

air resistance
a force opposing motion through air that grows stronger with speed

terminal velocity
the steady falling speed reached when air resistance balances weight

Chapter 5

Newton's Laws of Motion

PHYS1.PS2.4–2.8

4.1 · Force and the First Law

By the end of this section you can: define force; find the net force on an object; state Newton's first law; and explain inertia.

Chapters 1 through 3 described motion. Now we ask what causes it. The cause is a **force**, a push or a pull, measured in **newtons** (N). Forces are vectors, so they have direction, and often several act on one object at once. What matters for motion is the **net force**, written ΣF, the vector sum of all forces acting on the object. The net force is the single push that would have the same effect as all of them together.

Newton's first law says: an object at rest stays at rest, and an object in motion stays in motion at constant velocity, unless a net force acts on it. Objects resist changes to their motion, and that resistance is called **inertia**. A hockey puck sliding on smooth ice keeps gliding in a straight line because almost no net force acts on it. The old idea that motion needs a continuing push is wrong. Motion continues on its own; it is *change* in motion that needs a force.

Misconception check: a moving object does not need a force to keep moving. This is the single most common misconception in physics, because on Earth friction and air quietly slow everything down, hiding the truth. Remove those forces, as in deep space or on frictionless ice, and motion simply continues forever. Force is required to *change* velocity, not to maintain it.

Three students pull a rope tied to a sled. Two pull right with 40 N and 25 N; one pulls left with 30 N. What is the net force on the sled? Take right as positive and add: $\Sigma F = (+40) + (+25) + (-30) = +35$ N.

$$\Sigma F = 40 + 25 - 30 = \boxed{+35 \text{ N (to the right)}}$$

The three real forces behave exactly like a single 35 N pull to the right. By the first law, if this net force were zero the sled would keep whatever velocity it had; because it is not zero, the sled's motion will change.

A box is pushed with 20 N to the right, while a 12 N friction force and a 5.0 N shove both act to the left. Find the net force. Right positive, add them all:

$$\Sigma F = (+20) + (-12) + (-5.0) = \boxed{+3.0 \text{ N (right)}}$$

Three forces reduce to one small 3.0 N push. The box will speed up to the right, but only gently.

A crate slides across a floor at a steady 2.0 m/s while a person pushes it with 40 N and friction pushes back with 40 N. What is the net force, and what does the crate do? $\Sigma F = (+40) + (-40) = \boxed{0}$. With zero net force, the first law takes over: the crate keeps its constant velocity of 2.0 m/s. Balanced forces do not mean stopped; they mean *unchanging* motion.

From the engineer's desk: this is Statics. When the forces on something balance so the net force is zero, engineers call that **equilibrium**, and the whole branch of engineering built on it is **Statics**. It is what keeps a bridge, a building, or a crane standing safely under load: every force is answered by an equal and opposite one, so nothing accelerates. A civil or structural engineer spends much of a career making sure $\Sigma F = 0$ (and that the torques balance too) for structures that must never move. Every equilibrium problem in this chapter is your first Statics problem.

Check yourself: 4.1

1. What is the net force on an object?
   The vector sum of all forces acting on it, ΣF.
2. State Newton's first law in your own words.
   An object keeps doing what it is doing, at rest or moving at constant velocity, unless a net force changes it.
3. What is inertia?
   An object's resistance to a change in its motion; more mass means more inertia.
4. Two forces, 12 N east and 12 N west, act on a box. What is the net force, and what does the box's motion do?
   Net force is zero, so the box keeps constant velocity (first law): if it was at rest it stays at rest; if it was moving it keeps moving at the same speed.

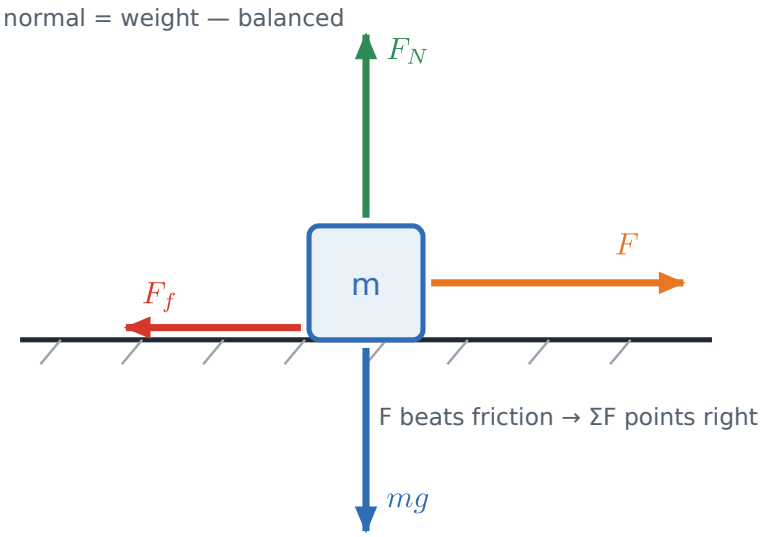
4.2 · Free-Body Diagrams

By the end of this section you can: draw a free-body diagram; and use it to find the net force on an object.

The most useful skill in this whole unit is drawing a **free-body diagram**: a simple sketch showing

every force acting *on* one chosen object, drawn as arrows pointing away from a dot that represents the object. Get the diagram right and the physics almost solves itself. Get it wrong, and no amount of algebra will save you.

A few forces show up again and again. **Weight** (the pull of gravity) always points straight down. The **normal force** F_N is the support a surface pushes back with, always perpendicular to the surface. **Friction** F_f acts along a surface, opposing sliding. **Tension** T is the pull of a rope or cable, directed along the rope. Applied pushes and pulls point whichever way they are applied. Draw only forces acting on your object, never forces it exerts on other things.



A free-body diagram. *Every force acting on the box, drawn from a single dot. Up-down forces here balance; the left-right forces decide whether it accelerates.*

Once the diagram is drawn, Newton's second law is applied one direction at a time. Add the vertical forces to get the vertical net force, add the horizontal forces to get the horizontal net force, and handle each axis on its own, exactly the component thinking from Chapter 2.

Check yourself: 4.2

1. What does a free-body diagram show?
   Every force acting on one chosen object, drawn as arrows from a single point.
2. Which direction does the normal force point?
   Perpendicular to the surface, pushing away from it.
3. Should a free-body diagram include forces the object exerts on other things?
   No, only forces acting on the object itself.

4.3 · The Second Law: ΣF = ma

By the end of this section you can: state and apply Newton's second law; relate net force, mass, and acceleration; and solve for any one of the three.

Newton's second law is the engine of the whole course, and it fits on one line:

$$\Sigma F = ma$$

The net force on an object equals its mass times its acceleration. Read what it says. A net force produces an acceleration in the same direction as the force. For the same force, a heavier object accelerates less, because mass is inertia and inertia resists change. And if the net force is zero, the acceleration is zero, which is just Newton's first law falling out as a special case. Units line up too: one newton is exactly the force that gives 1 kg an acceleration of 1 m/s^2, so 1 N = 1 kg · m/s^2.

From the engineer's desk: this is Dynamics. The moment the forces stop balancing, $\Sigma F = ma$ takes over and things accelerate. The engineering branch that studies motion under unbalanced forces is **Dynamics**, the heart of mechanical engineering: engines, robots, vehicles, roller coasters, anything that speeds up, slows down, or turns. Every $\Sigma F = ma$ problem in this chapter is a first taste of Dynamics. Here is the tidy way I want you to remember the pair: **Statics keeps the world standing still; Dynamics makes it move.** One is civil and structural engineering, the other is mechanical engineering, and both are just Newton's laws with a job to do.

A net force of 12 N acts on Sparky, whose mass is 3.0 kg. What is his acceleration? **Given:** $\Sigma F = 12$ N, $m = 3.0$ kg. **Equation:** $a = \dfrac{\Sigma F}{m}$.

$$a = \frac{12\ \text{N}}{3.0\ \text{kg}} = \boxed{4.0\ \text{m/s}^2}$$

Units check: N/kg is (kg · m/s^2)/kg = m/s^2, an acceleration. If Sparky carried more mass, the same 12 N would give a smaller acceleration.

A 5.0 kg cart feels a 20 N push forward and a 8.0 N friction force backward. Find its acceleration. First combine the forces into one net force: $\Sigma F = 20 - 8.0 = 12$ N forward. Then apply the second law:

$$a = \frac{\Sigma F}{m} = \frac{12\ \text{N}}{5.0\ \text{kg}} = \boxed{2.4\ \text{m/s}^2\ \text{forward}}$$

Always find the *net* force before dividing by mass. A common slip is to plug in the 20 N push and forget the friction. The second law only ever uses the total.

What force is needed to accelerate a 1200 kg car from rest to 27 m/s (about 60 mph) in 9.0 s? First the acceleration, from Chapter 1: $a = \dfrac{v_f - v_i}{t} = \dfrac{27 - 0}{9.0} = 3.0$ m/s^2. Then the force:

$$\Sigma F = ma = (1200)(3.0) = \boxed{3600 \text{ N}}$$

This is a good example of how the two halves of the course join up: kinematics gives the acceleration, and the second law turns it into a force.

Check yourself: 4.3

1. State Newton's second law as an equation and in words.
   $\Sigma F = ma$: the net force equals mass times acceleration, and acceleration points the same way as the net force.
2. The same force is applied to a bowling ball and a soccer ball. Which accelerates more?
   The soccer ball, because it has less mass, so less inertia to overcome.
3. A 2.0 kg cart accelerates at 3.0 m/s^2. What net force acts on it?
   $\Sigma F = ma = (2.0)(3.0) = 6.0$ N.
4. A 4.0 kg block feels a 30 N pull forward and 6.0 N of friction. Find its acceleration.
   $\Sigma F = 30 - 6.0 = 24$ N; $a = 24/4.0 = 6.0$ m/s^2.

4.4 · Weight, Mass, and the Normal Force

By the end of this section you can: distinguish mass from weight; compute weight from mass; and find the normal force, including when an object accelerates vertically.

People use "mass" and "weight" as if they mean the same thing, but in physics they are sharply different. **Mass** is the amount of matter in an object, measured in kilograms, and it never changes no matter where you go. **Weight** is the force of gravity *on* that mass, measured in newtons, and it does change with location. Weight is just the second law applied to gravity:

$$W = mg$$

Sparky's 3.0 kg mass weighs about $(3.0)(9.8) = 29$ N on Earth, but only about 5 N on the Moon, where g is one-sixth as strong. His mass is the same in both places; only his weight changed.

Sitting on a table, Sparky does not fall through it, so the table must push up on him. That upward push is the **normal force** F_N. On flat ground with nothing else pushing vertically, the normal force simply balances the weight, so $F_N = mg$. But normal force is not always equal to weight. Press down on the object, tip the surface, or accelerate up or down, and it changes. The normal force is whatever it needs to be to keep the object from sinking into the surface, no more and no less.

A 60 kg person stands on a scale in an elevator. What does the scale read (the normal force) when the elevator accelerates upward at 2.0 m/s²? Two vertical forces act on the person: the normal force F_N up and the weight mg down. Taking up as positive, the second law gives:

$$\Sigma F = F_N - mg = ma \quad \Rightarrow \quad F_N = m(g + a)$$

$$F_N = 60(9.8 + 2.0) = 708 \text{ N}$$

At rest the scale would read $mg = \boxed{588 \text{ N}}$. Accelerating upward, it reads 708 N, which is why you feel heavier as an elevator starts up. The normal force is not equal to the weight here, because the person is accelerating.

What is the weight of an 8.0 kg object on Earth? What is its mass on the Moon? Weight is the pull of gravity: $W = mg = (8.0)(9.8) = \boxed{78 \text{ N}}$. Mass, the amount of matter, does not change with location, so on the Moon its **mass is still 8.0 kg**. Only its weight would change (to about 13 N), because the Moon's gravity is weaker.

A 5.0 kg box rests on a level table. (a) What is the normal force? (b) Now you press straight down on it with an extra 10 N. What is the normal force? (a) With no vertical acceleration, up and down balance: $F_N = mg = (5.0)(9.8) = 49$ N. (b) The table must now support the weight *plus* your push: $F_N = mg + 10 = 49 + 10 = \boxed{59 \text{ N}}$. The normal force adjusts to whatever it takes to keep the box from accelerating through the table. It is rarely a fixed number.

Misconception check: mass and weight are not the same. Mass (kg) is how much stuff you are; weight (N) is how hard gravity pulls on that stuff. A bathroom scale reads the normal force it pushes back with, which equals your weight only when you are not accelerating vertically. On the Moon you would weigh less while being made of exactly the same amount of matter.

Check yourself: 4.4

1. What is the difference between mass and weight?
   Mass is the amount of matter (kg) and never changes; weight is the gravitational force on it (N) and changes with location.
2. What is the weight of a 10 kg object on Earth?
   $W = mg = (10)(9.8) = 98$ N.
3. Does the normal force always equal an object's weight?
   No. Only on level ground with no other vertical forces and no vertical acceleration.
4. The same elevator now accelerates *downward* at 2.0 m/s². What does the scale read for the 60 kg person?
   $F_N = m(g - a) = 60(9.8 - 2.0) = 468$ N — you feel lighter.

4.5 · Friction

By the end of this section you can: describe static and kinetic friction; compute a friction force from the normal force; and decide whether an object will start to move.

Friction is a force that opposes sliding between two surfaces in contact. It comes in two kinds. **Static friction** holds a still object in place, and it adjusts itself to match whatever is trying to move the object, up to a maximum. **Kinetic friction** acts once the object is already sliding, and it is generally a bit weaker than the peak static value, which is why a heavy box lurches into motion once it finally breaks free. Both depend on how hard the surfaces are pressed together, the normal force, and on the roughness of the pair of surfaces, captured by a **coefficient of friction** μ:

$$F_f = \mu F_N$$

Notice what friction does *not* depend on: speed and surface area barely matter for everyday problems. It is the pressing force and the materials that count. Friction is also a two-faced character. It steals energy and wears out machines, but without it you could not walk, drive, or hold a pencil.

Sparky pushes a 20 kg crate across level ground where the coefficient of kinetic friction is 0.30. What friction force opposes him? On level ground the normal force equals the weight: $F_N = mg = (20)(9.8) = 196$ N. Then:

$$F_f = \mu F_N = (0.30)(196) = \boxed{59 \text{ N}}$$

If Sparky pushes with exactly 59 N, the crate glides at constant speed (zero net force). To speed it up, he must push harder than 59 N.

A 10 kg box sits on the floor with $\mu_s = 0.40$ and $\mu_k = 0.30$. You push horizontally with 30 N. Does it move? If you then push with 50 N, what is its acceleration? First find the *maximum* static friction, the most the floor can resist before the box breaks free. With $F_N = mg = 98$ N:

$$F_{f,\max} = \mu_s F_N = (0.40)(98) = \boxed{39 \text{ N}}$$

Your 30 N push is less than 39 N, so static friction simply matches it and **the box does not move**. Push with 50 N, which beats 39 N, and it breaks free. Now kinetic friction takes over: $F_f = \mu_k F_N = (0.30)(98) = 29$ N, so

$$a = \frac{\Sigma F}{m} = \frac{50 - 29}{10} = 2.1 \text{ m/s}^2$$

The lesson: compare your push to the maximum static friction first. Below it, nothing moves; above it, use kinetic friction to find the acceleration.

A 6.0 kg box slides across a floor where $\mu_k = 0.30$. Find the kinetic friction force, and the acceleration if nothing else pushes it. Normal force on level ground: $F_N = mg = (6.0)(9.8) = 58.8$ N. Friction:

$$F_f = \mu_k F_N = (0.30)(58.8) = \boxed{18 \text{ N}}$$

With only friction acting, $\Sigma F = -18$ N, so $a = \Sigma F/m = -18/6.0 = -2.9$ m/s^2. The box slows at 2.9 m/s^2 until it stops. Friction alone is what brings a sliding object to rest.

Check yourself: 4.5

1. What is the difference between static and kinetic friction?
   Static friction holds a still object and adjusts up to a maximum; kinetic friction acts on a sliding object and is usually a bit weaker.
2. What two things determine the friction force?
   The normal force pressing the surfaces together and the coefficient of friction for that pair of surfaces.
3. Find the kinetic friction on a box with $F_N = 50$ N and $\mu_k = 0.20$.
   $F_f = \mu_k F_N = (0.20)(50) = 10$ N.
4. A 5.0 kg box has $\mu_s = 0.50$. What is the least horizontal force that will start it moving?
   $F_{f,\max} = \mu_s mg = (0.50)(5.0)(9.8) = 24.5$ N; a push just over 24.5 N starts it.

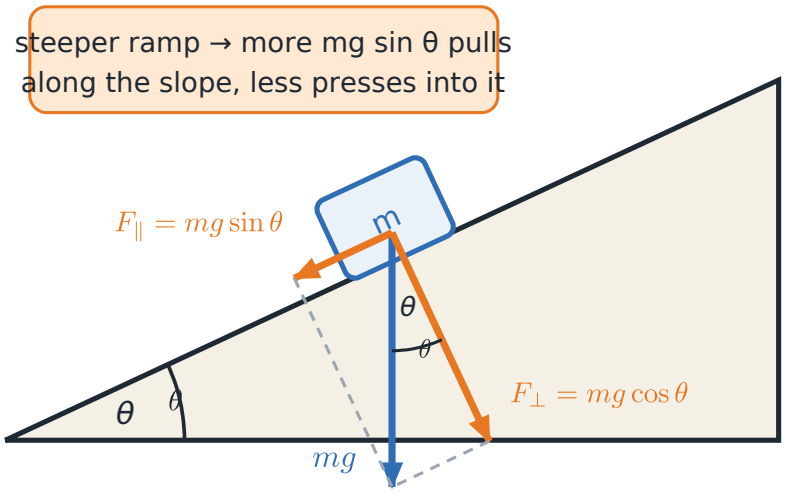
4.6 · Inclined Planes

By the end of this section you can: resolve weight into components on a ramp; find the normal force on an incline; and find the acceleration of an object sliding up or down, with and without friction.

Ramps show up everywhere, from wheelchair access to ski slopes, and they are the classic place to use the component thinking from Chapter 2. The trick is to tilt your axes to match the ramp: one axis *along* the surface and one *perpendicular* to it. Gravity still points straight down, so on a ramp at angle θ the weight splits into two parts, one pulling the object down the slope and one pressing it into the surface:

$$F_{\parallel} = mg\sin\theta \qquad F_{\perp} = mg\cos\theta$$

The part along the ramp, $mg\sin\theta$, is what makes the object slide. The part into the ramp, $mg\cos\theta$, is balanced by the normal force, so on an incline $F_N = mg\cos\theta$, *not* the full weight. That smaller normal force is why friction is weaker on a slope than on flat ground.



Weight on a ramp. *Gravity (mg, down) splits into a part along the slope (mg sinθ, which slides the block) and a part into the slope (mg cosθ, balanced by the normal force).*

A 2.0 kg block is released on a frictionless ramp tilted at 30°. What is its acceleration down the slope? Along the ramp, the only force is the sliding component of gravity, $mg\sin\theta$. So the second law along the slope reads:

$$\Sigma F = mg\sin\theta = ma \quad \Rightarrow \quad a = g\sin\theta$$

$$a = (9.8)\sin 30^\circ = (9.8)(0.50) = \boxed{4.9\ \text{m/s}^2}$$

Notice the mass cancels: on a frictionless ramp, every object slides with the same acceleration $g\sin\theta$, a cousin of the free-fall result from Chapter 1. A steeper ramp (larger θ) gives a larger acceleration, up to g itself when the ramp is vertical.

The same 2.0 kg block slides down a 30° ramp, but now the coefficient of kinetic friction is 0.20. Find its acceleration. Two forces act along the ramp now: gravity pulling it down the slope, $mg\sin\theta$, and friction resisting the slide, $F_f = \mu F_N = \mu\, mg\cos\theta$. The net force along the slope is their difference:

$$ma = mg\sin\theta - \mu\, mg\cos\theta \quad \Rightarrow \quad a = g(\sin\theta - \mu\cos\theta)$$

$$a = 9.8\,(\sin 30^\circ - 0.20\cos 30^\circ) = 9.8\,(0.500 - 0.173) = \boxed{3.2\ \text{m/s}^2}$$

Friction shaved the acceleration from 4.9 down to 3.2 m/s^2. If the ramp were shallow enough that $\mu\cos\theta$ matched $\sin\theta$, the block would slide at constant speed or not start at all.

A 4.0 kg block slides down a 20° ramp where $\mu_k = 0.15$. Find the normal force and the acceleration. Normal force (perpendicular part of the weight): $F_N = mg\cos\theta = (4.0)(9.8)\cos 20° = 37$ N. **Acceleration** along the slope:

$$a = g(\sin\theta - \mu\cos\theta) = 9.8(\sin 20° - 0.15\cos 20°) = 9.8(0.342 - 0.141) = \boxed{2.0 \text{ m/s}^2}$$

The normal force is well below the block's full 39 N weight, because only the cosine part presses into the ramp.

Check yourself: 4.6

1. On a ramp at angle θ, which weight component makes an object slide, and which sets the normal force?
   $mg\sin\theta$ slides it (along the ramp); $mg\cos\theta$ presses into the ramp and equals the normal force.
2. Why is an object's acceleration down a frictionless ramp independent of its mass?
   Because $a = g\sin\theta$; the mass cancels out of $mg\sin\theta = ma$.
3. Find the acceleration of a block down a frictionless 20° ramp.
   $a = g\sin 20° = 9.8(0.342) = 3.4$ m/s^2.
4. Why is the normal force on a ramp less than the object's full weight?
   Only the perpendicular part of gravity, $mg\cos\theta$, presses into the surface, and $\cos\theta < 1$ for any real slope.

4.7 · Tension, Pulleys, and Connected Objects

By the end of this section you can: analyze objects connected by ropes and pulleys; and find the shared acceleration and the tension.

Tension is the pull a rope, string, or cable exerts, always directed along the rope and away from the object it pulls. An ideal rope has one useful property: it passes the same tension through to both ends, even around a frictionless pulley, which simply changes the pull's direction without changing its size. When two objects are tied together, they must move together, so they share one **acceleration**, and the rope between them carries one tension.

These problems have a reliable recipe. Because the connected objects share an acceleration, treat them as one system to find that acceleration: the net outside force divided by the total mass. Then look at just one object to find the tension in the rope.

A 3.0 kg block sits on a frictionless table, tied by a string over a pulley at the edge to a 2.0 kg mass hanging off the side. Find the acceleration of the system and the tension in the string. The whole system. The only outside force driving the motion is gravity on the hanging mass, $m_2 g$. The total mass being moved is $m_1 + m_2$. So:

$$a = \frac{m_2\, g}{m_1 + m_2} = \frac{(2.0)(9.8)}{3.0 + 2.0} = \boxed{3.9 \text{ m/s}^2}$$

One object for the tension. Look at the table block: the only horizontal force on it is the string's tension, so $T = m_1 a$:

$$T = m_1 a = (3.0)(3.9) = 12 \text{ N}$$

Check it against the hanging mass: its weight $m_2 g = 19.6$ N minus the tension 12 N gives a net of 7.6 N, and $7.6/2.0 = 3.9$ m/s^2, the same acceleration. Consistent.

Two masses, 3.0 kg and 5.0 kg, hang from a string over a frictionless pulley. Find the acceleration and the tension. Treat the pair as one system. The driving force is the difference in the two weights; the mass being moved is their sum:

$$a = \frac{(m_2 - m_1)\, g}{m_1 + m_2} = \frac{(5.0 - 3.0)(9.8)}{8.0} = \boxed{2.5 \text{ m/s}^2}$$

For the tension, look at the lighter mass, which accelerates upward: the tension must beat its weight by enough to accelerate it, so $T - m_1 g = m_1 a$, giving

$$T = m_1(g + a) = 3.0(9.8 + 2.5) = 37 \text{ N}$$

Checking on the heavier mass: $m_2 g - T = 5.0(9.8) - 37 = 12$ N, and $12/5.0 = 2.5$ m/s^2. The books balance, so the answer holds.

Misconception check: the tension in the connecting rope is *not* simply the weight of the hanging mass. If it were, there would be no net force and nothing would accelerate. The tension is always somewhere between the two weights, exactly the value that gives both objects the same acceleration. Solve for it; do not assume it.

A 3.0 kg block and a 2.0 kg block sit connected by a string on a frictionless table. A 10 N force pulls the 3.0 kg block horizontally. Find the acceleration of the pair and the tension in the connecting string. The whole system. One 10 N force pulls a total mass of 5.0 kg:

$$a = \frac{F}{m_1 + m_2} = \frac{10}{3.0 + 2.0} = \boxed{2.0 \text{ m/s}^2}$$

One block for the tension. The only force on the trailing 2.0 kg block is the string, so:

$$T = m_2\, a = (2.0)(2.0) = 4.0 \text{ N}$$

The string only has to accelerate the back block, so it carries less than the full 10 N pull. Same recipe as the pulley: system first for a, one object for T.

Check yourself: 4.7

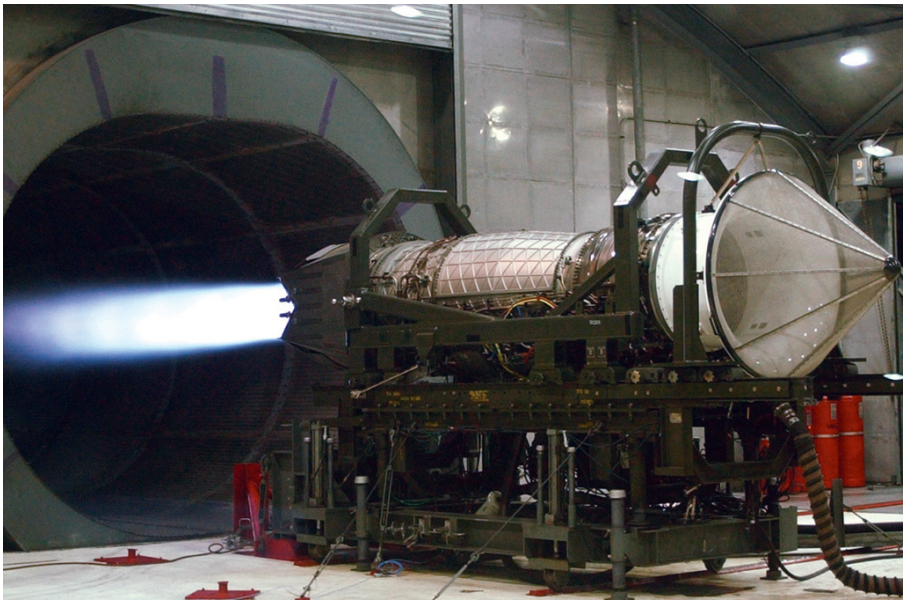
1. What is tension, and which way does it act?
   The pull of a rope, directed along the rope and away from the object it pulls.
2. Two connected objects that move together share what two quantities?
   The same acceleration, and (through the connecting rope) one tension.
3. In an Atwood machine with equal masses, what is the acceleration?
   Zero: with $m_1 = m_2$ the driving force $(m_2 - m_1)g$ is zero, so the system stays at rest or moves at constant velocity.

4.8 · The Third Law: Action and Reaction

By the end of this section you can: state Newton's third law; identify action-reaction pairs; and explain why they do not cancel.

Newton's third law says: for every action, there is an equal and opposite reaction. Whenever object A pushes on object B, object B pushes back on A with a force equal in size and opposite in direction. Forces always come in pairs. You push on the ground to walk, and the ground pushes you forward. A rocket pushes hot gas out the back, and the gas pushes the rocket forward. A swimmer pushes water backward, and the water pushes the swimmer ahead.

Here is the subtle part that trips everyone up. The two forces in an action-reaction pair act on *different objects*, so they never cancel each other. Your feet push the Earth back and the Earth pushes you forward; those act on two different bodies. Only forces on the *same* object can cancel. This is exactly why a free-body diagram shows just one object at a time. If action and reaction canceled, nothing could ever accelerate, and clearly things do.



Action and reaction. This fighter-jet engine, bolted to a test stand, hurls exhaust one way — and the exhaust pushes back on the engine just as hard the other way. Two forces, two objects, no cancellation. In flight, that reaction force is what shoves the plane forward.

Misconception check: action-reaction forces do not cancel, because they act on two different objects. Students often try to "cancel" the Earth's pull on a book with the book's pull on the Earth, or a horse's pull on a cart with the cart's pull on the horse, and conclude nothing can move. The fix: only forces acting on the *same* object can add to zero. The pair members always sit on different objects.

Faith & Science "In Him all things hold together" (Colossians 1:17). Newton's third law says you cannot push on the world without the world pushing back on you, in equal measure. Every force you exert returns a force to you. There is a homely echo of it in the way Scripture describes a life: what we give out has a way of coming back, and we move forward in the world not by standing apart from it but by pressing against it and being met. The physics is not a proof of the proverb. It is a nice picture of a world built so that action and response belong together, where you get somewhere by engaging, not by withdrawing.

Practice Problems

Work these on your own, then open the solutions to check yourself. Draw a free-body diagram first, and use $g = 9.8$ m/s^2.

1. A 6.0 kg cart feels a net force of 24 N. Find its acceleration.
2. What is the weight of a 12 kg object on Earth?
3. Forces of 30 N right, 18 N left, and 4.0 N left act on a box. Find the net force.

4. A 10 kg crate slides on a floor with $\mu_k = 0.25$. Find the kinetic friction force.
5. A block is released on a frictionless 25° ramp. Find its acceleration down the slope.
6. A 70 kg person stands on a scale in an elevator accelerating upward at 2.0 m/s^2. What does the scale read?
7. Two masses, 3.0 kg and 5.0 kg, hang over a frictionless pulley (an Atwood machine). Find the acceleration.
8. A rocket pushes exhaust gas downward. In one sentence, use Newton's third law to explain why the rocket rises.

Solutions to the Practice Problems

1. $a = \dfrac{\Sigma F}{m} = \dfrac{24}{6.0} = 4.0$ m/s^2.
2. $W = mg = (12)(9.8) = 118$ N.
3. Right positive: $30 - 18 - 4.0 = 8.0$ N to the right.
4. $F_N = mg = (10)(9.8) = 98$ N; $F_f = \mu_k F_N = (0.25)(98) = 24.5$ N.
5. $a = g\sin\theta = 9.8\sin 25^\circ = 4.1$ m/s^2.
6. $F_N = m(g + a) = 70(9.8 + 2.0) = 826$ N.
7. $a = \dfrac{(m_2 - m_1)g}{m_1 + m_2} = \dfrac{(5.0 - 3.0)(9.8)}{8.0} = 2.45$ m/s^2.
8. The rocket pushes the gas down (action); by the third law the gas pushes the rocket up with an equal and opposite force (reaction), and that upward force lifts it.

Challenge Problems

The hard ones. Each of these takes more than one step and more than one idea; some borrow tools from earlier chapters. Work them in GUESS order (Given, Unknown, Sketch, Equation, Substitute, Solve) before opening the solutions.

1. **The Hard One: Incline + Pulley + Friction.** A 4.0 kg block rests on a 30° incline where $\mu_k = 0.20$. A string runs from the block up the slope, over a pulley, to a 5.0 kg mass hanging straight down. Find the acceleration of the system and the tension in the string.

Scan to watch: Atwood Machine on an Incline (worked example)

2. **The Crate in the Elevator.** A 10 kg crate sits on the floor of an elevator; $\mu_s = 0.50$. While the elevator accelerates upward at 2.0 m/s^2, what is the largest horizontal push the crate can take before it starts to slide?

Scan to watch: The Crate in the Elevator (apparent weight)

3. **Push Down or Pull Up?** Sparky moves a 20 kg crate across a floor ($\mu_k = 0.30$) with an 80 N force angled 25° from the horizontal. Compare the acceleration if he pushes down along the handle versus pulls up on a rope. Which is better, and why?

Scan to watch: Push Down or Pull Up? (friction race)

4. **The Constant-Speed Clue.** A block slides down a 30° ramp at constant velocity. Find the coefficient of kinetic friction. (Notice what you are NOT given: the mass.)

Scan to watch: The Constant-Speed Clue (find mu with no scale)

Solutions to the Challenge Problems

1. **Incline + Pulley + Friction.** Perpendicular: $N = m_1 g\cos\theta = 34$ N, $f = \mu_k N = 6.8$ N. The hanging weight (49 N) beats the parallel component (19.6 N), so the block slides up and friction points down the slope. Block: $T - m_1 g\sin\theta - f = m_1 a$; hanging: $m_2 g - T = m_2 a$. Add them (tension cancels): $49 - 19.6 - 6.8 = 9.0a$, so $a = \mathbf{2.5\ m/s^2}$ and $T = m_2(g - a) = \mathbf{36\ N}$.

2. **The Crate in the Elevator.** Vertical first: the elevator's acceleration raises the normal force, $N = m(g + a) = 10(11.8) = 118$ N. Friction's ceiling rises with it: $F_{max} = \mu_s N = \mathbf{59\ N}$ (versus 49 N in a parked elevator). Upward acceleration presses the crate into the floor harder, and friction gets stronger for free.

3. **Push Down or Pull Up?** Pushing: $N = 196 + 80\sin 25° = 230$ N, $f = 69$ N, $a = (72.5 - 69)/20 = \mathbf{0.18\ m/s^2}$. Pulling: $N = 196 - 34 = 162$ N, $f = 49$ N, $a = (72.5 - 49)/20 = \mathbf{1.2\ m/s^2}$. Same effort, and pulling wins by a factor of six, because the upward component lifts weight off the floor and starves friction.

4. **The Constant-Speed Clue.** Constant velocity means $\Sigma F = 0$ along the slope: $mg\sin\theta = \mu_k mg\cos\theta$. The mass cancels, leaving $\mu_k = \tan\theta = \tan 30° = \mathbf{0.58}$. "Constant speed" is not filler; it is the entire problem. This is also how you measure μ in lab: tilt until it slides steadily.

Chapter 4 Summary

- A force is a push or pull, in newtons. The net force ΣF is the vector sum of all forces on an object.

- Newton's first law: an object keeps its velocity (including rest) unless a net force acts. Inertia is the resistance to changing motion, and it grows with mass.
- A free-body diagram shows every force acting on one object. Apply the second law one axis at a time.
- Newton's second law: $\Sigma F = ma$. Always find the net force before dividing by mass. Zero net force means zero acceleration.
- Mass (kg) never changes; weight is $W = mg$ (N) and changes with location. The normal force is a surface's support, equal to mg only on level ground with no vertical acceleration.
- Friction is $F_f = \mu F_N$. Static friction holds a still object up to a maximum $\mu_s F_N$; kinetic friction $\mu_k F_N$ acts while sliding. Compare a push to the maximum static friction to see if motion begins.
- On a ramp, weight splits into $mg\sin\theta$ (along, slides the object) and $mg\cos\theta$ (into the surface, sets F_N). A frictionless ramp gives $a = g\sin\theta$; friction subtracts $\mu\, mg\cos\theta$.
- Connected objects share one acceleration and one rope tension. Solve the whole system for the acceleration, then one object for the tension.
- Newton's third law: forces come in equal, opposite pairs acting on different objects, so pair members never cancel.

Key terms

force
a push or pull, measured in newtons (N); a vector

net force (ΣF)
the vector sum of all forces acting on an object

Newton's first law
an object keeps constant velocity (or stays at rest) unless a net force acts

inertia
an object's resistance to a change in motion, tied to its mass

free-body diagram
a sketch of all forces acting on one object, drawn as arrows from a point

Newton's second law
$\Sigma F = ma$; net force equals mass times acceleration

mass
the amount of matter in an object (kg); does not change with location

weight
the gravitational force on an object, $W = mg$ (N); changes with location

normal force (F_N)
the perpendicular support force a surface exerts on an object

friction (F_f)
a force opposing sliding between surfaces, $F_f = \mu F_N$

coefficient of friction (μ)
a number capturing how rough a pair of surfaces is (static μ_s, kinetic μ_k)

tension (T)
the pull of a rope or cable, directed along the rope

Newton's third law
for every force there is an equal and opposite force on a different object

equilibrium
the state of zero net force ($\Sigma F = 0$), so an object stays at rest or at constant velocity

Statics
the engineering study of bodies in equilibrium (civil and structural engineering); $\Sigma F = 0$

Dynamics
the engineering study of bodies accelerating under unbalanced forces (mechanical engineering); $\Sigma F = ma$

Chapter 6

Circular Motion & Gravitation

PHYS1.PS2.3, 2.9, 2.13

5.1 · Motion in a Circle

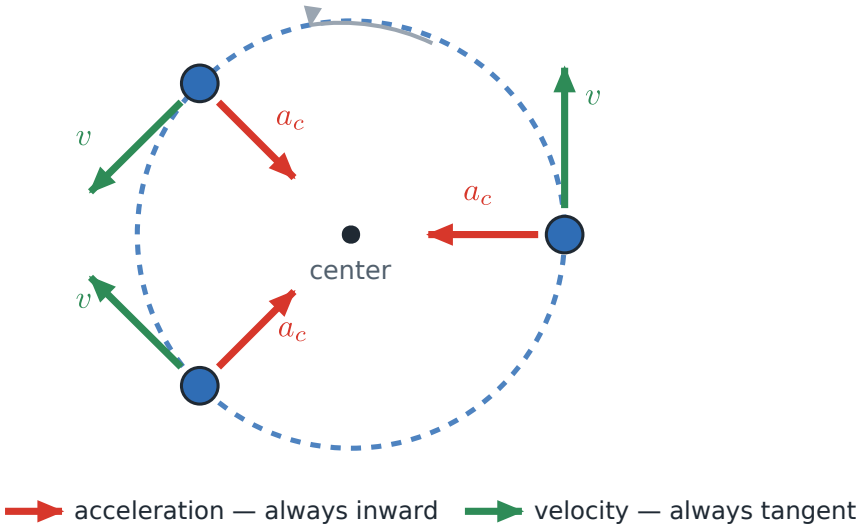
By the end of this section you can: explain why circular motion is accelerated motion; and describe the direction and size of centripetal acceleration.

Swing a ball on a string in a circle at a steady speed. Is it accelerating? Your instinct says no, because the speed is not changing. But acceleration is any change in *velocity*, and velocity is a vector that includes direction. The ball's direction is changing every instant, so its velocity is changing, so it is accelerating the whole way around. This is **uniform circular motion**: constant speed, constantly changing direction, constant acceleration.

Which way does that acceleration point? Not along the circle, but straight toward the center. We call it **centripetal acceleration**, from Latin for "center-seeking." Its size depends on how fast the object moves and how tight the circle is:

$$a_c = \frac{v^2}{r}$$

Faster speed or a tighter radius means a larger center-pointing acceleration. The velocity always aims along the circle (tangent), while the acceleration always aims inward, perpendicular to it. That constant inward pull is what bends the straight-line path into a circle.



Center-seeking. *The velocity points along the circle; the acceleration points to the center. It is the inward acceleration that keeps bending the motion into a curve.*

A car rounds a curve of radius 50 m at 20 m/s. Find its centripetal acceleration.

$$a_c = \frac{v^2}{r} = \frac{20^2}{50} = \boxed{8.0\ \text{m/s}^2}$$

That is nearly the acceleration of gravity, aimed sideways toward the center of the curve. The passengers feel it as a firm push toward the inside of the turn.

Sparky whirls a ball on a 0.50 m string at 4.0 m/s. Find its centripetal acceleration.

$$a_c = \frac{v^2}{r} = \frac{4.0^2}{0.50} = \boxed{32\ \text{m/s}^2}$$

A tight circle produces a large center-seeking acceleration even at a modest speed, because the radius is small.

An object moves in a circle of radius 4.0 m with a centripetal acceleration of 25 m/s². How fast is it moving? Rearrange $a_c = v^2/r$ to solve for v:

$$v = \sqrt{a_c\, r} = \sqrt{(25)(4.0)} = \boxed{10\ \text{m/s}}$$

The same relationship runs backward: given any two of a_c, v, and r, you can find the third.

5.2 · Angular Quantities

By the end of this section you can: measure rotation in radians; relate arc length to angle; and use angular velocity and angular acceleration, connecting them to the straight-line quantities from Chapter 1.

So far we have described a spinning object with its ordinary (linear) speed. But there is a second, often simpler, way to describe going around: by the *angle* swept out. Every point on a spinning wheel travels a different linear distance, but they all sweep the same angle in the same time, so angular quantities describe the whole rotating object at once.

We measure rotation angle θ in **radians** rather than degrees, because radians connect angle directly to distance. One full turn is 2π radians. The **arc length** s, the actual distance a point travels along the circle, is simply the radius times the angle:

$$s = r\theta$$

How fast the angle changes is the **angular velocity** ω (the Greek letter omega), measured in radians per second. It is the rotational cousin of ordinary velocity. And because one full turn (2π radians) takes one period T:

$$\omega = \frac{\Delta\theta}{t} = \frac{2\pi}{T}$$

The linear (tangential) speed of a point and the angular speed of the wheel are tied together by the radius. A point farther from the center covers more distance per turn, so it moves faster:

$$v = r\omega$$

Finally, when the spin rate itself changes, the object has an **angular acceleration** α (alpha), the rotational cousin of linear acceleration:

$$\alpha = \frac{\Delta\omega}{t}$$

Rotation mirrors straight-line motion. Every linear idea from Chapter 1 has an angular twin: position x becomes angle θ, velocity v becomes angular velocity ω, and acceleration a becomes angular acceleration α. The linear and angular pictures are linked at every step by the radius: $s = r\theta$, $v = r\omega$, and $a = r\alpha$. Learn one and you nearly know the other.

A bicycle wheel of radius 0.33 m spins at 2.0 turns per second. Find its angular velocity, and the linear speed of a point on the rim. Two turns per second means the period is $T = 0.50$ s per turn. Then:

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.50} = \boxed{12.6 \text{ rad/s}}$$

The rim's linear speed follows from the radius:

$$v = r\omega = (0.33)(12.6) = 4.2 \text{ m/s}$$

That is about a brisk jog, which is reasonable for a wheel turning twice a second. A point halfway to the center would share the same ω but move at half the linear speed.

A spinning disk speeds up from rest to an angular velocity of 12 rad/s in 4.0 s. Find its angular acceleration.

$$\alpha = \frac{\Delta\omega}{t} = \frac{12 - 0}{4.0} = \boxed{3.0 \text{ rad/s}^2}$$

Just like linear acceleration, this says the spin rate grows by 3.0 rad/s every second. The rotational world mirrors the straight-line world step for step.

A grinding wheel of radius 0.30 m turns at an angular velocity of 10 rad/s. How fast is a point on the rim moving?

$$v = r\omega = (0.30)(10) = \boxed{3.0 \text{ m/s}}$$

The rim speed depends on the radius, which is why the outer edge of a large wheel can move dangerously fast even at a modest spin rate.

Check yourself: 5.2

1. Why do we measure rotation in radians instead of degrees?
   Radians connect angle directly to distance: $s = r\theta$. One full turn is 2π radians.
2. Two points on a spinning disk, one near the center and one at the rim, have the same what? Different what?
   Same angular velocity ω; different linear speed $v = r\omega$ (the rim moves faster).
3. A record turns once every 1.8 s. Find its angular velocity.
   $\omega = 2\pi/T = 2\pi/1.8 = 3.5$ rad/s.

5.3 · Centripetal Force

By the end of this section you can: apply Newton's second law to circular motion; and identify what real force provides the centripetal force in each situation.

An inward acceleration requires, by Newton's second law, an inward net force. We call it the **centripetal force**, and it obeys the same $\Sigma F = ma$ as everything else:

$$F_c = \frac{mv^2}{r}$$

Here is the idea students most need to hear: centripetal force is not a new kind of force. It is a *job* that some ordinary force is doing. The word "centripetal" tells you the direction (inward) and the role (keeping the object on the circle), but the actual force is always something familiar. For a ball on a string, tension provides it. For a car rounding a curve, friction provides it. For the Moon circling Earth, gravity provides it. Always ask, "what real force is pointing inward here?" That force is the centripetal force.

Misconception check: there is no outward "centrifugal force" throwing you out of a turn. When a car turns, your body simply tries to continue straight (inertia, Newton's first law), and the door pushes you inward to make you turn with the car. The feeling of being flung outward is your inertia, not a real outward force. Every force in circular motion points inward. If you draw an outward arrow, you have drawn a force that is not there.

Sparky (mass 3.0 kg) is swung on a 1.5 m string at 6.0 m/s. What is the tension in the string? The tension is the centripetal force, so:

$$F_c = \frac{mv^2}{r} = \frac{(3.0)(6.0)^2}{1.5} = \frac{(3.0)(36)}{1.5} = \boxed{72\text{ N}}$$

Spin him faster or shorten the string, and the required tension climbs quickly, because speed is squared. Push it too far and the string snaps, after which Sparky flies off along a straight tangent line, not outward.

A 1200 kg car rounds a flat curve of radius 50 m at 15 m/s. How much friction force must the tires provide? If the most friction available is 6000 N, is the car safe? Friction is the centripetal force here, so the required amount is:

$$F_c = \frac{mv^2}{r} = \frac{(1200)(15)^2}{50} = \frac{270{,}000}{50} = \boxed{5400\text{ N}}$$

The tires can supply up to 6000 N, which is more than the 5400 N needed, so the car makes the turn. Notice what happens if the driver speeds up: the needed force grows with v^2, so at about 16.6 m/s the demand would exceed 6000 N and the car would slide off the curve. This is exactly why speed limits drop on sharp bends.

A 0.50 kg ball is whirled on a 1.2 m string at 5.0 m/s. Find the centripetal force (the string tension).

$$F_c = \frac{mv^2}{r} = \frac{(0.50)(5.0)^2}{1.2} = \boxed{10.4 \text{ N}}$$

The string must supply 10.4 N of inward pull to keep the ball on its circle. Spin it faster and the tension climbs with the square of the speed, until eventually the string can no longer hold.

Check yourself: 5.3

1. Is centripetal force a new type of force? Explain.
   No. It is the role played by an ordinary inward force, such as tension, friction, or gravity.
2. What provides the centripetal force for a car rounding a flat curve?
   Friction between the tires and the road.
3. When a string breaks, which way does the object fly?
   Straight along the tangent line it had at that instant, not outward, because no force then pulls it inward.
4. A 0.20 kg ball whirls on a 0.80 m string at 4.0 m/s. Find the tension.
   $F_c = mv^2/r = (0.20)(16)/0.80 = 4.0$ N.

5.4 · Universal Gravitation

By the end of this section you can: state Newton's law of universal gravitation; and describe how gravitational force depends on mass and distance.

Newton's great insight was that the force pulling an apple to the ground and the force holding the Moon in orbit are the same force. Every mass in the universe attracts every other mass. This is the **law of universal gravitation**:

$$F_g = \frac{G\,m_1 m_2}{r^2}$$

The force grows with each mass, so bigger objects pull harder, which is why planets and stars dominate. And it shrinks with the *square* of the distance between the centers, so moving twice as far apart weakens the pull to one quarter. The constant $G = 6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$ is tiny, which is why you feel no pull toward the person next to you. Gravity is actually the weakest of nature's forces. It only rules the heavens because planets and stars pile up enormous mass, and because it never cancels out, only adds up.

Where g comes from. The familiar $g = 9.8\ \text{m/s}^2$ is not a separate fact. It is universal gravitation evaluated at Earth's surface: plug Earth's mass and radius into the law and out comes 9.8. The same equation predicts a different g on the Moon or Mars. One law quietly stands behind every falling object in this whole book.

Two objects attract each other with a gravitational force of 40 N. If you move them twice as far apart, what is the new force? Force depends on $1/r^2$. Doubling r multiplies the force by $1/2^2 = 1/4$:

$$F_{new} = 40 \times \tfrac{1}{4} = \boxed{10 \text{ N}}$$

The inverse-square rule is dramatic: a modest increase in distance causes a large drop in force. Triple the distance and the force falls to one ninth.

Estimate the gravitational force between two 70 kg students standing 1.0 m apart.

$$F_g = \frac{G\,m_1 m_2}{r^2} = \frac{(6.67 \times 10^{-11})(70)(70)}{(1.0)^2} \approx \boxed{3.3 \times 10^{-7} \text{ N}}$$

That is under a millionth of a newton, far too small to feel or even to measure without delicate equipment. This is why gravity seems important only near enormous masses like a planet. The force is real; it is just fantastically weak between everyday objects.

Find the gravitational force between two 5.0 kg bowling balls whose centers are 0.50 m apart.

$$F_g = \frac{G\,m_1 m_2}{r^2} = \frac{(6.67 \times 10^{-11})(5.0)(5.0)}{(0.50)^2} = \boxed{6.7 \times 10^{-9} \text{ N}}$$

Even sitting close together, the pull between them is billionths of a newton, far too faint to notice. Gravity only becomes a major player when at least one mass is astronomical.

Check yourself: 5.4

1. How does gravitational force change if you double the distance between two masses?
   It drops to one quarter, because force depends on $1/r^2$.
2. Why don't you feel a gravitational pull toward everyday objects around you?
   Gravity is extremely weak (G is tiny), so ordinary masses produce forces far too small to notice.
3. Where does the value $g = 9.8$ m/s^2 come from?
   From applying universal gravitation at Earth's surface, using Earth's mass and radius.

5.5 · Orbits, Kepler, and Weightlessness

By the end of this section you can: explain an orbit as continuous free fall; state Kepler's laws in plain terms; and explain why astronauts feel weightless.

An **orbit** is one of the most elegant ideas in physics, and Newton pictured it with a cannon. Fire a cannonball horizontally and it curves to the ground. Fire it faster and it lands farther away. Fire

it fast enough, and the ground curves away beneath it just as fast as it falls, so it never lands. It is perpetually falling toward Earth and perpetually missing. That is an orbit: continuous free fall with enough sideways speed to keep circling. Gravity is the centripetal force, doing the same job the string did for Sparky. Setting gravity equal to the centripetal requirement gives the period of an orbit around a central mass m_s:

$$T = 2\pi\sqrt{\frac{r^3}{G\,m_s}}$$

Kepler's laws

Long before Newton, Johannes Kepler found three patterns in how the planets actually move, and universal gravitation later explained all three:

- **First law (orbits are ellipses).** Planets travel in ellipses, not perfect circles, with the Sun at one focus.
- **Second law (equal areas).** A planet sweeps out equal areas in equal times, which means it moves faster when it is closer to the Sun and slower when farther away.
- **Third law (period and distance).** The square of a planet's period is proportional to the cube of its orbital radius, which is exactly what the period equation above says.

A satellite orbits just above Earth's surface, at radius $r = 6.4 \times 10^6$ m. Using Earth's mass $m_s = 6.0 \times 10^{24}$ kg, find its orbital period.

$$T = 2\pi\sqrt{\frac{(6.4 \times 10^6)^3}{(6.67 \times 10^{-11})(6.0 \times 10^{24})}} \approx \boxed{5.1 \times 10^3 \text{ s}}$$

That is about 85 minutes, close to the roughly 90-minute orbit of the real International Space Station. The same law that drops a pencil sets the clock for a spacecraft circling the planet.

How fast must a satellite move to circle Earth at radius $r = 6.8 \times 10^6$ m? (Earth's mass $m_s = 6.0 \times 10^{24}$ kg.) For a circular orbit, gravity provides exactly the centripetal force, $\frac{G m_s m}{r^2} = \frac{m v^2}{r}$. The satellite's mass cancels, leaving:

$$v = \sqrt{\frac{G\,m_s}{r}} = \sqrt{\frac{(6.67 \times 10^{-11})(6.0 \times 10^{24})}{6.8 \times 10^6}} = \boxed{7.7 \times 10^3 \text{ m/s}}$$

That is about 7.7 km/s, roughly 17,000 mph. An orbit is not slow floating; it is a headlong fall fast enough to keep missing the ground.

This also explains the puzzle of the "weightless" astronaut. Astronauts on the space station are not beyond gravity; gravity there is nearly as strong as on the ground. They feel weightless because they and their station are all in free fall together, falling around the Earth at the same rate. There is

nothing pushing up on them, no normal force, so they feel no weight. It is the same floating feeling as the top of a roller-coaster drop, stretched out forever. We call it **apparent weightlessness**, because the weight is still there; it is just no longer being resisted.

Astronaut Bruce McCandless, 1984, flying free with no tether and no cable, about 100 meters from the shuttle. Nothing is holding him up. Nothing needs to. He is falling around the Earth at the same rate the shuttle is, which is exactly what an orbit is. This chapter explains why he never comes down.

Faith & Science "The heavens declare the glory of God, and the sky above proclaims his handiwork" (Psalm 19:1). It is worth sitting with how strange and steady an orbit is. The Moon has been falling toward the Earth for billions of years and has never once hit it, held in a balance so fine that we can predict eclipses centuries ahead. The same law that drops a pencil holds the planets in their courses. For the psalmist, that dependable order is a reason for praise, not because the math needs God to work, but because a world this orderly invites the question of who made it orderly. Studying the heavens carefully has always been, for many, a way of looking up.

Check yourself: 5.5

1. What is an orbit, in terms of falling?
   Continuous free fall with enough sideways speed to keep missing the ground, so the object circles instead of landing.

2. Kepler's second law says a planet moves faster when it is where?
   Closer to the Sun (so it sweeps equal areas in equal times).
3. Why do astronauts feel weightless in orbit?
   They are in free fall along with their spacecraft, so no normal force pushes up on them; the weight is unresisted, not gone.

Practice Problems

Work these on your own, then open the solutions to check yourself. Use $G = 6.67 \times 10^{-11}$ N$\cdot$ m^2/kg^2 where needed.

1. A cyclist rounds a circular track of radius 30 m at 15 m/s. Find the centripetal acceleration.
2. A wheel turns at 45 revolutions per minute. Find its angular velocity in rad/s.
3. A 1000 kg car rounds a curve of radius 80 m at 20 m/s. Find the centripetal force the tires must supply.
4. Find the gravitational force between two 2.0 kg masses whose centers are 1.0 m apart.
5. An object moves in a circle of radius 3.0 m with a centripetal acceleration of 12 m/s^2. Find its speed.
6. If two masses are moved three times as far apart, what happens to the gravitational force between them?
7. In one or two sentences, explain why an astronaut on the space station feels weightless even though gravity there is nearly as strong as on the ground.

Solutions to the Practice Problems

1. $a_c = \dfrac{v^2}{r} = \dfrac{15^2}{30} = 7.5$ m/s^2.
2. $T = \dfrac{60}{45} = 1.33$ s; $\omega = \dfrac{2\pi}{T} = 4.7$ rad/s.
3. $F_c = \dfrac{mv^2}{r} = \dfrac{(1000)(20)^2}{80} = 5000$ N.
4. $F_g = \dfrac{G(2.0)(2.0)}{(1.0)^2} = 2.7 \times 10^{-10}$ N.
5. $v = \sqrt{a_c\, r} = \sqrt{(12)(3.0)} = 6.0$ m/s.
6. Force depends on $1/r^2$, so tripling the distance divides the force by $3^2 = 9$: it drops to one ninth.
7. The astronaut and the station are both in free fall around Earth at the same rate, so nothing pushes up on the astronaut (no normal force). The weight is unresisted, not gone, which feels like weightlessness.

Challenge Problems

The hard ones. Each of these takes more than one step and more than one idea; some borrow tools from earlier chapters. Work them in GUESS order (Given, Unknown, Sketch, Equation, Substitute, Solve) before opening the solutions.

1. **The Vertical Circle.** Sparky (3.0 kg) is whirled on the same 1.5 m string from Worked Example 5.3a, but now in a vertical circle. (a) What is the slowest speed he can have at the top without the string going slack? (b) At the bottom he is moving 9.0 m/s; find the tension there.

Scan to watch: The Vertical Circle (worked example)

2. **The Penny on the Turntable.** A penny rests on a turntable spinning at 45 revolutions per minute. The coefficient of static friction is 0.30. How far from the center can the penny sit without sliding off?

Scan to watch: Penny on a Turntable (worked example)

3. **The Satellite That Never Moves.** Some satellites hover over one spot on the equator, which requires an orbital period of exactly one day. At what orbital radius must such a satellite fly, and how high is that above the ground? ($m_s = 6.0 \times 10^{24}$ kg, $R_E = 6.4 \times 10^6$ m)

Scan to watch: The Satellite That Never Moves (geostationary orbit)

4. **Predicting the Moon.** The ISS orbits at $r_1 = 6.8 \times 10^6$ m with a period of 92 minutes. The Moon orbits at $r_2 = 3.84 \times 10^8$ m. Using Kepler's third law and nothing else, predict how long the Moon takes to orbit the Earth, in days.

Scan to watch: Predicting the Moon with Kepler's Third Law

Solutions to the Challenge Problems

1. **The Vertical Circle.** (a) At the top both tension and weight point toward the center: $T+mg = mv^2/r$. The slack limit is $T = 0$, so $mg = mv_{min}^2/r$ and $v_{min} = \sqrt{gr} = \sqrt{(9.8)(1.5)} =$ **3.8 m/s**. (b) At the bottom, up is toward the center: $T - mg = mv^2/r$, so $T = m(v^2/r + g) = 3.0(54 + 9.8) =$ **191 N**, about 6.5 times Sparky's weight, versus 72 N in the horizontal circle. At the bottom the string fights gravity and turns the corner at the same time.

2. **The Penny on the Turntable.** Convert: $\omega = 45 \times 2\pi/60 = 4.7$ rad/s. Friction is the only inward force, and its ceiling is $\mu_s N = \mu_s mg$. At the edge of slipping: $\mu_s mg = m\omega^2 r_{max}$. The mass cancels: $r_{max} = \mu_s g/\omega^2 = (0.30)(9.8)/(4.7)^2 =$ **0.13 m, about 13 cm**. A penny or a bowling ball: same answer.

3. **The Satellite That Never Moves.** Solve the period equation backward. Square both sides of $T = 2\pi\sqrt{r^3/Gm_s}$ and isolate: $r^3 = Gm_sT^2/4\pi^2 = 7.6 \times 10^{22}$ m^3. Cube root (not square root!): $r = 4.2 \times 10^7$ **m**. Altitude: $h = r - R_E = 3.6 \times 10^7$ **m, about 36,000 km up**. Every TV and weather satellite shares this one ring, because only one radius gives a one-day orbit.

4. **Predicting the Moon.** Both orbit the same Earth, so the constants cancel in Kepler's third law: $(T_2/T_1)^2 = (r_2/r_1)^3$. Radius ratio: 56.5. Then $T_2/T_1 = \sqrt{56.5^3} = 424$, so $T_2 = 424 \times$ 92 min $\approx$ 39,000 min $\approx$ **27 days**. The real Moon takes 27.3 days. Two numbers from a space station just predicted the month.

Chapter 5 Summary

- Uniform circular motion has constant speed but changing direction, so it is accelerated. Centripetal acceleration points to the center, $a_c = v^2/r$.
- Rotation is described by angle θ (radians), arc length $s = r\theta$, angular velocity $\omega = 2\pi/T$, and angular acceleration $\alpha = \Delta\omega/t$. Linear and angular link through the radius: $v = r\omega$.
- An inward acceleration needs an inward net force, the centripetal force $F_c = mv^2/r$. It is not a new force but a role filled by tension, friction, or gravity. There is no outward centrifugal force.
- Every mass attracts every other: $F_g = Gm_1m_2/r^2$. Force grows with mass and falls with the square of distance. Gravity is weak but never cancels, so it rules at astronomical scale.
- An orbit is continuous free fall with enough sideways speed to keep circling; its period is $T = 2\pi\sqrt{r^3/Gm_s}$. Kepler's laws (ellipses, equal areas, period-cubed-radius) all follow from gravitation. Astronauts feel weightless because they are in free fall.

Key terms

uniform circular motion
motion in a circle at constant speed but continuously changing direction

centripetal acceleration
the center-pointing acceleration of circular motion, $a_c = v^2/r$

radian
the natural unit of angle; one full turn is 2π radians

arc length (s)
the distance traveled along a circle, $s = r\theta$

angular velocity ()
how fast the angle changes, $\omega = 2\pi/T$, in rad/s; $v = r\omega$

angular acceleration ()
the rate of change of angular velocity, $\alpha = \Delta\omega/t$

centripetal force
the inward net force that keeps an object moving in a circle, $F_c = mv^2/r$

law of universal gravitation
every mass attracts every other with $F_g = Gm_1m_2/r^2$

orbit
continuous free fall with enough sideways speed to keep circling a body

Kepler's laws
ellipatical orbits, equal areas in equal times, and $T^2 \propto r^3$

apparent weightlessness
the floating sensation of free fall, when no surface resists an object's weight

Chapter 7

Work, Energy & Power

PHYS1.PS3.2, 3.5, 3.6

6.1 · Work

By the end of this section you can: define work in the physics sense; compute it; and explain when a force does zero work.

In everyday speech, "work" means effort. In physics it means something narrower and more precise. **Work** is done only when a force moves an object through a distance in the direction of the force. It is force times distance:

$$W = Fd\cos\theta$$

Work is measured in **joules** (J), where one joule is one newton acting through one meter. Work is how energy gets transferred from one thing to another. When you do work on an object, you hand it energy; that is the deep reason this quantity matters, as the next sections will show.

The catch is the phrase "in the direction of the force." If the force is perpendicular to the motion, it does *no work at all*. Carry a heavy box straight across a level room and, in the physics sense, you do zero work on it: the force you apply is upward, but the motion is sideways. Your arms feel tired, but no energy went into the box's motion. This surprises people, and it is worth sitting with, because it is exactly the idea that makes circular motion and orbits make sense: gravity does no work on a circular orbit, because it always pulls perpendicular to the motion.

Misconception check: effort is not the same as physics work. Holding a barbell perfectly still is exhausting, but the barbell does not move, so the work done on it is zero. Pushing hard against a locked wall is the same. No motion in the direction of the force means no work, no matter how tired you get.

Sparky drags a sled 8.0 m by pulling on a rope with 50 N of force, held at 30° above the ground. How much work does he do? Only the part of the force along the motion counts, which is where the cosine comes in:

$$W = Fd\cos\theta = (50)(8.0)\cos 30^\circ = (400)(0.866) = \boxed{346\ \text{J}}$$

If he had pulled straight along the ground ($\theta = 0$), the full 400 J would count. The steeper the rope, the more of his pull is wasted lifting rather than dragging, and at 90° (straight up) the work on the sled's forward motion would be zero.

Sparky pushes a cart 6.0 m across the floor with a steady 40 N force in the direction of motion. How much work does he do? The force is along the motion ($\theta = 0$, $\cos 0 = 1$), so work is simply force times distance:

$$W = Fd = (40)(6.0) = \boxed{240 \text{ J}}$$

When the push lines up with the motion, the cosine drops out and work is at its maximum for that force.

How much work does it take to lift a 15 kg box straight up onto a 2.0 m shelf? You must push up with a force equal to the box's weight, mg, through the height:

$$W = mgh = (15)(9.8)(2.0) = \boxed{294 \text{ J}}$$

That 294 J does not vanish. It is stored as gravitational potential energy, the subject of Section 6.3.

Check yourself: 6.1

1. What three things must be true for a force to do work?
   There must be a force, the object must move, and the motion must have a component in the direction of the force.
2. You push a cart 4.0 m with a force of 25 N. How much work do you do?
   W = Fd = (25)(4.0) = 100 J.
3. Why does carrying a box across level ground do no work on it?
   The lifting force is vertical while the motion is horizontal, so the force does nothing in the direction of motion.

6.2 · Kinetic Energy

By the end of this section you can: define kinetic energy; compute it; and state the work-energy theorem.

Energy is the capacity to do work, and it also is measured in joules. The energy of motion is **kinetic energy**:

$$KE = \tfrac{1}{2}mv^2$$

Two features deserve attention. Kinetic energy grows with mass, so a truck carries more than a bike at the same speed. And it grows with the *square* of the speed, which has real consequences. Double a car's speed and its kinetic energy quadruples, which is why a crash at 60 mph is four times as violent as one at 30, not twice. That squared term is a matter of life and death on the road.

Work and kinetic energy are tied together by the **work-energy theorem**: the net work done on an object equals its change in kinetic energy, $W_{total} = \Delta KE$. Do positive work on something and you speed it up; do negative work (like friction or braking) and you slow it down. This is the bridge between the force ideas of Chapter 4 and the energy ideas of this one.

Sparky has a mass of 3.0 kg and moves at 4.0 m/s. What is his kinetic energy? **Equation:** $KE = \frac{1}{2}mv^2$. **Substitute:** $KE = \frac{1}{2}(3.0)(4.0)^2$. **Solve:** $KE = \frac{1}{2}(3.0)(16) = \boxed{24\text{ J}}$. If he doubled his speed to 8.0 m/s, his kinetic energy would become 96 J, four times as much, from the squared speed.

Find the kinetic energy of a 1200 kg car moving at 20 m/s.

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}(1200)(20)^2 = \boxed{240{,}000\text{ J}}$$

That is 240 kJ, all of which the brakes must remove to stop the car. Double the speed and this jumps to nearly a million joules, which is the physics behind long stopping distances.

A net force does 500 J of work on a 4.0 kg cart that starts from rest. How fast is it going afterward? By the work-energy theorem, the net work equals the change in kinetic energy, and since it started from rest, $W = \frac{1}{2}mv^2$:

$$v = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(500)}{4.0}} = \boxed{15.8\text{ m/s}}$$

Work went in, kinetic energy came out, and the speed followed. This is the bridge between forces and energy.

Check yourself: 6.2

1. What is kinetic energy, and what is its formula?
   The energy of motion, $\text{KE} = \frac{1}{2}\text{mv}^2$.
2. A car doubles its speed. What happens to its kinetic energy?
   It quadruples, because energy depends on speed squared.
3. State the work-energy theorem.
   The net work done on an object equals its change in kinetic energy: $\text{W}_{\text{net}} = \Delta\text{KE}$.

6.3 · Gravitational Potential Energy

By the end of this section you can: define gravitational potential energy; compute it; and explain what "stored" energy means.

Energy can also be stored, waiting to be released. **Potential energy** is stored energy an object has

because of its position or condition. The most important kind for us is **gravitational potential energy**, the energy stored by lifting something up:

$$PE_g = mgh$$

Lift an object of mass m to a height h, and you store mgh joules in it. Let it fall, and that stored energy converts to motion. A boulder at a cliff edge, a drawn bow, water behind a dam, and a raised hammer all hold potential energy, poised to do work the moment they are released. Height is measured from whatever level you choose to call zero; only the *change* in height matters, so pick a convenient floor and stick with it.

A coaster upside-down at the top of a loop, with no engine pushing it. Every bit of speed it has up there was bought with height it gave up on the way down, and every meter it climbs buys that speed back. This chapter is the accounting: energy traded, never lost. (And the loop itself? That was Chapter 5.)

How much gravitational potential energy does a 2.0 kg book have when it sits on a shelf 1.5 m above the floor?

$$PE_g = mgh = (2.0)(9.8)(1.5) = \boxed{29.4 \text{ J}}$$

Knock it off the shelf and that 29.4 J converts into kinetic energy on the way down, arriving as motion just before it lands.

A 10 kg mass stores 500 J of gravitational potential energy. How high is it? Rearrange $PE_g = mgh$ for height:

$$h = \frac{PE_g}{mg} = \frac{500}{(10)(9.8)} = \boxed{5.1 \text{ m}}$$

The same formula runs in any direction: give it two of the three quantities and it hands you the third.

How much potential energy does a 60 kg student gain climbing a staircase 3.0 m tall?

$$\Delta PE_g = mg\,\Delta h = (60)(9.8)(3.0) = \boxed{1764 \text{ J}}$$

Only the vertical rise matters, not the length of the staircase, because gravity only does work against the change in height.

6.4 · Springs and Elastic Potential Energy

By the end of this section you can: state Hooke's law; compute the force a spring exerts; and find the elastic potential energy stored in a stretched or compressed spring.

Gravity is not the only way to store energy. Stretch a rubber band, draw a bow, or compress a spring and you store energy in the material itself, ready to snap back. This is **elastic potential energy**, and springs are the cleanest example.

A spring resists being stretched or squeezed, and it pushes back harder the farther you move it. That relationship is **Hooke's law**: the restoring force is proportional to how far the spring is displaced from its natural length:

$$F_s = kx$$

Here x is the distance stretched or compressed, and k is the **spring constant**, a stiffness rating in newtons per meter. A stiff spring (large k) pushes back hard for even a small stretch; a floppy one (small k) barely resists. Pull twice as far and the spring pulls back twice as hard.

Because the force builds up gradually as you stretch it (rather than staying constant), the energy stored is not simply force times distance. Working it out gives the **elastic potential energy**:

$$PE_s = \tfrac{1}{2}kx^2$$

Like kinetic energy, it depends on the square of the displacement, so stretching a spring twice as far stores four times the energy. Release the spring and that stored energy becomes motion, exactly the way a dart gun, a trampoline, or a pole-vaulter's pole works.

A spring with spring constant $k = 200$ N/m is compressed 0.15 m. (a) How much force does it push back with? (b) How much energy is stored in it? (a) Hooke's law gives the force:

$$F_s = kx = (200)(0.15) = \boxed{30 \text{ N}}$$

(b) The stored elastic potential energy is:

$$PE_s = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(200)(0.15)^2 = \tfrac{1}{2}(200)(0.0225) = 2.25 \text{ J}$$

If this spring launched a toy, that 2.25 J would become the toy's kinetic energy, and you could find its launch speed with $\tfrac{1}{2}mv^2 = 2.25$ J. Springs are just another account in the same energy ledger.

A spring with $k = 400$ N/m is stretched 0.20 m. Find the force it pulls back with and the energy stored in it. Force (Hooke's law): $F_s = kx = (400)(0.20) = 80$ N. **Stored energy:**

$$PE_s = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(400)(0.20)^2 = \boxed{8.0 \text{ J}}$$

The force grows in proportion to the stretch, but the stored energy grows with the stretch squared, which is why pulling a bow the last few centimeters takes so much effort.

Check yourself: 6.4

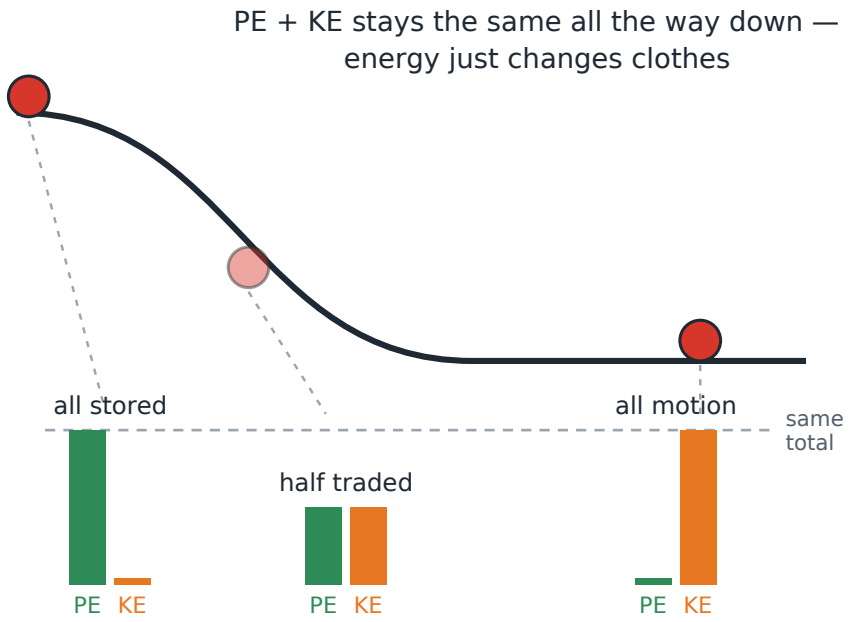
1. State Hooke's law in words.
   The force a spring pushes back with is proportional to how far it is stretched or compressed: $F_s = kx$.
2. What does the spring constant k tell you?
   How stiff the spring is, in newtons per meter. A large k means a stiff spring that resists hard.
3. A spring is stretched twice as far. What happens to the stored elastic PE?
   It becomes four times as large, because $PE_s = \tfrac{1}{2}kx^2$ depends on x squared.
4. A spring (k = 300 N/m) is stretched 0.10 m. Find the stored energy.
   $PE_s = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(300)(0.10)^2 = 1.5$ J.

6.5 · Conservation of Energy

By the end of this section you can: state the law of conservation of energy; and use it to solve motion problems without forces.

Here is one of the deepest and most useful laws in all of science, the **law of conservation of energy**: energy cannot be created or destroyed, only transformed from one form to another. The total amount of energy in a closed system stays constant. Energy is the universe's currency, and the books always balance.

Follow a roller coaster to see it in action. At the top of the first hill it is nearly still but high up, so its energy is almost all potential. As it drops, height turns into speed, potential energy pouring into kinetic energy, until at the bottom it is fast but low, mostly kinetic. Add the two together at any point and the total is the same (ignoring friction). This gives a powerful shortcut: you can find a speed from a height, or a height from a speed, without ever touching a force or a time. Set the total energy at one point equal to the total at another and solve.



Energy just changes clothes. *All potential at the top, all kinetic at the bottom, and the same total everywhere in between.*

Sparky (3.0 kg) starts from rest at the top of a 5.0 m ramp. Ignoring friction, how fast is he going at the bottom? (g = 9.8 m/s²) At the top: all potential, $PE_g = mgh = (3.0)(9.8)(5.0) = 147$ J, and KE = 0. At the bottom: all kinetic. Conservation says $KE_{bottom} = PE_{g,top} = 147$ J. Then $\frac{1}{2}mv^2 = 147$, so $v^2 = 2(147)/3.0 = 98$ and $v = \boxed{9.9 \text{ m/s}}$. Notice we never used a force or a time. Energy gave us the speed directly. (The mass even cancels, so any object reaches the same speed down the same frictionless ramp.)

When friction is in the picture

Real ramps are not frictionless, and this is where students get nervous, but the law does not break. Friction is a **nonconservative force**: it quietly drains mechanical energy and turns it into heat, so the energy is not destroyed, just moved to a form we can no longer get back as motion. The bookkeeping simply adds one line. The starting energy equals the ending energy plus whatever friction carried off:

$$PE_{top} = KE_{bottom} + W_{friction}$$

where the energy lost to friction is the friction force times the distance it acted, $W_{friction} = F_f\, d$. Gravitational PE and elastic PE are conservative (you always get them back); friction and air resistance are not. The total energy still balances perfectly. Some of it just left the mechanical account and showed up as warmth.

Same setup: Sparky (3.0 kg) slides down the 5.0 m-tall ramp, whose surface is 8.0 m long. This time friction does 40 J of work against him. How fast is he going at the bottom? Start with the stored energy at the top, $PE_g = mgh = 147$ J. Friction removes 40 J on the way down, so the kinetic energy at the bottom is what remains:

$$KE_{bottom} = PE_{top} - W_{friction} = 147 - 40 = \boxed{107 \text{ J}}$$

Then $\frac{1}{2}mv^2 = 107$, so $v^2 = 2(107)/3.0 = 71.3$ and $v = 8.4$ m/s. He arrives slower than the frictionless 9.9 m/s, exactly as we would expect. The 40 J did not vanish; it is now heat in the ramp and in Sparky.

Check yourself: 6.5

1. State the law of conservation of energy.
   Energy cannot be created or destroyed, only transformed; the total in a closed system stays constant.
2. On a frictionless roller coaster, where is kinetic energy greatest? Where is potential energy greatest?
   Kinetic energy is greatest at the lowest point; potential energy is greatest at the highest point.
3. When friction acts, is energy destroyed? Where does it go?
   No. Friction moves mechanical energy into heat, so it leaves the motion but the total energy still balances.
4. Why can energy conservation find a speed without using forces or time?
   Because the total energy is the same at both points, you set them equal and solve directly for the unknown.

6.6 · Power and Efficiency

By the end of this section you can: define power; compute it two ways; and describe efficiency.

Two engines can do the same work, but the stronger one does it faster. **Power** is the rate of doing work, work divided by time:

$$P = \frac{W}{t}$$

Power is measured in **watts** (W), where one watt is one joule per second. A 100-watt bulb uses 100 joules every second. Running up a flight of stairs and walking up it slowly take the same work (same weight, same height), but running takes less time, so it demands more power. Horsepower, the old unit still stamped on car engines, is just another way of stating the same rate of doing work.

There is a second, handy form of the power equation. Since work is force times distance, and distance over time is speed, power can also be written as force times velocity:

$$P = Fv$$

This is why a car needs far more engine power to hold a high speed than a low one: at greater speed, the same driving force delivers energy faster.

Sparky (weight 30 N) runs up a 4.0 m staircase in 5.0 s. How much power does he generate? The work he does is lifting his weight through the height: $W = Fd = (30)(4.0) = 120$ J. Power is that work over the time:

$$P = \frac{W}{t} = \frac{120}{5.0} = \boxed{24 \text{ W}}$$

If he walked up the same stairs in 10 s, he would do the same 120 J of work but only generate 12 W, half the power. Same work, more time, less power.

A car engine pushes with 5000 N of driving force while the car cruises at 25 m/s. How much power is that?

$$P = Fv = (5000)(25) = \boxed{125{,}000 \text{ W}}$$

That is 125 kW, about 168 horsepower. Because $P = Fv$, holding a high speed demands far more power than a low one, even for the same driving force.

A motor lifts a 50 kg load 10 m in 20 s. What is its power output? The work is lifting the weight through the height: $W = mgh = (50)(9.8)(10) = 4900$ J. Then:

$$P = \frac{W}{t} = \frac{4900}{20} = \boxed{245 \text{ W}}$$

A faster lift would do the same work in less time, and so demand more power from the motor.

No real machine turns all its input energy into useful output; some always leaks away, usually as heat from friction. **Efficiency** is the fraction that comes out useful, the useful output divided by the total input, written as a percent. A car engine is only about 25 to 35 percent efficient, because most of the fuel's energy leaves as heat and sound rather than motion. Efficiency is never 100 percent in the real world, a fact that connects straight to the next chapter's story about heat and to the reason perpetual-motion machines can never work.

Faith & Science , "For he will not much remember the days of his life because God keeps him occupied with joy in his heart" (Ecclesiastes 5:20). Conservation of energy says the total is never lost, only changed in form, and there is something quietly hopeful in that. Nothing simply vanishes; it is carried forward as something else. A believer can hear an echo of a larger promise, that the effort and love we pour out are not spent into nothing but held and transformed by the God who wastes nothing. The physics does not preach it. It only describes a world where the books always balance, which is a fitting stage for a story about a faithfulness that keeps account of every sparrow and every tear.

Practice Problems

Work these on your own, then open the solutions to check yourself. Use $g = 9.8$ m/s^2.

1. You push a box 8.0 m with a 25 N force in the direction of motion. Find the work done.
2. Find the kinetic energy of a 0.50 kg ball moving at 10 m/s.
3. Find the gravitational potential energy of a 5.0 kg mass raised 4.0 m.
4. A spring with $k = 200$ N/m is compressed 0.30 m. Find the elastic potential energy stored.
5. A cart starts from rest at the top of a frictionless 5.0 m ramp. Using energy conservation, find its speed at the bottom.
6. An engine does 600 J of work in 3.0 s. Find its power output.
7. A machine takes in 1000 J and delivers 800 J of useful work. Find its efficiency.

Solutions to the Practice Problems

1. $W = Fd = (25)(8.0) = 200$ J.
2. $KE = \frac{1}{2}mv^2 = \frac{1}{2}(0.50)(10)^2 = 25$ J.
3. $PE_g = mgh = (5.0)(9.8)(4.0) = 196$ J.

4. $PE_s = \frac{1}{2}kx^2 = \frac{1}{2}(200)(0.30)^2 = 9.0$ J.
5. All PE becomes KE: $mgh = \frac{1}{2}mv^2$, so $v = \sqrt{2gh} = \sqrt{2(9.8)(5.0)} = 9.9$ m/s (mass cancels).
6. $P = \dfrac{W}{t} = \dfrac{600}{3.0} = 200$ W.
7. Efficiency $= \dfrac{\text{useful out}}{\text{total in}} = \dfrac{800}{1000} = 80\%$.

Challenge Problems

The hard ones. Each of these takes more than one step and more than one idea; some borrow tools from earlier chapters. Work them in GUESS order (Given, Unknown, Sketch, Equation, Substitute, Solve) before opening the solutions.

1. **The Spring Cannon.** A spring with $k = 400$ N/m is compressed 0.25 m inside a vertical launcher holding a 0.50 kg ball. When released, how high above the launch point does the ball rise? (Ignore friction.)

Scan to watch: The Spring Cannon (energy conservation)

2. **The Water Slide Audit.** A 60 kg student starts from rest atop a water slide that drops 5.0 m over a 12 m surface, and exits at 7.0 m/s. (a) How much energy did friction take? (b) What was the average friction force?

Scan to watch: The Water Slide Audit (where did the energy go?)

3. **The Elevator Motor.** A motor lifts a 300 kg elevator car 12 m in 15 s at constant speed. (a) What useful power does the job require? (b) If the motor is 70% efficient, how much electrical power does it draw?

Scan to watch: The Elevator Motor (power and efficiency)

4. **Braking, the Energy Way.** A 1200 kg car moving at 25 m/s brakes with a steady 6000 N force. Use energy, not kinematics, to find the stopping distance. Then explain why doubling the speed quadruples the answer.

Scan to watch: Braking, the Energy Way (work-energy theorem)

Solutions to the Challenge Problems

1. **The Spring Cannon.** Energy ledger, spring to gravity: $\frac{1}{2}kx^2 = mgh$. Stored: $\frac{1}{2}(400)(0.25)^2 = 12.5$ J. So $h = 12.5/(0.50 \times 9.8) =$ **2.6 m**. No forces, no time; 12.5 joules in, 12.5 joules out as height.
2. **The Water Slide Audit.** $PE = mgh = 2940$ J; $KE = \frac{1}{2}(60)(7.0)^2 = 1470$ J. Friction took the difference: **1470 J, exactly half**. Its force acts along the 12 m surface: $f = 1470/12 \approx$ **120 N**. Gravity cares about the 5.0 m drop; friction cares about the 12 m path. Choosing the right distance for each is the whole skill.
3. **The Elevator Motor.** $W = mgh = (300)(9.8)(12) = 35{,}280$ J; $P = W/t \approx$ **2400 W** useful. Efficiency is useful out over total in: $P_{in} = 2352/0.70 \approx$ **3400 W**. The missing 1000 W becomes heat in the windings; efficiency is the exchange rate.
4. **Braking, the Energy Way.** The brakes' work destroys the KE: $Fd = \frac{1}{2}mv^2$, so $d = \frac{1}{2}(1200)(25)^2/6000 =$ **62.5 m**. KE grows with v^2, so doubling the speed quadruples the energy and the distance. And check: $6000/1200 = 5.0$ m/s^2, making this exactly Worked Example 1.5c solved by a different road. Two methods, one answer: that is how you know the physics is real.

Chapter 6 Summary

- Work is force times distance in the direction of the force, W = Fd cosθ, measured in joules. A force perpendicular to the motion does no work.
- Kinetic energy is the energy of motion, KE = $\frac{1}{2}$mv^2, growing with the square of speed. The work-energy theorem says $W_{net} = \Delta KE$.
- Gravitational potential energy is stored by height, PE_g = mgh. Only changes in height matter.
- Springs obey Hooke's law, F_s = kx, and store elastic potential energy $PE_s = \frac{1}{2}kx^2$. Both grow with the square of the stretch.
- Energy is conserved: it changes form but the total stays constant. Setting total energy equal at two points solves for a speed or height without forces or time. Friction is nonconservative: it moves mechanical energy into heat, so $PE_{top} = KE_{bottom} + W_{friction}$.
- Power is the rate of doing work, P = W/t = Fv, in watts. Efficiency is the useful fraction of energy out, always less than 100 percent because some energy leaks away as heat.

Key terms

work
force times distance in the direction of the force, W = Fd cosθ, in joules

joule (J)
the SI unit of work and energy; one newton acting through one meter

energy
the capacity to do work, measured in joules

kinetic energy
the energy of motion, $KE = \frac{1}{2}mv^2$

work-energy theorem
the net work on an object equals its change in kinetic energy

potential energy
stored energy due to position or condition

gravitational potential energy
energy stored by height, $PE_g = mgh$

Hooke's law
a spring's restoring force is proportional to its displacement, $F_s = kx$

spring constant (k)
a spring's stiffness, in newtons per meter

elastic potential energy
energy stored in a stretched or compressed spring, $PE_s = \frac{1}{2}kx^2$

nonconservative force
a force such as friction that removes mechanical energy as heat, so it cannot be recovered as motion

conservation of energy
energy is never created or destroyed, only transformed; the total stays constant

power
the rate of doing work, $P = W/t = Fv$, measured in watts

efficiency
the fraction of input energy that comes out as useful work, always under 100%

Chapter 8

Momentum & Impulse

PHYS1.PS2.6, 2.10, PS3.3

7.1 · Momentum

By the end of this section you can: define momentum; compute it; and explain why it is a vector.

Why is a slow freight train so much harder to stop than a fast bicycle? Speed alone does not answer it, and neither does mass alone. The quantity that combines them is **momentum**, mass times velocity:

$$p = mv$$

Momentum measures how much motion an object has, and how hard it is to stop. A heavy, fast object has a lot of it; a light or slow one has little. Because velocity is a vector, momentum is a vector too, pointing the same direction as the motion. That direction matters enormously in this chapter, because momentum going east and momentum going west can cancel, just like the vectors of Chapter 2. Momentum is measured in $\text{kg} \cdot \text{m/s}$, a unit with no special name of its own.

Compare the momentum of a 2000 kg truck moving at 3 m/s with a 60 kg cyclist moving at 10 m/s. Truck: $p = (2000)(3) = 6000\ \text{kg} \cdot \text{m/s}$. Cyclist: $p = (60)(10) = \boxed{600\ \text{kg} \cdot \text{m/s}}$. The truck has ten times the momentum despite moving much slower, because its mass dwarfs the cyclist's. This is why the slow truck is far harder to stop, and far more dangerous in a collision.

Find the momentum of a 1500 kg car moving at 25 m/s.

$$p = mv = (1500)(25) = \boxed{37{,}500\ \text{kg} \cdot \text{m/s}}$$

A large momentum like this is exactly why highway crashes are so violent and why stopping takes so long.

A 4.0 kg object has a momentum of 12 kg · m/s. How fast is it moving?

$$v = \frac{p}{m} = \frac{12}{4.0} = \boxed{3.0 \text{ m/s}}$$

The definition works in reverse just as easily: knowing two of p, m, and v gives the third.

Check yourself: 7.1

1. What is momentum, and what is its formula?
   The quantity of motion, p = mv, a vector pointing along the velocity.
2. Why is momentum a vector?
   Because velocity is a vector, so momentum has direction and can cancel when objects move opposite ways.
3. Find the momentum of a 0.15 kg baseball thrown at 40 m/s.
   p = (0.15)(40) = 6.0 kg · m/s.

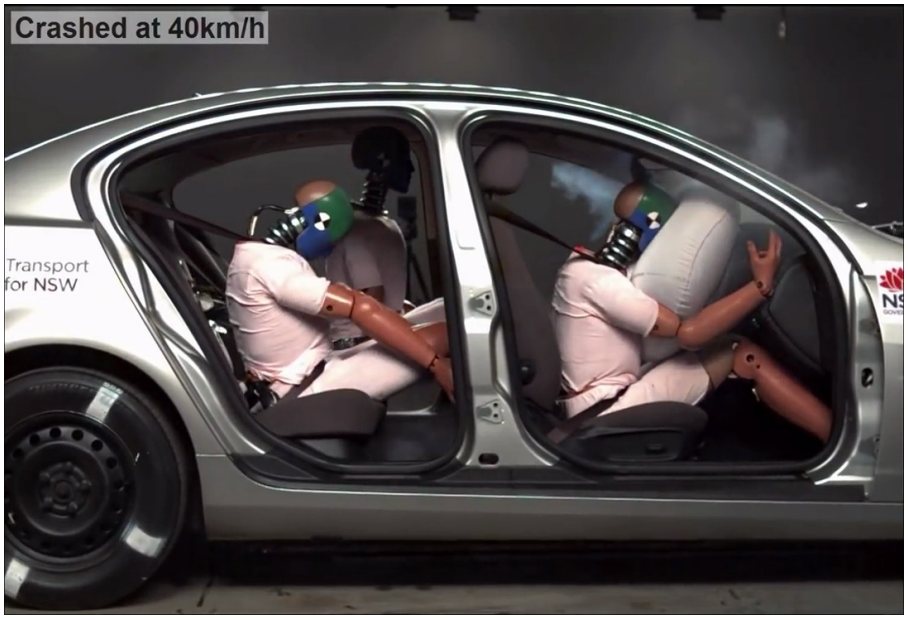
7.2 · Impulse and Crash Safety

By the end of this section you can: define impulse; state the impulse-momentum theorem; and explain how safety devices reduce force.

To change an object's momentum you must apply a force for some time. That combination, force multiplied by the time it acts, is the **impulse**, and it equals the change in momentum. This is the **impulse-momentum theorem**:

$$I = Ft = \Delta p = m\,\Delta v$$

Read the equation as a trade. To produce a certain change in momentum, you can use a big force over a short time or a small force over a long time. The *product* is what is fixed. This one idea explains nearly every safety device you have ever used. In a car crash your momentum must drop to zero no matter what, so Δp is fixed. Stopping in a hard, instant collision with the dashboard means a huge force. Stopping over a longer time, cushioned by a crumpling hood, an airbag, and a stretching seatbelt, spreads that same momentum change over more time, so the force on you is far smaller. Airbags do not reduce how much your momentum changes. They lengthen the time it takes, and that saves lives.



A 40 km/h crash test, frozen at the worst instant. The dummies' momentum must go to zero either way; the belt and the airbag exist only to stretch out the time it takes. Force is momentum change divided by time, and in a crash, time is everything. Challenge 7.2 puts numbers on exactly this picture.

The same idea, everywhere. A boxer rides a punch by moving away to lengthen the contact time. You bend your knees when you land a jump. A baseball glove is padded, and an egg survives a drop onto a pillow but not onto tile. In every case, extending the stopping time reduces the peak force for the same momentum change.

Sparky (3.0 kg) moving at 8.0 m/s is brought to rest. Compare the force if he stops in 0.10 s versus 2.0 s. Momentum change: $\Delta p = m\,\Delta v = (3.0)(0-8.0) = -24\ \text{kg·m/s}$ (the sign just means the force opposes his motion). Hard stop: $F = \frac{\Delta p}{t} = \frac{24}{0.10} = 240$ N. Soft stop: $F = \frac{24}{2.0} = \boxed{12\ \text{N}}$. The same momentum change, but the gentle stop cuts the force by a factor of twenty. Time is the lever that safety engineering pulls.

A 0.50 kg ball moving at 20 m/s is caught and stopped in 0.020 s. What average force does the catcher's hand feel? Impulse equals the change in momentum: $\Delta p = m\,\Delta v = (0.50)(20) = 10\ \text{kg}\cdot\text{m/s}$. Then:

$$F = \frac{\Delta p}{t} = \frac{10}{0.020} = \boxed{500\ \text{N}}$$

Catching "soft," by pulling the hand back to stretch the time, is how you keep that force from stinging.

A 1000 kg car moving at 20 m/s is stopped by a steady 5000 N braking force. How long does it take? $\Delta p = (1000)(20) = 20{,}000$ kg · m/s, and $Ft = \Delta p$, so:

$$t = \frac{\Delta p}{F} = \frac{20{,}000}{5000} = \boxed{4.0 \text{ s}}$$

A bigger braking force would stop the same car in less time, exactly the trade the impulse equation describes.

Check yourself: 7.2

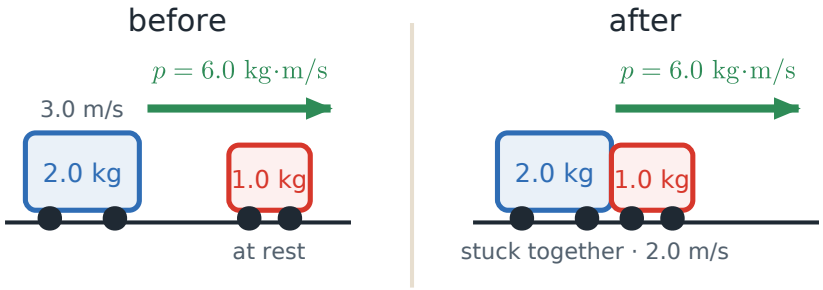
1. What is impulse, and what does it equal?
   Force times the time it acts, F · t, and it equals the change in momentum.
2. How does an airbag reduce the force on a passenger?
   It extends the stopping time, so the same momentum change happens over more time, lowering the force.
3. Why do you bend your knees when landing a jump?
   Bending lengthens the stopping time, reducing the peak force on your legs.

7.3 · Conservation of Momentum

By the end of this section you can: state the law of conservation of momentum; and use it to analyze collisions and explosions.

Here is the law that makes momentum so powerful. In any collision or push between objects, if no outside force interferes, the **total momentum before equals the total momentum after**. Momentum is conserved. It follows straight from Newton's third law: the two objects push on each other with equal and opposite forces for the same time, so they receive equal and opposite impulses, and the momentum one gains the other loses. The total never changes.

This lets you predict the outcome of a collision without knowing any of the messy forces during the crunch. Add up the momentum before, add up the momentum after, and set them equal, remembering that momentum is a vector so direction and sign count. The same law runs in reverse for explosions and recoil. A rifle and its bullet start with zero total momentum, so after firing, the small bullet's forward momentum must be matched by the heavy rifle's backward momentum. That backward kick is recoil, and it is conservation of momentum you can feel in your shoulder.



$$\Sigma p_{\text{before}} = \Sigma p_{\text{after}}$$
$$(2.0)(3.0) + 0 = (3.0)(2.0)$$
$$6.0 \text{ kg·m/s} = 6.0 \text{ kg·m/s} \checkmark$$

the green arrows are the SAME length — that is the whole law

The ledger balances. *Whatever momentum one cart loses in a collision, the other gains. The total is the same before and after.*

Sparky (3.0 kg) sits still on frictionless wheels and catches a 1.0 kg ball moving at 8.0 m/s. How fast do Sparky-plus-ball roll away together? Before: ball $(1.0)(8.0) = 8.0 \text{ kg} \cdot \text{m/s}$, Sparky 0. Total $= 8.0 \text{ kg} \cdot \text{m/s}$. **After:** they move together with combined mass 4.0 kg at velocity v. Conservation: $8.0 = (4.0)v$, so $v = \boxed{2.0 \text{ m/s}}$. Catching the ball shares its momentum across more mass, so the pair rolls off slower than the ball came in. The books balanced.

A 4.0 kg rifle fires a 0.020 kg bullet at 400 m/s. How fast does the rifle recoil? Before firing, the total momentum is zero, so the two momenta after must cancel: $m_{bullet}v_{bullet} = m_{rifle}v_{rifle}$.

$$v_{rifle} = \frac{(0.020)(400)}{4.0} = \boxed{2.0 \text{ m/s (backward)}}$$

The tiny bullet flies fast one way; the heavy rifle kicks slowly the other. Equal and opposite momenta from a zero total is conservation you feel in your shoulder.

A 2.0 kg cart moving at 3.0 m/s strikes a 1.0 kg cart at rest and they couple together. Find their shared velocity. Momentum before equals momentum after; the combined mass is 3.0 kg:

$$(2.0)(3.0) + 0 = (3.0)v \;\Rightarrow\; v = \frac{6.0}{3.0} = \boxed{2.0 \text{ m/s}}$$

Spreading the same momentum over more mass gives a slower shared speed, just like Sparky catching the ball.

Check yourself: 7.3

1. State the law of conservation of momentum.
   With no outside force, the total momentum before an interaction equals the total momentum after.
2. Why does a rifle recoil when fired?
   The total momentum starts at zero, so the bullet's forward momentum must be matched by the rifle's equal backward momentum.
3. What law is conservation of momentum a consequence of?
   Newton's third law: equal and opposite forces for equal times give equal and opposite momentum changes.

7.4 · Elastic and Inelastic Collisions

By the end of this section you can: distinguish elastic from inelastic collisions; and state what is conserved in each.

Momentum is conserved in *every* collision. Kinetic energy is not, and that difference sorts collisions into two families. In an **elastic collision**, the objects bounce apart cleanly and the total kinetic energy is also conserved. Billiard balls and the clicking spheres of a Newton's cradle come close. In an **inelastic collision**, the objects crumple, stick, or generate heat and sound, so some kinetic energy is lost to those other forms, even though momentum is still perfectly conserved. When the objects stick together and move as one, like Sparky catching the ball, it is a **perfectly inelastic collision**.

Misconception check: "kinetic energy is lost" in an inelastic collision does not break conservation of energy. The energy is not destroyed; it is converted into heat, sound, and the work of bending metal. Total energy is always conserved. It is specifically the *kinetic* energy that drops, because it changed into other forms. Momentum, meanwhile, is conserved in both kinds of collision.

A 1000 kg car moving at 15 m/s rear-ends a 1500 kg car stopped at a light. They lock together. How fast does the wreckage move just after impact? Momentum before equals momentum after, with a combined mass of 2500 kg:

$$(1000)(15) + 0 = (2500)v \;\Rightarrow\; v = \frac{15{,}000}{2500} = \boxed{6.0 \text{ m/s}}$$

Momentum was conserved, but check the kinetic energy: it fell from 112,500 J to 45,000 J. The missing 67,500 J became the crunch of bending metal and heat. That is exactly what "inelastic" means.

Two carts sit at rest with a compressed spring between them. When released, the 2.0 kg cart shoots left at 3.0 m/s. How fast does the 3.0 kg cart go, and in what direction? The total momentum starts at zero and must stay zero, so the two momenta cancel:

$$0 = (2.0)(-3.0) + (3.0)v \;\Rightarrow\; v = \frac{6.0}{3.0} = \boxed{2.0 \text{ m/s (right)}}$$

An explosion is just a collision run backward: the spring's stored energy became motion, but the momentum ledger still balances at zero.

Collisions in two dimensions

Not every collision happens along a single line. A cue ball striking another ball off-center, or two cars meeting at an intersection, sends objects off in different directions. The good news is that momentum is a vector, so the ROXY idea from Chapter 2 takes over: momentum is conserved *separately* in the x-direction and in the y-direction. You simply run conservation of momentum twice, once for each axis. Whatever total x-momentum the system had before, it still has after; the same is true for y.

A 1000 kg car moving east at 20 m/s is struck by a 1500 kg truck moving north at 10 m/s. They lock together. Which way and how fast do they slide off? Treat each axis on its own. Before the crash, all the east (x) momentum belongs to the car and all the north (y) momentum belongs to the truck:

$$p_x = (1000)(20) = \boxed{20{,}000 \text{ kg} \cdot \text{m/s},} \qquad p_y = (1500)(10) = \boxed{15{,}000 \text{ kg} \cdot \text{m/s}}$$

After they stick, the combined mass is 2500 kg carrying both totals. Solve each axis for the shared velocity:

$$v_x = \frac{20{,}000}{2500} = 8.0 \text{ m/s}, \qquad v_y = \frac{15{,}000}{2500} = 6.0 \text{ m/s}$$

Rebuild the resultant exactly as in Chapter 2: $v = \sqrt{8.0^2 + 6.0^2} = 10$ m/s, aimed $\theta = \tan^{-1}(6.0/8.0) = 37°$ north of east. The wreckage slides off diagonally, which is why investigators can read skid directions to reconstruct a crash.

Faith & Science , "Bear one another's burdens, and so fulfill the law of Christ" (Galatians 6:2). A collision shares momentum: what one body carries in, the whole system carries out together, and in a gentle collision the shock is spread so no single part bears it all at once. There is a picture here of how we are meant to meet the impacts of a life, not alone and rigid, taking the full force in an instant, but together and yielding, lengthening the blow so it can be borne. The physics is only a metaphor, and a loose one. But a world where impact is softened by sharing is a fitting home for a command to carry each other's weight.

Practice Problems

Work these on your own, then open the solutions to check yourself. Momentum is a vector, so mind your signs and directions.

1. Find the momentum of a 0.20 kg hockey puck moving at 15 m/s.
2. A 60 kg skater moving at 5.0 m/s is stopped in 0.50 s. Find the average force.
3. A 3.0 kg cart at 4.0 m/s strikes a 2.0 kg cart at rest and they stick together. Find their shared velocity.
4. A 600 kg cannon fires a 3.0 kg ball forward at 400 m/s. Find the cannon's recoil speed.
5. An object has 50 kg $\cdot$ m/s of momentum and a mass of 10 kg. Find its velocity.
6. In one or two sentences, explain how an airbag lowers the force on a passenger during a crash.

Solutions to the Practice Problems

1. $p = mv = (0.20)(15) = 3.0$ kg $\cdot$ m/s.
2. $\Delta p = (60)(5.0) = 300$ kg $\cdot$ m/s; $F = \Delta p/t = 300/0.50 = 600$ N.
3. $(3.0)(4.0) + 0 = (5.0)v \Rightarrow v = 12/5.0 = 2.4$ m/s.
4. Total momentum starts at zero: $(3.0)(400) = (600)v \Rightarrow v = 1200/600 = 2.0$ m/s backward.
5. $v = p/m = 50/10 = 5.0$ m/s.
6. The passenger's momentum must drop to zero either way, but the airbag stretches the stopping time; since $F = \Delta p/t$, a longer time means a smaller force.

Challenge Problems

The hard ones. Each of these takes more than one step and more than one idea; some borrow tools from earlier chapters. Work them in GUESS order (Given, Unknown, Sketch, Equation, Substitute, Solve) before opening the solutions.

1. **The Dart and the Block.** A 0.020 kg dart flying at 30 m/s embeds itself in a 0.98 kg wooden block hanging from strings. How high does the block (with dart) swing?

Scan to watch: The Dart and the Block (ballistic pendulum)

2. **Dashboard versus Airbag.** A 70 kg driver moving at 15 m/s is stopped in a crash. Compare the average force if the dashboard stops him in 0.050 s versus an airbag in 0.30 s. Express both in "times his body weight."

Scan to watch: Dashboard versus Airbag (impulse)

3. **Recoil on the Ice.** Sparky (60 kg) stands on ice ($\mu_k = 0.050$) and throws a 3.0 kg ball forward at 8.0 m/s. (a) How fast does he recoil? (b) How far does he slide before friction stops him?

Scan to watch: Recoil on the Ice (momentum conservation)

4. **T-Bone with an Energy Audit.** A 1200 kg car heading east at 15 m/s collides with an 800 kg car heading north at 20 m/s, and they lock together. (a) Find the speed and direction of the wreckage just after impact. (b) How much kinetic energy did the collision destroy?

Scan to watch: T-Bone Crash — a 2-D collision with an energy audit

Solutions to the Challenge Problems

1. **The Dart and the Block.** Two acts. Act 1, the collision, conserves momentum only: $(0.020)(30) = (1.0)v$, so $v = 0.60$ m/s. Act 2, the swing, conserves energy: $h = v^2/2g =$ **1.8 cm**. Check the energy: 9.0 J before the collision, 0.18 J after; the crunch destroyed 98% of it. Momentum survives collisions; kinetic energy does not. Knowing which law to use when IS the problem.

2. **Dashboard versus Airbag.** Either way, $\Delta p = (70)(15) = 1050$ kg $\cdot$ m/s. Dashboard: $F = 1050/0.050 = 21{,}000$ N $\approx$ **31 times his weight**. Airbag: $F = 1050/0.30 = 3500$ N $\approx$ **5 times his weight**. The airbag cannot change the momentum he must lose; it just buys time, and force is momentum over time.

3. **Recoil on the Ice.** (a) Momentum starts at zero: $0 = (3.0)(8.0) + (60)v$, so $v =$ **0.40 m/s backward**. (b) Friction decelerates him at $a = \mu_k g = 0.49$ m/s^2, and $d = v^2/2a =$ **about 16 cm**. One throw, three chapters: momentum for the recoil, $\Sigma F = ma$ for the friction, kinematics for the distance.

4. **T-Bone with an Energy Audit.** Conserve each direction separately: $p_x = 18{,}000$, $p_y = 16{,}000$ kg · m/s; total $p = 24{,}000$ kg · m/s. (a) $v = 24{,}000/2000 =$ **12 m/s at 42° north of east**. (b) $KE_{before} = 135{,}000 + 160{,}000 = 295{,}000$ J; $KE_{after} = \frac{1}{2}(2000)(12.0)^2 = 145{,}000$ J. Lost: **about 150,000 J, over half**, into bent metal, heat, and sound. Momentum: perfectly conserved. Kinetic energy: not even close.

Chapter 7 Summary

- Momentum is mass times velocity, p = mv, a vector measured in kg · m/s. It measures how much motion an object has and how hard it is to stop.
- Impulse is force times time and equals the change in momentum, F · t = Δp. Safety devices lengthen the stopping time to reduce the force for the same momentum change.
- With no outside force, total momentum is conserved: the total before equals the total after. This follows from Newton's third law and explains collisions, explosions, and recoil.
- Momentum is conserved in every collision. Kinetic energy is conserved only in elastic collisions; in inelastic ones some becomes heat and sound, and perfectly inelastic objects stick together.
- In two dimensions, momentum is conserved separately along x and along y. Run conservation on each axis, then rebuild the resultant velocity with the ROXY method from Chapter 2.

Key terms

momentum
the quantity of motion, p = mv; a vector in kg · m/s

impulse
force times the time it acts, F · t, equal to the change in momentum

impulse-momentum theorem
F · t = Δp; impulse equals change in momentum

conservation of momentum
with no outside force, total momentum before equals total after

elastic collision
a collision in which total kinetic energy is also conserved

inelastic collision
a collision in which some kinetic energy becomes heat, sound, or deformation

perfectly inelastic collision
an inelastic collision in which the objects stick together and move as one

Chapter 9

Thermal Energy & Heat Transfer

PHYS1.PS3.1, 3.4

8.1 · Temperature and Thermal Energy

By the end of this section you can: distinguish temperature from thermal energy; and explain both in terms of moving particles.

Everything is made of particles in constant motion, even a solid, whose atoms jiggle in place. That hidden motion is the key to heat. **Temperature** is a measure of the *average* kinetic energy of the particles in a substance. When particles move faster on average, the temperature is higher. **Thermal energy** is the *total* energy of all that particle motion added up across the whole object.

The difference between average and total is worth pausing on, because it explains a puzzle. A lit sparkler burns at thousands of degrees, yet its tiny glowing bits can land on your skin without harm, while a bathtub of warm water at a far lower temperature holds enough thermal energy to scald you. The sparkler's particles move fast (high temperature) but there are almost none of them (low total energy). The bathtub's particles move slower (lower temperature) but there are trillions upon trillions of them (huge total energy). Temperature is the average speed; thermal energy is the whole account.

Misconception check: temperature and thermal energy are not the same thing. Temperature is the average kinetic energy per particle; thermal energy is the total for all particles. A cup of boiling water is hotter than a warm swimming pool, but the pool holds vastly more thermal energy because it contains so much more matter. Do not confuse how fast the particles move with how many there are.

Check yourself: 8.1

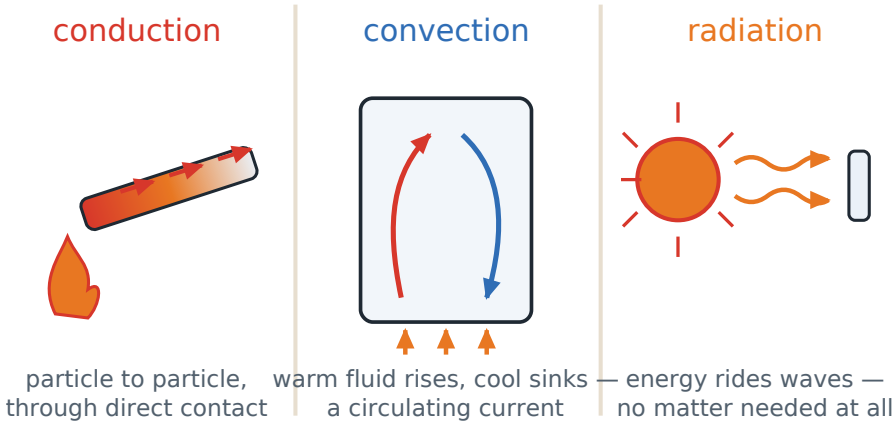
1. What does temperature measure?
   The average kinetic energy of the particles in a substance.
2. How is thermal energy different from temperature?
   Thermal energy is the total energy of all the particles; temperature is the average per particle.
3. Why can a warm swimming pool hold more thermal energy than a cup of boiling water?
   It has far more particles, so even at a lower average energy the total is much greater.

8.2 · Heat and How It Moves

By the end of this section you can: define heat; and describe conduction, convection, and radiation.

Heat is thermal energy on the move, flowing from a hotter object to a cooler one. Heat always travels in that direction on its own, from hot to cold, never the reverse without help, until the two reach the same temperature. Your hand does not "gain cold" from an ice cube; heat flows out of your hand into the ice. There are three ways heat gets from one place to another.

Conduction is heat moving through direct contact, as fast-jiggling particles bump their slower neighbors and pass the energy along. A metal spoon left in hot soup grows warm at the handle this way. **Convection** is heat carried by the bulk motion of a fluid, as warm fluid rises and cool fluid sinks, setting up a current. It is why a room heats from the radiator, why boiling water churns, and why weather happens. **Radiation** is heat carried by electromagnetic waves that need no material at all, which is how the sun's energy crosses empty space to warm your face and how a campfire warms you from across a gap of air.



Three paths for heat. *Conduction through touching particles, convection by a moving fluid, and radiation by waves that cross empty space.*

Check yourself: 8.2

1. What is heat, and which way does it flow?
   Thermal energy in transit; it flows from hotter to cooler until temperatures match.
2. Name the three methods of heat transfer.
   Conduction (through contact), convection (by fluid motion), and radiation (by waves through space).
3. How does the sun's heat reach Earth through empty space?
   By radiation, which needs no material to travel through.

8.3 · Specific Heat

By the end of this section you can: define specific heat; and use Q = mcΔT to compute heat.

Some things heat up easily; others stubbornly resist. Sand at the beach scorches your feet by noon while the ocean beside it stays cool, both under the same sun. The property that explains this is **specific heat**, the amount of heat needed to raise 1 kilogram of a substance by 1 degree Celsius. Water has an unusually high specific heat, so it soaks up a lot of heat with only a small temperature rise. Metals have low specific heat, so they heat and cool quickly.

The heat needed to change a substance's temperature follows a single equation:

$$Q = mc\,\Delta T$$

Here Q is the heat (in joules), m is the mass, c is the specific heat, and ΔT is the temperature change. More mass, a higher specific heat, or a bigger temperature change all mean more heat required. Water's stubbornness, its high c, is quietly one of the most important facts on Earth: it lets oceans store immense heat and moderate the planet's climate, and it lets the water inside your own body hold a steady temperature.

How much heat is needed to warm 2.0 kg of water from 20°C to 80°C? (c for water = 4184 J/kg · °C) Given: m = 2.0 kg, c = 4184 J/kg · °C, $\Delta T = 80 - 20 = 60$°C. **Equation:** $Q = mc\,\Delta T$. **Solve:** $Q = (2.0)(4184)(60) = 502{,}080$ J $\approx \boxed{5.0 \times 10^5 \text{ J}}$. That is half a million joules just to heat two liters of water, a direct measure of how much heat water can hold.

How much heat does it take to warm a 0.30 kg aluminum block from 25°C to 75°C? (c for aluminum = 900 J/kg · °C)

$$Q = mc\,\Delta T = (0.30)(900)(50) = \boxed{13{,}500 \text{ J}}$$

Compare this to water: heating the same mass of water the same amount would take over four times as much heat, because water's specific heat is so much larger. Metals warm up cheaply.

You add 5000 J of heat to 0.50 kg of water. How much does its temperature rise? Rearrange $Q = mc\,\Delta T$ for ΔT:

$$\Delta T = \frac{Q}{mc} = \frac{5000}{(0.50)(4184)} = \boxed{2.4 \text{ °C}}$$

Five thousand joules barely nudges the temperature, which is exactly why water is so good at soaking up heat without getting dangerously hot.

Check yourself: 8.3

1. What is specific heat?
   The heat needed to raise 1 kg of a substance by 1°C.
2. Why does the beach sand get much hotter than the ocean in the same sunlight?
   Sand has a low specific heat, so it heats quickly; water has a high specific heat, so it warms slowly.
3. Find the heat to warm 0.50 kg of water by 10°C. (c = 4184 J/kg · °C)
   Q = (0.50)(4184)(10) = 20,920 J ≈ 2.1×10^4 J.

8.4 · Calorimetry

By the end of this section you can: apply conservation of energy to heat exchange; and solve a calorimetry problem.

Drop a hot piece of metal into cool water and heat flows from the metal to the water until both settle at one shared temperature. **Calorimetry** is the accounting of that heat exchange, and it rests on conservation of energy from Chapter 6. If the container is well insulated, the heat lost by the hot object exactly equals the heat gained by the cold one. Nothing is created or destroyed; the energy simply moves:

$$Q_{\text{lost}} = Q_{\text{gained}}$$

This simple balance is how a lab actually measures specific heats and the energy in foods and fuels. Set the heat lost by one substance equal to the heat gained by the other, write each side as mcΔT, and solve for the unknown. It is conservation of energy wearing a thermometer, and it closes the loop on the whole unit: the energy that leaves the hot object is not gone, only relocated, exactly as Chapter 6 promised.

A 0.50 kg block of metal at 90°C is dropped into 1.0 kg of water at 20°C. They settle at 24°C. Find the metal's specific heat. (water c = 4184 J/kg · °C) Set heat lost by the metal equal to heat gained by the water:

$$m_{metal}\, c_{metal}\, \Delta T_{metal} = m_{water}\, c_{water}\, \Delta T_{water}$$

The water warms by 4°C and the metal cools by 66°C. Plug in:

$$(0.50)(c_{metal})(66) = (1.0)(4184)(4)$$

So $33\, c_{metal} = 16{,}736$, giving $c_{metal} = \boxed{507 \text{ J/kg} \cdot \text{°C}}$. That is close to iron, and the low value fits the lesson from Section 8.3: metals take little heat to change temperature, which is why the block gave up so much heat while cooling only the water a few degrees.

You mix 1.0 kg of water at 80°C with 1.0 kg of water at 20°C in an insulated container. What is the final temperature? Heat lost by the hot water equals heat gained by the cold water. Since both are water (same c) with equal masses, the specific heat and mass cancel:

$$mc(80 - T) = mc(T - 20) \;\Rightarrow\; 80 - T = T - 20 \;\Rightarrow\; T = \boxed{50\ ^\circ\text{C}}$$

Equal amounts of the same substance meet exactly in the middle. With different masses or different materials, you would keep the mc terms and solve for T.

A thermite reaction, and all three of this chapter's heat-transfer mechanisms in one photograph. The glow you could feel on your face from meters away is radiation. The plume rising off the top is convection. And the stand holding the crucible is getting hot the slow way: conduction. (The reaction itself is chemistry. Ask me about it sometime.)

Faith & Science , "Let the peace of Christ rule in your hearts" (Colossians 3:15). Heat always moves toward balance, flowing from where there is more to where there is less until everything shares one temperature, at rest. The universe leans toward that kind of settling, an evening-out that the physicists call equilibrium. It is only a picture, but a gentle one, of the peace Scripture describes: not emptiness, but a settled balance, a rest where striving quiets down. The physics simply shows us a world that tends toward stillness, which is a fitting home for a promise of peace that passes understanding.

8.5 · Phase Changes and Latent Heat

By the end of this section you can: explain why temperature holds steady during melting and boiling; and use Q = mL to find the heat of a phase change.

Heat an ice cube and something strange happens at 0°C. Keep adding heat, and for a while the temperature *stops rising* even though energy is still pouring in. All that energy goes into breaking the bonds that hold the solid together, not into speeding up the particles. Only once the ice has fully melted does the temperature climb again. The same pause happens at 100°C while liquid water boils into steam. These are **phase changes**, and the heat they swallow without any temperature change is called **latent heat** (latent means "hidden").

The heat needed to melt or boil a mass of material has its own simple equation:

$$Q = mL$$

where m is the mass and L is the **latent heat** of the substance, in joules per kilogram. There are two versions: the **latent heat of fusion** L_f for melting or freezing, and the **latent heat of vaporization** L_v for boiling or condensing. For water, $L_f = 3.34 \times 10^5$ J/kg and $L_v = 2.26 \times 10^6$ J/kg. Notice how much larger vaporization is: fully boiling water away takes far more energy than melting it, which is why steam burns are so severe and why sweating cools you so effectively.

Two kinds of heating, two equations. When temperature is *changing*, use $Q = mc\,\Delta T$. When the substance is *changing phase* at a constant temperature, use $Q = mL$. A full problem, like turning ice into steam, is really several steps stacked together, alternating between the two equations.

How much heat is needed to melt 0.20 kg of ice that is already at 0°C? ($L_f = 3.34 \times 10^5$ **J/kg)** The ice is already at its melting point, so no temperature change is involved, only the phase change. Use $Q = mL$:

$$Q = mL_f = (0.20)(3.34 \times 10^5) = \boxed{6.7 \times 10^4 \text{ J}}$$

That is 67,000 joules just to turn solid ice into liquid water at the same temperature. All of it goes into loosening the crystal structure, none into raising the temperature, which is exactly why a drink stays ice-cold until the last ice cube has melted.

How much heat is needed to boil 0.25 kg of water that is already at 100°C into steam? ($L_v = 2.26 \times 10^6$ **J/kg)**

$$Q = mL_v = (0.25)(2.26 \times 10^6) = \boxed{5.65 \times 10^5 \text{ J}}$$

Compare that to Example 8.5: boiling this quarter-kilogram takes far more energy than melting the same mass of ice, because L_v is nearly seven times L_f. This is why steam carries such a dangerous amount of energy.

Check yourself: 8.5

1. Why does temperature stay constant while ice melts?
   The incoming heat goes into breaking the bonds of the solid (the phase change), not into speeding up the particles, so the temperature does not rise until melting is complete.
2. Which equation do you use during a phase change, and which during a temperature change?
   Q = mL during a phase change (constant temperature); Q = mcΔT while the temperature is changing.
3. Find the heat to boil 0.10 kg of water already at 100°C (L_v = 2.26×10 J/kg).
   Q = mL_v = (0.10)(2.26×10) = 2.26×10^5 J.

Practice Problems

Work these on your own, then open the solutions to check yourself. For water, $c = 4184$ J/kg · °C, $L_f = 3.34 \times 10^5$ J/kg, and $L_v = 2.26 \times 10^6$ J/kg.

1. How much heat is needed to warm 1.5 kg of water by 30°C?
2. How much heat warms a 2.0 kg aluminum pan by 40°C? (c for aluminum = 900 J/kg · °C)
3. How much heat is needed to melt 0.50 kg of ice at 0°C?
4. You add 8368 J of heat to 1.0 kg of water. How much does its temperature rise?
5. You mix 2.0 kg of water at 60°C with 2.0 kg at 20°C. Find the final temperature.

6. Which takes more energy: melting 1 kg of ice or boiling 1 kg of water? Explain in one sentence.
7. By which method does heat from a campfire reach your face across a gap of air?

Solutions to the Practice Problems

1. $Q = mc\,\Delta T = (1.5)(4184)(30) = 188{,}280$ J $\approx 1.9 \times 10^5$ J.
2. $Q = (2.0)(900)(40) = 72{,}000$ J.
3. $Q = mL_f = (0.50)(3.34 \times 10^5) = 1.67 \times 10^5$ J.
4. $\Delta T = \dfrac{Q}{mc} = \dfrac{8368}{(1.0)(4184)} = 2.0$ °C.
5. Equal masses of water meet in the middle: $T = \dfrac{60 + 20}{2} = 40$ °C.
6. Boiling, because L_v ($2.26{\times}10$) is far larger than L_f ($3.34{\times}10^5$), so boiling 1 kg takes about seven times the energy of melting 1 kg.
7. Radiation, which crosses the gap of air as electromagnetic waves without needing to heat the air in between.

Challenge Problems

The hard ones. Each of these takes more than one step and more than one idea; some borrow tools from earlier chapters. Work them in GUESS order (Given, Unknown, Sketch, Equation, Substitute, Solve) before opening the solutions.

1. **From Freezer to Drink.** How much total heat turns 0.50 kg of ice at -10°C into water at 25°C? ($c_{ice} = 2100$ J/kg · °C, $L_f = 3.34 \times 10^5$ J/kg, $c_{water} = 4184$ J/kg · °C)

Scan to watch: From Freezer to Drink (the heating staircase)

2. **The Blacksmith's Test.** A 0.30 kg piece of iron ($c = 450$ J/kg · °C) at 95°C is dropped into 0.80 kg of water at 22°C in an insulated cup. Find the final temperature.

Scan to watch: The Blacksmith's Test (calorimetry)

3. **How Much Ice for the Cooler?** You want to cool 1.0 kg of water from 80°C to 20°C using ice at exactly 0°C. What mass of ice does the job? (Assume the meltwater also ends at 20°C.)

Scan to watch: How Much Ice for the Cooler? (the ice collects twice)

4. **Racing the Kettle.** A 1500 W kettle heats 1.2 kg of water from 20°C to boiling. Assuming all the energy goes into the water, how long does it take, and why is the real answer a little longer?

Scan to watch: Racing the Kettle (power meets specific heat)

Solutions to the Challenge Problems

1. **From Freezer to Drink.** Three steps on the heating staircase. Warm the ice: $(0.50)(2100)(10) = 10{,}500$ J. Melt it: $(0.50)(3.34 \times 10^5) = 167{,}000$ J. Warm the water: $(0.50)(4184)(25) = 52{,}300$ J. Total: 2.3×10^5 **J**. The melt costs more than both temperature changes combined, and the temperature never moves while it happens. That is latent heat.
2. **The Blacksmith's Test.** $Q_{lost} = Q_{gained}$ with the final temperature unknown on both sides: $(0.30)(450)(95 - T) = (0.80)(4184)(T - 22)$. Solve: $135(95 - T) = 3347(T - 22)$ gives $T \approx$ **25°C**. The hot iron moved the water three degrees. Water's giant specific heat makes it a thermal sponge; it wins every calorimetry fight it enters.
3. **How Much Ice for the Cooler?** The hot water pays: $(1.0)(4184)(60) = 251{,}040$ J. The ice collects twice, melting and then warming its meltwater to 20°C: $m(3.34 \times 10^5 + 4184 \times 20) = m(417{,}680)$. So $m \approx$ **0.60 kg of ice**. The common mistake is forgetting the second job: the meltwater does not stay at 0°C.
4. **Racing the Kettle.** The pile of joules: $Q = mc\Delta T = (1.2)(4184)(80) \approx 402{,}000$ J. Power is energy per second: $t = Q/P = 402{,}000/1500 \approx 270$ s $\approx$ **4.5 minutes**. The real kettle runs longer because some heat warms the kettle and the air; "all the energy goes into the water" was an efficiency assumption in disguise.

Chapter 8 Summary

- Temperature is the average kinetic energy of particles; thermal energy is the total of all their motion. A small hot object can have less thermal energy than a large warm one.
- Heat is thermal energy in transit, always flowing hot to cold. It moves by conduction (contact), convection (fluid motion), and radiation (waves through space).
- Specific heat is the heat needed to raise 1 kg by 1°C. Heat follows Q = mcΔT. Water's high specific heat lets it store great amounts of heat and moderate climate.
- Calorimetry applies conservation of energy: in an insulated system the heat lost by the hot object equals the heat gained by the cold one.

- During a phase change the temperature holds steady while heat breaks or forms bonds. That hidden heat is the latent heat, Q = mL, with separate values for fusion (melting) and vaporization (boiling).

Key terms

temperature
a measure of the average kinetic energy of particles

thermal energy
the total energy of all the particle motion in an object

heat
thermal energy flowing from a hotter object to a cooler one

conduction
heat transfer through direct contact of particles

convection
heat transfer by the bulk motion of a fluid

radiation
heat transfer by electromagnetic waves, needing no material

specific heat (c)
the heat needed to raise 1 kg of a substance by 1°C

calorimetry
measuring heat exchange using conservation of energy: heat lost equals heat gained

phase change
a change of state (melting, freezing, boiling, condensing) at constant temperature

latent heat (L)
the heat absorbed or released during a phase change, Q = mL; fusion for melting, vaporization for boiling

Appendix A — The Trigonometry You Need

This appendix exists for two kinds of students. Maybe you have not taken trigonometry yet. That is fine, and you are not behind: everything this course asks of trig fits in these few pages, and none of it is hard. Or maybe you took trig a while ago and just want the rust knocked off. Either way, this is not a new chapter to study. It is a toolbox to visit. Nothing here changes what we cover this year or when we cover it.

Here is the honest scope. Physics I uses trigonometry for exactly one job: solving right triangles. Splitting a velocity into pieces, finding the direction of a resultant, taking apart gravity on a ramp. That is it. No identities, no graphing waves, no law of cosines. If you can solve a right triangle and keep your angles straight, you are fully equipped.

A.0 · Start Here: An Eight-Question Self-Test

How to use this: try all eight without looking anything up. If you get all eight right, close this appendix. You are ready for every chapter in the book. Miss a few? The section numbers next to the answers tell you exactly where to go.

1. In a right triangle, the angle θ sits at the lower left corner and the right angle sits at the lower right. Which side is the hypotenuse? Which side is opposite θ?θ
2. Same triangle, but now use the other acute angle, ϕ, at the top. The side that was opposite θ is now what, relative to ϕ?
3. A ramp is 10 m long and makes a 30° angle with the ground. How high is the top of the ramp?
4. A velocity of 20 m/s points 53° above the horizontal. Find the horizontal component.
5. A right triangle has an opposite side of 3 and an adjacent side of 4. Find the angle θ.
6. A vector has components 6 east and 8 north. How big is the vector?
7. A vector has components $x = -4$ and $y = +3$. Your calculator says $\tan^{-1}(3/-4) \approx -37$. Is that the vector's direction?
8. A student computes $\sin 30$ and gets -0.988. What went wrong, and how do you know without checking the manual?

Answers to the Self-Test

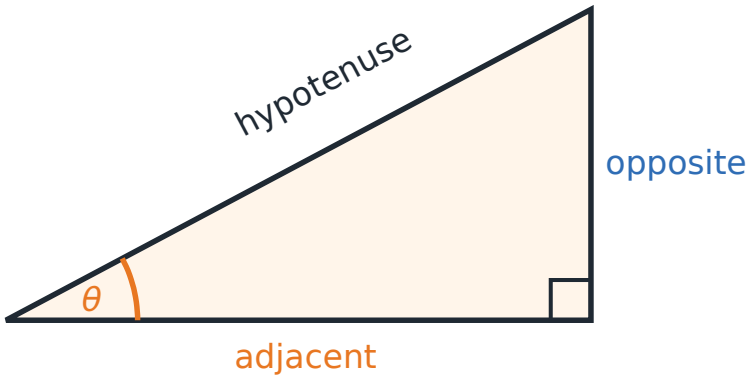
1. The hypotenuse is the longest side, across from the right angle. The side opposite θ is the one that does not touch θ: here, the vertical side on the right. (A.1)
2. Adjacent. Opposite and adjacent are not properties of the triangle. They depend on which angle you are using. (A.1)

3. $10 \sin 30 = 5.0$ m. (A.3)
4. $20 \cos 53 \approx 12$ m/s. (A.3)
5. $\theta = \tan^{-1}(3/4) \approx 37$. (A.4)
6. $\sqrt{6^2 + 8^2} = 10$. (A.5)
7. No. A negative x with a positive y puts the vector in the second quadrant, up and to the left. The calculator can only answer between -90 and $+90$, so it silently hands you the wrong quadrant. Add 180°: the true direction is 143. (A.7)
8. The calculator is in radian mode. You know because $\sin 30$ must be exactly 0.5, and the sine of an acute angle is never negative. (A.9)

A.1 · The Right Triangle, Named Properly

By the end of this section you can: name the three sides of a right triangle relative to a chosen angle, and explain why two of those names can swap.

Every right triangle has three sides, and physics needs you to name them correctly every time. One name is permanent and two are relative.



The names, relative to the angle θ. The hypotenuse never moves. The other two names depend on which angle you pick.

The **hypotenuse** is the longest side, always directly across from the right angle. That name never changes. The other two names are assigned relative to the angle you are working with. The **opposite** side is the one that does not touch your angle. The **adjacent** side is the one that touches your angle but is not the hypotenuse.

Misconception check: students love to memorize "the vertical side is the opposite side." It is not. Pick the other acute angle in the same triangle and the names trade places: what was opposite becomes adjacent. The names belong to the angle, not to the triangle and not to a direction on the page. Before you write any trig ratio, put your finger on the angle you are using and name the sides from there.

A.2 · The Three Ratios

By the end of this section you can: write the sine, cosine, and tangent of an angle as ratios of sides, and state the one association this whole appendix is built on.

For a given angle in a right triangle, the ratios between the sides are locked. It does not matter how big the triangle is. A 37° angle always produces the same shape, so the ratio of opposite to hypotenuse is always the same number. Those fixed ratios are the three trig functions:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

The classic memory aid is **SOH-CAH-TOA**, and it works. But I want you to hold the idea underneath it. Sine tells you what fraction of the hypotenuse shows up on the opposite side. Cosine tells you what fraction shows up on the adjacent side. For 37°, $\cos 37 \approx 0.80$ and $\sin 37 \approx 0.60$: the adjacent side is 80 percent as long as the hypotenuse, and the opposite side is 60 percent as long.

The one association to build: cosine goes with *adjacent*. Sine goes with *opposite*. Not "cosine goes with x." Later in this appendix, and all through the course, you will see components written as $x = R\cos\theta$. That formula is a special case that works because of how we measure the angle. The rule that never fails is the one in this box. Everything else in this appendix is this one relationship wearing different costumes.

A.3 · Finding a Side

By the end of this section you can: use sine and cosine to find an unknown side, which is the single most repeated move in this course.

Most of the time physics hands you the hypotenuse and an angle, and asks for the legs. A velocity and its direction, find the components. A weight and a ramp angle, find the piece along the ramp. The move is always the same: multiply the hypotenuse by the ratio that picks out the side you want.

A ball is kicked at 20 m/s at 53° above the horizontal. Find the horizontal and vertical components. 20 m/svx= 12 m/svy= 16 m/s53° The velocity is the hypotenuse. The horizontal component is adjacent to the 53° angle, so it takes the cosine. The vertical component is opposite, so it takes the sine:

$$v_x = 20\cos 53 \approx \boxed{12 \text{ m/s}} \qquad v_y = 20\sin 53 \approx \boxed{16 \text{ m/s}}$$

Check the sizes. The angle is above 45°, so the vertical piece should win, and it does. This exact move opens nearly every problem in Chapters 2 and 3.

A 4.0 kg box sits on a 30° ramp. Find the component of its weight along the ramp and the component pressing into the ramp. First, the habit this course teaches: rotate your axes so x runs along the ramp. Do that and something nice happens. The normal force, friction, and the box's motion all lie on an axis. Only one vector is left tilted, and it is gravity, which no longer points along y. θxymgmg sin θmg cos $\theta\theta$ The weight is $mg = (4.0)(9.8) = 39.2$ N, and it is the hypotenuse of a little triangle. The geometry puts the ramp angle between the weight vector and the new y-axis. Name the sides from that angle: the piece along the ramp is opposite, and the piece into the ramp is adjacent.

$$W_{\parallel} = 39.2\sin 30 = \boxed{19.6 \text{ N}} \qquad W_{\perp} = 39.2\cos 30 \approx \boxed{33.9 \text{ N}}$$

Notice what just happened. The along-the-ramp piece, the "x direction" in our rotated frame, took the *sine*. If you had memorized "x goes with cosine," this problem would feel like a betrayal. It is not. Cosine went with the adjacent side, exactly like it always does. The angle simply landed next to the y-axis this time. Cosine follows the angle, not the letter x.

A.4 · Finding an Angle

By the end of this section you can: use inverse trig functions to recover an angle from two known sides.

Sometimes the triangle is solved and the angle is the missing piece. You know a resultant's components and want its direction. For that you need the inverse functions, written $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$. Each one answers a question. $\tan^{-1}(0.75)$ asks: what angle has a tangent of 0.75?

A displacement has components 6 blocks east and 8 blocks north. What direction does it point? 6 east8 northresultantθ = ? Tangent is opposite over adjacent. Measuring the angle up from east, north is opposite and east is adjacent:

$$\theta = \tan^{-1}\left(\frac{8}{6}\right) \approx \boxed{53 \text{ north of east}}$$

In this course you will use $\tan^{-1}$ far more than the other two, because components hand you an opposite and an adjacent.

Calculator notes: the inverse functions live above the regular ones, reached with the 2nd or shift key. Make sure the calculator is in **degree mode**, and expect answers in degrees. One more habit: keep a digit or two extra during the work and round at the end.

A.5 · The Pythagorean Theorem

By the end of this section you can: find the hypotenuse from the two legs, which in physics language means finding a vector's magnitude from its components.

The oldest tool in the box still earns its keep every week:

$$c = \sqrt{a^2 + b^2}$$

In physics costume: a vector's magnitude is the square root of the sum of its squared components. Components 6 and 8 give $\sqrt{36 + 64} = 10$. This book leans on the clean triangle families on purpose, and you should learn to spot them: 3-4-5 (and its doubles, 6-8-10 and 9-12-15) and 5-12-13. When you see two legs from one of these families, you can write the hypotenuse without touching the calculator, and you get a free error check besides.

A.6 · Which Way Is Zero? Conventions for Angles

By the end of this section you can: state the standard convention for measuring angles, explain what it buys us, and correctly read direction phrases like "20° east of north."

An angle by itself is just a sweep. To turn it into a direction, everyone has to agree where zero points and which way the angle grows. Unless a problem clearly says otherwise, physics uses this agreement: **zero degrees points along the positive x-axis, and angles grow counterclockwise.** A positive angle rotates counterclockwise; a negative angle rotates clockwise. So 90° is straight up, 180° is the negative x direction, and 270° points straight down.

Why bother with a convention? Because it makes the work mechanical. With one shared reference, the component recipe is always the same two lines, $x = R\cos\theta$ and $y = R\sin\theta$, and every row of

a ROXY table gets built the same way with no re-deriving. Your answer of 143° means the same direction to me, to your calculator, and to the back of the book. And here is the consistency check that shows the pieces agree: at zero degrees a vector points along x and has no y at all, and sure enough $\cos 0 = 1$ and $\sin 0 = 0$. All x, no y. This will click completely in A.8, where you can watch it happen on the unit circle.

Navigation talk: "east of north"

Maps and navigation often set zero at north instead, and directions get spoken in phrases like "20° east of north." Students get this backwards constantly, so here is the rule: **the word after "of" is where you start. The word before it is which way you lean.** "20° east of north" means face north first, then rotate 20° toward east. This book's own answers use these phrases all the time, so make sure you can translate them.

NE20°"20° east of north"start at north, lean toward east

Quick check: can you parse these three?

1. "42° north of east" Face east, lean 42° toward north. In standard position, that is just 42°.
2. "12° east of north" Face north (90°), lean 12° toward east, which rotates you clockwise: $90° - 12° = 78°$ standard position.
3. "30° west of south" Face south (270°), lean 30° toward west (180°): $270° - 30° = 240°$ standard position.

A.7 · When the Angle Leaves the First Quadrant

By the end of this section you can: find the true direction of any vector, including the ones your calculator lies to you about.

Real resultants do not stay politely up and to the right. Sum a few vectors and you can land anywhere: x negative, y negative, or both. The angle formula is still $\tan^{-1}(\Sigma Y/\Sigma X)$, but now there is a trap in it, and Chapter 2 builds two whole challenge problems around it.

Here is the trap. Your calculator's $\tan^{-1}$ button can only answer between -90 and $+90$. It physically cannot say "second quadrant" or "third quadrant." Take components $\Sigma X = -30$ and $\Sigma Y = -40$. That vector points down and to the left, into the third quadrant. But $\tan^{-1}(-40/-30) = \tan^{-1}(1.33)$, and the calculator cheerfully reports 53°, pointing up and to the right. Off by exactly half a circle.

the truth: 233°calculator says 53°ΣX = −30, ΣY = −40

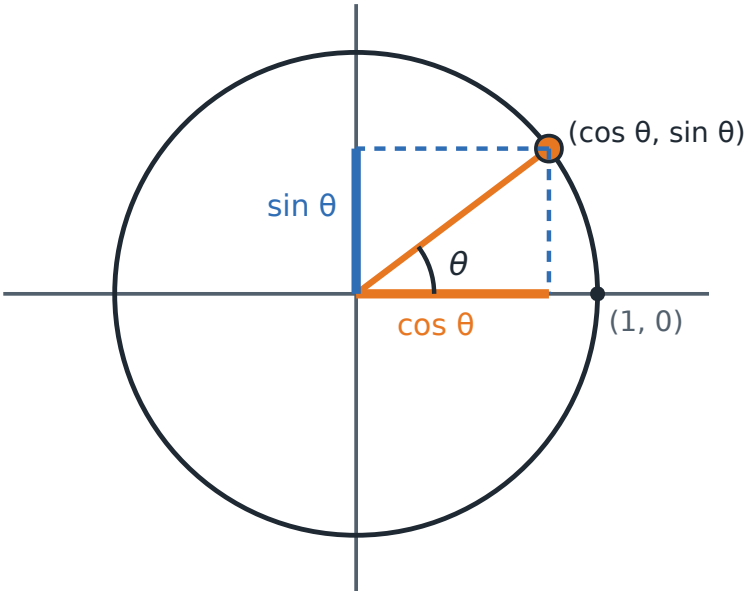
The quadrant rule: sketch ΣX and ΣY before you touch the inverse tangent, so you know which quadrant the truth lives in. Then: **if ΣX is negative, add 180° to the calculator's answer.** In the example above, $53° + 180° = 233°$, which really does point down-left. The calculator is not broken. It just answers a limited question, and finishing the job is your responsibility. Challenge Problems 2.4 and 2.5, and their videos, work this exact trap in full.

A.8 · Where Sine and Cosine Come From: The Unit Circle

By the end of this section you can: describe sine and cosine as the shadows of a rotating point, and use that picture to see every convention and sign rule in this appendix working at once.

So far, sine and cosine have been triangle ratios. Here is the deeper picture. Draw a circle with radius 1 and let a point walk counterclockwise around it, starting from the positive x-axis. At every moment, drop the point straight down to the x-axis and straight across to the y-axis. Two shadows.

Cosine is the x-shadow. Sine is the y-shadow.



The unit circle. The orange radius sweeps counterclockwise from (1, 0); its point casts an x-shadow (cosine) and a y-shadow (sine).

Scan to watch: Where Sine and Cosine Come From — the Unit Circle

Now watch this picture pay off everything we set up earlier. Start the point at zero degrees. It sits at (1, 0): the x-shadow is 1 and the y-shadow is 0, which is exactly the $\cos 0 = 1$, $\sin 0 = 0$ check from A.6. Let it sweep counterclockwise and watch the shadows trade: x shrinks while y grows, until at 90° the point sits at (0, 1). Keep going into the second quadrant and the x-shadow goes *negative*. You can see it cross to the left of the y-axis. The per-quadrant signs that A.7's rule bookkeeps are not memorized facts anymore. They are visible.

One more gift from this picture. The radius is 1, and the two shadows are the legs of a right triangle whose hypotenuse is that radius. Pythagoras then says:

$$\sin^2\theta + \cos^2\theta = 1$$

That famous identity is nothing but the Pythagorean theorem in disguise. Check it at 37°: $0.60^2 + 0.80^2 = 0.36 + 0.64 = 1$. Exactly.

A.9 · Radians: The Arc-Length Angle

By the end of this section you can: decide when a problem wants degrees and when it wants radians, and convert between them.

Here is the decision rule, and it is the whole section in one sentence. **Degrees are for the sweep of an angle. Radians are for when the problem involves arc length,** meaning actual distance traveled along a curve.

Try this question: how far does a wheel move forward when it rotates one revolution? That is not really an angle question. It is a distance question, an arc-length question, so it is a radian question. The radian is built so that arc length is as easy as it can possibly be:

$$s = r\theta \qquad (\theta \text{ in radians})$$

Angle in radians, times radius, equals arc length. Period. One full revolution is 2π radians, so a wheel of radius 0.40 m rolls forward $s = (0.40)(2\pi) \approx 2.5$ m per revolution. Try that formula with degrees and it lies to you; the radian is the unit that makes it true. In Chapter 5 you will meet angular speed ω measured in rad/s, and now you know why. It is hidden arc length per second, which is exactly why $v = r\omega$ converts spin into speed with a single multiplication.

Converting is one proportion: $180 = \pi$ radians. So 45° is $\pi/4 \approx 0.79$ rad, and 3.0 rad is $3.0 \times (180/\pi) \approx 172$.

The number one calculator disaster in this course is sitting in radian mode while working a degree problem. The poison is that the calculator does not warn you. It happily returns $\sin 30 = -0.988$, a nonsense answer wearing a confident face. Your defense is knowing a few anchor values cold: $\sin 30 = 0.5$, $\cos 60 = 0.5$, $\sin 90 = 1$. If a familiar angle returns a strange number, or any acute angle returns a negative sine, check the mode before you check anything else.

A.10 · Practice Problems

How to use these: each problem is tagged with the chapter where you will do this exact move for real. Work them in GUESS spirit: sketch first, name the sides from the angle, then compute.

1. **[Ch2]** A right triangle has θ at the lower left and the right angle at the lower right. Name the three sides. Then name them again using the top angle instead.
2. **[Ch2]** In a 3-4-5 right triangle, let θ be the angle whose opposite side is 3. Write $\sin\theta$, $\cos\theta$, and $\tan\theta$ as decimals.
3. **[Ch4]** A ramp is 8.0 m long and rises at 25°. How high is its top end above the ground?
4. **[Ch3]** A ball is launched at 15 m/s at 40° above the horizontal. Find both components of its velocity.
5. **[Ch4]** A 2.0 kg block rests on a 20° incline. Find the component of its weight along the incline and the component pressing into it.
6. **[Ch2]** A right triangle has an opposite side of 5 and an adjacent side of 12. Find the angle and the hypotenuse, calculator only allowed for the angle.
7. **[Ch2]** A vector has components 9 m east and 12 m north. Find its magnitude without a calculator.
8. **[Ch2]** A resultant has $\Sigma X = -6$ and $\Sigma Y = +6$. Find its true direction in standard position.
9. **[Ch2]** Translate "30° west of south" into a standard-position angle.
10. **[Ch5]** A bicycle wheel has a radius of 0.40 m. How far does the bike move forward in one full revolution of the wheel?
11. **[Ch5]** Convert 45° to radians and 3.0 radians to degrees.
12. **[Ch0]** A student computes $\cos 60$ and gets -0.952. Diagnose the error and state the correct value.

Solutions to the Practice Problems

1. From θ: hypotenuse is the slanted side across from the right angle; opposite is the vertical right side; adjacent is the bottom. From the top angle: hypotenuse is unchanged, but the bottom is now opposite and the vertical side is now adjacent. The names follow the angle.
2. $\sin\theta = 3/5 = 0.6$, $\cos\theta = 4/5 = 0.8$, $\tan\theta = 3/4 = 0.75$.
3. The height is opposite the 25° angle: $8.0\sin 25 \approx 3.4$ m.
4. $v_x = 15\cos 40 \approx 11$ m/s (adjacent), $v_y = 15\sin 40 \approx 9.6$ m/s (opposite).
5. $W = mg = 19.6$ N. Along: $19.6\sin 20 \approx 6.7$ N. Into: $19.6\cos 20 \approx 18$ N. Rotated axes, angle lands by the new y-axis, so the along-ramp piece takes the sine.
6. $\theta = \tan^{-1}(5/12) \approx 23$. The hypotenuse is 13, because 5-12-13 is one of the clean families.
7. 9-12-15 is 3-4-5 tripled: magnitude 15 m.
8. The calculator says $\tan^{-1}(6/-6) = -45$. But $\Sigma X < 0$, so add 180°: the true direction is 135°, up and to the left, matching the sketch.
9. Face south (270°), lean 30° toward west: 240° in standard position.
10. One revolution is 2π radians, so $s = r\theta = (0.40)(2\pi) \approx 2.5$ m.
11. $45 = \pi/4 \approx 0.79$ rad. $3.0 \text{ rad} \times (180/\pi) \approx 172$.
12. Radian mode. $\cos 60$ must be exactly 0.500, and a familiar angle returning a strange number is the tell.

Key Terms

hypotenuse
the longest side of a right triangle, across from the right angle; its name never changes

opposite
the side that does not touch the angle you are using

adjacent
the side that touches your angle but is not the hypotenuse

sine
opposite over hypotenuse; the fraction of the hypotenuse that shows up on the opposite side

cosine
adjacent over hypotenuse; the fraction that shows up on the adjacent side

tangent
opposite over adjacent

inverse trig function
recovers the angle from a known ratio, e.g. $\tan^{-1}$

standard position
the shared convention: zero degrees along positive x, angles growing counterclockwise, 0 to 360°

bearing phrase
a direction like "20° east of north": start at the word after "of," lean toward the word before it

unit circle
a circle of radius 1; a point walking it has x-shadow cosine and y-shadow sine

radian
the arc-length angle: angle in radians times radius equals distance along the arc

Appendix B — The GUESS Method on One Page

Chapter 0 teaches this method and every worked example in the book models it. This page is the short version, the one to have open while you work. The method is not decoration. It is how you show your thinking, and showing your thinking is most of your grade. A right answer with no visible reasoning is a guess. A wrong answer with clear reasoning is worth partial credit and, more importantly, can be fixed.

Scan to watch: The GUESS Method and the Silent S

The five steps

G**Given.** List every quantity the problem hands you, with its symbol, value, and unit. Convert units now, not later. Hidden givens count: "starts from rest" means $v_i = 0$; "constant speed" means $a = 0$.

U**Unknown.** Write what you are solving for, as a symbol with a question mark. If there are two unknowns, write both. This one line keeps you from solving the wrong problem.

Sketch, the silent S. GUESS is spelled with five letters but worked in six steps. The Sketch is the silent S: you do not hear it in the name, but I had better see it on your paper. Before any equation, draw the situation. This step is so important it gets its own section below.

E**Equation.** Choose the equation whose ingredients match your Given and Unknown lists, and write it down in symbols first, no numbers. If it is a force problem, start from the sum of the forces, never from a pre-rearranged formula.

S**Substitute.** Now the numbers go in, each with its unit. Keep an extra digit or two during the work and round at the end.

S**Solve.** Do the algebra, box the answer, and then check it twice: do the units come out right, and does the size make sense? A car should not decelerate at 500 m/s^2. A thrown ball should not land at 300 m/s. Catch nonsense before it leaves your paper.

What a good sketch includes

Before Chapter 4 (motion problems): a simple picture of the situation with the knowns labeled on it, the unknown marked with a question mark, a coordinate axis, and an arrow showing which direction you chose as positive. **From Chapter 4 on** (force problems): the sketch upgrades to a **free-body diagram**. Draw the object as a labeled box. Draw every force as an arrow whose tail sits ON the object, pointing in the direction the force acts, even a push. Label each arrow with its symbol. A force you have not solved for yet gets a symbol only, never a made-up value.

The force-equation rule

Always start force equations from the sum of the forces in a direction. Then ask: is there acceleration in that direction? **No acceleration → statics:** $\Sigma F = 0$. Example, vertical direction: $\Sigma F_y = 0$: $F_N + F\sin\theta - mg = 0$, and then solve for F_N. **Acceleration → dynamics:** $\Sigma F = ma$. Example, horizontal direction: $\Sigma F_x = ma$: $F\cos\theta - F_f = ma$, and then solve for a.

Do not jump straight to a rearranged formula like $F_N = mg - F\sin\theta$. Derive it from $\Sigma F_y = 0$ every time. The habit feels slow for exactly two weeks, and then it becomes the reason hard problems do not scare you.

The thirty-second checklist

1. Every given listed, units converted, hidden givens caught?
2. Unknown written as a symbol?
3. Sketch drawn, axes chosen, positive direction marked? FBD if forces are involved?
4. Equation written in symbols before numbers? Force equations started from ΣF?
5. Substituted with units attached?
6. Answer boxed, units checked, size sanity-checked?

Six yeses and your paper shows real physics, whatever the final number says.

The method, run once

A car is moving at 24 m/s when the driver brakes, slowing at 6.0 m/s^2. How far does the car travel before it stops? Given: $v_i = 24$ m/s, $a = -6.0$ m/s^2 (negative because it opposes the motion). And the hidden given: "before it stops" means $v_f = 0$. **Unknown:** $d = ?$ in meters. **Sketch (the silent S):** a car on a line, velocity arrow pointing forward, a shorter deceleration arrow pointing backward, and the stopping distance marked off with a question mark. Positive direction: the way the car moves. **Equation:** no time given and none asked for, so choose the kinematics equation without t. Then solve it for d in symbols, before a single number goes in:

$$v_f^2 = v_i^2 + 2ad \;\Rightarrow\; 2ad = v_f^2 - v_i^2 \;\Rightarrow\; d = \frac{v_f^2 - v_i^2}{2a}$$

Substitute, units attached:

$$d = \frac{(0)^2 - (24\text{ m/s})^2}{2(-6.0\text{ m/s}^2)}$$

Solve:

$$d = \frac{-576\text{ m}^2/\text{s}^2}{-12\text{ m/s}^2} = \boxed{48\text{ m}}$$

Check: the units work out to meters, and 48 m is about half a football field, a believable panic stop from highway speed. Boxed, checked, done.

What it looks like on your paper

Here is the same problem worked by hand, the way it should look on your own page. Notice the parts, not the penmanship: every step visible, the sketch present, units everywhere, the answer boxed.

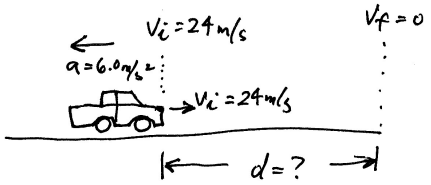
Given: $V_i = 24\ m/s$

$a = -6.0\ m/s^2$

$V_f = 0\ m/s$ (implied because of word "stop")

Unknown: $d = ?$ (meters)

Sketch:



Equations: use $V_f^2 = V_i^2 + 2ad$ (because no t)

$$\Rightarrow 2ad = V_f^2 - V_i^2$$

$$\Rightarrow d = \frac{V_f^2 - V_i^2}{2a}$$

Substitute:

$$d = \frac{(0\ m/s)^2 - (24\ m/s)^2}{2(-6\ m/s^2)}$$

Solve:

$$d = \frac{-(+24\ m/s)^2}{-12\ m/s^2} = \frac{576\ m^2/s^2}{12\ m/s^2}$$

$$= \boxed{48\ m}$$

The GUESS method on real paper. Your page does not need to be pretty. It needs to be complete.

Appendix C — The Symbols, Decoded

Physics borrows letters from the Greek alphabet, and nobody stops class to tell you how to pronounce them or why the same letter keeps changing jobs. This page does both. It covers what this book actually uses, plus the two spare angle letters every physics student eventually meets. If a symbol is not here, you will not need it this year.

The Greek letters

Symbol	Name	Say it	What it means in this book
θ	theta	THAY-tuh	an angle. Always an angle, and always the first choice.
	phi	FYE	a second angle, when θ is already taken. A triangle's two acute angles are often θ and .
	beta	BAY-tuh	a third angle, when θ and are both busy. You will see it more in later courses than in t
Δ	Delta	DEL-tuh	"the change in." $\Delta x = x_{final} - x_{initial}$. Not a quantity itself; it modifies whatever follo
Σ	Sigma	SIG-muh	"the sum of." ΣF means every force added up, the net force.
μ	mu	MYOO	coefficient of friction. μ_s for static, μ_k for kinetic. A pure number, no units.
	omega	oh-MAY-guh	angular speed, in rad/s. The rotational cousin of v.
π	pi	PIE	3.14159..., the circle constant. Shows up whenever something goes around.
	alpha	AL-fuh	angular acceleration, in rad/s^2. The rotational cousin of a. (Other books also use as ye

Same letter, two jobs

The five classic mix-ups. Physics reuses letters, and context is the only thing that tells you which job a letter is doing. These five cause real lost points. Read this table twice.

Letter	Job one	Job two
m	mass, in kilograms: $m = 2.0$ kg	meters, the unit: $x = 5.0$ m
g / G	$g = 9.8$ m/s^2, free-fall acceleration near Earth's surface	$G = 6.67 \times 10^{-11}$, the universal gravitation constant (C
N	the newton, unit of force: $F = 50$ N	the normal force, written F_N
T	tension in a rope, in newtons (Ch4)	period of one revolution or orbit, in seconds (Ch5)
W	weight, $W = mg$, in newtons (Ch4)	work, $W = Fd\cos\theta$, in joules (Ch6)

One more letter that moonlights: **s** is the unit for seconds, the speed in Chapter 0's $s = d/t$, and the arc length in Chapter 5's $s = r\theta$. As always, the units and the chapter tell you which job it is doing.

Notation you will see everywhere

Notation	Say it	What it means
$v_i,\ v_f$	vee-initial, vee-final	the value at the start and at the end of the problem. $v_i = 0$ means "
$v_x,\ v_y$	vee-ex, vee-why	the x and y components of v: the pieces of a vector along each axis.
$\vec{F}$	(arrow on top)	a vector: a quantity with a direction, not just a size.
$\approx$	approximately	"approximately equal to." Honest rounding, not sloppiness.
$\Rightarrow$	therefore / which gives	chains one line of algebra to the next: "this leads to that." Mathema
$\mathrm{m/s^2}$	meters per second squared	the unit of acceleration: how much the velocity (m/s) changes each s
3.0×10^8	three point zero times ten to the eighth	scientific notation: a digit part times a power of ten (Ch0).

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appendix-b/guess-worked-paper.jpg
App B worked example by hand. Rob's own scan (handwritten GUESS solution of Example B.1, braking car, 48 m) **License:** n/a.