

Gr 12

2025

PHYSICAL SCIENCE

**PHYSICS
REVISION BOOK**



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CREDITS

The following question papers were used to compile this book:
 Department of Basic Education, *National Senior Certificate*
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HOW TO USE THIS DOCUMENT

Dear grade 12 Learner

1. This document was compiled as an extra resource to help you to perform well in physical sciences.
2. Firstly, you must make sure that you study the terms and definitions provided for each topic. Theory always forms part of any test or examination, and you should ensure that you obtain full marks for ALL theory questions. Always be prepared to write a test on terms and definitions as soon as a topic is completed in class. Frequently revise terms and definitions of topics already completed so that you know them by the time you are sitting for a test or an examination.
3. Answer all the questions on a certain topic in your homework book as soon as the topic is completed. Numerical answers are given at the back of this book. Use them to guide you about the correctness of your answers. If you differ from a given answer, you may want to check the correctness of your answer. In the case of vectors, only the magnitude of the answer is given. Interpret the direction in terms of your choice of direction. A separate book with fully worked out answers is available. Your teacher will decide when he/she will hand out that specific booklet.
4. If you have the answer book, DO NOT look at the answers before attempting the questions. First try it yourself. Compare your answers with the given answers. Mark your work with a pencil and do corrections for your incorrect answers. If you do not know how to answer a question, the answers are there to guide you. Acquaint yourself with the way in which a particular type of question should be answered. Answers supplied are from memoranda used to mark the questions in previous years.
5. Your teacher can, for example, give you two of the questions in this document as homework. The following day he/she will just check whether you answered them and whether you marked your answers. The teacher will only discuss those questions in which you do not understand the answers supplied in the document. Therefore, a lot of time will be saved, depending on when you receive the answer booklet.
6. The answers are meant to help you to prepare for your tests and examinations. If you choose to copy answers into your homework book without trying them out yourself, you will be losing the developmental aspect of trying to solve problems yourself!
7. Work through all the questions and answers of a particular topic before you sit for an examination, even if you answered the questions before.
8. Any additional resource is only of help when used correctly. Ensure that you make use of all help provided in the correct way to enable you to be successful. All the best and may you perform very well in physical sciences.

PROBLEM-SOLVING

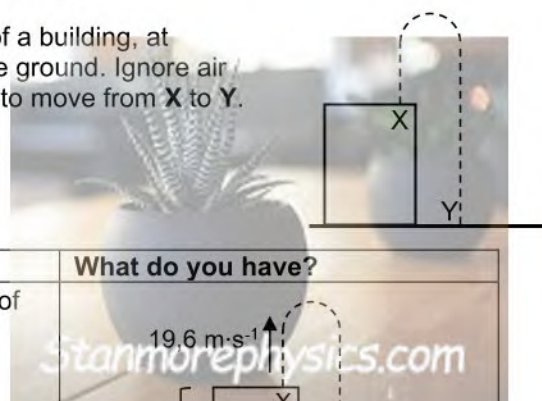
It is very important to develop your problem-solving skills. In physics almost all questions involve problem-solving and a way to solve almost any question is a simple step-by-step strategy.

1	Read the question carefully to analyse it.
2	Analyse the given diagram or draw your own diagram, force diagram or free-body diagram.
If a calculation is involved:	
3	List applicable variables;
4	Choose relevant equations (Start with basic formula);
5	Show which is your positive or negative direction;
6	Substitute values;
7	Do the calculations; and
8	Interpret the final answer if required.
If an explanation is involved:	
The normal flow of steps from three to eight is not so formal as with calculations. However, you still go through the problem-solving process by identifying relevant equations, based on the relevant scientific laws and principles, and then see how the different variables are affected. Use this to steer you in the direction of the answer you must give.	

Let's give a few examples of how this strategy is applied.

Example 1: Vertical motion

An object is projected vertically upwards from point **X**, on the roof of a building, at $19,6 \text{ m} \cdot \text{s}^{-1}$ and strikes the ground at point **Y**. **X** is $102,9 \text{ m}$ above the ground. Ignore air resistance and calculate the velocity at **Y** as well as the time taken to move from **X** to **Y**.

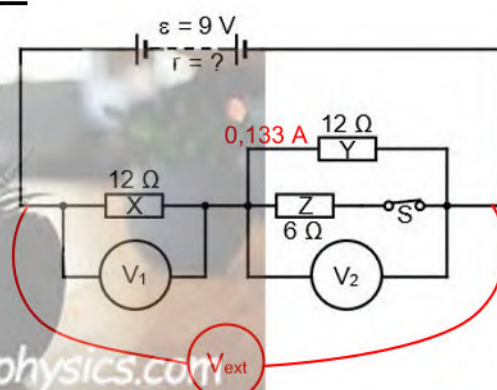


Steps	Critical thinking	What do you have?
1 & 2	- A diagram is given, but the initial velocity and height of the building may be added to aid understanding. - Equations of motion can be used to solve vertical motion problems.	
3	- List the variables of the equations of motion. Add values that you know. - Note how arrows are used to indicate the directions of the vectors. Initial velocity is upwards. Acceleration and displacement from X to Y are downwards. - Nothing is mentioned about "positive" or "negative" at this stage. Once you have chosen your positive or negative direction, the applicable signs are allocated when you do the calculations.	$v_i = 19,6 \text{ m} \cdot \text{s}^{-1} \uparrow$ $v_f = ?$ $\Delta y = 102,9 \text{ m} \downarrow$ $a = 9,8 \text{ m} \cdot \text{s}^{-2} \downarrow$ $\Delta t = ?$
4	- The known variables are marked with ✓. Unknown variables are marked with ovals. - Remember that an equation (one equation) can only be solved if there is one unknown variable. - Equations (1) and (4) have two unknowns. - Equation (2) has one unknown (v_f); therefore, this is a good starting point to calculate v_f. Subsequently, Δt can be calculated with either equation (1) or equation (4). - Equation (3) has one unknown as well (Δt); however, it is a quadratic equation that is slightly more difficult to	$v_f = v_i + a\Delta t \dots\dots\dots (1)$ $v_f^2 = v_i^2 + 2a\Delta y \dots\dots\dots (2)$ $\Delta y = v_i\Delta t + \frac{1}{2}a\Delta t^2 \dots\dots\dots (3)$ $\Delta y = \left(\frac{v_i + v_f}{2}\right)\Delta t \dots\dots\dots (4)$

	solve than equation (2), but still suitable to calculate Δt . Subsequently, v_f can be calculated with either equation (1) or equation (4).	
5	- If the question does not tell you which direction to consider as positive, you may choose a convenient direction yourself. - For this example, downward is considered as positive.	Show your direction somewhere near your solution. An arrow with a positive or negative sign next to it, or a short description in words, is fine.
6, 7 and 8	- Start with equation (2). - Note how the vector answer(*) is interpreted by writing down the magnitude, unit and direction of v_f. - Use equation (1) to calculate Δt. It is not necessary to interpret Δt, because it can only have a positive value. OR - Start with equation (2) and then use equation (4) to calculate Δt. (*) Note that if "speed" or "magnitude of the velocity" at Y was asked and you started with equation (2), your interpretation would be "Speed = $49 \text{ m} \cdot \text{s}^{-1}$" or "Magnitude of velocity = $49 \text{ m} \cdot \text{s}^{-1}$".	$v_f^2 = v_i^2 + 2a\Delta y$ $= (-19,6)^2 + 2(9,8)(102,9)$ $v_f = +49 \text{ m} \cdot \text{s}^{-1}$ $\therefore v_f = 49 \text{ m} \cdot \text{s}^{-1}; \text{down}$ $v_f = v_i + a\Delta t$ $49 = -19,6 + (9,8)\Delta t$ $\Delta t = 7 \text{ s}$ OR $\Delta y = \left(\frac{v_i + v_f}{2} \right) \Delta t$ $102,9 = \left(\frac{-19,6 + 49}{2} \right) \Delta t$ $\Delta t = 7 \text{ s}$

Example 2: Electric circuits

Consider the circuit diagram on the right. Two resistors, Y and Z of 12Ω and 6Ω respectively, are connected in parallel. A closed switch S is connected in series with Z. A third resistor, X, of 12Ω is connected in series with Y and Z. Voltmeters 1 and 2 are connected across X, Y and Z as shown. The current in Y is $0,133 \text{ A}$. The battery has an unknown internal resistance r and its emf is 9 V . Calculate the following: (1) The reading on V_2 ; (2) the reading on V_1 ; and (3) the internal resistance, r , of the battery.



Steps	Critical thinking	What do you have?
1 & 2	- A circuit diagram is given. If any information in the question is not indicated on the diagram, add it to the diagram. In this case it is the branch current in Y, which is a crucial piece of information. - A good habit is to add information to the circuit diagram as you solve the questions. This makes it more "visual" and helps to see the relationships between different numbers.	The following are important facts at this stage: V_1 is only applicable to X. V_2 is only applicable to the parallel set of resistors Y and Z; also, to each of them. - If V_{ext} represents the voltmeter reading across all three resistors, $V_{ext} = V_1 + V_2$. - Due to the internal resistance, $V_{emf} = V_{ext} + V_{int}$. - It is easy to determine the current in Z from the following: V_2 is the same for Y and Z. $\therefore V_2 = R_Y I_Y = R_Z I_Z$ $R_Y \text{ is } 2R_Z$ $I_Y \text{ is } \frac{1}{2}I_Z$

		As you solve the questions in steps six and seven, more arguments will surface.
3	In contrast with the example on vertical motion, it is usually better to write down the different known values on the circuit diagram itself than making a list. The main reason is to make it easier to see the relationships between the variables of the circuit. However, if you are more comfortable with a list, draw up a list.	Refer to the circuit diagram.
5	In addition to this, we do not use "directions" in electric circuit calculations, but it is important to pick the correct variables for each calculation.	
4	We are dealing with potential difference, emf, current, resistance, etc. in this question. Hence, the equation on the right is mainly used, but correctly applied to each solution .	$R = \frac{V}{I}$
	Note how the three variables R , V and I are applied to deal with the "same situation in each solution". This way of thinking is described in detail in the section on electric circuits under "A simple way of how a circuit (direct current) works" where the examples of calculations are given.	
6 & 7	For question (1): Calculate V_2 - Current in Y is known. - Resistance of Y is known.	$R_Y = \frac{V_2}{I_Y}$ $12 = \frac{V_2}{0,133}$ $V_2 = 1,6 \text{ V}$ Remember to enter the reading on V_2 on your circuit diagram.
	For question (2): Calculate V_1 - You need the resistance of X (known) and the current in X (unknown) to calculate V_1. - The current in X is the main current. If you know both branch currents in Y and Z, you have the main current by adding them. - The first step is then to determine the branch current in Z.	We can use $I_Z = 2 \times 0,133 = 0,266 \text{ A}$ as explained before, or the following calculation: $R_Z = \frac{V_2}{I_Z}$ $6 = \frac{1,6}{I_Z}$ $I_Z = 0,267 \text{ A}$ $I_X = I_T = 0,133 + 0,267 = 0,4 \text{ A}$ Remember to write down I_Z and I_T on your circuit diagram. $R_X = \frac{V_1}{I_X}$ $12 = \frac{V_1}{0,4}$ $V_1 = 4,8 \text{ V}$ Remember to write down V_1 on your circuit diagram.
	For question (3): Calculate r - You need the potential drop across the battery (V_{int}) and the main current in the battery to calculate the internal resistance r. - To get V_{int} you must subtract the external potential difference V_{ext} from the emf.	$V_{ext} = V_1 + V_2 = 4,8 + 1,6 = 6,4 \text{ V}$ $V_{int} = V_{emf} - V_{ext}$ $= 9 - 6,4$ $= 2,6 \text{ V}$ $r = \frac{V_{int}}{I_T}$

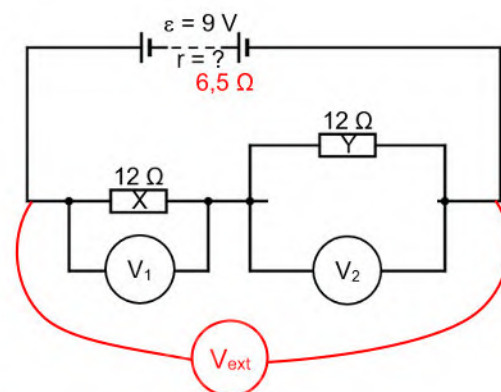
		$\frac{2,6}{0,4}$ $= 6,5 \Omega$
	For those who like to use the formula $\varepsilon = I(R + r)$ to calculate question (3), the following: - Note the meaning of the symbols: I is the total current. R and r are total external resistance and internal resistance respectively. ε is the emf of the battery. - This formula is the same as $\varepsilon = IR + Ir$, where: IR is the same as V_{ext}; and Ir is the same as V_{int}. - After you have calculated questions (1) and (2) as shown above, you know the total current and the V_{ext}, from which question (3) can be solved easily.	$\varepsilon = I(R + r)$ $= IR + Ir$ $= V_{ext} + Ir$ $= V_1 + V_2 + Ir$ $\therefore 9 = 4,8 + 1,6 + 0,4r$ $r = 6,5 \Omega$
8	In electric circuit questions we do not deal with vectors and scalars in the same sense as with vertical motion for example. Hence, this step is usually not required in calculations.	

Example 3: Electric circuits

Let's use the same circuit as in example 2 above, but change it into a question in which you must **explain** how the reading on each of V_1 and V_2 change if switch **S** is opened.

Example 3 Part 1

Let's first look at **calculations** to address this type of situation before we address the explanation type of answer. From the previous example, we know that the internal resistance, r , of the battery is $6,5 \Omega$. Opening the switch is the same as ignoring resistor **Z**. Therefore, the circuit diagram looks like the one on the right-hand side. The following set of calculations (six steps) can be done to determine the reading on each of V_1 and V_2 .



Total resistance: 1 $R_T = R_X + R_Y + r$ $= 12 + 12 + 6,5$ $= 30,5 \Omega$	Total current: 2 $R_T = \frac{V_{emf}}{I_T}$ $30,5 = \frac{9}{I_T}$ $I_T = 0,295 \text{ A}$	Reading on V_1: 3 $R_X = \frac{V_1}{I_T}$ $12 = \frac{V_1}{0,295}$ $V_1 = 3,54 \text{ V}$
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Potential difference across battery: 4 $r = \frac{V_{int}}{I_T}$ $6,5 = \frac{V_{int}}{0,295}$ $V_{int} = 1,92 \text{ V}$	External potential difference: 5 $V_{ext} = V_{emf} - V_{int}$ $= 9 - 1,92$ $= 7,08 \text{ V}$	Reading on V_2: 6 $V_1 + V_2 = V_{ext}$ $3,54 + V_2 = 7,08$ $V_2 = 3,54 \text{ V}$
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Note: It is easy to see that V_2 must be the same as V_1 after step three, because equal resistors in series carry the same current.

Let's look at a comparison between the readings for a closed (example 2) and an open switch (example 3 part 1).

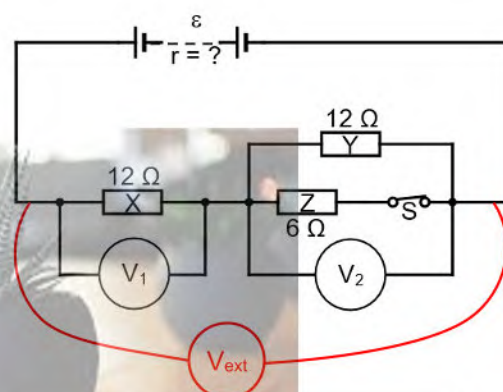
	S closed	S open	Comparison
V_1	4,8 V	3,54 V	Smaller
V_2	1,6 V	3,54 V	Greater
I_T	0,4 A	0,295 A	Smaller
V_{ext}	6,4 V	7,08 V	Greater
V_{int}	2,6 V	1,92 V	Smaller

We can now go back to discuss the approach to an **explanation**. The question might be something like this:

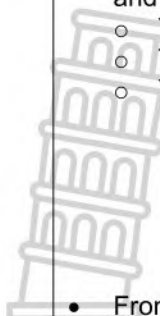
Example 3 Part 2

Consider the circuit diagram on the right. Two resistors, **Y** and **Z** of $12\ \Omega$ and $6\ \Omega$ respectively, are connected in parallel. A closed switch **S** is connected in series with **Z**. A third resistor, **X**, of $12\ \Omega$ is connected in series with **Y** and **Z**. Voltmeters **1** and **2** are connected across **X**, **Y** and **Z** as shown, and each one displays a reading. The battery has an internal resistance r and an emf ε . Switch **S** is then **opened**. How does the reading on each of **V**₁ and **V**₂ change? Explain your answers.

Note that you have only the values of the resistors; no currents, internal resistance or emf. The challenge is to explain the situation in terms of fixed scientific facts.



Steps	Critical thinking	What do you have
All	- If a switch is opened, it generally affects: - The resistance of a circuit; - The main current, and branch currents if any; and V₁ and V₂ (and other variables such as power, energy, etc.). - In an explanation, one usually follows the above-mentioned order, but each question or situation must be analysed based on the factors that are present. - When the switch is opened in this circuit, the following happens: - Resistor Z is effectively removed; - The total resistance increases; and - The total current decreases (more resistance; smaller current). On the right are a few calculations to show you why the total resistance increases from the closed switch to the open switch. This corresponds with the result of calculation 1 in example 3, part 1.	With S closed: Effective resistance of Y and Z: $\frac{1}{R_p} = \frac{1}{R_Y} + \frac{1}{R_Z}$ $= \frac{1}{12} + \frac{1}{6}$ $\therefore R_p = 4\ \Omega$ The total external resistance is: $R_{ext} = R_X + R_p = 12 + 4 = 16\ \Omega$ With S open: Only Y is present and the total external resistance is: $R_{ext} = R_X + R_Y = 12 + 12 = 24\ \Omega$ The total resistance of the circuit is the sum of internal and external resistance; hence, R_T increases.
All	What is the effect on the voltmeter readings? - Let's consider V₁: - The resistance of X is the same as before; - The current in X is the total current; and - The total current is less than before. - Note how only two of the three variables in the equation on the right have changed. This makes it easy to explain the effect on V₁ if the current changes. - Let's consider V₂: - The resistance between the arms of V₂ increases. - Current is problematic. With a closed switch, resistor Y carries a branch current. With an open switch, it carries the total current, which is less than the total current with a closed switch. The problem is: <i>Is this current less than or greater than the branch current with a closed switch?</i> Without values you cannot really tell. - This presents us with a problem. If you do not know what happens to one of the variables, you cannot tell what happens to V₂. Therefore, we must use another argument to determine the effect on V₂.	$V_1 = R_X I_T$ ← Smaller ↑ Same V₁ is smaller than before. See steps 2 and 3 of example 3, part 1 to guide you. This equation can also be written as a direct proportionality: $V_1 \propto I_T$ $V_2 = RI$ ← ?? ↑ Greater Effect on V₂ cannot be explained.

All	 - Let's consider the internal resistance, the total current, and the potential drop across the battery. - The internal resistance remains the same(*); - The total current decreases; and - You can explain the effect on V_{int}. - From the emf of the battery and the potential drop across the battery, you have: - The potential drop across the battery is smaller; - The emf is the same; and - You can explain the effect on the external potential difference V_{ext}. - Lastly, the effect on V_2 can be explained from: V_{ext} is greater than before; and V_1 is smaller than before. (*) The internal resistance and emf are properties of the battery we assume remain constant.	$V_{int} = rI_T \leftarrow \text{Smaller}$ ↑ Same V_{int} is smaller than before. See step 4 of example 3, part 1 to guide you. The direct proportionality in this case is: $V_{int} \propto I_T$ $V_{ext} = V_{emf} - V_{int} \leftarrow \text{Smaller}$ ↑ Same V_{ext} is greater than before. See step 5 of example 3, part 1 to guide you. $V_{ext} = V_2 + V_1 \leftarrow \text{Smaller}$ ↑ Greater V_2 is greater than before. See step 6 of example 3, part 1 to guide you.
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You have completed all the "thinking" and must present your answer now. Here is an example of such an answer. Note how the critical parts of the thinking process are written down with proof of each argument.

When the switch is opened, the **total resistance increases**.
 The **total current decreases**.
 R_X is the same as before. From $V_1 = R_X I_T$ (or $V_1 \propto I_T$), V_1 decreases.
 r is the same as before. From $V_{int} = r I_T$ (or $V_{int} \propto I_T$), V_{int} decreases.
 V_{emf} is the same as before. From $V_{ext} = V_{emf} - V_{int}$, V_{ext} increases.
 From $V_{ext} = V_2 + V_1$, V_2 increases.

ADDITION OF VECTORS: A GOOD APPROACH TO USE

The equation of motion $v_f = v_i + a\Delta t$ teaches us important principles about the **addition of vectors** that you must be aware of.

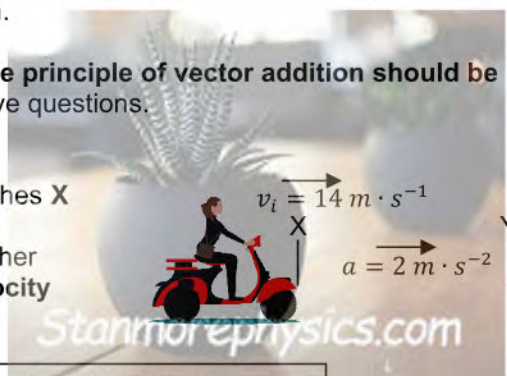
Let's analyse this equation of motion.

- The term $a\Delta t$ comes from the formula $a = \frac{\Delta v}{\Delta t}$ or $a\Delta t = \Delta v$.
- $a\Delta t$ is the product of a vector (a) and a scalar (t). Such a product is a **vector quantity** according to mathematical theory. Hence, $a\Delta t$, or change in velocity (Δv), is a vector.
- $v_f = v_i + a\Delta t$ is the same as $v_f = v_i + \Delta v$.
- Hence, this equation consists of **three** vectors: final velocity (v_f), initial velocity (v_i) and change in velocity ($a\Delta t$). v_f is the **resultant** of v_i and $a\Delta t$, or one can say that v_i and $a\Delta t$ are the **components** of v_f .
- Final velocity is equal to initial velocity **plus** the change in velocity. Note that the change in velocity is **added** (**NOT subtracted**) to the initial velocity to get the final velocity. Therefore: In the equation $v_f = v_i + a\Delta t$ the **plus sign represents addition. It does not indicate direction.**

Examples 1 and 2 below, with their solutions, **demonstrate how the principle of vector addition should be applied.** Teach yourself to use this good, scientific approach to solve questions.

Example 1

A learner on her scooter is moving from left to right. When she reaches X the magnitude of her velocity is $14 \text{ m} \cdot \text{s}^{-1}$. Three seconds later she reaches Y. Calculate her velocity at Y in magnitude and direction if her acceleration from X to Y has a magnitude of $2 \text{ m} \cdot \text{s}^{-2}$. **Note that velocity and acceleration have the same directions.**



Solution 1: Left to right is negative.

$$\begin{aligned} v_f &= v_i + a\Delta t \\ &= -14 + (-2)(3) \\ &= -20 \text{ m} \cdot \text{s}^{-1} \\ \therefore v_f &= 20 \text{ m} \cdot \text{s}^{-1} \text{ left to right} \end{aligned}$$

Solution 2: Left to right is positive.

$$\begin{aligned} v_f &= v_i + a\Delta t \\ &= +14 + (+2)(3) \\ &= +20 \text{ m} \cdot \text{s}^{-1} \\ \therefore v_f &= 20 \text{ m} \cdot \text{s}^{-1} \text{ left to right} \end{aligned}$$

Characteristics of these calculations

	Solution 1	Solution 2
Initial velocity (v_i)	It is from left to right, which is the negative direction; a negative sign is used in front of the "14".	It is from left to right, which is the positive direction; a positive sign* is used in front of the "14".
The operator Vector addition	Encircled in the calculation. It is a positive showing you add the two vectors v_i and $a\Delta t$.	Encircled in the calculation. It is a positive showing you add the two vectors v_i and $a\Delta t$.
Change in velocity ($a\Delta t$)	Acceleration (a) is from left to right; hence $a\Delta t$ is from left to right, which is the negative direction. Use a negative sign in front of the "2" because Δt is always a positive scalar quantity.	Acceleration (a) is from left to right; hence $a\Delta t$ is from left to right, which is the positive direction. Use a positive sign* in front of the "2" because Δt is always a positive scalar quantity.
The answer, final velocity (v_f)	It is negative ; hence, it is from X to Y.	It is positive ; hence, it is from X to Y.
Interpretation of the final answer	It is stated as a magnitude and direction based on your choice of directions. In mathematical terms, the magnitude of the velocity vector is the absolute value of the calculation's answer; $ -20 = 20$.	As for solution 1 except that $ +20 = 20$.
Directions of velocity and acceleration	The same. In such a case $v_f > v_i$.	The same. In such a case $v_f > v_i$.
		(*) If no sign is shown, it is assumed to be positive.

Very important: Note that solution 1 is **NOT DONE** as shown on the right-hand side. Although you get the same answer, one **does not use "subtraction" of vectors** (meaning the encircled sign is a negative) **in an equation** to cater for the negative acceleration in this case. **Stay with the principle of vector addition, even for negative vectors.**

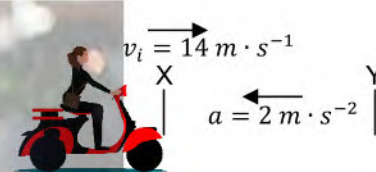
Solution 1: Left to right is negative.

$$\begin{aligned} v_f &= v_i \ominus a\Delta t \\ &= -14 \ominus 2(3) \\ &= -20 \text{ m} \cdot \text{s}^{-1} \\ \therefore v_f &= 20 \text{ m} \cdot \text{s}^{-1} \text{ left to right} \end{aligned}$$

Example 2

Let's use the same information as in example 1, except that the learner accelerates in the opposite direction to velocity.

A learner on her scooter is moving from left to right. When she reaches X the magnitude of her velocity is $14 \text{ m} \cdot \text{s}^{-1}$. Three seconds later she reaches Y. Calculate her velocity at Y in magnitude and direction if her acceleration from Y to X has a magnitude of $2 \text{ m} \cdot \text{s}^{-2}$. **Note that velocity and acceleration have opposite directions.**



Solution 3: Left to right is negative.

$$\begin{aligned} v_f &= v_i \oplus a\Delta t \\ &= -14 \oplus (+2)(3) \\ &= -8 \text{ m} \cdot \text{s}^{-1} \\ \therefore v_f &= 8 \text{ m} \cdot \text{s}^{-1} \text{ left to right} \end{aligned}$$

Solution 4: Left to right is positive.

$$\begin{aligned} v_f &= v_i \oplus a\Delta t \\ &= +14 \oplus (-2)(3) \\ &= +8 \text{ m} \cdot \text{s}^{-1} \\ \therefore v_f &= 8 \text{ m} \cdot \text{s}^{-1} \text{ left to right} \end{aligned}$$

Characteristics of these calculations

	Solution 3	Solution 4
Initial velocity (v_i)	It is from left to right, which is the negative direction; a negative sign is used in front of the "14".	It is from left to right, which is the positive direction; a positive sign* is used in front of the "14".
The operator Vector addition	Encircled in the calculation. It is a positive showing you add the two vectors v_i and $a\Delta t$.	Encircled in the calculation. It is a positive showing you add the two vectors v_i and $a\Delta t$.
Change in velocity ($a\Delta t$)	Acceleration (a) is from right to left; hence $a\Delta t$ is from right to left, which is the positive direction. Use a positive sign* in front of the "2" because Δt is always a positive scalar quantity.	Acceleration (a) is from right to left; hence $a\Delta t$ is from right to left, which is the negative direction. Use a negative sign in front of the "2" because Δt is always a positive scalar quantity.
The answer, final velocity (v_f)	It is negative ; hence, it is from X to Y.	It is positive ; hence, it is from X to Y.
Interpretation of the final answer	It is stated as a magnitude and direction based on your choice of directions. In mathematical terms, the magnitude of the velocity vector is the absolute value of the calculation's answer; $ -8 = 8$	As for solution 3 except that $ +8 = 8$.
Directions of velocity and acceleration	Opposite. In such a case $v_f < v_i$.	Opposite. In such a case $v_f < v_i$.

(*) If no sign is shown, it is assumed to be positive.

Very important: As before, **do not use "subtraction" of vectors** (the encircled sign is a negative) **in an equation** such as for solution 4 to cater for the negative acceleration in this case. **Stay with the principle of vector addition, even for negative vectors.**

Solution 4: Left to right is positive.

$$\begin{aligned} v_f &= v_i \ominus a\Delta t \\ &= +14 \ominus (2)(3) \\ &= +8 \text{ m} \cdot \text{s}^{-1} \\ \therefore v_f &= 8 \text{ m} \cdot \text{s}^{-1} \text{ left to right} \end{aligned}$$

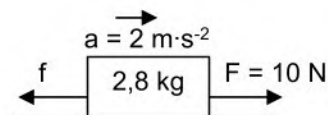
From the above-mentioned, remember:

- Use the rule of **vector addition (vector sum)** in **vector equations**;
- **Substitute vector quantities** with their **signs for direction**; and
- **Do not use vector subtraction** to cater for negative directions.

The equation of motion $v_f = v_i + a\Delta t$ **appears on the data sheet** in the **correct form** as the **addition of vectors** to give the resultant vector v_f . But what happens if you use an equation **where you must construct the components of a resultant vector yourself**? Let's consider Newton's second law of motion as an example. Only the basic equation $F_{net} = ma$ appears on the data sheet. Depending on the situation, the net force (i.e. the resultant force) may consist of only one component, or more than one component. In the following example F_{net} consists of two components.

Example 3

A horizontal force F of 10 N to the right is applied to a box with a mass of 2,8 kg. As a result, the box accelerates along a rough, horizontal floor at $2 \text{ m}\cdot\text{s}^{-2}$ to the right. Calculate the magnitude and direction of the (kinetic) frictional force on the box.



Solution 5: Left to right is positive.

$F_{net} = ma$ Basic formula on data sheet
 $F + f = ma$ **Vector addition of the components of F_{net} .**
 $(+10) + f = (2,8)(+2)$ Substitution with correct signs for F and a
 $f = -4,4 \text{ N}$ Final answer (**negative**)
 $\therefore f = 4,4 \text{ N}; \text{right to left}$ Interpretation based on sign of final answer

Solution 6: Left to right is negative.

$F_{net} = ma$ Basic formula on data sheet
 $F + f = ma$ **Vector addition of the components of F_{net} .**
 $(-10) + f = (2,8)(-2)$ Substitution with correct signs for F and a
 $f = +4,4 \text{ N}$ Final answer (**positive**)
 $\therefore f = 4,4 \text{ N}; \text{right to left}$ Interpretation based on sign of final answer

Note the following:

- In both calculations, the sign of the final numerical answer gives the direction of the vector.
- You allow the algebra to give the correct scientific information about the direction of the vector answer.

The following is a **bad habit!** Note that **vector addition is not used** for the components of F_{net} .

Solution 7: Left to right is positive.

$F_{net} = ma$ Basic formula on data sheet
 $F - f = ma$ **No vector addition** of the components of F_{net} .
 The sign for f is **chosen by yourself** based on your knowledge that the direction of f is opposite to the direction of F .
 $(+10) - f = (2,8)(+2)$ Substitution with correct signs for F and a
 $f = +4,4 \text{ N}$ Final answer (**positive**)
 $\therefore f = 4,4 \text{ N}; \text{right to left}$ Interpretation **cannot be based on the sign** of the final answer, because you have already chosen the negative in the formula.

Note the following:

- You cannot use the sign of the final numerical answer to get the direction of the vector.
- You do NOT allow the algebra to give you the correct, scientific information about the direction of the vector answer.

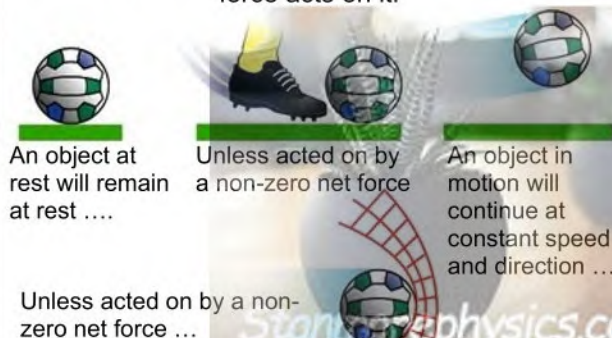
Using the method in solution 7 is not a good, scientific approach and **must be avoided**. In simple cases you may get the correct answer, but there are cases where you **cannot guess** the correct direction and the algebra is then the only way to give you the correct answer. **If you do not use vector addition (vector sum) in the formula, you could get the wrong answer.**

NEWTON'S LAWS

Newton's Laws of Motion

Newton's First Law

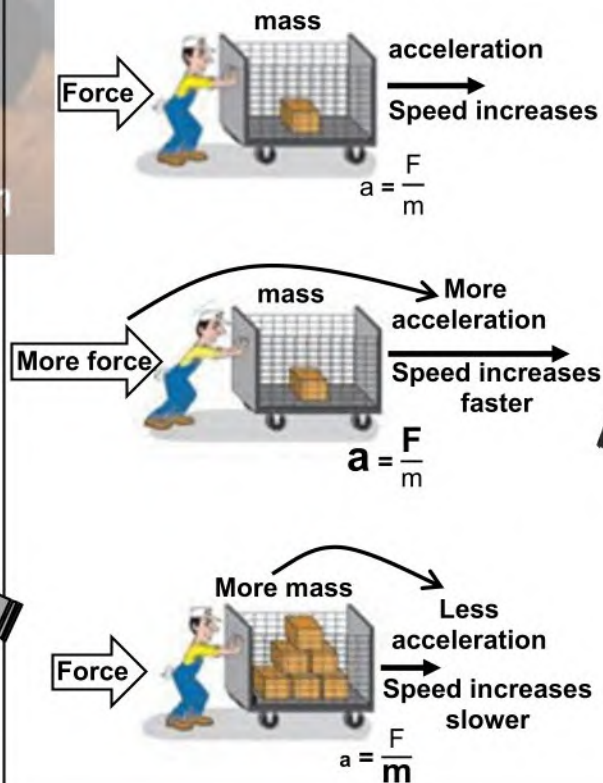
A body will remain in its state of rest or motion at constant velocity unless a non-zero resultant/net force acts on it.



Newton's Second Law

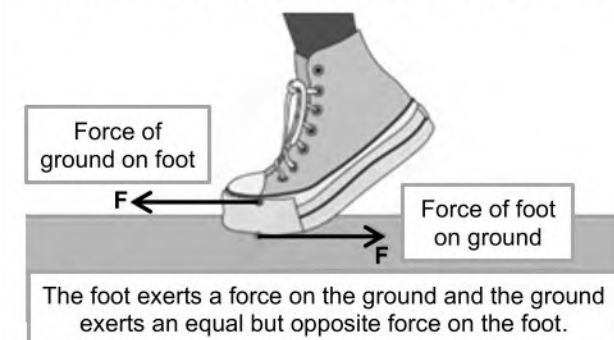
When a **resultant/net force** acts on an object, the **object will accelerate** in the **direction of the force** at an **acceleration directly proportional to the force** and **inversely proportional to the mass** of the object.

In symbols: $F_{\text{net}} = ma$

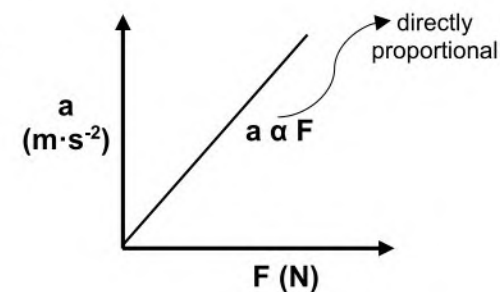
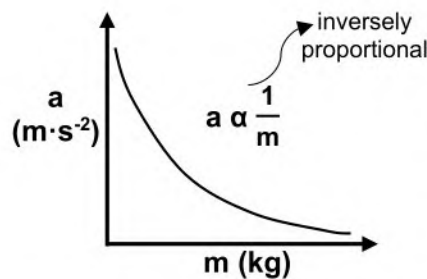


Newton's Third Law

When object **A** exerts a force on object **B**, object **B** **SIMULTANEOUSLY** exerts a force equal in magnitude but opposite in direction on object **A**.

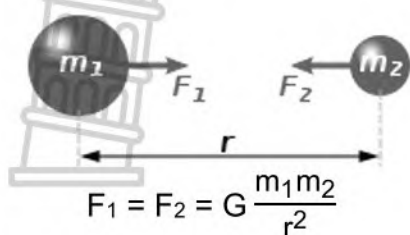


Action-reaction force pairs: The two forces act on different bodies and cannot cancel!

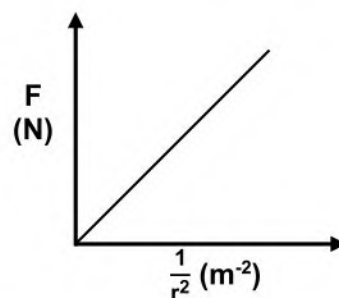
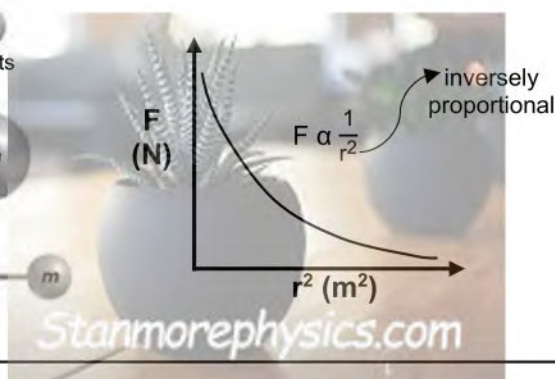
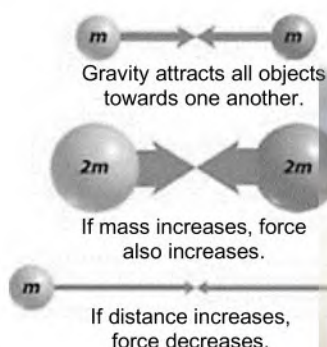


Newton's Law of Universal Gravitation

Every object in the universe attracts every other object with a force that is **directly proportional to the product of their masses** and **inversely proportional to the square of the distance between their centres**.



- F_1 : The force that m_2 exerts on m_1 (in N)
- F_2 : The force that m_1 exerts on m_2 (in N)
- G is the universal gravitational constant ($6,674 \times 10^{-11} \text{ N} \cdot \text{m}^2 \cdot \text{kg}^{-2}$)
- m_1 is the first mass (in kg)
- m_2 is the second mass (in kg)
- r is the distance between the centers of the masses (in m).



TERMS AND DEFINITIONS

Acceleration	The rate of change of velocity.
Free-body diagrams	This is a diagram that shows the relative magnitudes and directions of forces acting on a body/particle that has been isolated from its surroundings.
Kinetic frictional force (f_k)	The force that opposes the motion of a MOVING object relative to a surface.
Mass	The amount of matter in a body measured in kilogram (kg).
Maximum static frictional force f_s^{max}	The static frictional force is a maximum f_s^{max} just before the object starts to move across the surface.
Newton's first law of motion	A body will remain in its state of rest or motion at constant velocity unless a non-zero resultant/net force acts on it.
Inertia	The resistance of a body to a change in its state of rest or uniform motion in a straight line. Mass is a measure of an object's inertia.
Newton's second law of motion	When a resultant/net force acts on an object, the object will accelerate in the direction of the force and the acceleration is directly proportional to the force and inversely proportional to the mass of the object. In symbols: $F_{net} = ma$
Newton's third law of motion	When object A exerts a force on object B, object B SIMULTANEOUSLY exerts an oppositely directed force of equal in magnitude on object A.
Newton's law of universal gravitation	Each body in the universe attracts every other body with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres. In symbols: $F = \frac{Gm_1m_2}{r^2}$
Normal force	The force or the component of a force which a surface exerts on an object with which it is in contact, and which is perpendicular to the surface.
Static frictional force (f_s)	The force that opposes the tendency of motion of a STATIONARY object relative to a surface.
Weight	The gravitational force the earth exerts on any object on or near its surface.
Weightlessness	The sensation experienced when all contact forces are removed, i.e. no external objects touch one's body.

Drawing of free-body diagrams

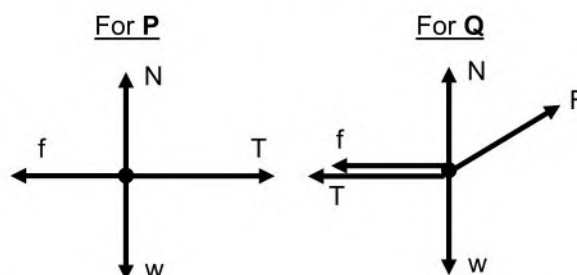
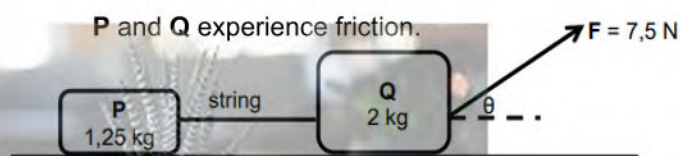
- Draw a dot to represent the object.
- Starting on the dot, draw arrows to represent each force.
- Each arrow's tail must start on the dot; this means the arrowhead must point away from the dot.
- The arrows must represent the direction of the forces.
- Do not draw components of forces in a free-body diagram; draw only the forces!
- Identify each force by writing the name or symbol of the force next to the arrow. It is good to use the symbols that are normally used for specific forces, such as "f" for friction, "N" for a normal force, etc. See the memoranda of final papers for more examples.



Scenario



Free-body diagrams



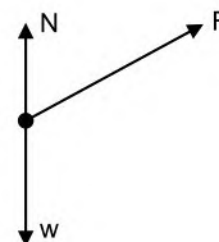
Examples of how to solve Newton's second law of motion questions

Problem 1: A single object moving on a horizontal plane **without** friction.

A 15 kg cement block is pulled to the right along a smooth surface with a force of 100 N, which makes an angle of 14° with the horizontal. Calculate the magnitude of the normal force and the acceleration of the cement block. The effects of friction may be ignored.

Analyse the question and draw a free-body diagram:

- The applied force acts at an angle and therefore the normal force is not equal to the weight of the block. The magnitude of the vertical component of the applied force plus the magnitude of the normal force equals the magnitude of the weight.
- The forces in the vertical plane, which are the normal force, weight and the vertical component of the applied force, do not affect the horizontal motion. Only one force, the horizontal component of the applied force, influence horizontal motion.



Variables:

$$m = 15 \text{ kg}$$

$$F = 100 \text{ N}$$

Solution:

Normal force; take up as positive:

$$F_{\text{net}} = N + F(\text{vertical}) + w = 0$$

[Vector sum for the vertical net force]

$$N + (100)(\sin 14^\circ) - (15)(9,8) = 0$$

[Substitution]

$$N = 122,81 \text{ N}$$

[Numerical answer]

Horizontal acceleration; take to the right as positive:

$$F_{\text{net}} = ma$$

[Basic formula]

$$F(\text{horizontal}) = ma$$

[The net force has only one component]

$$(100)(\cos 14^\circ) = 15a$$

[Substitution]

$$a = 6,47 \text{ m} \cdot \text{s}^{-2}$$

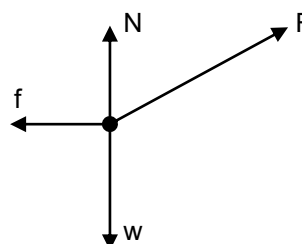
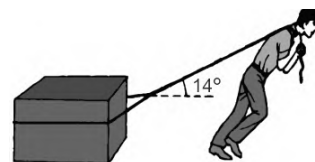
[Numerical answer]

$$\therefore a = 6,47 \text{ m} \cdot \text{s}^{-2}; \text{ to the right}$$

[Interpretation]

Problem 2: A single object moving on a horizontal plane **with** friction.

A 15 kg cement block is pulled across the floor with a force of 100 N, which forms an angle of 14° with the horizontal. The coefficient of kinetic friction between the block and the floor is 0,4. Calculate the acceleration of the cement block.



Analyse the question and draw a free-body diagram:

- The applied force acts at an angle and therefore the normal force is not equal to the weight of the block. The magnitude of the vertical component of the applied force plus the magnitude of the normal force equals the magnitude of the weight.
- The forces in the vertical plane, which are the normal force, weight and the vertical component of the applied force, do not affect the horizontal motion. Only two forces, friction and the horizontal component of the applied force, influence horizontal motion.
- Although the normal force is not asked in this question, it is needed to calculate the frictional force.

Variables:

$$m = 15 \text{ kg}$$

$$F = 100 \text{ N} \nearrow$$

$$\mu = 0,4$$

Solution:

Normal force; take up as positive: $F_{net} = N + F(\text{vertical}) + w = 0$ $N + (100)(\sin 14^\circ) - (15)(9,8) = 0$ $N = 122,808 \text{ N}$	[Vector sum for the vertical net force] [Substitution] [Numerical answer]	Frictional force: $f = \mu N$ $= (0,4)(122,808)$ $= 49,123 \text{ N}$
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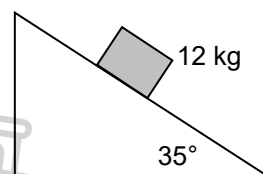
Horizontal acceleration; take to the right as positive: $F_{net} = ma$ $F(\text{horizontal}) + f = ma$ $(100)(\cos 14^\circ) - 49,123 = 15a$ $a = 3,19 \text{ m} \cdot \text{s}^{-2}$ $\therefore a = 3,19 \text{ m} \cdot \text{s}^{-2}; \text{ to the right}$	[Basic formula] [Vector sum for horizontal net force] [Substitution] [Numerical answer] [Interpretation]
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Problem 3: A single object moving on an inclined plane **without** friction.

An inclined surface makes an angle of 35° with the horizontal. Due to an applied force, F , parallel to the surface, the object with a mass of 12 kg accelerates at $1,5 \text{ m} \cdot \text{s}^{-2}$. Ignore all frictional forces and calculate the magnitude and direction of F if the acceleration is:

3.1 upwards, along the surface; and

3.2 downward, along the surface.



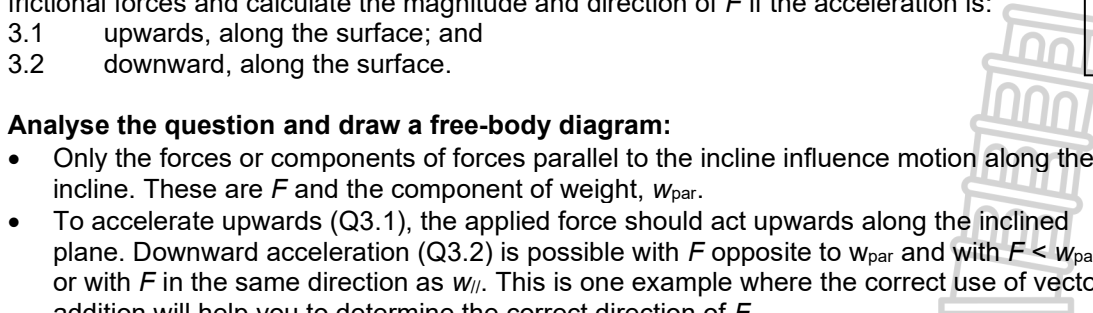
Analyse the question and draw a free-body diagram:

- Only the forces or components of forces parallel to the incline influence motion along the incline. These are F and the component of weight, w_{par} .
- To accelerate upwards (Q3.1), the applied force should act upwards along the inclined plane. Downward acceleration (Q3.2) is possible with F opposite to w_{par} and with $F < w_{\text{par}}$, or with F in the same direction as w_{par} . This is one example where the correct use of vector addition will help you to determine the correct direction of F .
- In Q3.1, the direction of the acceleration is opposite to that of w_{par} , which is down the incline. In Q3.2, the direction of the acceleration is the same as that of w_{par} .

Variables:

$$m = 12 \text{ kg}$$

$$a = 1,5 \text{ m} \cdot \text{s}^{-2} \text{ (Q3.1 } \nearrow \text{) (Q3.2 } \searrow \text{)}$$



Solution:

Q3.1; take upwards along the incline as positive:

$$F_{\text{net}} = ma$$

$$F + w_{\text{par}} = ma$$

$$F - (12)(9,8)(\sin 35^\circ) = (12)(1,5)$$

$$F = 85,45 \text{ N}$$

$\therefore F = 85,45 \text{ N}$; upwards along the incline

[Basic formula]

[Vector sum of components]

[Substitution]

[Numerical answer]

[Interpretation]

Q3.2; take upwards along the incline as positive:

$$F_{\text{net}} = ma$$

$$F + w_{\text{par}} = ma$$

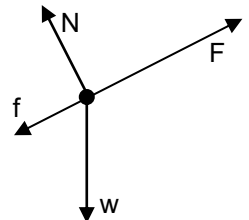
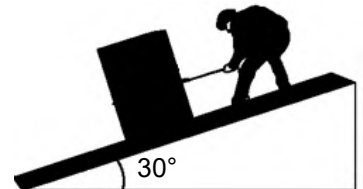
$$F - (12)(9,8)(\sin 35^\circ) = (12)(-1,5)$$

$$F = 49,45 \text{ N}$$

$\therefore F = 49,45 \text{ N}$; upwards along the incline

Problem 4: A single object moving on an inclined plane *with* friction.

Richard pulls a crate, mass 20 kg, with the help of a rope up along an inclined plane, making an angle of 30° with the horizontal. The tension in the rope is 147 N and the coefficient of kinetic friction between the crate and the inclined plane is 0,1. Calculate the acceleration of the crate.



Analyse the question and draw a free-body diagram:

- The normal force is needed to calculate the frictional force. The magnitude of the normal force is equal to the magnitude of the component of weight perpendicular to the inclined plane ($w_{\perp}$).
- Three forces, friction, applied force (same as tension) and the component of weight parallel to the incline (w_{par}), influence the motion parallel to the incline.
- Remember that $w_{\perp} = mg \cos 30^\circ$ and $w_{\text{par}} = mg \sin 30^\circ$.

Variables:

$$m = 20 \text{ kg}$$

$$F = 147 \text{ N} \nearrow$$

$$\mu = 0,1$$

Solution:

Normal force; take perpendicular to the incline as positive:

$$F_{\text{net}} = N + w_{\perp} = 0$$

$$N - (20)(9,8)(\cos 30^\circ) = 0$$

$$N = 169,741 \text{ N}$$

[Vector sum for perpendicular net force]

[Substitution]

[Numerical answer]

Frictional force:

$$f = \mu N$$

$$= (0,1)(169,741)$$

$$= 16,974 \text{ N}$$

Acceleration; take up and parallel to the incline as positive:

$$F_{\text{net}} = ma$$

$$F + w_{\text{par}} + f = ma$$

$$147 - (20)(9,8)(\sin 30^\circ) - 16,974 = 20a$$

$$a = 1,60 \text{ m} \cdot \text{s}^{-2}$$

$$\therefore a = 1,60 \text{ m} \cdot \text{s}^{-2}; \text{ up along the incline}$$

[Basic formula]

[Vector sum for net force]

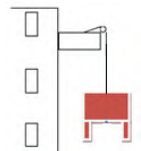
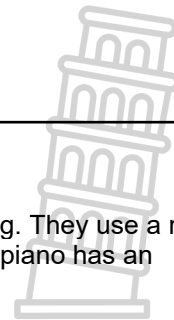
[Substitution]

[Numerical answer]

[Interpretation]

Problem 5: A single object moving in the vertical plane.

A company needs to lift a 320 kg piano to the top floor of an apartment building. They use a rope and pulley system to pull the piano up. Calculate the tension in the rope if the piano has an acceleration of $0,45 \text{ m} \cdot \text{s}^{-2}$.



Analyse the question and draw a free-body diagram:

- The rope exerts an upward force F on the piano. F must be greater than w to accelerate the piano. Hence, F_{net} is upwards; therefore, acceleration is upwards. The force F must be large enough to support the weight of the piano, and to give it an upward acceleration.
- The magnitude of F is equal to the magnitude of the tension T in the rope. However, tension does not have direction. It is a scalar (check how the answer is given below).
- No normal force is included in the free-body diagram. The normal force is a force due to the contact between two surfaces and the piano is not resting on any surface.

Variables:

$$m = 320 \text{ kg}$$

$$a = 0,45 \text{ m} \cdot \text{s}^{-2} \uparrow$$

Solution:

For downwards as positive (it is less complicated if upward is chosen as positive, but let's practise vector directions):

$$F_{\text{net}} = ma \quad [\text{Basic formula}]$$

$$T + w = ma \quad [\text{Vector sum for net force}]$$

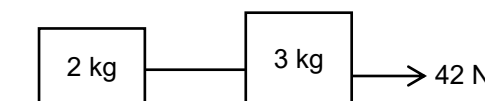
$$T + (320)(9,8) = (320)(-0,45) \quad [\text{Substitution}]$$

$$T = -3\,280 \text{ N} \quad [\text{Numerical answer}]$$

$$\therefore \text{Magnitude of } T = 3\,280 \text{ N} \quad [\text{Interpretation}]$$

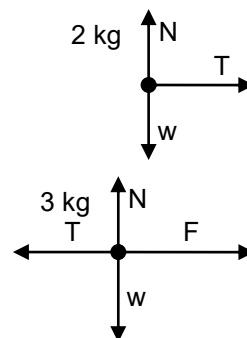
Problem 6: Two bodies, joined by a light, inextensible rope, are moving along a flat, horizontal surface **without** friction.

Two blocks, masses 2 kg and 3 kg, are joined by a light, inelastic rope and move on a smooth, horizontal surface under the influence of a horizontal force F of 42 N as shown. Calculate the tension in the rope.



Analyse the question and draw free-body diagrams:

- Both objects experience the same acceleration in the same direction.
- The tension in the rope is considered to be the same everywhere in the rope. Hence, the magnitude of the force exerted by the rope on the 2 kg block (to the right) is equal to the magnitude of the force exerted by the rope on the 3 kg block (to the left). Note that the two forces are opposite in direction. Therefore, when you substitute the forces (tension), they should get opposite signs.
- Newton's second law of motion must be applied to **each block separately**.
- Vector addition is applied as usual to each block. However, and this is **very important**, in this case you have **two unknown variables** (tension and acceleration). But you have **two objects**; hence, you have **two equations**. In such a case, you apply the mathematics for the solution of **simultaneous equations**. Treat the **unknown variables like numbers** and **substitute those variables with their correct signs** for direction.



Variables:

$$m = 2 \text{ kg \& } 3 \text{ kg}$$

$$F = 42 \text{ N} \rightarrow$$

Solution:

For the 2 kg block; take to the right as positive:

$$F_{\text{net}} = ma \quad [\text{Basic formula}]$$

$$T = ma \quad [T \text{ is the net force}]$$

$$T = 2a \quad [\text{Substitution; } T \text{ and } a \text{ with positive signs}]$$

Solve the simultaneous equations:

$$42 = 5a \quad [\text{Eliminate } T.]$$

$$a = 8,4 \text{ m} \cdot \text{s}^{-2} \quad [\text{Numerical answer for acceleration}]$$

For the 3 kg block; take to the right as positive:

$$F_{\text{net}} = ma \quad [\text{Basic formula}]$$

$$F + T = ma \quad [\text{Vector sum for the net force}]$$

$$42 - T = 3a \quad [\text{Substitution; } T \text{ negative, 42 and } a \text{ positive}]$$

Substitute a into one of the previous equations to solve for T :

$$T = 2a$$

$$T = (2)(8,4) \quad [\text{Substitution}]$$

$$T = 16,8 \text{ N} \quad [\text{Numerical answer}]$$

OR

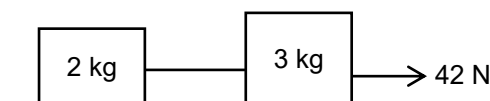
$$42 - T = 3a$$

$$42 - T = (3)(8,4) \quad [\text{Substitution}]$$

$$T = 16,8 \text{ N} \quad [\text{Numerical answer}]$$

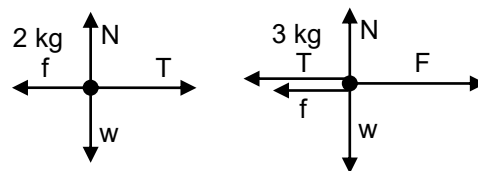
Problem 7: Two bodies, joined by a light, inextensible rope, are moving along a flat, horizontal surface **with** friction.

Two blocks, masses 2 kg and 3 kg, are joined by a light, inelastic rope and move on a rough, horizontal surface under the influence of a horizontal force F of 42 N as shown. The coefficients of kinetic friction between the surface and the blocks are 0,1 and 0,15 respectively. Calculate the tension in the rope.



Analyse the question and draw free-body diagrams:

- Follow the same strategies as in problem 6, except that friction is present now.
- Different frictional forces act on the two blocks. Therefore, calculate each frictional force separately.



Variables:

$m = 2 \text{ kg}$ & $\mu = 0,1$
 $m = 3 \text{ kg}$ & $\mu = 0,15$
 $F = 42 \text{ N} \rightarrow$

Solution:

Frictional force on 2 kg block:

$$\begin{aligned} f &= \mu N \\ &= (0,1)(2)(9,8) \\ &= 1,96 \text{ N} \end{aligned}$$

Frictional force on 3 kg block:

$$\begin{aligned} f &= \mu N \\ &= (0,15)(3)(9,8) \\ &= 4,41 \text{ N} \end{aligned}$$

For the 2 kg block; take to the right as positive:

$$\begin{aligned} F_{\text{net}} &= ma && \text{[Basic formula]} \\ T + f &= ma && \text{[Vector sum for net force]} \\ T - 1,96 &= 2a && \text{[Substitution; } f \text{ negative, } T \text{ and } a \text{ positive]} \end{aligned}$$

For the 3 kg block; take to the right as positive:

$$\begin{aligned} F_{\text{net}} &= ma && \text{[Basic formula]} \\ F + T + f &= ma && \text{[Vector sum for net force]} \\ 42 - T - 4,41 &= 3a && \text{[Substitution; } f \text{ and } T \text{ negative, } 42 \text{ and } a \text{ positive]} \end{aligned}$$

Solve the simultaneous equations:

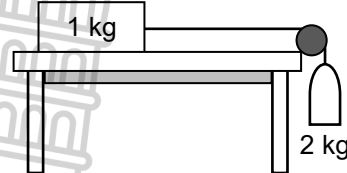
$$\begin{aligned} -1,96 + 42 - 4,41 &= 5a && \text{[Eliminate } T.] \\ a &= 7,126 \text{ m} \cdot \text{s}^{-2} && \text{[Numerical answer for acceleration]} \end{aligned}$$

Substitute a into one of the previous equations to solve for T :

$$\begin{aligned} T - 1,96 &= 2a && 42 - T - 4,41 = 3a \\ T - 1,96 &= (2)(7,126) && \text{[Substitution]} \quad \text{OR} \quad 42 - T - 4,41 = (3)(7,126) && \text{[Substitution]} \\ T &= 16,21 \text{ N} && \text{[Numerical answer]} \quad T = 16,21 \text{ N} && \text{[Numerical answer]} \end{aligned}$$

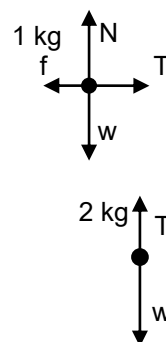
Problem 8: Two bodies are joined by a light, inextensible rope. One moves on a horizontal plane **with** friction, and the second one is suspended vertically from a rope over a light, frictionless pulley.

A 1 kg mass piece on a rough, horizontal surface is connected to a 2 kg mass piece by a light, inextensible rope running over a frictionless pulley. When the 1 kg mass piece moves, the coefficient of kinetic friction between the mass piece and the surface is 0,13. Calculate the tension in the rope.



Analyse the question and draw free-body diagrams:

- The magnitude of the acceleration is the same for both mass pieces. However, the directions of the two accelerations are not the same due to the setup with the pulley. The 1 kg mass piece accelerates to the right and the 2 kg mass piece accelerates downwards. In choosing directions, you may refer to right as positive (1 kg) and downward as positive (2 kg). Even better is to say you "take the direction of motion as positive". This phrase covers both mass pieces.
- Only the 1 kg mass piece experiences a frictional force.
- As in problem 6, the tension in the rope is considered to be the same everywhere in the rope. Hence, the magnitude of the force exerted by the rope on the 1 kg mass piece (to the right) is equal to the magnitude of the force exerted by the rope on the 2 kg mass piece (upwards), but the directions are not the same. As before, when you substitute the forces (tension), they should get opposite signs.



- Apply Newton's second law of motion separately to each mass piece and solve the simultaneous equations as done before, but with the variables applicable to this problem.

Variables:

$$m = 1 \text{ kg} \text{ \& } \mu = 0,13$$

$$m = 2 \text{ kg}$$

$$W_2 \text{ kg} = (2)(9,8) \downarrow$$

Solution:

Frictional force on the 1 kg mass piece:

$$\begin{aligned} f &= \mu N \\ &= (0,13)(1)(9,8) \\ &= 1,274 \text{ N} \end{aligned}$$

For the 1 kg object; direction of motion is positive: For the 1 kg object; direction of motion is positive:

$$\begin{aligned} F_{\text{net}} &= ma && \text{[Basic formula]} \\ T + f &= ma && \text{[Vector sum for net force]} \\ T - 1,274 &= 1a && \text{[Substitution; } f \text{ negative, } T \text{ and } a \text{ positive]} \end{aligned}$$

$$\begin{aligned} F_{\text{net}} &= ma && \text{[Basic formula]} \\ w + T &= ma && \text{[Vector sum for net force]} \\ (2)(9,8) - T &= 2a && \text{[Substitution; } w \text{ and } a \text{ positive, } T \text{ negative]} \end{aligned}$$

Solve the simultaneous equations:

$$\begin{aligned} -1,274 + (2)(9,8) &= 3a && \text{[Eliminate } T.] \\ a &= 6,109 \text{ m} \cdot \text{s}^{-2} && \text{[Numerical answer for acceleration]} \end{aligned}$$

Substitute a into one of the previous equations to solve for T :

$$\begin{aligned} T - 1,274 &= 1a && (2)(9,8) - T = 2a \\ T - 1,274 &= (1)(6,109) && \text{[Substitution]} \quad \text{OR} \quad 19,6 - T = (2)(6,109) && \text{[Substitution]} \\ T &= 7,38 \text{ N} && \text{[Numerical answer]} \quad T = 7,38 \text{ N} && \text{[Numerical answer]} \end{aligned}$$

Problem 9: Two bodies are joined by a light, inextensible rope. One moves on a horizontal plane **without** friction, and the second one is suspended vertically from a rope over a light, frictionless pulley.

A 1 kg mass piece on a smooth, horizontal surface is connected to a 2 kg mass piece by a light, inextensible rope running over a frictionless pulley. Calculate the tension in the rope.

Analyse the question and draw free-body diagrams:

- The same arguments of problem 8 are valid, except that the 1 kg mass piece does not experience friction. Hence, this is less complicated.
- The free-body diagram for the 1 kg mass piece does not include friction.

Variables:

$$m = 1 \text{ kg}$$

$$m = 2 \text{ kg}$$

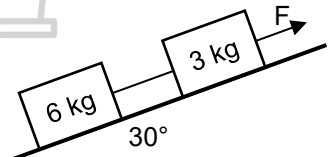
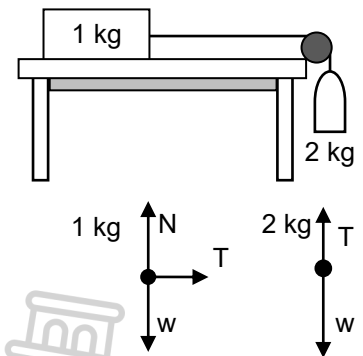
Solution:

Refer to the solution of problem 8. The only difference is that friction is not included in the equation for $F_{\text{net}} = ma$ for the 1 kg mass piece. The answers for acceleration and tension are $6,53 \text{ m} \cdot \text{s}^{-2}$ and $6,53 \text{ N}$ respectively.

Problem 10: Two bodies, joined by a light, inextensible rope, are moving on an inclined plane **with** friction.

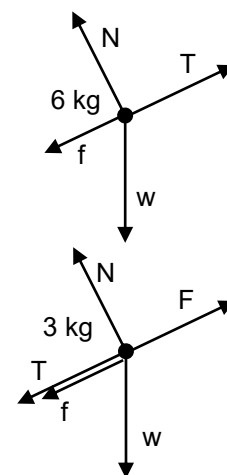
Two objects, masses 6 kg and 3 kg, are connected by a light, inelastic rope. The objects are pulled upwards along an inclined plane, which makes an angle of 30° with the horizontal, by a force F , which is parallel to the inclined plane. The coefficients of kinetic friction for the objects are 0,2 and 0,1 respectively. Calculate:

- the tension in the string if the objects accelerate up the inclined plane at $4 \text{ m} \cdot \text{s}^{-2}$; and
- force F if the objects move up the inclined plane at CONSTANT VELOCITY.



Analyse the question and draw free-body diagrams:

- The two objects experience the same acceleration in Q10.1. In Q10.2 the acceleration for the two objects is also the same, but different from Q10.1.
- Only the forces or components of forces parallel to the inclined plane influence motion parallel to the inclined plane. In the case of the 6 kg object, these are T , f and the component of weight parallel to the inclined plane (w_{par}). In the case of the 3 kg object these are F , T , f and w_{par} .
- To calculate N , which you need to calculate f , the component of weight perpendicular to the inclined plane ($w_{\perp}$) is used.
- Newton's second law of motion is applied separately to each object as before.



For Q10.1:

- Note that you have enough information to calculate T , which is the only unknown variable in the case of the 6 kg object. Two equations with two unknowns each are usually obtained and solved for connected objects, but here the acceleration is known, and simultaneous equations are not needed.
- Be on the lookout for cases like this where something about connected objects can be calculated without using simultaneous equations.

For Q10.2:

- The objects move up the incline at constant velocity. This means that the acceleration is zero, which also means the net force acting on the system is zero.
- Note that the tension in the string in this case is different from the tension in Q10.1 where the acceleration is not zero.
- Once again, if acceleration is known, you can calculate T for the 6 kg object and use the same value for the 3 kg object. Remember that the forces exerted by the rope on the two objects are equal but opposite in direction.
- The frictional force on the 6 kg object, calculated in Q10.1, can be used in Q10.2. Friction is not influenced by the change in acceleration from Q10.1 to Q10.2.

Variables:

For Q10.1

$$\begin{aligned} m &= 6 \text{ kg} \text{ \& } \mu = 0,2 \\ m &= 3 \text{ kg} \text{ \& } \mu = 0,1 \\ a &= 4 \text{ m}\cdot\text{s}^{-2} \end{aligned}$$

For Q10.2

$$\begin{aligned} m &= 6 \text{ kg} \text{ \& } \mu = 0,2 \\ m &= 3 \text{ kg} \text{ \& } \mu = 0,1 \\ a &= 0 \text{ m}\cdot\text{s}^{-2} \end{aligned}$$

Solution Q10.1:

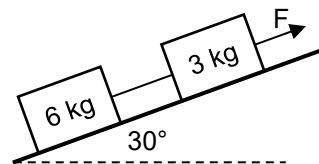
Frictional force on the 6 kg object: $f = \mu N$ $= (0,2)(6)(9,8)(\cos 30^\circ)$ $= 10,184 \text{ N}$	For the 6 kg object; up along incline is positive: $F_{\text{net}} = ma$ [Basic formula] $T + f + w_{\text{par}} = ma$ [Vector sum for net force] $T - 10,184 - (6)(9,8)(\sin 30^\circ) = (6)(4)$ [Substitution] $T = 63,584 \text{ N}$ [Numerical answer]
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Solution Q10.2:

For the 6 kg object; up along incline is positive: $F_{\text{net}} = ma$ [Basic formula] $T + f + w_{\text{par}} = ma$ [Vector sum for net force] $T - 10,184 - (6)(9,8)(\sin 30^\circ) = (6)(0)$ [Substitution] $T = 39,584 \text{ N}$ [Numerical answer]	
Frictional force on the 3 kg object: $f = \mu N$ $= (0,1)(3)(9,8)(\cos 30^\circ)$ $= 2,546 \text{ N}$	For the 3 kg object; up along incline is positive: $F_{\text{net}} = ma$ [Basic formula] $F + T + f + w_{\text{par}} = ma$ [Vector sum for net force] $F - 39,584 - 2,546 - (3)(9,8)(\sin 30^\circ) = (3)(0)$ [Substitution] $F = 56,83 \text{ N}$ [Numerical answer] $\therefore F = 56,83 \text{ N; up along incline}$ [Interpretation]

Problem 11: Two bodies, joined by a light, inextensible rope, are moving on an inclined plane **without** friction.

Two objects, masses 6 kg and 3 kg, are connected by a light, inelastic rope. The objects are pulled upwards along a smooth inclined plane, which makes an angle of 30° with the horizontal, by a force F , which is parallel to the inclined plane.



Calculate:

- 11.1 the tension in the string if the objects accelerate up the inclined plane at $4 \text{ m}\cdot\text{s}^{-2}$; and
 11.2 force F if the objects move up the inclined plane at CONSTANT VELOCITY.

Analyse the question and draw free-body diagrams:

- The same arguments of problem 10 are valid, except that the objects do not experience friction. Hence, this is less complicated.
- The free-body diagrams do not include friction.

Variables:

For Q11.1

$$m = 6 \text{ kg}$$

$$m = 3 \text{ kg}$$

$$a = 4 \text{ m}\cdot\text{s}^{-2}$$

For Q11.2

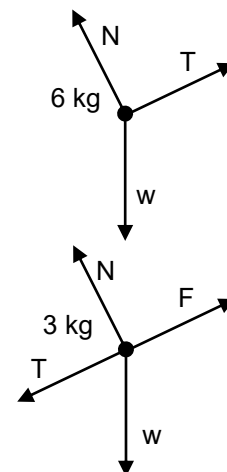
$$m = 6 \text{ kg}$$

$$m = 3 \text{ kg}$$

$$a = 0 \text{ m}\cdot\text{s}^{-2}$$

Solution:

Refer to the solution of problem 10. The only difference is that friction is not included in the equations for $F_{\text{net}} = ma$. In Q11.1 the answer for T is 53,4 N. In Q11.2 the answer for the magnitude of F is 44,1 N.



Problem 12: Two bodies, joined by a light, inextensible string, are suspended vertically from a rope over a frictionless pulley of whom the mass can be ignored.

Two objects, masses of 2 kg and 4 kg, are suspended from a light, inelastic rope over a frictionless pulley. Ignore the mass of the pulley. Calculate the:

- 12.1 acceleration of the objects; and
 12.2 tension in the rope.

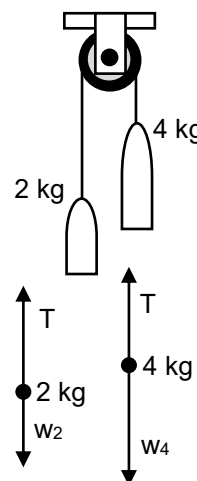
Analyse the question and draw free-body diagrams:

- Because the pulley rotates easily (is frictionless), the tension in the rope is the same on both sides. Because the rope is light and does not stretch, the magnitude of the acceleration is the same for both objects.
- The 2 kg object accelerates upwards; the 4 kg object accelerates downward. In problems like this one, and those in problems 8 and 9, it is convenient to consider the direction of motion as positive.
- The magnitude of the force exerted by the rope on the 2 kg object (upward) is equal to the magnitude of the force exerted by the rope on the 4 kg object (also upward). Note that the two forces act in the same direction, but if the direction of motion is chosen as positive, "upward" in the case of the 2 kg object is positive, while "upward" in the case of the 4 kg object is negative. Therefore, when you substitute the forces (tension), they should get opposite signs.
- Apply Newton's second law of motion separately to each mass piece and solve the simultaneous equations, if any, as done before, but with the variables applicable to this problem.

Variables:

$$m_2 = 2 \text{ kg}$$

$$m_4 = 4 \text{ kg}$$



Solution:

For the 2 kg object; direction of motion is positive:

$$F_{net} = ma$$

[Basic formula]

$$T + w_2 = m_2 a$$

[Vector sum for net force]

$$+T - (2)(9,8) = 2a$$

[Substitution; T and a positive, w_2 negative]

$$T - 19,6 = 2a$$

For the 4 kg object; direction of motion is positive:

$$F_{net} = ma$$

[Basic formula]

$$w_4 + T = m_4 a$$

[Vector sum for net force]

$$(4)(9,8) - T = 4a$$

[Substitution; w_4 and a positive, T negative]

$$39,2 - T = 4a$$

Solve the simultaneous equations:

$$-19,6 + 39,2 = 6a$$

[Eliminate T .]

$$a = 3,267 \text{ m} \cdot \text{s}^{-2}$$

[Numerical answer for acceleration]

Substitute a into one of the previous equations to solve for T :

$$T - 19,6 = 2a$$

$$39,2 - T = 4a$$

$$T - 19,6 = (2)(3,267)$$

[Substitution]

OR

$$39,2 - T = (4)(3,267)$$

[Substitution]

$$T = 26,13 \text{ N}$$

[Numerical answer]

$$T = 26,13 \text{ N}$$

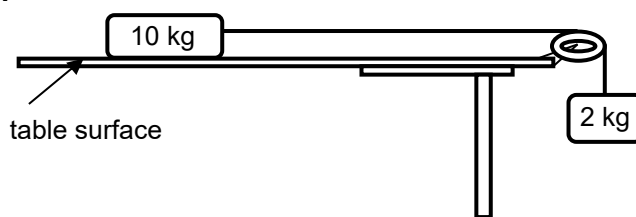
[Numerical answer]



Typical questions

QUESTION 1

The diagram shows a 10 kg block lying on a flat, rough, horizontal surface of a table. The block is connected by a light, inextensible string to a 2 kg block hanging over the side of the table. The string runs over a light, frictionless pulley. The blocks are **stationary**.

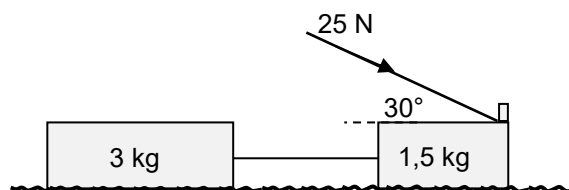


- 1.1 State Newton's FIRST law of motion in words. (2)
 - 1.2 Write down the magnitude of the NET force acting on the 10 kg block. (1)
- When a 15 N force is applied vertically downwards on the 2 kg block, the 10 kg block accelerates to the right at $1,2 \text{ m}\cdot\text{s}^{-2}$.
- 1.3 Draw a free-body diagram for the 2 kg block when the 15 N force is applied to it. (3)
 - 1.4 Calculate the coefficient of kinetic friction between the 10 kg block and the surface of the table. (7)
 - 1.5 How does the value, calculated in QUESTION 1.4, compare with the value of the coefficient of STATIC friction for the 10 kg block and the table? Write down only LARGER THAN, SMALLER THAN or EQUAL TO. (1)
 - 1.6 If the 10 kg block had a larger surface area in contact with the surface of the table, how would this affect the coefficient of kinetic friction calculated in QUESTION 1.4? Assume that the rest of the system remains unchanged. Write down only INCREASES, DECREASES or REMAINS THE SAME. Give a reason for the answer. (2)

[16]

QUESTION 2

A learner constructs a push toy using two blocks with masses 1,5 kg and 3 kg respectively. The blocks are connected by a massless, inextensible cord. The learner then applies a force of 25 N at an angle of 30° to the 1,5 kg block by means of a light rigid rod, causing the toy to move across a flat, rough, horizontal surface, as shown in the diagram.



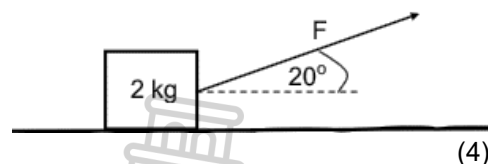
The coefficient of kinetic friction (μ_k) between the surface and each block is 0,15.

- 2.1 State Newton's Second Law of Motion in words. (2)
- 2.2 Calculate the magnitude of the kinetic frictional force acting on the 3 kg block. (3)
- 2.3 Draw a labelled free-body diagram showing ALL the forces acting on the 1,5 kg block. (5)
- 2.4 Calculate the magnitude of the: (3)
- 2.4.1 Kinetic frictional force acting on the 1,5 kg block (5)
- 2.4.2 Tension in the cord connecting the two blocks (5)

[18]

QUESTION 3

- 3.1 A crate of mass 2 kg is being pulled to the right across a rough horizontal surface by constant force F . The force F is applied at an angle of 20° to the horizontal, as shown in the diagram.



- 3.1.1 Draw a labelled free-body diagram showing ALL the forces acting on the crate. (4)

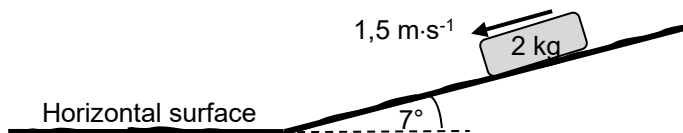
A constant frictional force of 3 N acts between the surface and the crate. The coefficient of kinetic friction between the crate and the surface is 0,2. Calculate the magnitude of the:

- 3.1.2 Normal force acting on the crate (3)
- 3.1.3 Force F (4)
- 3.1.4 Acceleration of the crate (3)
- 3.2 A massive rock from outer space is moving towards the Earth. (2)
- 3.2.1 State Newton's Law of Universal Gravitation in words. (2)
- 3.2.2 How does the magnitude of the gravitational force exerted by the Earth on the rock change as the distance between the rock and the Earth becomes smaller? Choose from INCREASES, DECREASES or REMAINS THE SAME. Give a reason for the answer. (2)

[18]

QUESTION 4

A small object of mass 2 kg is sliding at a constant velocity of $1,5 \text{ m}\cdot\text{s}^{-1}$ down a rough plane inclined at 7° to the horizontal surface. At the bottom of the plane, the object continues sliding onto a rough horizontal surface and eventually comes to a stop. The



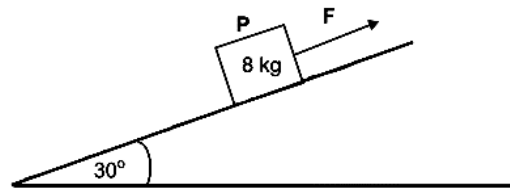
coefficient of kinetic friction between the object and both the inclined and the horizontal surfaces is the same.

- 4.1 Write down the magnitude of the net force acting on the object. (1)
- 4.2 Draw a labelled free-body diagram for the object while it is on the inclined plane. (3)
- 4.3 Calculate the:
 - 4.3.1 Magnitude of the frictional force acting on the object while it is sliding down the inclined plane (3)
 - 4.3.2 Coefficient of kinetic friction between the object and the surfaces (3)
 - 4.3.3 Distance the object travels on the horizontal surface before it comes to a stop (5)

[15]

QUESTION 5

- 5.1 An 8 kg block, **P**, is being pulled by constant force **F** up a rough inclined plane at an angle of 30° to the horizontal, at **CONSTANT SPEED**. Force **F** is parallel to the inclined plane, as shown in the diagram.

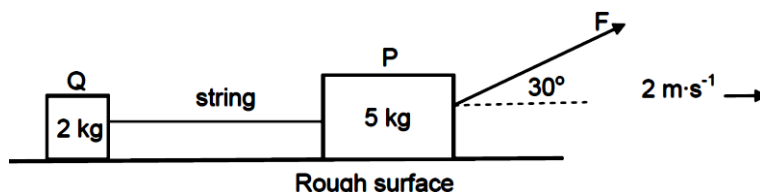


- 5.1.1 State Newton's First Law in words. (2)
- 5.1.2 Draw a labelled free-body diagram for block **P**. (4)
- 5.1.3 The kinetic frictional force between the block and the surface of the inclined plane is 20,37 N. Calculate the magnitude of force **F**. (5)
- 5.1.4 Force **F** is now removed, and the block **ACCELERATES** down the plane. The kinetic frictional force remains 20,37 N. Calculate the magnitude of the acceleration of the block. (4)
- 5.2 A 200 kg rock lies on the surface of a planet. The acceleration due to gravity on the surface of the planet is $6,0 \text{ m}\cdot\text{s}^{-2}$.
 - 5.2.1 State *Newton's Law of Universal Gravitation* in words. (2)
 - 5.2.2 Calculate the mass of the planet if its radius is 700 km. (4)

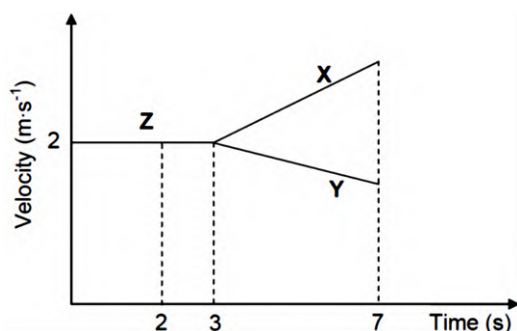
[21]

QUESTION 6

Two boxes, **P** and **Q**, resting on a rough horizontal surface, are connected by a light inextensible string. The boxes have masses 5 kg and 2 kg respectively. A constant force **F**, acting at an angle of 30° to the horizontal, is applied to the 5 kg box, as shown. The two boxes now move to the right at a **constant speed** of $2 \text{ m}\cdot\text{s}^{-1}$.



- 6.1 State Newton's First Law of Motion in words. (2)
- 6.2 Draw a labelled free-body diagram for box **Q**. (4)



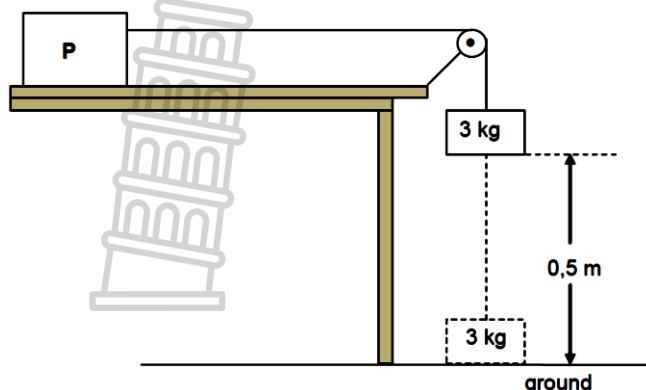
Box **P** experiences a constant frictional force of 5 N and box **Q** a constant frictional force of 3 N.

- 6.3 Calculate the magnitude of force **F**. (6)
- The string connecting **P** and **Q** suddenly breaks after 3 s while force **F** is still being applied. Learners draw the velocity-time graph for the motion of **P** and **Q** before and after the string breaks, as shown alongside.
- 6.4 Write down the time at which the string breaks. (4)
- 6.5 Which portion (**X**, **Y** or **Z**) of the graph represents the motion of box **Q**, after the string breaks? Use the information in the graph to fully support the answer. (4)

[17]

QUESTION 7

Block **P**, of unknown mass, is placed on a rough horizontal surface. It is connected to a second block of mass 3 kg, by a light inextensible string passing over a light, frictionless pulley, as shown. Initially the system of masses is held stationary with the 3 kg block, 0,5 m above the ground. When the system is released the 3 kg block moves vertically downwards and strikes the ground after 3 s. Ignore the effects of air resistance.



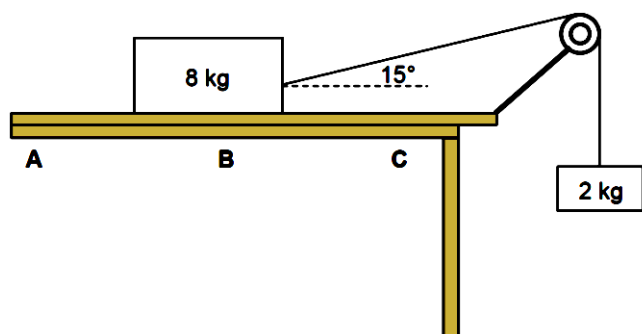
experienced by block **P** is 27 N.

- 7.4 Draw a labelled free-body diagram for block **P**.
- 7.5 Calculate the mass of block **P**.

- 7.1 Define the term *acceleration* in words. (2)
- 7.2 Calculate the magnitude of the *acceleration* of the 3 kg block using equations of motion. (3)
- 7.3 Calculate the magnitude of the tension in the string. (3)

The magnitude of the kinetic frictional force

QUESTION 8



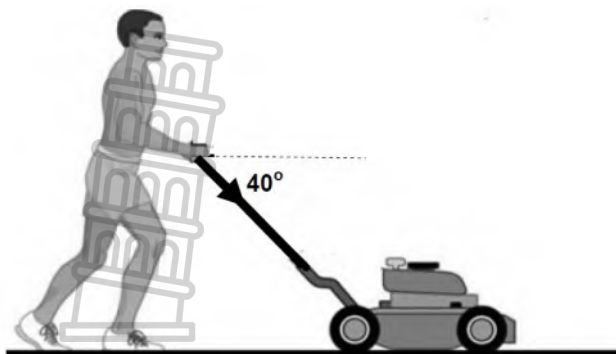
A block, of mass 8 kg, is placed on a rough horizontal surface. The 8 kg block, which is connected to a 2 kg block by means of a light inextensible string passing over a light frictionless pulley, starts sliding from point **A**, as shown.

- 8.1 State Newton's Second Law in words. (2)
- 8.2 Draw a labelled free-body diagram for the 8 kg block. (4)
- 8.3 When the 8 kg block reaches point **B**, the angle between the string and the horizontal is 15° and the acceleration of the system is $1,32 \text{ m}\cdot\text{s}^{-2}$.
 - 8.3.1 Give a reason why the system is NOT in equilibrium. (1)
 - 8.3.2 Use the 2 kg mass to calculate the tension in the string. (3)
 - 8.3.3 Calculate the kinetic frictional force between the 8 kg block and the horizontal surface. (4)
- 8.4 As the 8 kg block moves from **B** to **C**, the kinetic frictional force between the 8 kg block and the horizontal surface is not constant. Give a reason for this statement. (1)
- 8.5 The horizontal surface on which the 8 kg block is moving, is replaced by another horizontal surface made from a different material. Will the kinetic frictional force, calculated in QUESTION 8.3.3 above, change? Choose from: YES or NO. Give a reason for the answer. (2)

[17]



QUESTION 9 (June 2019)



9.1 A person pushes a lawn mower of mass 15 kg at a **constant speed** in a straight line over a flat grass surface with a force of 90 N. The force is directed along the handle of the lawn mower. The handle has been set at an angle of 40° to the horizontal. Refer to the diagram.

- 9.1.1 Draw a labelled free-body diagram for the lawn mower. (4)
- 9.1.2 Why is it CORRECT to say that the moving lawn mower is in equilibrium? (1)
- 9.1.3 Calculate the magnitude of the frictional force acting between the lawn mower and the grass. (3)

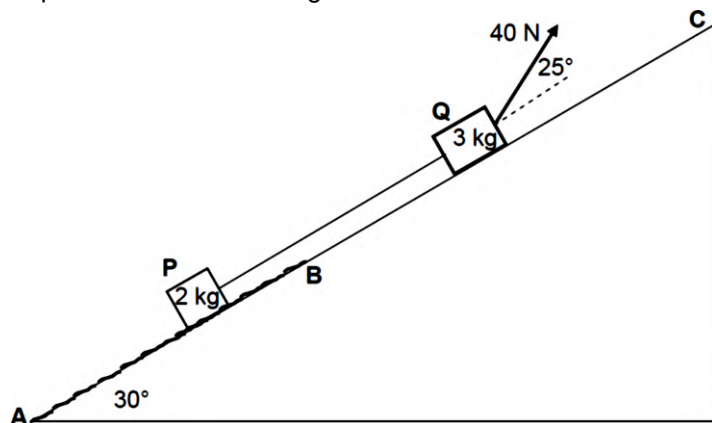
The lawn mower is now brought to a stop.

- 9.1.4 Calculate the magnitude of the constant force that must be applied through the handle in order to accelerate the lawn mower *from rest* to $2 \text{ m} \cdot \text{s}^{-1}$ in a time of 3 s. Assume that the frictional force between the lawn mower and grass remains the same as in QUESTION 9.1.3. (6)
- 9.2 Planet Y has a radius of $6 \times 10^5 \text{ m}$. A 10 kg mass weighs 20 N on the surface of planet Y. Calculate the mass of planet Y. (4)

[18]

QUESTION 10

Block P, of mass 2 kg, is connected to block Q, of mass 3 kg, by a light inextensible string. Both blocks are on a plane inclined at an angle of 30° to the horizontal. Block Q is pulled by a constant force of 40 N at an angle of 25° to the incline. Block P moves on a rough section, AB, of the incline, while block Q moves on a frictionless section, BC, of the incline. See diagram. An average constant frictional force of 2,5 N acts on block P as it moves from A to B up the incline.



- 10.1 State Newton's Second Law in words. (2)
- 10.2 Draw a labelled free-body diagram for block P. (4)
- 10.3 Calculate the magnitude of the acceleration of block P while block P is moving on section AB. (8)
- 10.4 If block P has now passed point B, how will its acceleration compare to that

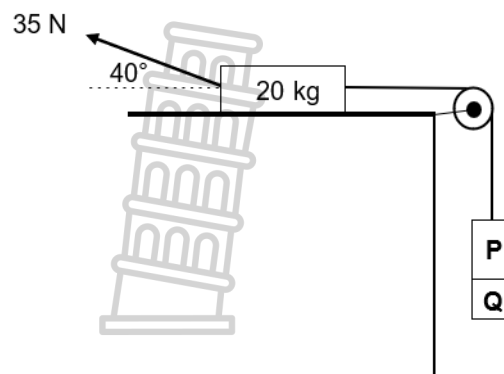
calculated in QUESTION 10.3? Choose from GREATER THAN, SMALLER THAN or EQUAL TO. Give a reason for the answer. (2)

[16]

QUESTION 11

A 20 kg block, resting on a rough horizontal surface, is connected to blocks P and Q by a light inextensible string moving over a frictionless pulley. Blocks P and Q are glued together and have a combined mass of m . A force of 35 N is now applied to the 20 kg block at an angle of 40° with the horizontal, as shown. The 20 kg block experiences a frictional force of magnitude 5 N as it moves to the RIGHT at a CONSTANT SPEED.

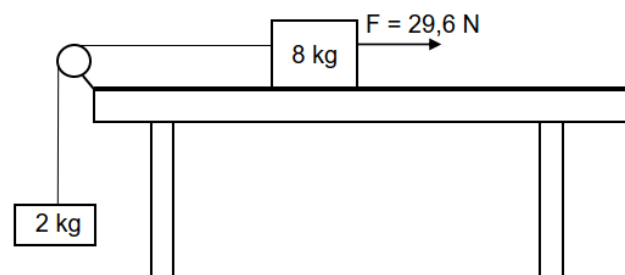
- 11.1 Define the term *normal force*. (2)
- 11.2 Draw a labelled free-body diagram of the 20 kg block. (5)
- 11.3 Calculate the combined mass m of the two blocks. (5)
- 11.4 At a certain stage of the motion, block Q breaks off and falls down. How will EACH of the following be affected when this happens?
- 11.4.1 The tension in the string. Choose from INCREASES, DECREASES or REMAINS THE SAME. (1)
- 11.4.2 The velocity of the 20 kg block. Explain the answer. (3)



[16]

QUESTION 12

An 8-kg block, on a rough horizontal surface, is connected to a 2-kg block by a light inextensible string passing over a frictionless pulley, as shown. The 8-kg block moves at a **constant speed** when pulled by a 29,6 N horizontal force to the right. The frictional force acting on the 8-kg block is 10 N.



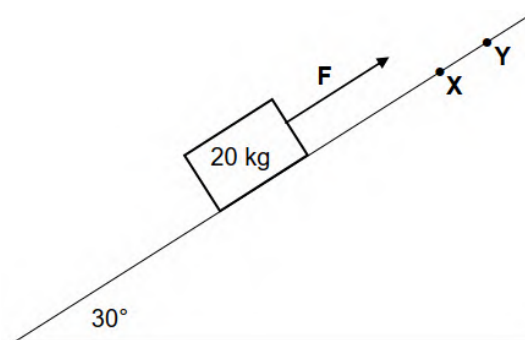
- 12.1 State *Newton's Second Law of Motion* in words. (2)
- 12.2 Draw a labelled free-body diagram for the 8-kg block. (5)
- 12.3 Calculate the tension in the string. (3)
- 12.4 The 29,6 N horizontal force is now increased to 50 N. Apply Newton's Second Law to EACH block and calculate the:
 - 12.4.1 Magnitude of the acceleration of the 8-kg block (5)
 - 12.4.2 Tension in the string (2)

[17]

QUESTION 13

A 20 kg block is placed on a rough surface inclined at 30° to the horizontal. A constant force **F**, acting parallel to the surface, is applied on the block so that the block moves up the incline at a **CONSTANT VELOCITY** of $2 \text{ m}\cdot\text{s}^{-2}$. Refer to the diagram. A constant kinetic frictional force of 18 N acts on the block.

- 13.1 State *Newton's First Law* in words. (2)
- 13.2 Draw a labelled free-body diagram for the block. (4)
- 13.3 Calculate the magnitude of force **F**. (4)



Force **F** is removed when the block reaches point **X** on the surface. The block continues to move up the surface and comes to rest momentarily at point **Y**. Assume that the kinetic frictional force acting on the block remains at 18 N as it moves from point **X** to point **Y**.

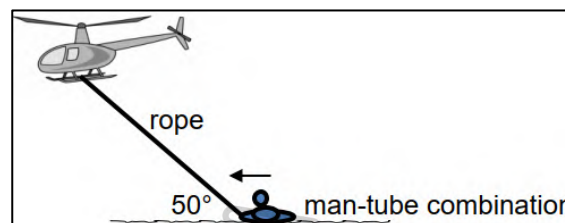
- 13.4 Write down the net force acting on the block as it moves from **X** to **Y**. (2)
- 13.5 Calculate the distance between points **X** and **Y**. (4)

[16]

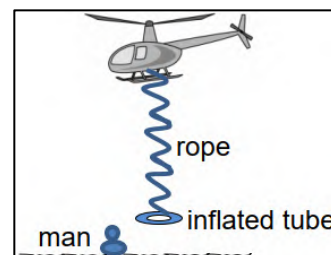
QUESTION 14

A man faces difficulty while swimming in a dam. During the rescue operation, an inflated tube attached to a helicopter by a rope is dropped from the helicopter.

The man, of mass 70 kg, holds onto the inflated tube of mass 4 kg, while the helicopter is flying horizontally at a **CONSTANT** speed. An average frictional force of 300 N is exerted on the man-tube combination while they are dragged horizontally along the surface of the water by the helicopter. The rope makes an angle of 50° with the surface of the water, as shown in the diagram. Assume that the rope is inextensible and massless, and the water of the dam does not flow.



- 14.1 State *Newton's First Law of Motion* in words. (2)
- 14.2 Draw a free-body diagram of the man-tube combination while they are being dragged. (4)
- 14.3 Calculate the tension in the rope. (4)
- 14.4 How will the answer to QUESTION 14.3 change if the helicopter **ACCELERATES** while dragging the man? The frictional force and the angle between the rope and the surface of the water remain the same. Choose from **INCREASES**, **DECREASES** or **NO CHANGE**. Give a reason for the answer. (2)
- 14.5 In another rescue operation, the inflated tube of mass 4 kg is dropped from the stationary helicopter, and it strikes the water at a speed of $16 \text{ m}\cdot\text{s}^{-1}$. The tube sinks vertically downwards into the water to a depth of 0,8 m and then rises to the surface. The rope hangs loosely. Calculate the magnitude of the average upward force exerted on the inflated tube while it is sinking. Assume that the average upward force is constant for the motion. (5)

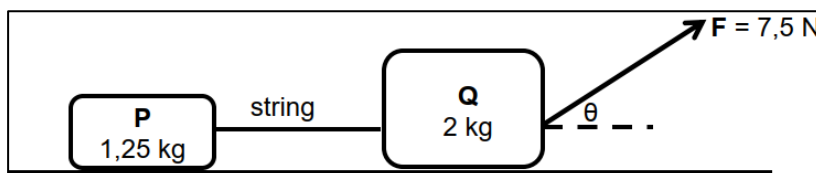


[17]

QUESTION 15

Crate **P** of mass 1,25 kg is connected to another crate, **Q**, of mass 2 kg by a light inextensible string. The two crates are placed on a rough horizontal surface.

A constant force F of magnitude 7,5 N, acting at angle θ to the horizontal, is applied on crate **Q** as shown in the diagram. The crates accelerate at $0,1 \text{ m}\cdot\text{s}^{-2}$ to the right. Crate **P** experiences a constant frictional force of 1,8 N and crate **Q** experiences a constant frictional force of 2,2 N.



15.1 State *Newton's Second Law of Motion* in words. (2)

15.2 Draw a labelled free-body diagram for crate **P**. (4)

15.3 Calculate the magnitude of:

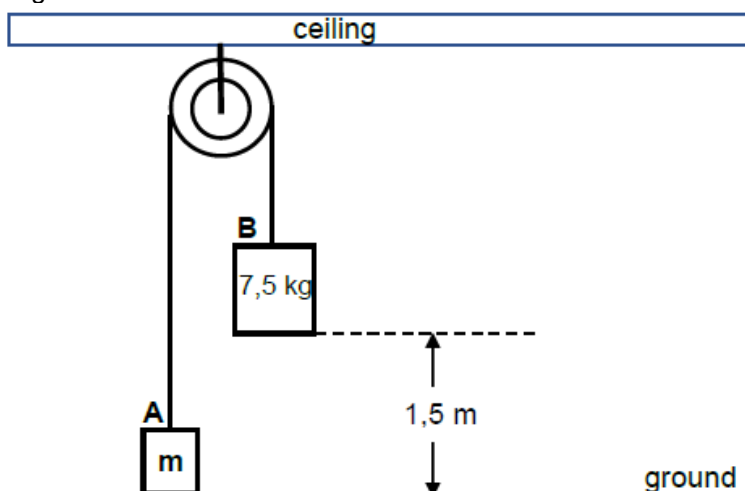
15.3.1 The tension in the string (4)

15.3.2 Angle θ (3)

[13]

QUESTION 16 (June 2023)

Block **A** of mass m is connected to block **B** of mass 7,5 kg by a light inextensible rope passing over a frictionless pulley. Block **B** is initially held at a height of 1,5 m above the ground, while block **A** is initially stationary on the ground, as shown in the diagram below.



When block **B** is released, it moves vertically downwards and strikes the ground with a velocity of $3,41 \text{ m}\cdot\text{s}^{-1}$. Ignore the effects of friction.

16.1 Show, by means of a calculation, that the magnitude of the acceleration of block **B** was $3,88 \text{ m}\cdot\text{s}^{-2}$ while the block was moving vertically downwards. (3)

16.2 Draw a labelled free-body diagram showing ALL the forces acting on block **B** immediately after it was released. (2)

16.3 State *Newton's Second Law of Motion* in words. (2)

16.4 Calculate the value of m by applying *Newton's Second Law* to EACH BLOCK while they are in motion. (5)

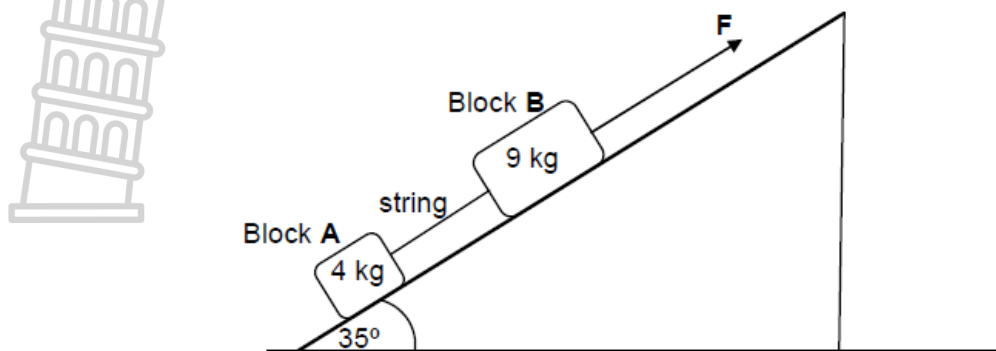
16.5 Calculate the maximum height above the ground reached by block **A**. (5)

[17]



QUESTION 17 (November 2023)

Two blocks, **A**, of mass 4 kg, and **B**, of mass 9 kg, are connected by a light inextensible string. The blocks are held at rest on a plane which is inclined at an angle of 35° with the horizontal. A constant force, **F**, acting parallel to the plane, is applied to block **B**, as shown in the diagram below. The blocks now accelerate up the plane at $2 \text{ m} \cdot \text{s}^{-2}$.

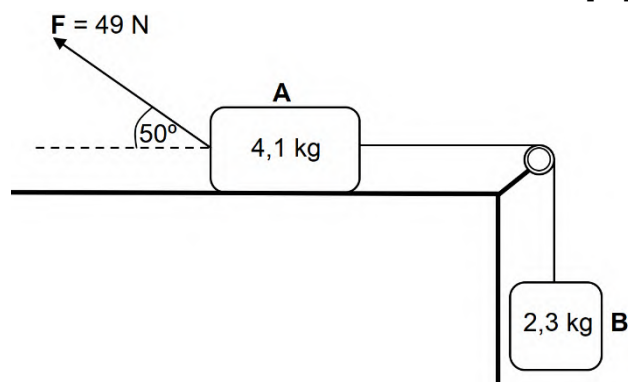


- 17.1 State Newton's Second Law of Motion in words. (2)
- The kinetic frictional forces acting on blocks **A** and **B** are 5,88 N and 13,23 N respectively.
- 17.2 Draw a labelled free-body diagram showing all the forces acting on block **A**. (4)
- 17.3 Calculate the magnitude of:
- 17.3.1 The tension in the string (4)
- 17.3.2 Force **F** (3)
- 17.4 The angle that the plane makes with the horizontal is now decreased.
- 17.4.1 How will this change the kinetic frictional force acting on block **A**? Choose from INCREASES, DECREASES or REMAINS THE SAME. (1)
- 17.4.2 Explain the answer to QUESTION 17.4.1. (2)

[16]

QUESTION 18 (June 2024)

Block **A** of mass 4,1 kg is connected to block **B** of mass 2,3 kg by a light inextensible string passing over a frictionless pulley. Block **A** is at rest on a rough horizontal table and block **B** hangs vertically, as shown in the diagram. A force **F** of magnitude 49 N is applied on block **A** at an angle of 50° to the horizontal, causing block **A** to accelerate TO THE LEFT from rest along the table. The coefficient of kinetic friction between the surface of the table and block **A** is 0,35.

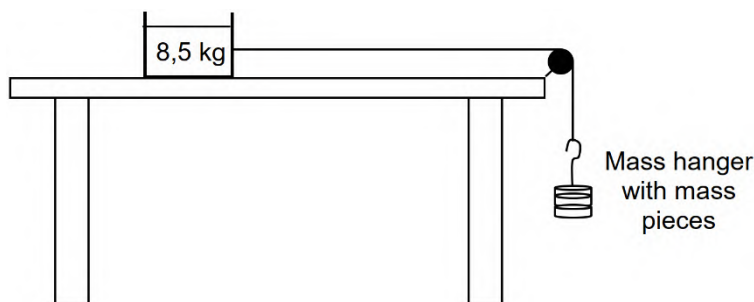


- 18.1 State Newton's Second Law of Motion in words. (2)
- 18.2 Draw a labelled free-body diagram showing all the forces acting on block **A** while it accelerates to the left. (5)
- 18.3 Calculate the magnitude of the:
- 18.3.1 Kinetic frictional force exerted on block **A** (3)
- 18.3.2 Acceleration of block **A**, by applying Newton's Second Law to each block separately (5)

[15]

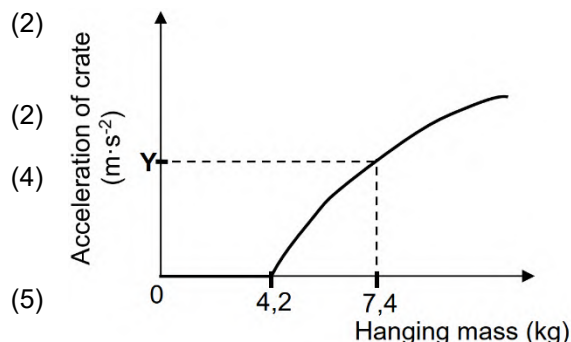
QUESTION 19 (November 2024)

In an experiment, a crate of mass 8,5 kg, lying stationary on a rough horizontal table, is connected to a mass hanger by means of a light inextensible string passing over a frictionless pulley, as shown in the diagram. Mass pieces are added to the mass hanger and the acceleration of the crate is measured. The experiment is repeated several times by adding different masses to increase the hanging mass each time. Ignore the effects of air friction.



The results obtained were used to draw a sketch graph.

- 19.1 Define the term *static friction*. (2)
- 19.2 Draw a labelled free-body diagram showing ALL the HORIZONTAL forces acting on the crate JUST BEFORE it starts moving. (2)
- 19.3 Calculate the: (4)
 - 19.3.1 Coefficient of static friction (μ_s)
 - 19.3.2 Magnitude of the acceleration represented by Y on the graph if the coefficient of kinetic friction between the crate and the table is 0,40
- 19.4 A 5 kg block is now placed inside the crate and the experiment is repeated. How will this affect the maximum static frictional force now experienced by the crate? Choose from INCREASES, DECREASES or REMAINS THE SAME. Give a reason for the answer. (2)



(2)
[15]



VERTICAL MOTION

TERMS AND DEFINITIONS	
1-D motion	One-dimensional motion / Linear motion / Motion in one line.
Acceleration	The rate of change of velocity. Symbol: a Unit: meters per second squared ($\text{m}\cdot\text{s}^{-2}$)
Gravitational acceleration (g)	The acceleration of a body due to the force of attraction of the earth.
Displacement	Change in position. Symbol: Δx (horizontal displacement) or Δy (vertical displacement) Unit: meters (m)
Free fall	Motion during which the only force acting on an object is the gravitational force.
Gravitational force	A force of attraction of one body on another due to their masses.
Position	Where an object is relative to a reference point. Symbol: x (horizontal position) or y (vertical position) Unit: meters (m)
Projectile	An object which has been given an initial velocity and then it moves under the influence of the gravitational force only.
Velocity	The rate of change of position. Symbol: v Unit: meters per second ($\text{m}\cdot\text{s}^{-1}$)

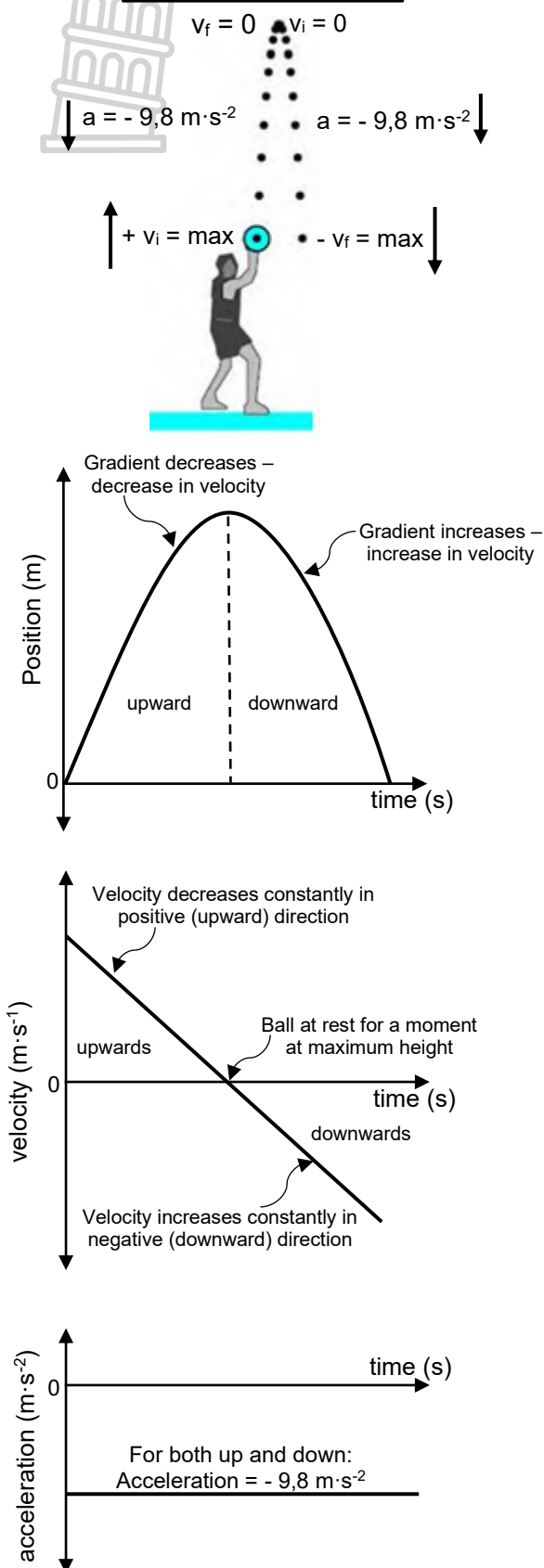
Equations of motion and vertical motion

- The equations of motion for vertical motion:
 - $v_f^2 = v_i^2 + 2a\Delta y$ v_i : initial velocity in $\text{m}\cdot\text{s}^{-1}$
 - $\Delta y = v_i\Delta t + \frac{1}{2}a\Delta t^2$ v_f : final velocity in $\text{m}\cdot\text{s}^{-1}$
 - $v_f = v_i + a\Delta t$ Δy : displacement in m
 - $\Delta y = \left(\frac{v_f + v_i}{2}\right)\Delta t$ Δt : time in s
 - a : acceleration in $\text{m}\cdot\text{s}^{-2}$
- The equations of motion apply to intervals where motion is:
 - Only upward
 - Only downward
 - Both upward and downward
- When solving problems involving equations of motion, it is important to:
 - Write a list of known values and then determine which equation needs to be used to find out the unknown value.
 - Be consistent about which direction you take as positive.
For example, you may choose to take the downward direction as positive and the upward direction as negative. Therefore, all vector values, such as velocity and displacement in the downward direction will be allocated a positive value and vector quantities in the upward direction will be allocated a negative value. You should do this so that the vector nature of the velocities, displacements and acceleration is taken into consideration.
 - An object in freefall accelerates at $9,8 \text{ m}\cdot\text{s}^{-2}$ downward both while it is moving upward and while it is moving downward. Therefore, in freefall, acceleration will always have a value of $+9,8 \text{ m}\cdot\text{s}^{-2}$, whether the object is moving upward or downward if the downward direction is taken as positive.

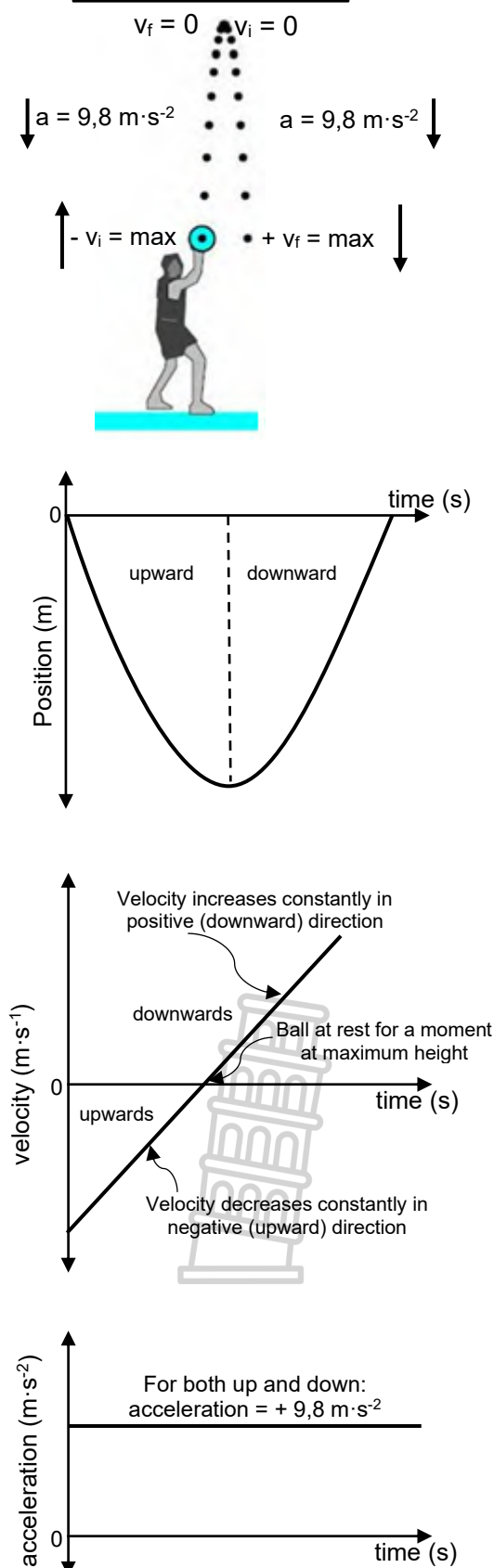
Vertical Motion represented by graphs

A ball is thrown vertically upwards and returns to the same position from where it was thrown.

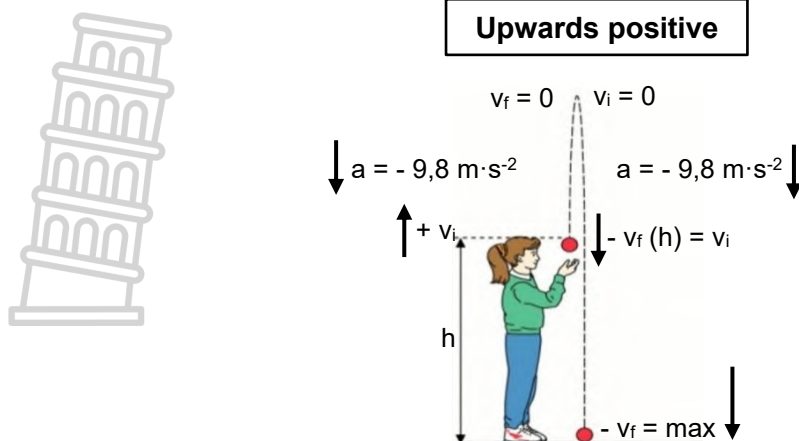
Upwards positive



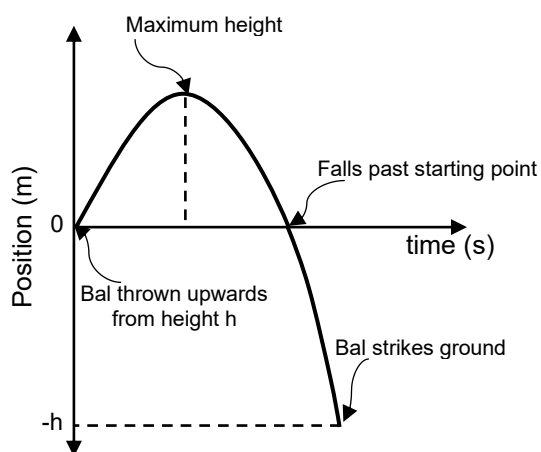
Upwards negative



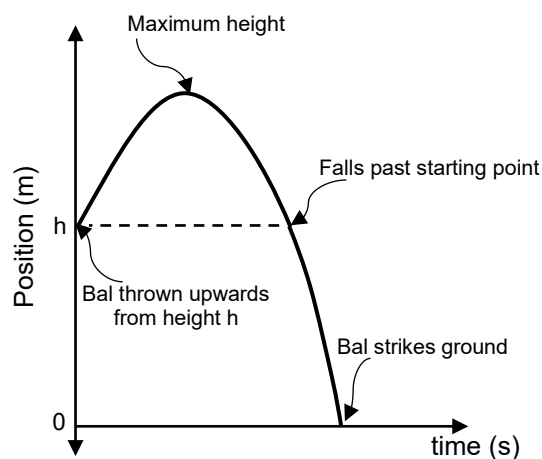
A ball is thrown upwards from a height h and returns to the ground.



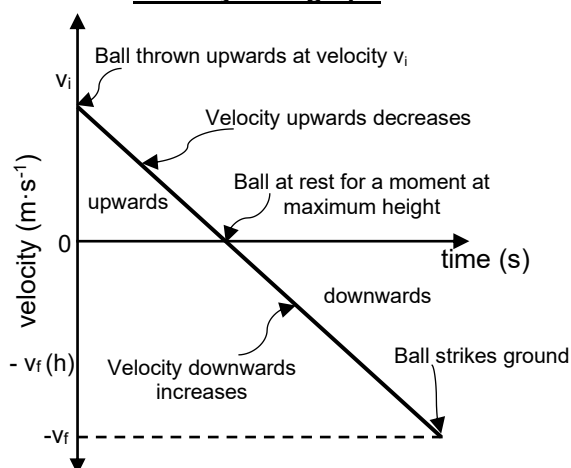
Height h is taken as zero reference
Position-time graph



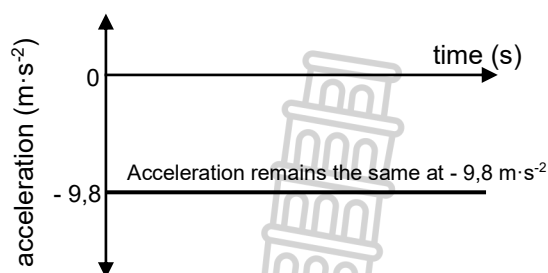
Ground is taken as zero reference
Position-time graph



Velocity-time graph

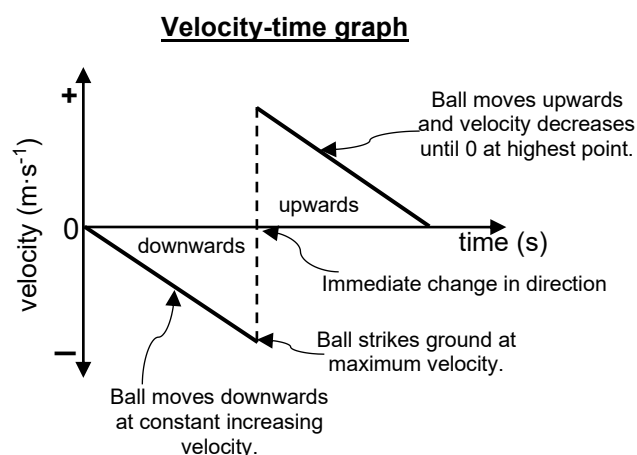
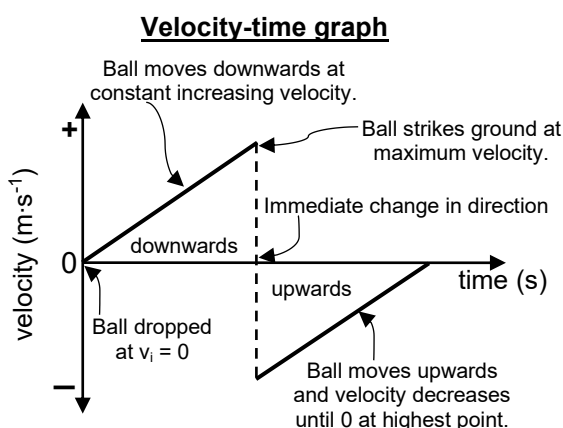
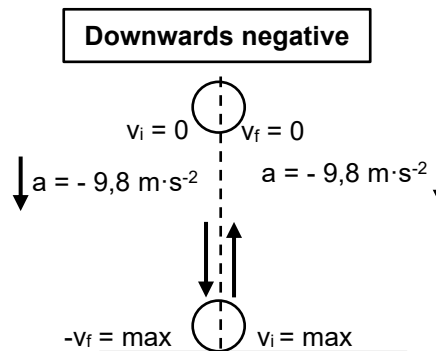
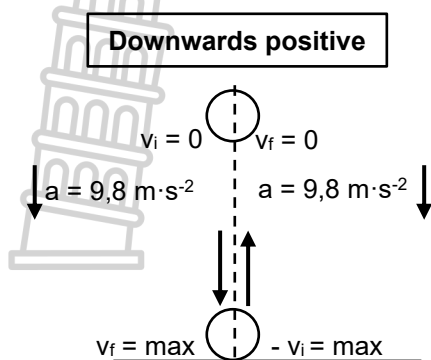


Acceleration-time graph

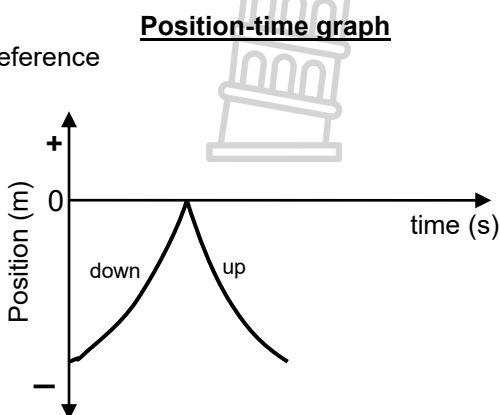
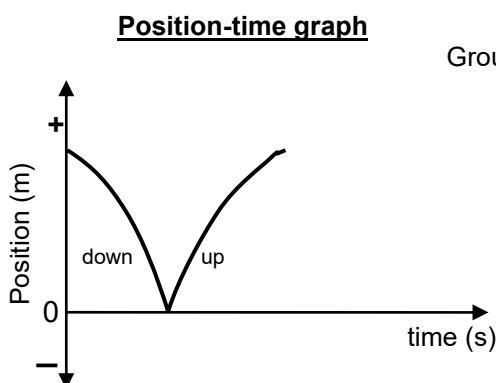
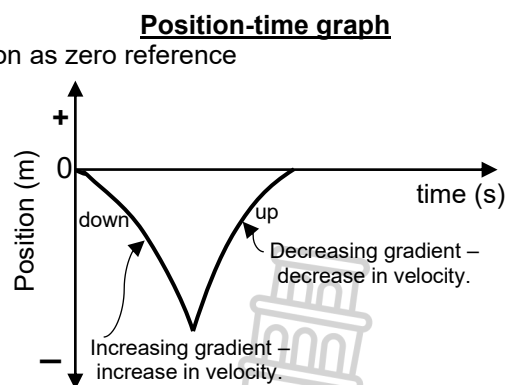
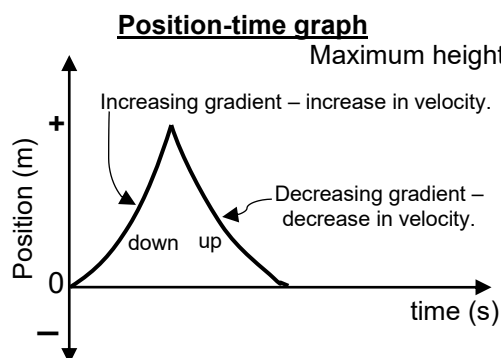


Bouncing ball

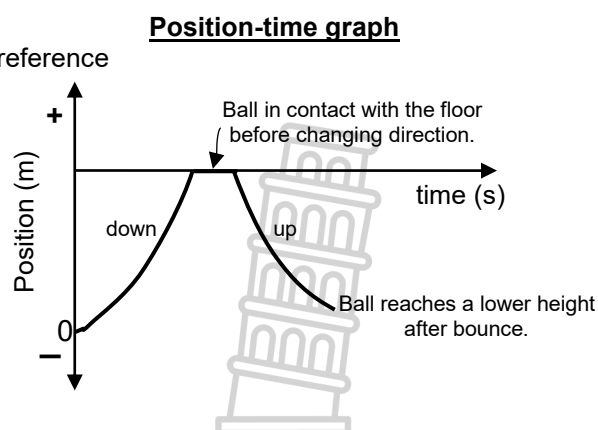
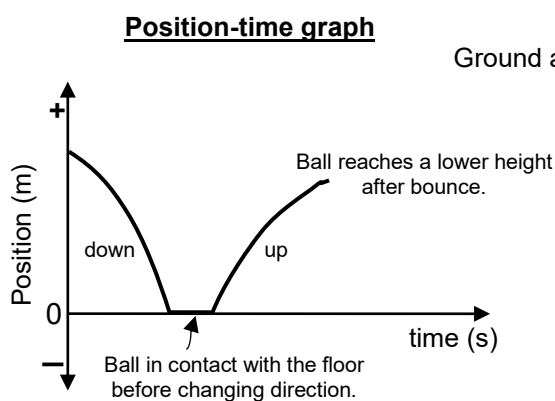
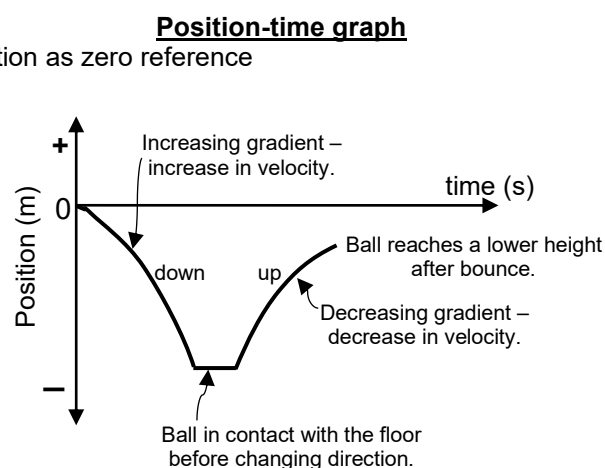
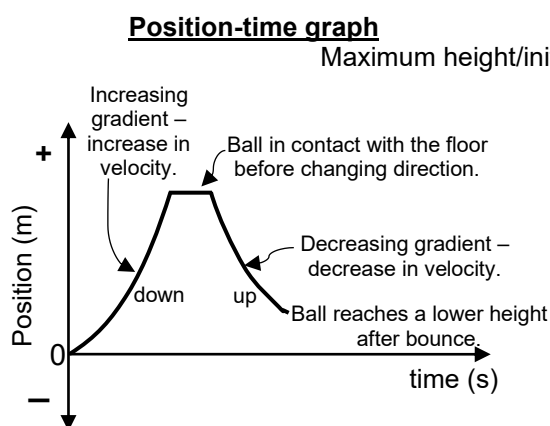
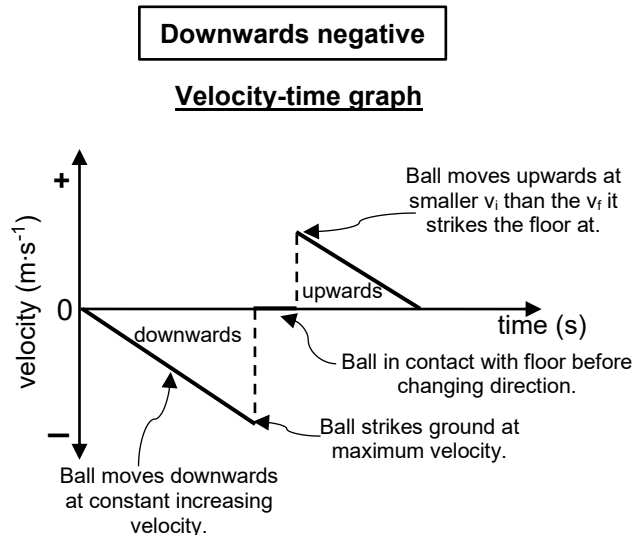
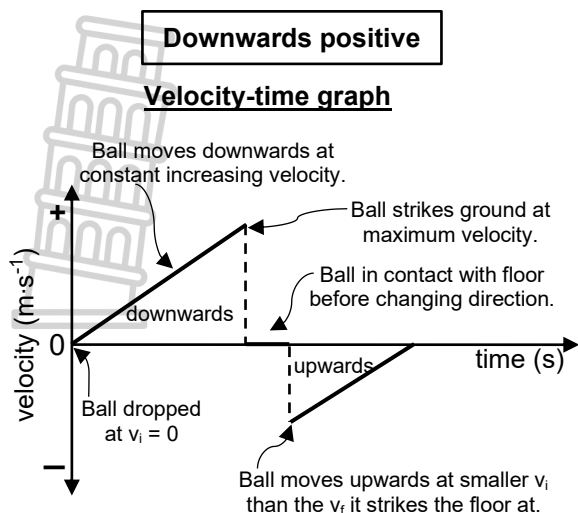
A HARD tennis ball is dropped vertically downwards to the ground and then bounces back.



Gradient of the graphs: Constant acceleration of $9,8 \text{ m} \cdot \text{s}^{-2}$ downwards represented by parallel lines



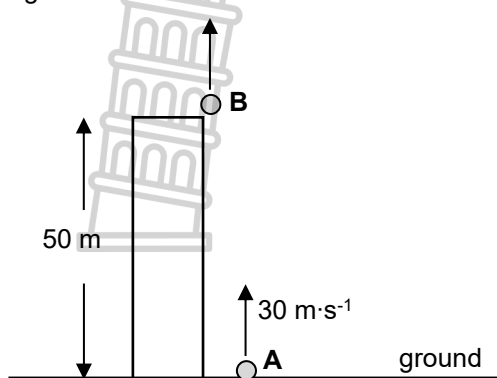
A SOFT tennis ball is dropped vertically downwards to the ground and bounces back.



Typical questions

QUESTION 1

Ball **A** is projected vertically upwards from the ground, near a tall building, with a speed of $30 \text{ m}\cdot\text{s}^{-1}$. Ignore the effects of air friction.

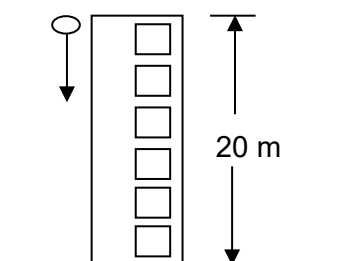


- 1.1 Explain what is meant by a *projectile*. (2)
- 1.2 Calculate the total time that ball **A** will be in the air. (4)
- 1.3 Calculate the distance travelled by ball **A** during the last second of its fall. (4)
- 1.4 TWO SECONDS after ball **A** is projected upwards, ball **B** is projected vertically upwards from the roof of the same building. The roof the building is 50 m above the ground. Both balls **A** and **B** reach the ground *at the same time*. Refer to the diagram. Ignore the effects of air friction. Calculate the speed at which ball **B** was projected upwards from the roof. (4)
- 1.5 Sketch velocity-time graphs for the motion of both balls **A** and **B** on the *same set of axes*. Clearly label the graphs for balls **A** and **B** respectively. Indicate the following on the graphs: (4)

- (a) Time taken by both balls **A** and **B** to reach the ground
- (b) Time taken by ball **A** to reach its maximum height

(4)
[18]

QUESTION 2



A ball is dropped from the top of a building 20 m high. Ignore the effects of air resistance.

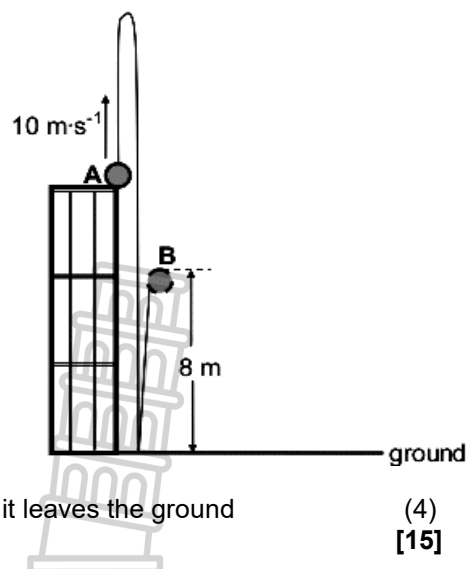
- 2.1 Define the term *free fall*. (4)
- 2.2 Calculate the speed at which the ball hits the ground. (4)
- 2.3 Calculate the time it takes the ball to reach the ground. (3)
- 2.4 Sketch a velocity-time graph for the motion of the ball (no values required). (2)

[11]

QUESTION 3

A ball is projected vertically upwards with a speed of $10 \text{ m}\cdot\text{s}^{-1}$ from point **A**, which is at the top edge of a building. The ball hits the ground after 3 s. It is in contact with the ground for 0,2 s and then bounces vertically upwards, reaching a maximum height of 8 m at point **B**. See the diagram. Ignore the effects of friction.

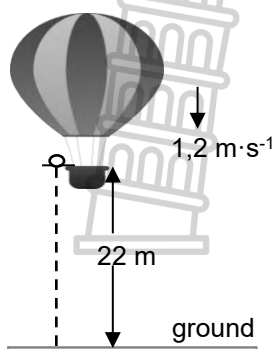
- 3.1 Why is the ball considered to be in free fall during its motion? (2)
- 3.2 Calculate the: (3)
 - 3.2.1 Height of the building (3)
 - 3.2.2 Speed with which the ball hits the ground (3)
 - 3.2.3 Speed with which the ball leaves the ground (3)
- 3.3 Draw a velocity versus time graph for the complete motion of the ball from **A** to **B**. Show the following on the graph: (4)
 - The magnitude of the velocity with which it hits the ground
 - The magnitude of the velocity with which it leaves the ground
 - The time taken to reach the ground, as well as the time at which it leaves the ground



[15]

QUESTION 4

A hot-air balloon moves vertically downwards at a constant velocity of $1,2 \text{ m}\cdot\text{s}^{-1}$. When it reaches a height of 22 m from the ground, a ball is dropped from the balloon. Refer to the diagram.



Assume that the dropping of the ball has no effect on the speed of the hot-air balloon. Ignore air friction for the motion of the ball.

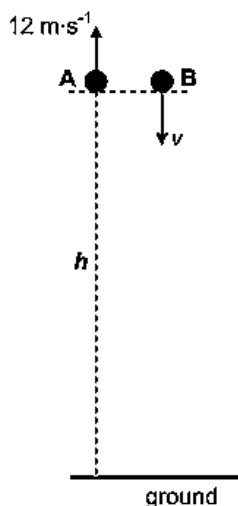
- 4.1 Explain the term projectile motion. (2)
- 4.2 Is the hot-air balloon in free fall? Give a reason for the answer. (2)
- 4.3 Calculate the time it takes for the ball to hit the ground after it is dropped. (4)

When the ball lands on the ground, it is in contact with the ground for 0,3 s and then it bounces vertically upwards with a speed of $15 \text{ m}\cdot\text{s}^{-1}$.

- 4.4 Calculate how high the balloon is from the ground when the ball reaches its maximum height after the first bounce. (6)
- [14]**

QUESTION 5

Stone **A** is projected vertically upwards at a speed of $12 \text{ m}\cdot\text{s}^{-1}$ from a height h above the ground. Ignore the effects of air resistance.



- 5.1 Calculate the time taken for stone **A** to reach its maximum height. (3)

At the same instant that stone **A** is projected upwards, stone **B** is thrown vertically downwards from the same height at an unknown speed, v . Refer to the diagram. When stone **A** reaches its maximum height, the speed of stone **B** is $3v$.

- 5.2 Calculate the speed, v , with which stone **B** is thrown downwards. (4)

At the instant stone **A** passes its initial position on its way down, stone **B** hits the ground.

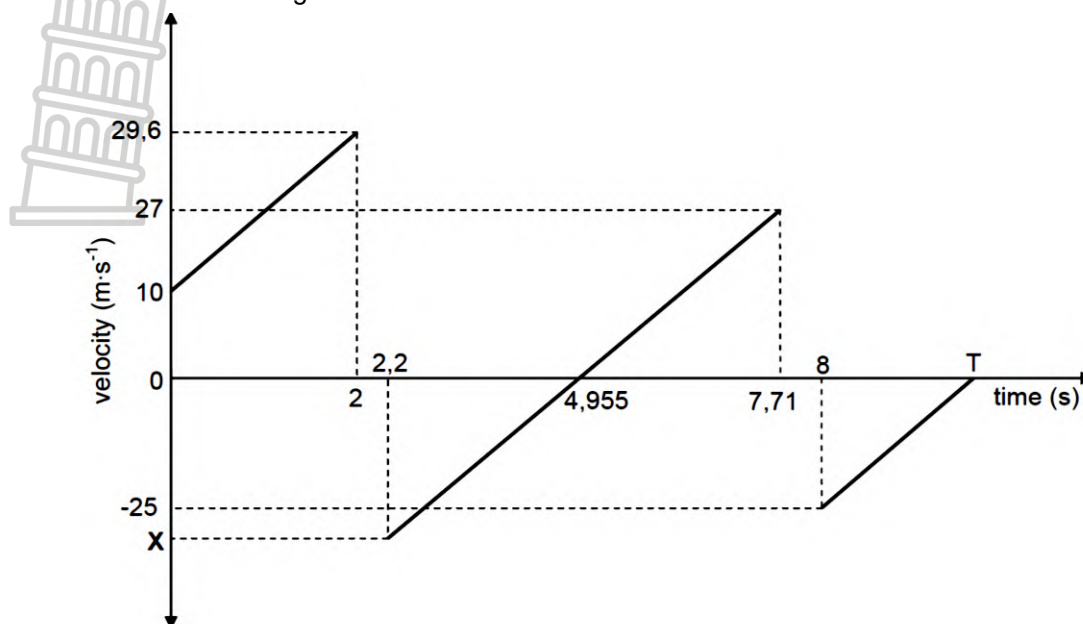
- 5.3 Calculate the height h . (3)

- 5.4 Sketch velocity-time graphs for the complete motions of stones **A** and **B** on the same set of axes. Label your graphs for stones **A** and **B** clearly. Show the time taken for stone **A** to reach its maximum height AND the velocity with which stone **B** is thrown downwards on the graphs. (4)
- [14]**



QUESTION 6

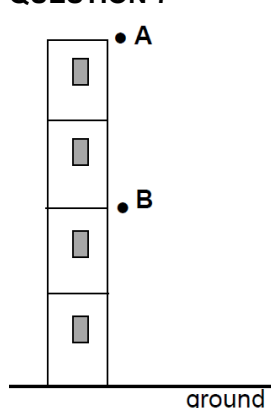
A ball is thrown **vertically downwards** from the top of a building and bounces a few times as it hits the ground. The velocity-time graph below describes the motion of the ball from the time it is thrown, up to a certain time **T**. Take downwards as the positive direction and the ground as zero reference. The graph is NOT drawn to scale. The effects of air friction are ignored.



- 6.1 Write down the speed with which the ball is thrown downwards. (1)
- 6.2 ALL parts of the graph have the same gradient. Give a reason for this. (2)
- 6.3 Calculate the height from which the ball is thrown. (3)
- 6.4 Calculate the time (**T**) shown on the graph. (4)
- 6.5 Write down the:
 - 6.5.1 Time that the ball is in contact with the ground at the first bounce (1)
 - 6.5.2 Time at which the ball reaches its maximum height after the first bounce (2)
 - 6.5.3 Value of **X** (1)
- 6.6 Is the collision of the ball with the ground elastic or inelastic? Give a reason for the answer using information in the graph. (2)

[16]

QUESTION 7



In the diagram shown, point **A** is at the top of a building. Point **B** is exactly **halfway** between the point **A** and the ground. Ignore air resistance.

- 7.1 Define the term *free fall*. (2)

A ball of mass 0,4 kg is dropped from point **A**. It passes point **B** after 1 s.

- 7.2 Calculate the height of point **A** above the ground. (3)

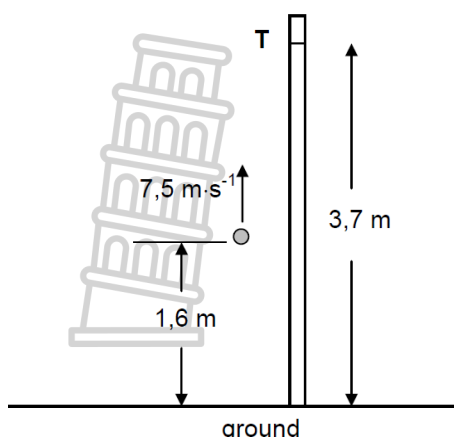
When the ball strikes the ground, it is in contact with the ground for 0,2 s and then bounces vertically upwards, reaching a maximum height at point **B**.

- 7.3 Calculate the magnitude of the velocity of the ball when it strikes the ground. (3)

- 7.4 Calculate the magnitude of the average net force exerted on the ball while it is in contact with the ground. (6)

[14]

QUESTION 8



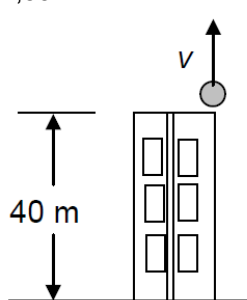
In a competition, participants must attempt to throw a ball vertically upwards past point **T**, marked on a tall vertical pole. Point **T** is 3,7 m above the ground. Point **T** may, or may not, be the highest point during the motion of the ball. One participant throws the ball vertically upwards at a velocity of $7,5 \text{ m} \cdot \text{s}^{-1}$ from a point that is 1,6 m above the ground, as shown in the diagram. Ignore the effects of air resistance.

- 8.1 In which direction is the net force acting on the ball while it moves towards point **T**? Choose from: UPWARDS or DOWNWARDS. Give a reason for the answer. (2)
- 8.2 Calculate the time taken by the ball to reach its highest point. (3)
- 8.3 Determine, by means of a calculation, whether the ball will pass point **T** or not. (6)
- 8.4 Draw a velocity-time graph for the motion of the ball from the instant it is thrown upwards until it reaches its highest point. Indicate the following on the graph:
 - The initial velocity and final velocity
 - Time taken to reach the highest point

[13]

QUESTION 9 (June 2019)

A ball is thrown vertically upwards, with velocity v , from the edge of a roof of a 40 m tall building. The ball takes 1,53 s to reach its maximum height. Ignore air resistance.



- 9.1 Define the term *free fall*. (2)
- 9.2 Calculate the:
 - 9.2.1 Magnitude of the initial velocity v of the ball (3)
 - 9.2.2 Maximum height reached by the ball above the edge of the roof (3)
- 9.3 Take the edge of the roof as reference point. Determine the position of the ball relative to the edge of the roof after 4 s. (3)
- 9.4 Will any of the answers to QUESTIONS 9.2 and 9.3 change if the height of the building is 30 m? Choose from YES or NO. Give a reason for the answer. (3)

[14]

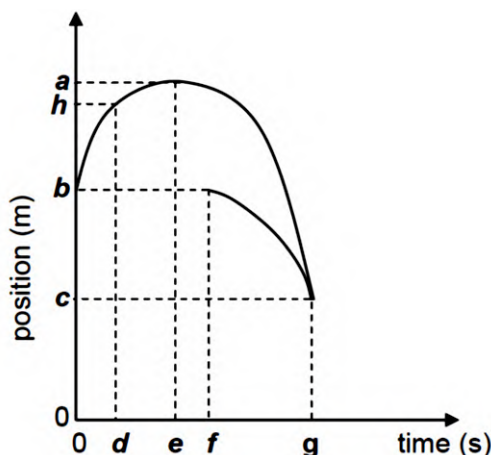
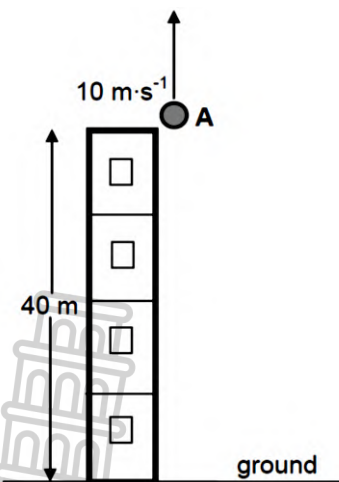
QUESTION 10

Stone **A** is thrown vertically upwards with a speed of $10 \text{ m} \cdot \text{s}^{-1}$ from the edge of the roof of a 40 m high building, as shown in the diagram. Ignore the effects of air friction. Take the ground as reference.

- 10.1 Define the term *free fall*. (2)
- 10.2 Calculate the maximum HEIGHT ABOVE THE GROUND reached by stone **A**. (4)
- 10.3 Write down the magnitude and direction of the acceleration of stone **A** at this maximum height. (2)

Stone **B** is dropped from rest from the edge of the roof, x seconds after stone **A** was thrown upwards.

- 10.4 Stone **A** passes stone **B** when the two stones are 29,74 m above the ground. Calculate the value of x . (6)



- 10.5 The graphs of position versus time for part of the motion of both stones are shown alongside.

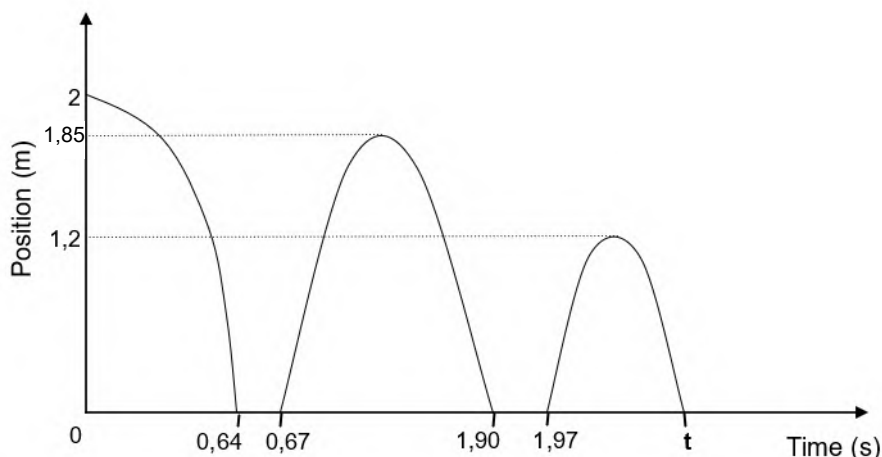
Which of labels **a** to **h** on the graphs above represents EACH of the following?

- 10.5.1 The time at which stone **A** has a positive velocity (1)
- 10.5.2 The maximum height reached by stone **A** (1)
- 10.5.3 The time when stone **B** was dropped (1)
- 10.5.4 The height at which the stones pass each other (1)

[18]

QUESTION 11

A small ball is dropped from a height of 2 m and bounces a few times after landing on a cement floor. Ignore air friction. The position-time graph, not drawn to scale, represents the motion of the ball.



11.1 Define the term *free fall*. (2)

11.2 Use the graph and determine:

11.2.1 The time that the ball is in contact with the floor before the first bounce

11.2.2 The time it takes the ball to reach its maximum height after the first bounce (2)

11.2.3 The speed at which the ball leaves the floor at the first bounce (3)

11.2.4 Time t indicated on the graph (6)

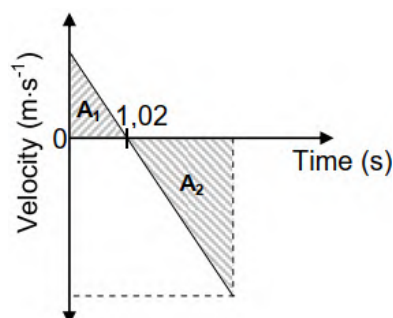
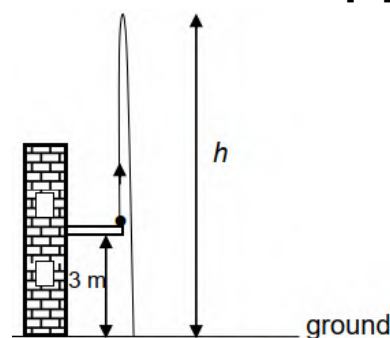
[15]

QUESTION 12

A ball, of mass 0,06 kg, is thrown vertically upwards from the balcony of a building, 3 m above the ground. The ball reaches a maximum height h above the ground, as shown in the diagram. Ignore the effects of air resistance.

12.1 Name the force acting on the ball while it is in free fall. (1)

The velocity-time graph below represents the motion of the ball from the instant it is thrown upwards until it hits the ground.



12.2 Write down the acceleration of the ball at time $t = 1,02$ s. (2)

12.3 Consider the areas A_1 and A_2 shown in the graph above. Write down the numerical value represented by the DIFFERENCE in areas A_1 and A_2 . (1)

12.4 Calculate the:

12.4.1 Speed at which the ball is thrown upwards (3)

12.4.2 Height h (4)

12.5 After hitting the ground, the ball bounces vertically upwards and reaches a new maximum height in 1,1 s. Calculate the work done by the ground on the ball while the ball is in contact with the ground. (6)

[17]

QUESTION 13

A hot-air balloon is moving upwards at a **CONSTANT UNKNOWN** speed.

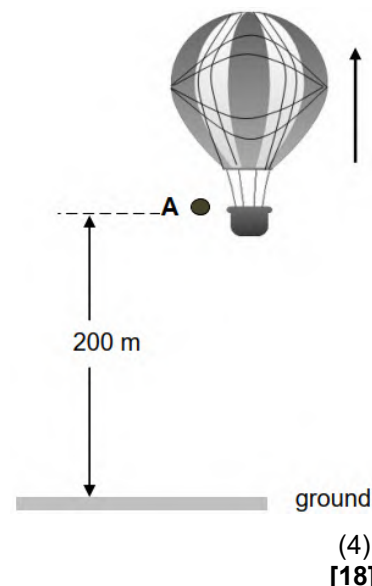
- 13.1 Is the hot air balloon in free fall? Choose from YES or NO. Give a reason for the answer. (2)

When the balloon is 200 m above the ground, a small stone **A** is dropped from the balloon. See the diagram. Another small stone **B** is dropped 5 s later from the balloon while the balloon is still moving upwards at constant velocity. Stone **A** strikes the ground at a speed of $62,68 \text{ m}\cdot\text{s}^{-1}$. Ignore air resistance.

- 13.2 Calculate the:

- 13.2.1 Speed of the hot air balloon (3)
 13.2.2 Time it takes stone **A** to strike the ground (3)
 13.2.3 Distance between the hot-air balloon and stone **B** at the instant when stone **A** strikes the ground (6)

- 13.3 On the same set of axes, draw position-time graphs for both the hot-air balloon and stone **A** from the moment the stone is dropped until it strikes the ground. Use the ground as zero reference. Label your graphs **BALLOON** and **A**. (4)



QUESTION 14

A small disc, **C**, is thrown vertically upwards at a speed of $15 \text{ m}\cdot\text{s}^{-1}$ from the edge of the roof of a building of height 30 m. AFTER 0,5 s, a small ball **B** is shot vertically upwards from the foot of the building at a speed of $40 \text{ m}\cdot\text{s}^{-1}$ in order to hit disc **C**. Ignore the effects of air resistance.

- 14.1 Explain the term projectile.

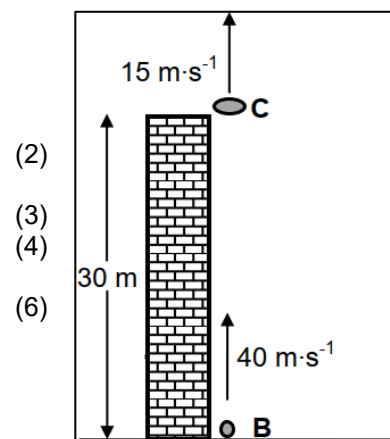
- 14.2 Calculate the:

- 14.2.1 Time taken by disc **C** to reach its maximum height (2)
 14.2.2 Maximum height above the ground reached by disc **C** (3)

- 14.3 Calculate the time from the moment that disc **C** was thrown upwards until the time ball **B** hits the disc. (4)

- 14.4 On the same set of axes, sketch graphs of velocity versus time for disc **C** and ball **B** from the moment that disc **C** was thrown upwards until ball **B** hits disc **C**. Label the graph for ball **B** as **B** and the graph for disc **C** as **C**. Clearly indicate the following on the graphs: (6)

- The initial velocities of ball **B** and disc **C**
- The time at which ball **B** was shot upward
- The time at which disc **C** reaches its maximum height
- The time at which ball **B** hits disc **C**



QUESTION 15

A ball is thrown vertically upwards from the top of a building of height 25 m with a velocity of $12 \text{ m}\cdot\text{s}^{-1}$. On its way down, the ball passes a door which has a height of 1,9 m and then strikes the ground, as shown in the diagram. Ignore the effects of air friction.

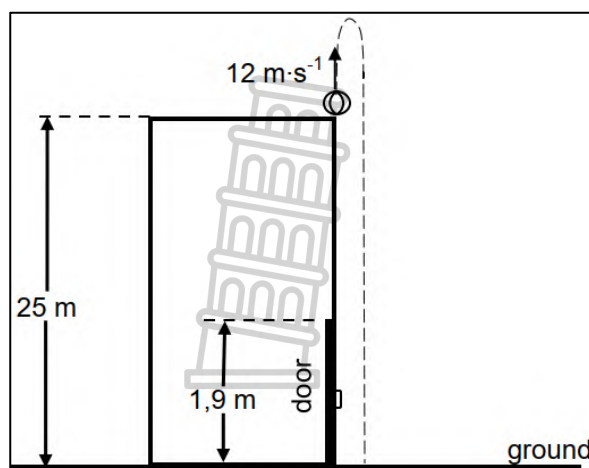
- 15.1 Define the term *free fall*. (2)

- 15.2 Calculate the:

- 15.2.1 Time taken for the ball to reach its maximum height (3)
 15.2.2 Velocity with which the ball strikes the ground (4)
 15.2.3 Time it took the ball to move from the top of the door to the ground (4)

- 15.3 Draw a velocity versus time graph for the motion of the ball from the moment that the ball is thrown upwards until it strikes the ground. Use the ground as zero reference. Clearly indicate the following on your graph:

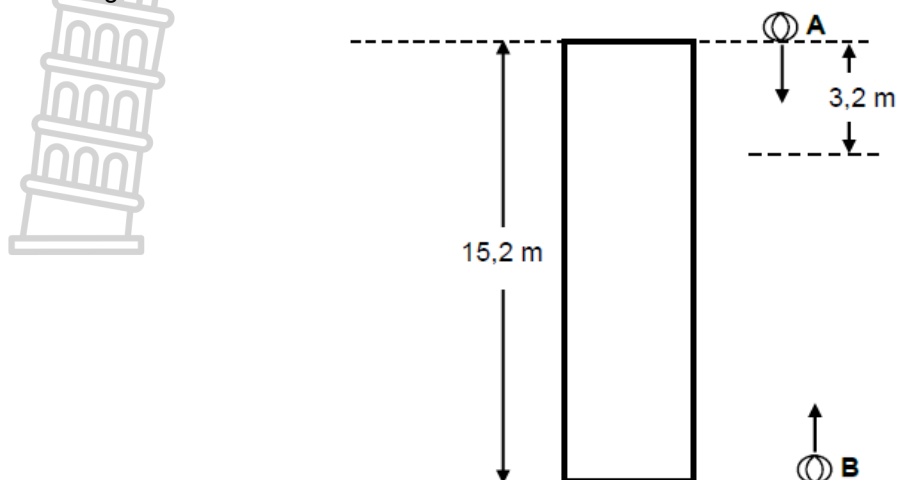
- The velocity with which the ball was thrown upward
- Time taken by the ball to reach its maximum height
- The velocity with which the ball strikes the ground



(3)
[16]

QUESTION 16 (June 2023)

Ball **A** is dropped from rest from the top of a building 15,2 m high. After ball **A** has fallen 3,2 m, a second ball **B** is projected vertically upwards from the ground. After a while, the two balls strike the ground at the SAME time. Ignore the effects of air resistance.

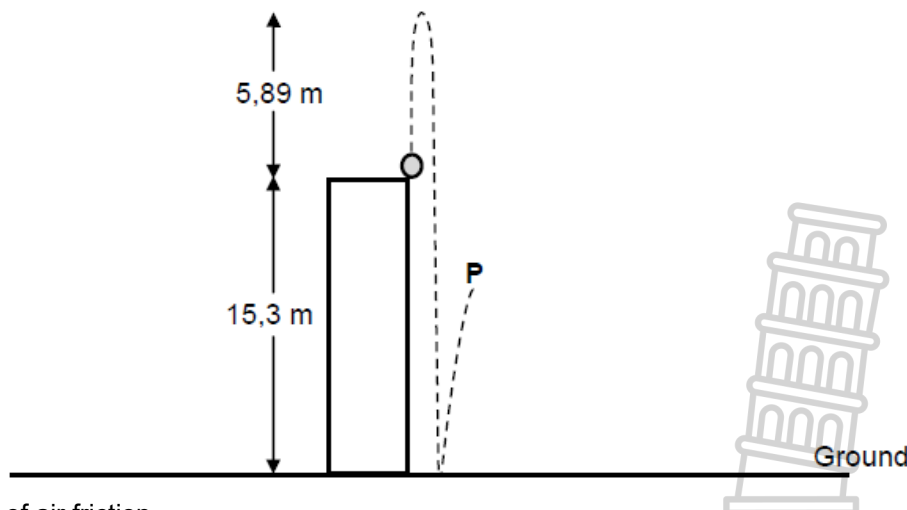


- 16.1 Define the term *free fall*. (2)
- 16.2 Calculate the: (2)
- 16.2.1 Time taken for ball **A** to strike the ground (3)
- 16.2.2 Magnitude of the velocity with which ball **B** was projected from the ground (5)
- 16.3 On the same system of axes, draw position-time graphs to show the motions of both ball **A** and ball **B** from the instant ball **A** is dropped until the time it reaches the ground. Take the ground as the zero position. Label the graphs **A** and **B**. Clearly indicate the following on the graphs: (5)
- The starting time for each ball
 - The initial position of each ball
 - The time when the balls strike the ground

[15]

QUESTION 17 (November 2023)

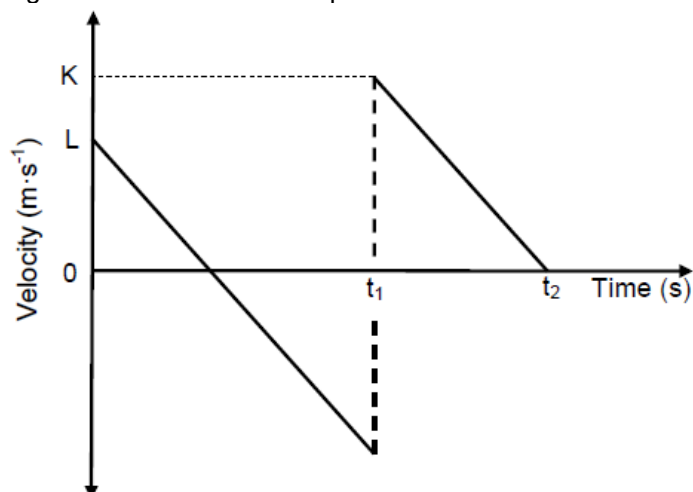
A ball of mass 0,5 kg is thrown vertically upwards from the top edge of a building which is 15,3 m high. The ball reaches a maximum height of 5,89 m above the top of the building. The ball strikes the ground and bounces vertically upwards, reaching a maximum height at point **P**, as shown in the diagram below.



Ignore the effects of air friction.

- 17.1 Define the term *free fall*. (2)
- 17.2 Using only EQUATIONS OF MOTION, calculate the speed at which the ball was projected upwards. (3)
- 17.3 After the collision with the ground, the ball leaves the ground with a speed of 11,92 m·s⁻¹. Calculate the: (3)
- 17.3.1 Amount of kinetic energy lost by the ball during the collision with the ground (5)
- 17.3.2 Time taken for the ball to reach point **P** after leaving the ground (3)

- 17.4 The velocity-time graph for the motion of the ball from the instant it is projected upwards from the top edge of the building until the time it reaches point P is shown below.



Write down the numerical values indicated by EACH of the following:

17.4.1 K

17.4.2 L

17.4.3 $t_2 - t_1$

(1)

(1)

(1)

[16]

QUESTION 18 (June 2024)

A hot-air balloon moves vertically downwards at a constant velocity of $3,4 \text{ m} \cdot \text{s}^{-1}$. When the balloon is 15 m above the ground, a small ball is dropped from the balloon. Refer to the diagram. The ball strikes the ground and bounces vertically upwards. The hot-air balloon continues to move downwards at the same constant velocity. Ignore the effects of air friction acting on the ball.

- 18.1 Define the term *free fall*.

(2)

The sketch graph (not drawn to scale) represents the positions of the ball relative to the ground from the time the ball is dropped until the time it reaches its maximum height after the first bounce.

- 18.2 Was the ball in free fall between t_1 and t_2 seconds?

Write down either YES or NO.

(1)

- 18.3 Use only EQUATIONS OF MOTION to calculate:

18.3.1 The value of t_1 indicated on the graph

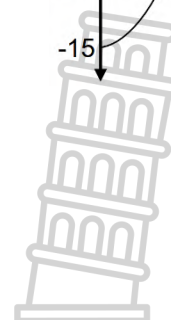
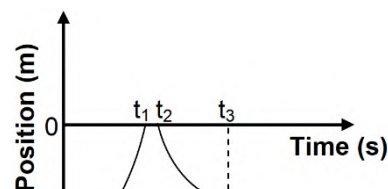
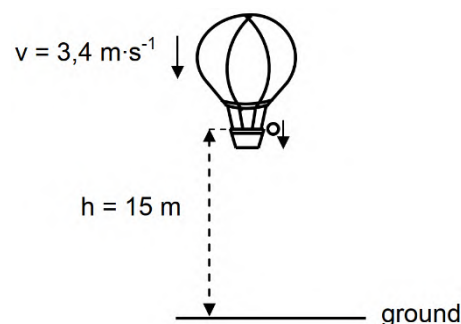
(3)

18.3.2 The height of the hot-air balloon above the ground at the instant when the ball struck the ground

(4)

- 18.4 The ball was in contact with the ground for 0,2 s and left the ground with a vertical upward velocity of $7,2 \text{ m} \cdot \text{s}^{-1}$. Use only EQUATIONS OF MOTION to calculate the value of t_3 indicated on the graph.

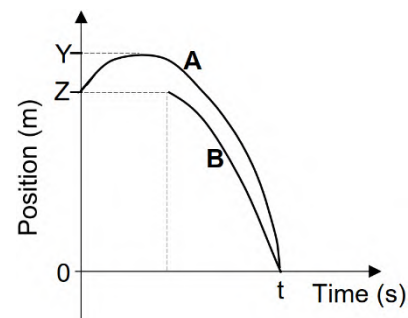
(4)



[14]

QUESTION 19 (November 2024)

Ball **A** is thrown vertically upwards at $12 \text{ m}\cdot\text{s}^{-1}$ from the top of a building. Two seconds after ball **A** was thrown upwards, ball **B** is thrown vertically downwards at $5,4 \text{ m}\cdot\text{s}^{-1}$ from the top of the same building. Both balls, **A** and **B**, strike the ground at time t seconds. Ignore the effects of air friction. The position-time sketch graphs for both balls are shown on the right.



19.1 Using EQUATIONS OF MOTION ONLY, calculate the value of EACH of the following, as shown in the graphs:

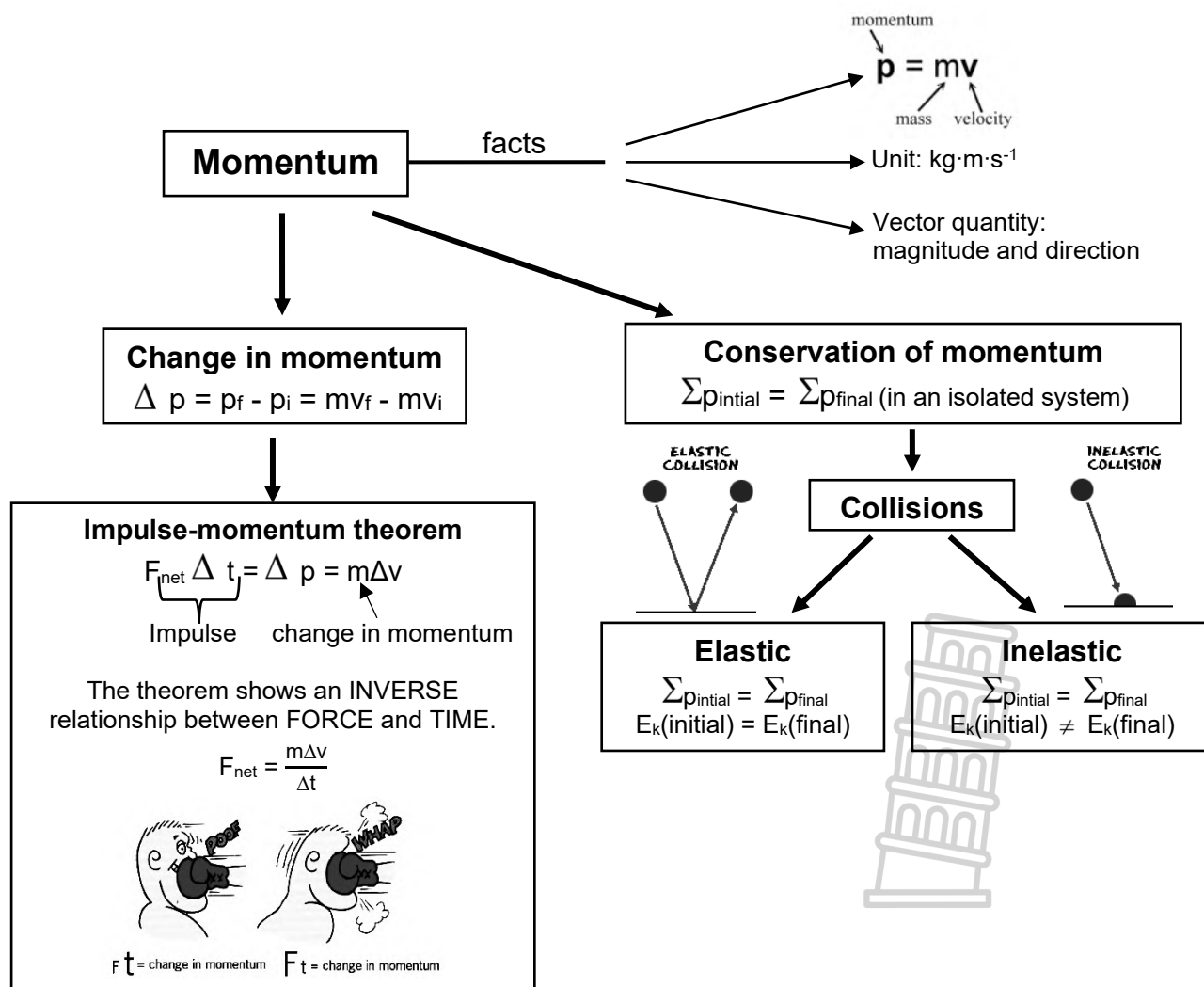
- 19.1.1 t (5)
 19.1.2 Z (3)
 19.1.3 Y (4)

19.2 On the same set of axes, sketch the velocity-time graphs for ball **A** and ball **B** while they are in free fall. Label the graphs **A** and **B** for ball **A** and ball **B** respectively. Clearly indicate the following on the graphs:

- The initial velocity of each ball
- The time at which each ball is thrown
- The time at which the balls strike the ground

(4)
[16]

MOMENTUM AND IMPULSE



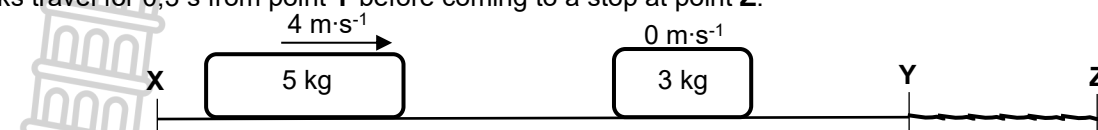
TERMS AND DEFINITIONS	
Contact forces	Contact forces arise from the physical contact between two objects (e.g. a soccer player kicking a ball).
Non-contact forces	Non-contact forces arise even if two objects do not touch each other (e.g. the force of attraction of the earth on a parachutist even when the earth is not in direct contact with the parachutist).
Momentum	The product of an object's mass and its velocity. In symbols: $p = mv$ Unit: $\text{kg}\cdot\text{m}\cdot\text{s}^{-1}$
Newton's second law of motion in terms of momentum	The net (or resultant) force acting on an object is equal to the rate of change of momentum of the object in the direction of the net force. In symbols: $F_{\text{net}} = \frac{\Delta p}{\Delta t}$
Principle of conservation of linear momentum	The total linear momentum of an isolated system remains constant (is conserved). In symbols: $\Sigma p_{\text{before}} = \Sigma p_{\text{after}}$
Isolated system	A system on which the net external force is zero.
Impulse	The product of the resultant/net force acting on an object and the time the resultant/net force acts on the object. In symbols: $\text{Impuls} = F_{\text{net}}\Delta t$ Unit: $\text{N}\cdot\text{s}$
Impulse-momentum theorem	In symbols: $F_{\text{net}}\Delta t = m\Delta v = m(v_f - v_i)$ Unit: $\text{N}\cdot\text{s}$ or $\text{kg}\cdot\text{m}\cdot\text{s}^{-1}$
Elastic collision	A collision in which both total momentum and total kinetic energy are conserved.
Inelastic collision	A collision during which kinetic energy is not conserved.



Typical questions

QUESTION 1

The diagram below shows two sections, **XY** and **YZ**, of a horizontal, flat surface. Section **XY** is smooth, while section **YZ** is rough. A 5 kg block, moving with a velocity of $4 \text{ m}\cdot\text{s}^{-1}$ to the right, collides head-on with a stationary 3 kg block. After the collision, the two blocks stick together and move to the right, past point **Y**. The combined blocks travel for 0,3 s from point **Y** before coming to a stop at point **Z**.

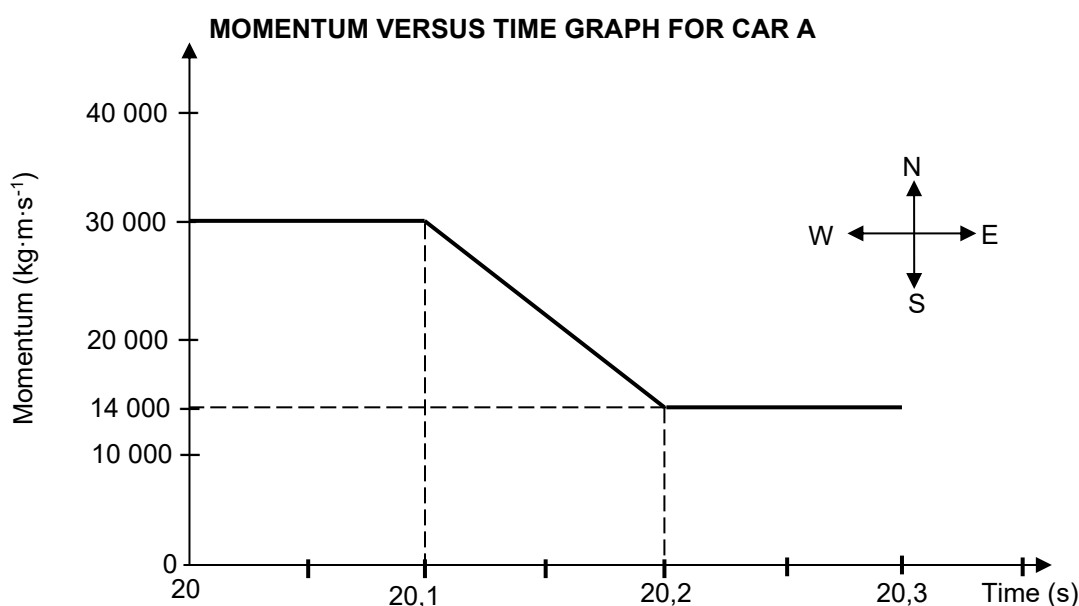


- 1.1 State the *principle of conservation of linear momentum* in words. (2)
- 1.2 Calculate the magnitude of the:
 - 1.2.1 Velocity of the combined blocks at point **Y** (4)
 - 1.2.2 Net force acting on the combined blocks when they move through section **YZ** (4)

[10]

QUESTION 2

The graph below shows how the momentum of car **A** changes with time *just before* and *just after* a head-on collision with car **B**. Car **A** has a mass of 1 500 kg, while the mass of car **B** is 900 kg. Car **B** was travelling at a constant velocity of $15 \text{ m}\cdot\text{s}^{-1}$ west before the collision. Take east as positive and consider the system as isolated.



- 2.1 What do you understand by the term *isolated system* as used in physics? (1)

Use the information in the graph to answer the following questions.

- 2.2 Calculate the:
 - 2.2.1 Magnitude of the velocity of car **A** just before the collision (3)
 - 2.2.2 Velocity of car **B** just after the collision (5)
 - 2.2.3 Magnitude of the net average force acting on car **A** during the collision (4)

[13]

QUESTION 3

A teacher demonstrates the principle of conservation of linear momentum using two trolleys. The teacher first places the trolleys, **A** and **B**, some distance apart on a flat frictionless horizontal surface, as shown in the diagram. The mass of trolley **A** is 3,5 kg and that of trolley **B** is 6,0 kg. Trolley **A** moves towards trolley **B** at constant velocity. The table below shows the position of trolley **A** for time intervals of 0,4 s before it collides with trolley **B**.

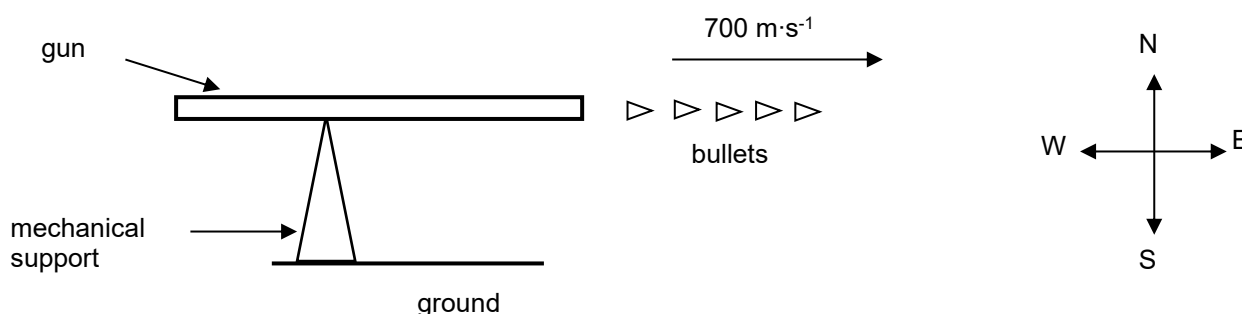


RELATIONSHIP BETWEEN POSITION AND TIME FOR TROLLEY A				
Position of trolley A (m)	0	0,2	0,4	0,6
Time (s)	0	0,4	0,8	1,2

- 3.1 Use the table above to prove that trolley **A** is moving at constant velocity before it collides with trolley **B**. (3)
- 3.2 State the *principle of conservation of linear momentum* in words. (2)
- 3.3 At time $t = 1,2$ s, trolley **A** collides with stationary trolley **B**. The collision time is 0,5 s after which the two trolleys move off together. Calculate the magnitude of the average net force exerted on trolley **B** by trolley **A**. (6)

QUESTION 4

- 4.1 Define the term *impulse* in words. (2)
- 4.2 The diagram below shows a gun mounted on a mechanical support which is fixed to the ground. The gun is capable of firing bullets rapidly in a horizontal direction. Each bullet travels at a speed of $700 \text{ m}\cdot\text{s}^{-1}$ in an easterly direction when it leaves the gun. (Take the initial velocity of a bullet, before being fired, as zero.)



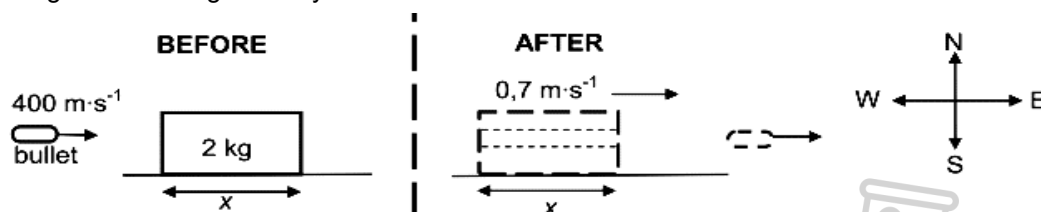
The gun fires 220 bullets per minute. The mass of each bullet is $0,03 \text{ kg}$. Calculate the:

- 4.2.1 Magnitude of the momentum of each bullet when it leaves the gun (3)
- 4.2.2 The net average force that each bullet exerts on the gun (5)
- 4.3 Without any further calculation, write down the net average horizontal force that the mechanical support exerts on the gun. (2)

[12]

QUESTION 5

A 2 kg block is at rest on a smooth, frictionless, horizontal table. The length of the block is x . A bullet of mass $0,015 \text{ kg}$, travelling east at $400 \text{ m}\cdot\text{s}^{-1}$, strikes the block and passes straight through it with constant acceleration. Refer to the diagram below. Ignore any loss of mass of the bullet and the block.



- 5.1 State the *principle of conservation of linear momentum* in words. (2)

The block moves eastwards at $0,7 \text{ m}\cdot\text{s}^{-1}$ after the bullet has emerged from it.

- 5.2 Calculate the magnitude of the velocity of the bullet immediately after it emerges from the block. (4)
- 5.3 If the bullet takes $0,002 \text{ s}$ to travel through the block, calculate the length, x , of the block. (5)

[11]

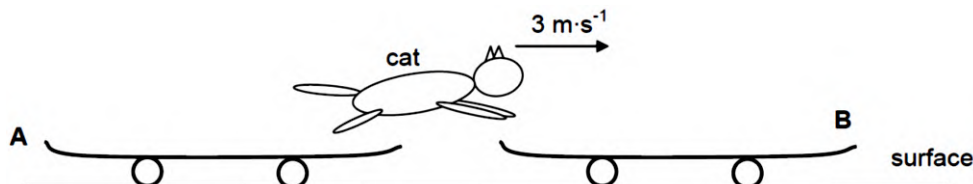
QUESTION 6

The diagram below shows two skateboards, **A** and **B**, initially at rest, with a cat standing on skateboard **A**. The skateboards are in a straight line, one in front of the other and a short distance apart. The surface is flat, frictionless and horizontal.



- 6.1 State the principle of conservation of linear momentum in words. (2)

EACH skateboard has a mass of 3,5 kg. The cat, of mass 2,6 kg, jumps from skateboard **A** with a horizontal velocity of $3 \text{ m} \cdot \text{s}^{-1}$ and lands on skateboard **B** with the same velocity. Refer to the diagram below.



- 6.2 Calculate the velocity of skateboard **A** just after the cat has jumped from it. (5)

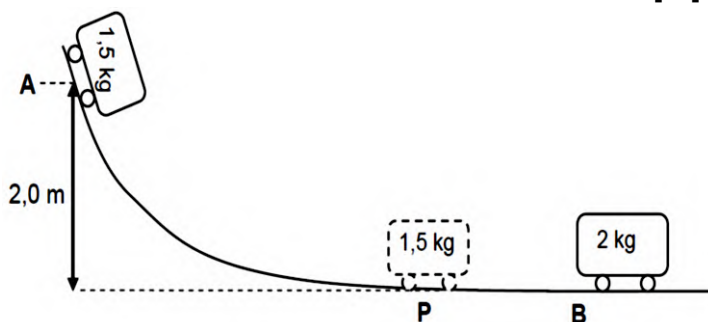
- 6.3 Immediately after the cat has landed, the cat and skateboard **B** move horizontally to the right at $1,28 \text{ m} \cdot \text{s}^{-1}$. Calculate the magnitude of the impulse on skateboard **B** as a result of the cat's landing. (3)

[10]

QUESTION 7

A trolley of mass 1,5 kg is held stationary at point **A** at the top of a frictionless track. When the 1,5 kg trolley is released, it moves down the track. It passes point **P** at the bottom of the incline and collides with a stationary 2 kg trolley at point **B**. Refer to the diagram. Ignore air resistance and rotational effects.

- 7.1 Use the principle of conservation of mechanical energy to calculate the speed of the 1,5 kg trolley at point **P**. (4)



When the two trolleys collide, they stick together and continue moving with constant velocity.

- 7.2 The principle of conservation of linear momentum is given by the incomplete statement below. In a/an ... system, the ... linear momentum is conserved.

Rewrite the complete statement and fill in the missing words or phrases. (2)

- 7.3 Calculate the speed of the combined trolleys immediately after the collision. (4)

- 7.4 Calculate the distance travelled by the combined trolleys in 3 s after the collision. (3)

[13]

QUESTION 8

Initially a girl on roller skates is at rest on a smooth horizontal pavement. The girl throws a parcel, of mass 8 kg, horizontally to the right at a speed of $4 \text{ m} \cdot \text{s}^{-1}$. Immediately after the parcel has been thrown, the girl-roller-skate combination moves at a speed of $0,6 \text{ m} \cdot \text{s}^{-1}$. Ignore the effects of friction and rotation.



- 8.1 Define the term *momentum* in words. (2)

- 8.2 Will the girl-roller-skate combination move TO THE RIGHT or TO THE LEFT after the parcel is thrown? NAME the law in physics that can be used to explain your choice of direction. (2)

The total mass of the roller skates is 2 kg.

- 8.3 Calculate the mass of the girl. (5)

- 8.4 Calculate the magnitude of the impulse that the girl-roller-skate combination is experiencing while the parcel is being thrown. (3)

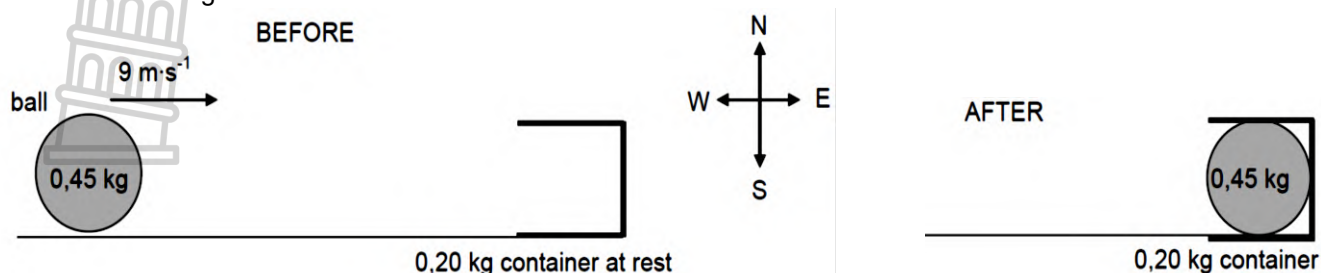
- 8.5 Without any further calculation, write down the change in momentum experienced by the parcel while it is being thrown. (2)

[14]

QUESTION 9 (June 2019)

A soccer player kicks a ball of mass $0,45 \text{ kg}$ to the east. The ball travels horizontally at a velocity of $9 \text{ m}\cdot\text{s}^{-1}$ along a straight line, without touching the ground, and enters a container lying at rest on its side, as shown in the diagram below. The mass of the container is $0,20 \text{ kg}$.

The ball is stuck in the container after the collision. The ball and container now move together along a straight line towards the east. Ignore friction and rotational effects.

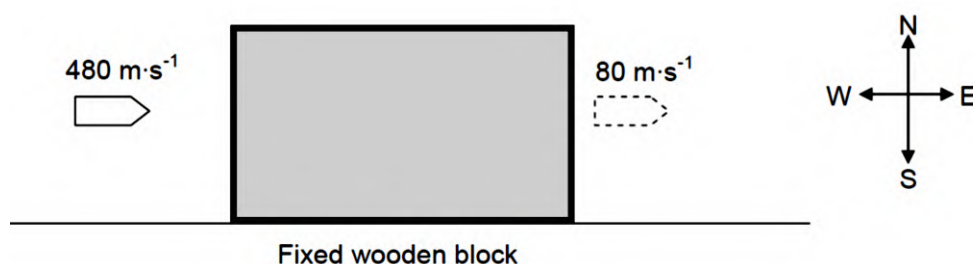


- 9.1 State the *principle of conservation of linear momentum* in words. (2)
- 9.2 Calculate the magnitude of the velocity of the ball-container system immediately after the collision. (4)
- 9.3 Determine, by means of a suitable calculation, whether the collision between the ball and container is elastic or inelastic. (5)

[11]

QUESTION 10

A bullet moves east at a velocity of $480 \text{ m}\cdot\text{s}^{-1}$. It hits a wooden block that is fixed to the floor. The bullet takes $0,01 \text{ s}$ to move through the stationary block and emerges from the block at a velocity of $80 \text{ m}\cdot\text{s}^{-1}$ east. See the diagram below. Ignore the effects of air resistance. Consider the block-bullet system as an isolated system.

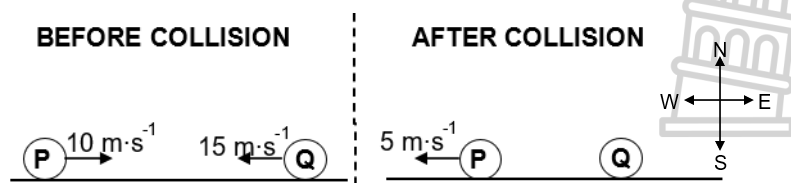


- 10.1 Explain what is meant by an *isolated system* as used in physics. (2)
- 10.2 The magnitude of the momentum of the bullet before it enters the block is $24 \text{ kg}\cdot\text{m}\cdot\text{s}^{-1}$. Calculate the:
 - 10.2.1 Mass of the bullet (3)
 - 10.2.2 Average net force exerted by the wooden block on the bullet (5)

[10]

QUESTION 11

Ball **P** of mass $0,16 \text{ kg}$, moving east at a speed of $10 \text{ m}\cdot\text{s}^{-1}$, collides head-on with another ball **Q** of mass $0,2 \text{ kg}$, moving west at a speed of $15 \text{ m}\cdot\text{s}^{-1}$. After the collision, ball **P** moves west at a speed of $5 \text{ m}\cdot\text{s}^{-1}$, as shown in the diagram below. Ignore the effects of friction and the rotational effects of the balls.



- 11.1 Define the term *momentum* in words. (2)
- 11.2 Calculate the:
 - 11.2.1 Velocity of ball **Q** after the collision (5)
 - 11.2.2 Magnitude of the impulse on ball **P** during the collision (3)

[10]

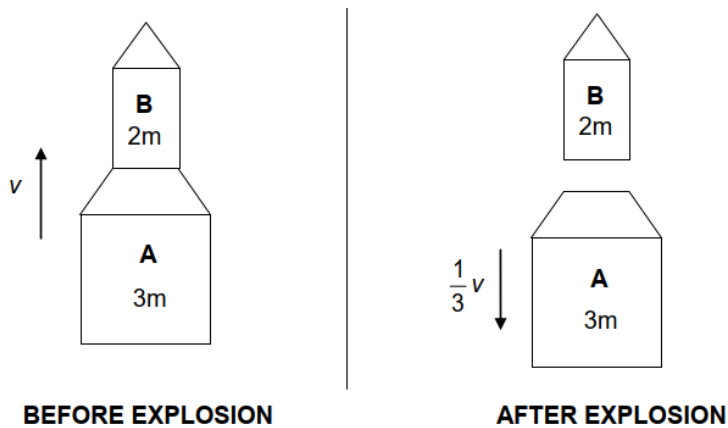
QUESTION 12

A simple rocket system consists of two parts, **A** of mass $3m$, and **B** of mass $2m$, as shown in the diagram. **B** is stacked on top of **A**.

- 12.1 State the *principle of conservation of momentum* in words. (2)

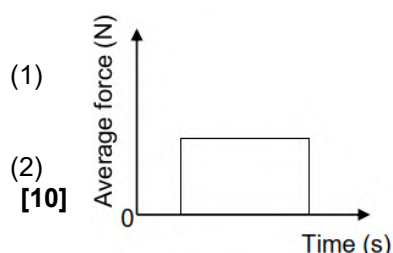
The rocket is travelling vertically upwards at a constant speed v when an internal explosion causes **A** to move DOWNWARDS at a speed $\frac{1}{3}v$. Ignore ALL external forces on the rocket.

- 12.2 Calculate the velocity of **B** in terms of v immediately after the internal explosion. (5)



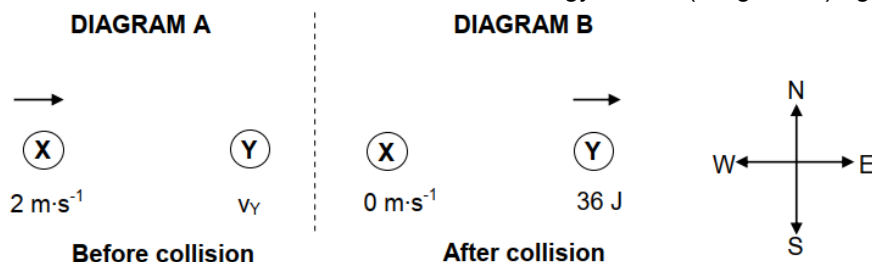
The graph shows the average force exerted by **A** on **B** during the internal explosion as a function of time.

- 12.3 Name the physical quantity represented by the area under the graph. (1)
12.4 Redraw the graph in your ANSWER BOOK. On the same set of axes, sketch the graph of the average force that **B** exerts on **A** as a function of time. (2)



QUESTION 13

A ball **X**, of mass 10 kg , is moving eastwards with a velocity of $2 \text{ m}\cdot\text{s}^{-1}$. It collides ELASTICALLY with another ball **Y**, of mass 2 kg which was moving with an unknown velocity v_Y (Diagram **A**). Immediately after the collision, ball **X** comes to rest and ball **Y** moves eastwards with a kinetic energy of 36 J (Diagram **B**). Ignore friction.

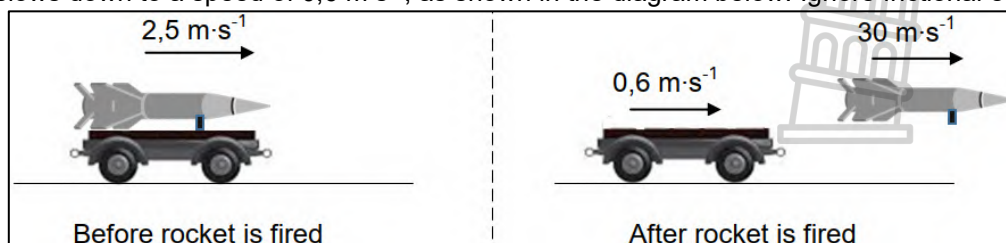


- 13.1 Explain the meaning of the term *elastic collision*. (2)
13.2 Calculate velocity v_Y . (5)
13.3 The balls were in contact with each other for $0,1 \text{ s}$ during the collision. Calculate the magnitude of the force that ball **X** exerted on ball **Y** during the collision. (3)

[10]

QUESTION 14

- 14.1 What is meant by an isolated system in physics? (2)
14.2 During an experiment, a rocket of unknown mass is mounted on a toy cart of mass 20 kg . The cart-rocket combination moves at a constant speed of $2,5 \text{ m}\cdot\text{s}^{-1}$ along a horizontal floor. At a certain instant, the rocket is fired horizontally in the direction of motion at a speed of $30 \text{ m}\cdot\text{s}^{-1}$. As a result, the cart slows down to a speed of $0,6 \text{ m}\cdot\text{s}^{-1}$, as shown in the diagram below. Ignore frictional effects.

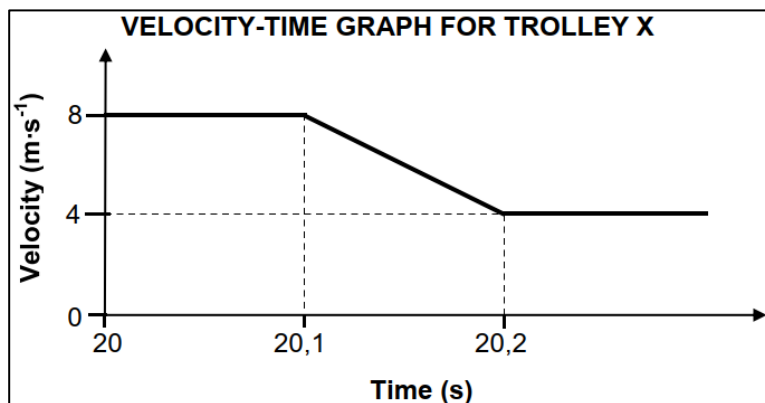


- 14.2.1 Use relevant physics principles to explain why the firing of the rocket will slow down the cart. (2)
14.2.2 Calculate the mass of the rocket at the instant the rocket was fired from the toy cart. (5)

[9]

QUESTION 15

Trolley X of mass 1,2 kg travels at $8 \text{ m}\cdot\text{s}^{-1}$ east and collides with trolley Y of mass 0,5 kg which is initially at rest. Ignore all frictional effects. The velocity-time graph shows the velocity of trolley X before, during and after the collision with trolley Y.



- 15.1 State the *principle of conservation of linear momentum*. (2)
- 15.2 Calculate the magnitude of the:
 - 15.2.1 Velocity of trolley Y immediately after the collision (4)
 - 15.2.2 Average net force that trolley X exerts on trolley Y during the collision (3)
- 15.3 Is the collision ELASTIC or INELASTIC? Explain the answer by means of suitable calculations. (5)

[14]

QUESTION 16 (June 2023)

Trolley A of mass 7,2 kg moves to the right at $0,4 \text{ m}\cdot\text{s}^{-1}$ in a straight line on a horizontal floor. It collides with a stationary trolley B of mass 5,3 kg. After the collision, the trolleys lock together and move to the right, as shown in the diagram below. Ignore any frictional effects.

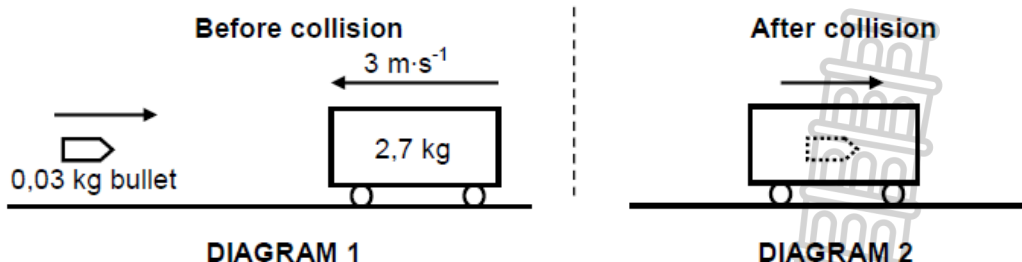


- 16.1 State the *principle of conservation of linear momentum* in words. (2)
- 16.2 Calculate the magnitude of the:
 - 16.2.1 Velocity of the trolleys immediately after the collision (3)
 - 16.2.2 Average net force exerted by trolley A on trolley B during the collision, if the collision time is 0,02 s (3)

[8]

QUESTION 17 (November 2023)

A wooden trolley of mass 2,7 kg moves to the left with a constant velocity of $3 \text{ m}\cdot\text{s}^{-1}$. A bullet of mass 0,03 kg is fired horizontally from the left towards the trolley (see DIAGRAM 1). The bullet strikes the trolley and comes to rest inside the trolley in 0,02 s. The average net force exerted by the trolley on the bullet during this time is 591 N. The bullet-trolley combination now moves to the right (see DIAGRAM 2.)



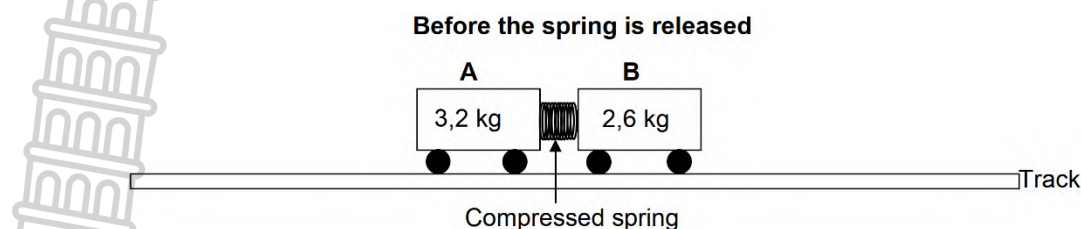
Ignore all frictional and rotational effects.

- 17.1 Write down the magnitude and direction of the average net force that the bullet exerts on the trolley. (1)
- 17.2 Calculate the magnitude of the velocity with which the bullet strikes the trolley. (4)
- 17.3 State the principle of conservation of linear momentum in words. (2)
- 17.4 Calculate the magnitude of the velocity of the bullet-trolley combination after the collision. (4)

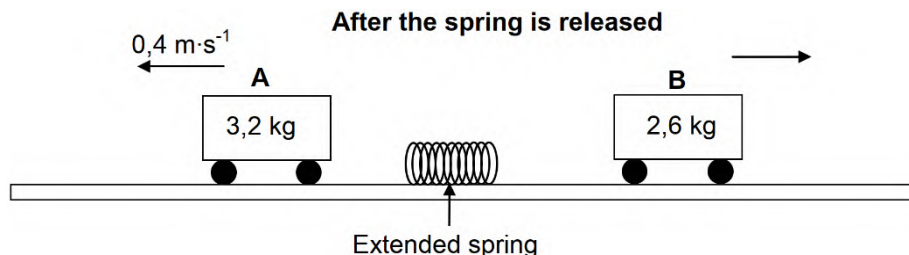
[11]

QUESTION 18 (June 2024)

Two trolleys **A** and **B** of mass $3,2 \text{ kg}$ and $2,6 \text{ kg}$ respectively are held at rest on a straight horizontal, frictionless track, with a compressed spring between them, as shown in the diagram below.



After the trolleys are released, the spring extends to its natural length and then falls onto the track. Trolley **A** now moves with a constant velocity of $0,4 \text{ m}\cdot\text{s}^{-1}$ to the left, while trolley **B** moves with a constant unknown velocity to the right. Trolley **B** reaches the end of the track after $1,3 \text{ s}$.

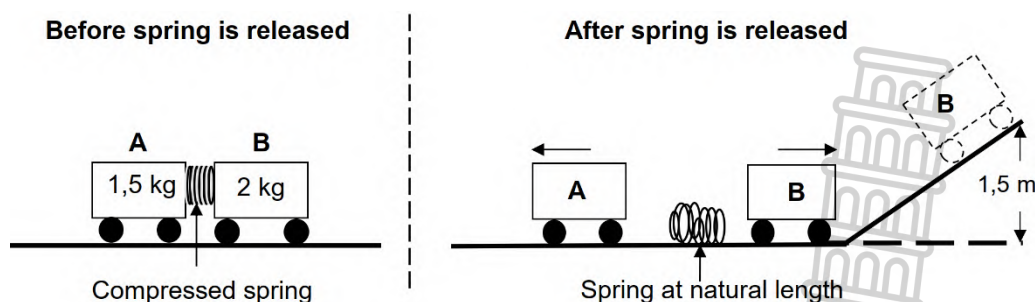


- 18.1 State the *principle of conservation of linear momentum* in words. (2)
- 18.2 Calculate the distance travelled by trolley **B** in $1,3 \text{ s}$. (5)
- 18.3 The average force exerted by the extended spring on each trolley while they were in contact with the spring was $4,2 \text{ N}$. Calculate the time it took the spring to extend to its natural length. (3)
- 18.4 Trolley **B** is now replaced by trolley **C**, which has a larger mass. The same compressed spring is placed between trolleys **A** and **C**. The trolleys are then released. The average force exerted by the extended spring on the trolleys remains at $4,2 \text{ N}$ for the same period of time as calculated in QUESTION 18.3. How does the magnitude of the velocity of trolley **C** compare to the magnitude of the velocity of trolley **B** after the spring has fallen to the track? Write only GREATER THAN, LESS THAN or EQUAL TO. Explain the answer. (3)

[13]

QUESTION 19 (November 2024)

Two trolleys, **A** and **B**, of masses of $1,5 \text{ kg}$ and 2 kg respectively, are held in a stationary position on a straight, horizontal, frictionless track, with a compressed spring between them. The trolleys are released and the spring takes t seconds to return to its natural length. The spring then falls to the ground. Trolley **A** moves to the left, while trolley **B** moves to the right and then up a frictionless inclined plane, rising to a maximum vertical height of $1,5 \text{ m}$, as shown in the diagram below.

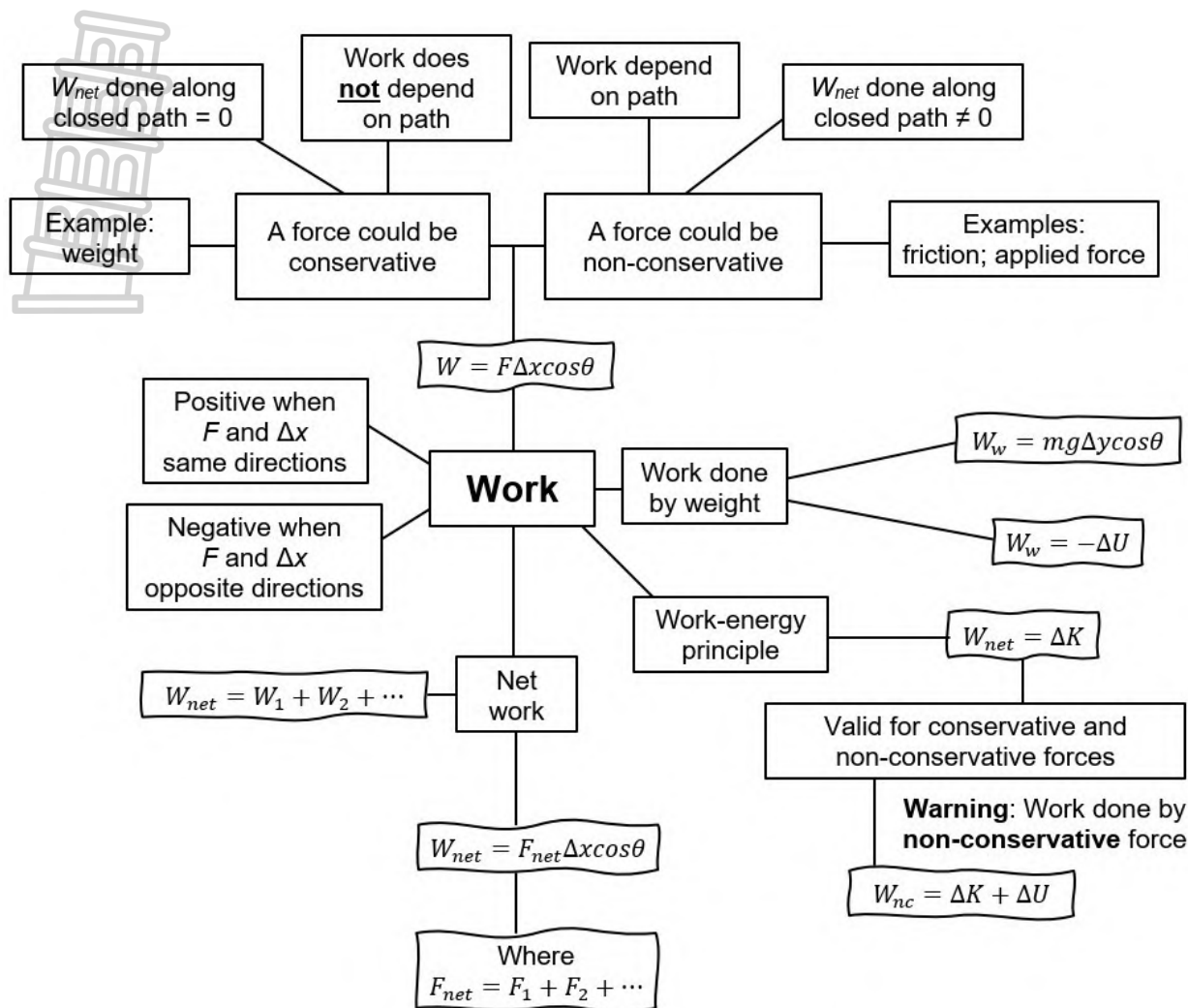


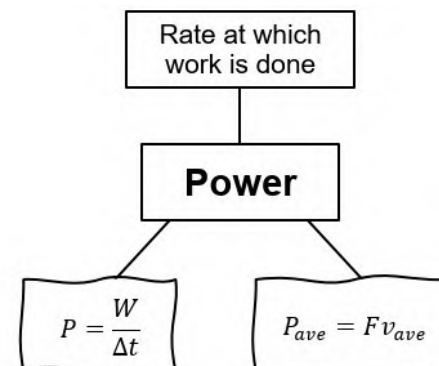
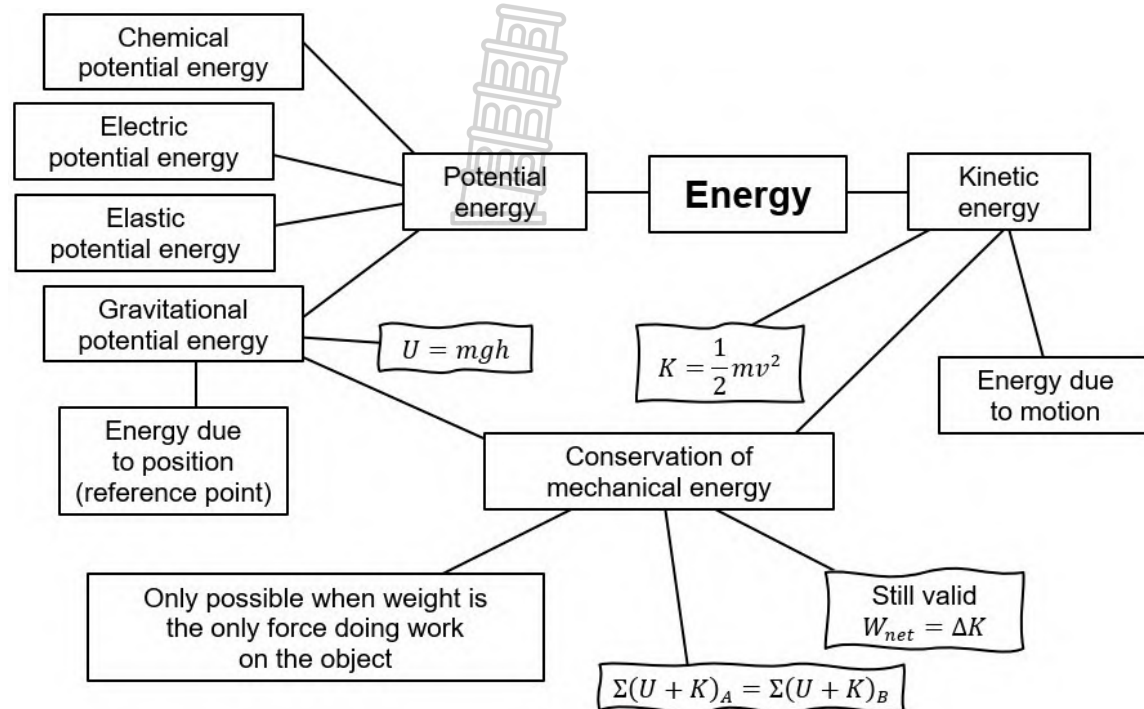
Ignore the rotational effects of the wheels.

- 19.1 Write down the *principle of conservation of mechanical energy* in words. (2)
- 19.2 Calculate the speed of trolley **B** at the bottom of the inclined plane. (4)
- 19.3 For the t seconds that the spring takes to return to its natural length:
 - 19.3.1 Calculate the change in momentum of trolley **B** (3)
 - 19.3.2 Write down the change in momentum of trolley **A** (1)
- 19.4 Calculate the speed of trolley **A** after t seconds. (2)

[12]

WORK, ENERGY AND POWER





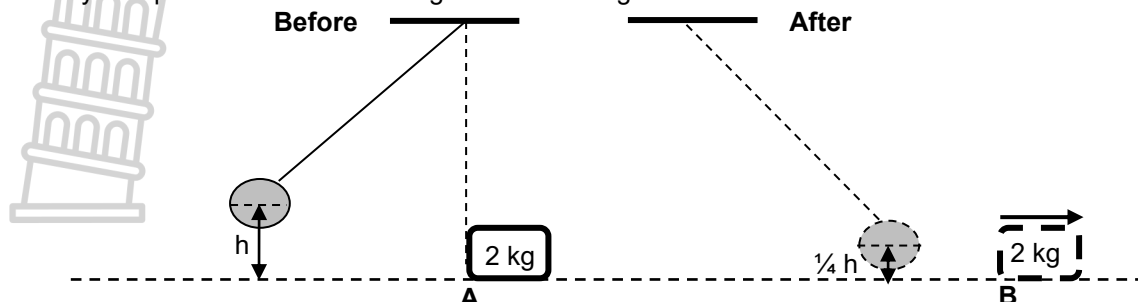
TERMS AND DEFINITIONS	
Work	The work done on an object by a constant force F is $F\Delta x \cos\theta$, where F is the magnitude of the force, x the magnitude of the displacement and θ the angle between the force and the displacement. In symbols: $W = F\Delta x \cos\theta$
Positive net work	The kinetic energy of the object increases.
Negative net work	The kinetic energy of the object decreases.
Work-energy theorem	The net work done on an object is equal to the change in the object's kinetic energy OR the work done on an object by a net (or resultant) force is equal to the change in the object's kinetic energy. In symbols: $W_{net} = \Delta K = K_f - K_i$
Principle of conservation of mechanical energy	The total mechanical energy (sum of gravitational potential energy and kinetic energy) in an isolated system remains constant. In symbols: $E_M(initial) = E_M(final)$ OR $(E_p + E_k)_{initial} = (E_p + E_k)_{final}$
Isolated system	A system is isolated when the net external force (excluding the gravitational force) acting on the system is zero.
Conservative force	A force for which the work done in moving an object between two points is independent of the path taken. Examples are gravitational force, the elastic force in a spring and electrostatic forces (coulomb forces).
Non-conservative force	A force for which the work done in moving an object between two points depends on the path taken. Examples are frictional force, air resistance, tension in a chord, etc.
Power	The rate at which work is done or energy is expended. In symbols: $P = \frac{W}{\Delta t}$ Unit: watt (W)



Typical questions

QUESTION 1

A pendulum with a bob of mass 5 kg is held stationary at a height h metres above the ground. When released, it collides with a block of mass 2 kg which is stationary at point **A**. The bob swings past **A** and comes to rest momentarily at a position $\frac{1}{4}h$ above the ground. The diagrams below are NOT drawn to scale.

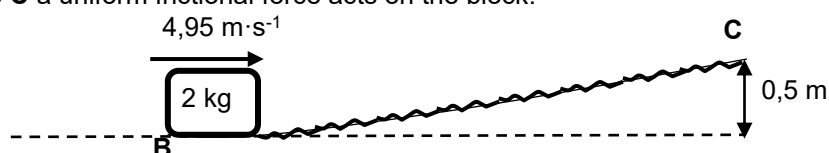


Immediately after the collision the 2 kg block begins to move from **A** to **B** at a constant speed of $4,95 \text{ m}\cdot\text{s}^{-1}$. Ignore frictional effects and assume that no loss of mechanical energy occurs during the collision.

1.1 Calculate the kinetic energy of the block immediately after the collision. (3)

1.2 Calculate the height h . (4)

The block moves from point **B** at a velocity of $4,95 \text{ m}\cdot\text{s}^{-1}$ up a rough inclined plane to point **C**. The speed of the block at point **C** is $2 \text{ m}\cdot\text{s}^{-1}$. Point **C** is 0,5 m above the horizontal, as shown in the diagram below. During its motion from **B** to **C** a uniform frictional force acts on the block.



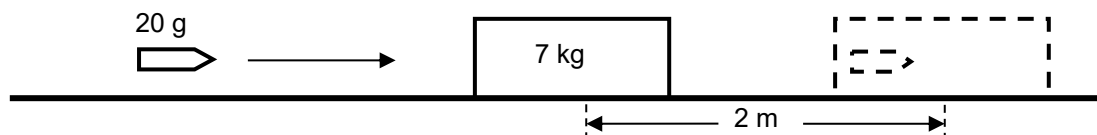
1.3 State the work-energy theorem in words. (2)

1.4 Use energy principles to calculate the work done by the frictional force when the 2 kg block moves from point **B** to point **C**. (4)

[13]

QUESTION 2

The diagram below shows a bullet of mass 20 g that is travelling horizontally. The bullet strikes a stationary 7 kg block and becomes embedded in it. The bullet and block together travel on a rough horizontal surface a distance of 2 m before coming to a stop.



2.1 Use the work-energy theorem to calculate the magnitude of the velocity of the bullet-block system immediately after the bullet strikes the block, given that the frictional force between the block and surface is 10 N. (5)

2.2 State the principle of conservation of linear momentum in words. (2)

2.3 Calculate the magnitude of the velocity with which the bullet hits the block. (4)

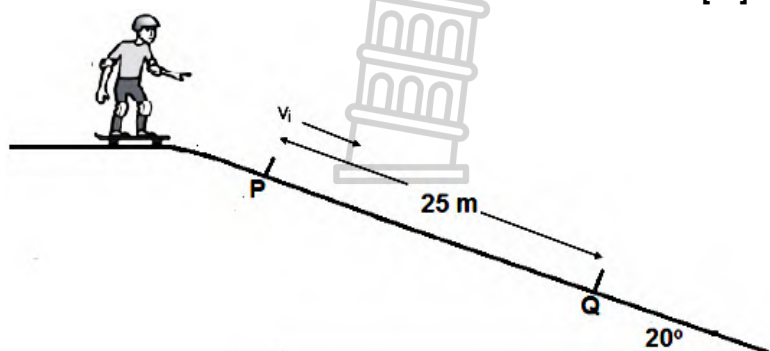
[11]

QUESTION 3

The diagram shows a boy skateboarding on a ramp which is inclined at 20° to the horizontal. A constant frictional force of 50 N acts on the skateboard as it moves from **P** to **Q**.

Consider the boy and the skateboard as a single unit of mass 60 kg. Ignore the effects of air friction.

3.1 Draw a labelled free-body diagram, showing ALL the forces acting on the boy-skateboard unit while moving down the ramp from **P** to **Q**. (3)



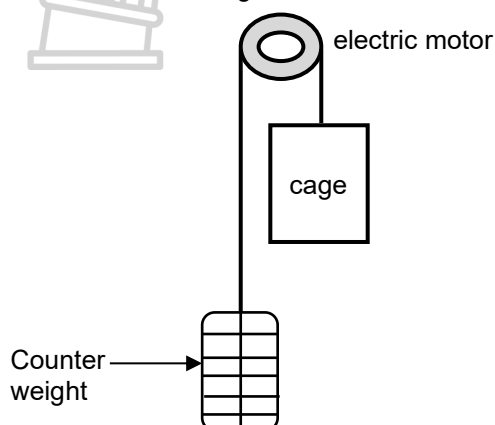
Points **P** and **Q** on the ramp are 25 m apart. The skateboarder passes point **P** at a speed v_i and passes point **Q** at a speed of $15 \text{ m}\cdot\text{s}^{-1}$. Ignore rotational effects due to the wheels of the skateboard.

- 3.2 State the work-energy theorem in words. (2)
 3.3 Use energy principles to calculate the speed v_i of the skateboarder at point **P**. (5)
 3.4 Calculate the average power dissipated by the skateboarder to overcome friction between **P** and **Q**. (4)

[14]

QUESTION 4

A lift arrangement comprises an electric motor, a cage and its counterweight. The counterweight moves vertically downwards as the cage moves upwards. The cage and counterweight move at the same constant speed. Refer to the diagram below.



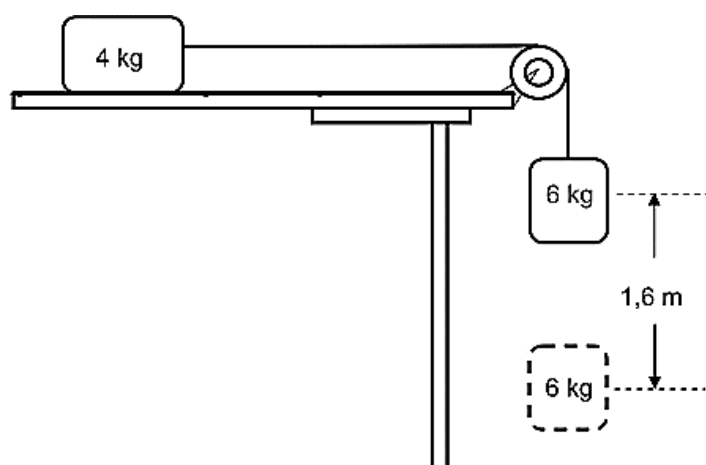
The cage, carrying passengers, moves vertically upwards at a constant speed, covering 55 m in 3 minutes. The counterweight has a mass of 950 kg. The total mass of the cage and passengers is 1 200 kg.

The electric motor provides the power needed to operate the lift system. Ignore the effects of friction.

- 4.1 Define the term *power* in words. (2)
 4.2 Calculate the work done by the:
 4.2.1 Gravitational force on the cage (3)
 4.2.2 Counterweight on the cage (2)
 4.3 Calculate the average power required by the motor to operate the lift arrangement in 3 minutes. Assume that there are no energy losses due to heat and sound. (6)

[13]

QUESTION 5



In the diagram below, a 4 kg block lying on a rough horizontal surface is connected to a 6 kg block by a light inextensible string passing over a light frictionless pulley. Initially the blocks are HELD AT REST.

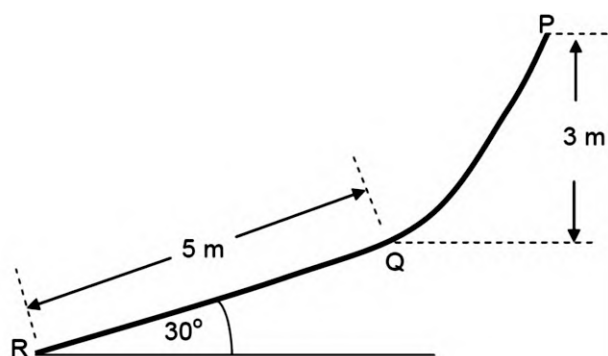
- 5.1 State the work-energy theorem in words. (2)
 When the blocks are released, the 6 kg block falls through a vertical distance of 1,6 m.
 5.2 Draw a labelled free-body diagram for the 6 kg block. (2)
 5.3 Calculate the work done by the gravitational force on the 6 kg block. (3)
 The coefficient of kinetic friction between the 4 kg block and the horizontal surface is 0,4. Ignore the effects of air resistance.

- 5.4 Use energy principles to calculate the speed of the 6 kg block when it falls through 1,6 m while still attached to the 4 kg block. (5)

[12]

QUESTION 6

A slide, **PQR**, at an amusement park consists of a curved *frictionless* section, **PQ**, and a section, **QR**, which is rough, straight and inclined at 30° to the horizontal. The starting point, **P**, is 3 m above point **Q**. The straight section, **QR**, is 5 m long.



A learner, with mass 50 kg, starting from rest at **P**, slides down section **PQ**, then continues down the straight section, **QR**.

- 6.1 State the *law of conservation of mechanical energy* in words. (2)
 6.2 Calculate the speed of the learner at **Q**. (4)
 6.3 Draw a labelled free-body diagram for the learner while he/she is on section **QR**. (3)

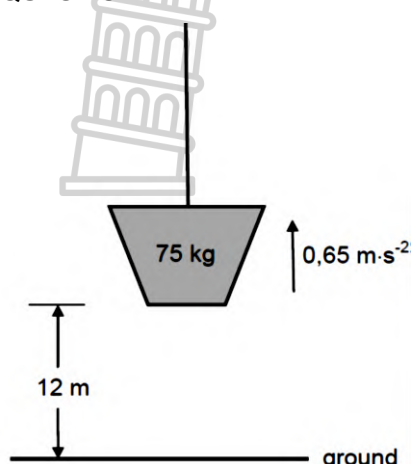
The coefficient of kinetic friction (μ_k) between the learner and the surface **QR** is 0,08.

6.4 Calculate the magnitude of the kinetic frictional force acting on the learner when the learner is on section **QR**. (3)

6.5 Use energy principles to calculate the speed of the learner at point **R**. (5)

[17]

QUESTION 7



A load of mass 75 kg is initially at rest on the ground. It is then pulled vertically upwards at a constant acceleration of $0,65 \text{ m}\cdot\text{s}^{-2}$ by means of a light inextensible rope. Refer to the diagram below. Ignore air resistance, rotational effects and the mass of the rope.

7.1 Draw a labelled free-body diagram for the load while it moves upward. (2)

7.2 Name the non-conservative force acting on the load. (1)

7.3 Calculate the work done on the load by the gravitational force when the load has reached a height of 12 m. (3)

7.4 State the *work-energy theorem* in words. (2)

7.5 Use the work-energy theorem to calculate the speed of the load when it is at a height of 12 m. (5)

[13]

QUESTION 8

The diagram, not drawn to scale, shows a vehicle with a mass of 1 500 kg starting from rest at point **A** at the bottom of a rough incline. Point **B** is 200 m vertically above the horizontal. The total work done by force **F** that moves the vehicle from point **A** to point **B** in 90 s is $4,80 \times 10^6 \text{ J}$.



8.1 Define the term non-conservative force. (2)

8.2 Is force **F** a conservative force? Choose from: YES or NO. (1)

8.3 Calculate the average power generated by force **F**. (3)

The speed of the vehicle when it reaches point **B** is $25 \text{ m}\cdot\text{s}^{-1}$.

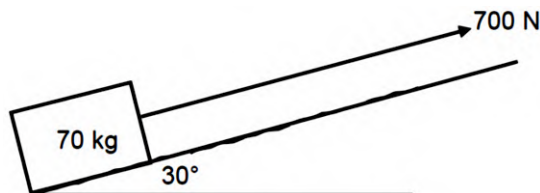
8.4 State the *work-energy theorem* in words. (2)

8.5 Use **energy principles** to calculate the total work done on the vehicle by the frictional forces. (5)

[13]

QUESTION 9 (June 2019)

A 70 kg box is initially at rest at the bottom of a **ROUGH** plane inclined at an angle of 30° to the horizontal. The box is pulled up the plane by means of a light inextensible rope, held parallel to the plane, as shown in the diagram below. The force applied to the rope is 700 N.



9.1 What is the name given to the force in the rope? (1)

9.2 Give a reason why the mechanical energy of the system will NOT be conserved as the box is pulled up the plane. (1)

The box is pulled up over a distance of 4 m along the plane. The kinetic frictional force between the box and the plane is 178,22 N.

9.3 Draw a labelled free-body diagram for the box as it moves up the plane. (4)

9.4 Calculate the work done on the box by the frictional force over the 4 m. (3)

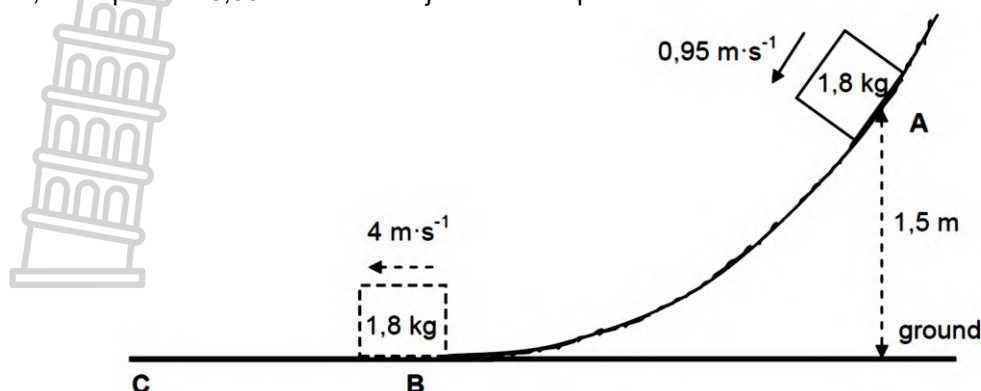
9.5 Use energy principles to calculate the speed of the box after it has moved 4 m. (5)

9.6 When the box is 4 m up the incline, the rope accidentally breaks, causing the box to slide back down to the bottom of the inclined plane. What will be the total work done by friction when the box moves up and then down to the bottom of the inclined plane? (1)

[15]

QUESTION 10

An object of mass $1,8 \text{ kg}$ slides down a rough curved track and passes point **A**, which is $1,5 \text{ m}$ above the ground, at a speed of $0,95 \text{ m}\cdot\text{s}^{-1}$. The object reaches point **B** at the bottom of the track at a speed of $4 \text{ m}\cdot\text{s}^{-1}$.



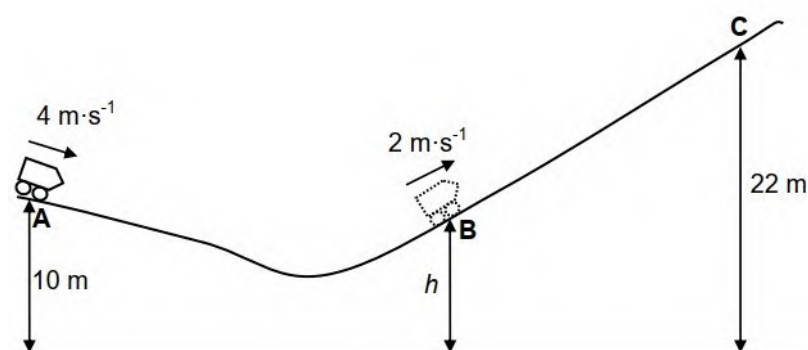
- 10.1 Define the term *conservative force*. (2)
- 10.2 Name the conservative force acting on the object. (1)
- 10.3 Is mechanical energy conserved as the object slides from point **A** to point **B**? Choose from YES or NO. Give a reason for the answer. (2)
- 10.4 Calculate the gravitational potential energy of the object when it was at point **A**. (3)
- 10.5 Using energy principles, calculate the work done by friction on the object as it slides from point **A** to point **B**. (4)
- 10.6 Surface **BC** in the diagram above is frictionless. What is the value of the net work done on the object as it slides from point **B** to point **C**? (1)

[13]

QUESTION 11

A roller-coaster car of mass 200 kg , with the engine switched off, travels along track **ABC**, which has a rough surface, as shown in the diagram. At point **A**, which is 10 m above the ground, the speed of the car is $4 \text{ m}\cdot\text{s}^{-1}$.

At point **B**, which is at a height h above the ground, the speed of the car is $2 \text{ m}\cdot\text{s}^{-1}$. During the motion from point **A** to point **B**, $3,40 \times 10^3 \text{ J}$ of energy is used to overcome friction. Ignore rotational effects due to the wheels of the car.



- 11.1 Define the term *non-conservative force*. (2)
- 11.2 Calculate the change in the kinetic energy of the car after it has travelled from point **A** to point **B**. (3)
- 11.3 Use energy principles to calculate the height h . (4)
- 11.4 On reaching point **B**, the car's engine is switched on in order to move up the incline to point **C**, which is 22 m above the ground. While moving from point **B** to point **C**, the car travels for 15 s at a constant speed of $2 \text{ m}\cdot\text{s}^{-1}$ while an average frictional force of 50 N acts on it. Calculate the power delivered by the engine to move the car from point **B** to point **C**. (5)

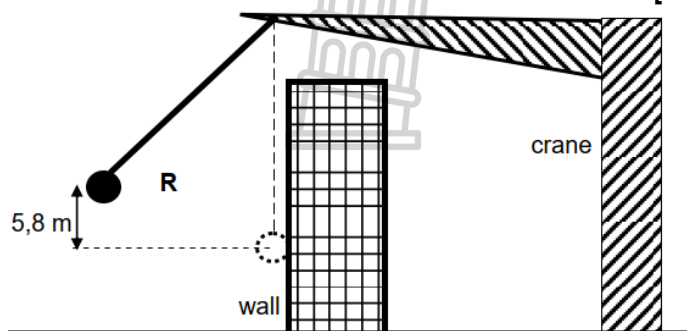
[14]

QUESTION 12

A demolition ball is used by a crane to break the wall of a building. The demolition ball, of mass $1\,250 \text{ kg}$, is lifted by the crane to a point **R** at a height of $5,8 \text{ m}$ above its lowest position in 60 s . Ignore air friction.

- 12.1 Define the term *power* in words. (2)
- 12.2 Calculate the average power dissipated by the crane in lifting the demolition ball to point **R**. (3)

The demolition ball is released from point **R** and strikes the wall at the lowest point of its swing. The ball then moves $0,25 \text{ m}$ HORIZONTALLY into the wall before coming to rest.



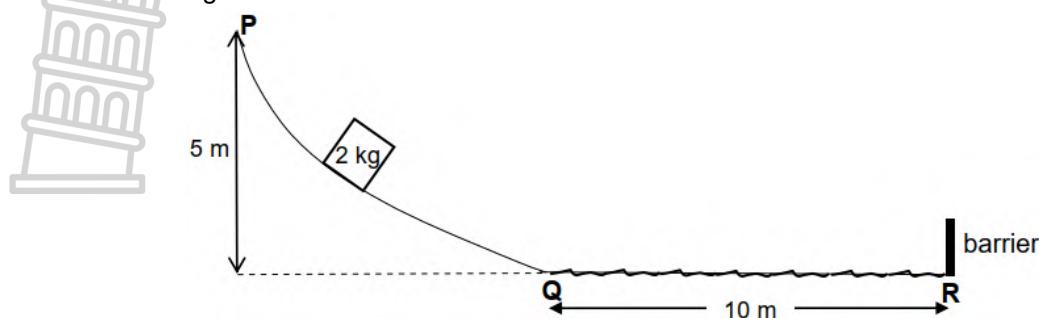
- 12.3 Define the term *conservative force*. (2)
- 12.4 Is the force which the wall exerts on the ball a CONSERVATIVE or a NON-CONSERVATIVE force? (1)
- 12.5 State the energy conversion that takes place during the downward swing of the demolition ball. (1)

- 12.6 Using energy principles, calculate the magnitude of the average force exerted on the ball while it moves into the wall. (5)

[14]

QUESTION 13

A 2 kg box is released from rest at point **P**, 5 m above the ground. It slides down a smooth frictionless curved track **PQ**. See the diagram.

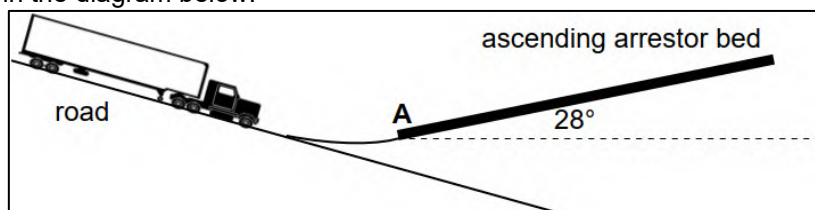


- 13.1 State the *principle of conservation of mechanical energy* in words. (2)
 13.2 Use the PRINCIPLE OF CONSERVATION OF MECHANICAL ENERGY to calculate the speed of the box when it reaches point **Q**. (3)
 13.3 The box passes point **Q** and moves 10 m on a rough horizontal surface before striking a barrier at point **R** at a speed of $4 \text{ m}\cdot\text{s}^{-1}$. Use ENERGY PRINCIPLES to calculate the magnitude of the average frictional force acting on the box as it moves from **Q** to **R**. (4)
 13.4 The barrier exerts an impulse of $14 \text{ N}\cdot\text{s}$ to the LEFT on the box when the box strikes the barrier. Calculate the change in kinetic energy of the box after striking the barrier. (5)

[14]

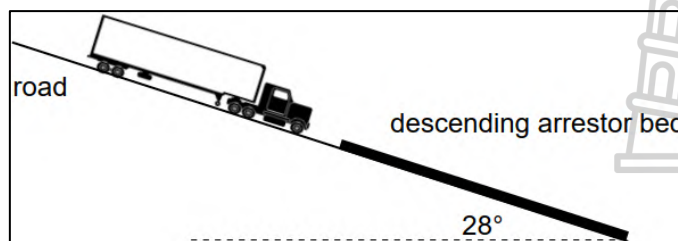
QUESTION 14

Arrestor beds are used to help moving trucks to come to a stop when their brakes fail. The driver of a 30 000 kg truck driving down a steep road drives onto an ASCENDING arrestor bed inclined at 28° to the horizontal, as shown in the diagram below.



- 14.1 State the *work-energy theorem* in words. (2)
 The truck with failed brakes passes point **A** at the beginning of the arrestor bed at a speed of $33 \text{ m}\cdot\text{s}^{-1}$. The average frictional force on the truck is 31 000 N while the truck moves up the arrestor bed. Ignore the rotational effects of the wheels.
 14.2 Give a reason why the net work done on the truck, while moving on the arrestor bed, is negative. (1)
 14.3 Use ENERGY PRINCIPLES to calculate the minimum length of the arrestor bed needed to bring the truck to a stop. (5)

The diagram below shows the same truck entering a DESCENDING arrestor bed inclined at 28° to the horizontal. The initial speed of the truck and the average frictional force on the truck are $33 \text{ m}\cdot\text{s}^{-1}$ and 31 000 N respectively.

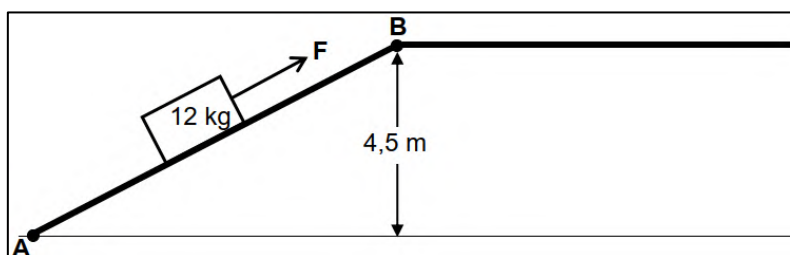


- 14.4 Which arrestor bed, ASCENDING or DESCENDING, will be able to stop the truck in a shorter distance? Explain the answer in terms of the forces acting on the truck. (3)

[11]

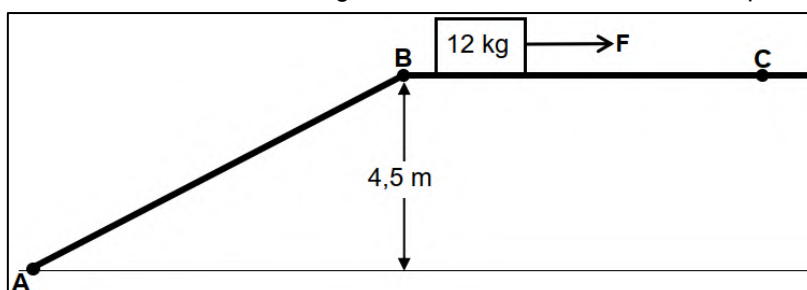
QUESTION 15

A 12 kg block is initially at rest at point **A** at the bottom of a ROUGH inclined plane. The block is pulled up the incline by a constant force F acting parallel to the incline. The block reaches point **B**, which is at a vertical height of 4,5 m above the horizontal, with a speed of $2,25 \text{ m}\cdot\text{s}^{-1}$. See the diagram below.



- 15.1 Define the term *non-conservative force*. (2)
- 15.2 Draw a labelled free-body diagram for the block when it is pulled up the inclined plane. (4)
- 15.3 Calculate the total work done on the block by the NON-CONSERVATIVE forces when the block moved from point **A** to point **B**. (4)

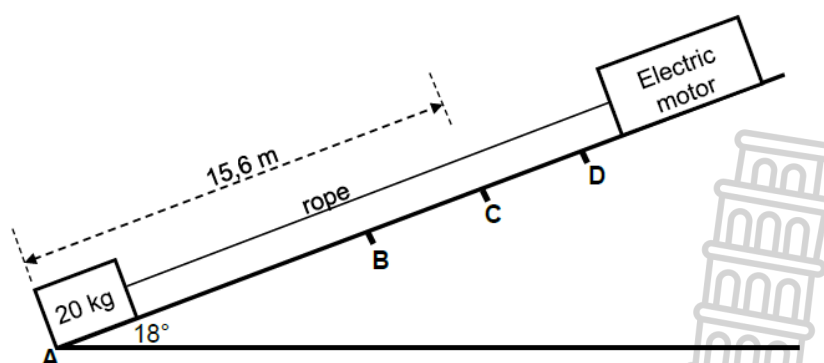
The same constant force F now moves the block at a CONSTANT VELOCITY across a rough horizontal surface from point **B** to point **C**, as shown below. Force F acts parallel to the horizontal surface. The magnitude of the constant frictional force acting on the block while moving from point **B** to point **C** is 42 N LARGER than the magnitude of the constant frictional force acting on the block when it moves from point **A** to point **B**.



- 15.4 Calculate the distance from point **A** to point **B**. (5)
- [15]**

QUESTION 16 (June 2023)

An electric motor pulls a 20 kg crate from rest at point **A** up an inclined plane by means of a light inextensible rope. The inclined plane makes an angle of 18° with the horizontal. **B**, **C** and **D** are points on the inclined plane and the distance between points **A** and **C** is 15,6 m, as shown in the diagram below.



The motor exerts a constant force of 96,8 N parallel to the inclined plane on the rope. A constant frictional force of 13,5 N acts on the crate as it moves on the inclined plane.

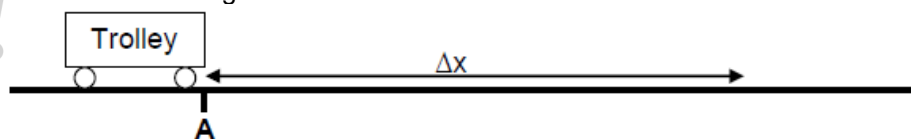
- 16.1 Define a *non-conservative force*. (2)
- 16.2 Use ENERGY PRINCIPLES to calculate the speed of the crate when it reaches point **C**. (5)
- 16.3 Calculate the minimum average power dissipated by the electric motor to pull the crate from point **A** to point **C**. (3)

When the crate reaches point **C**, the rope breaks. The crate continues moving up the inclined plane, comes to a stop at point **D**, and then slides down the plane past point **B**.

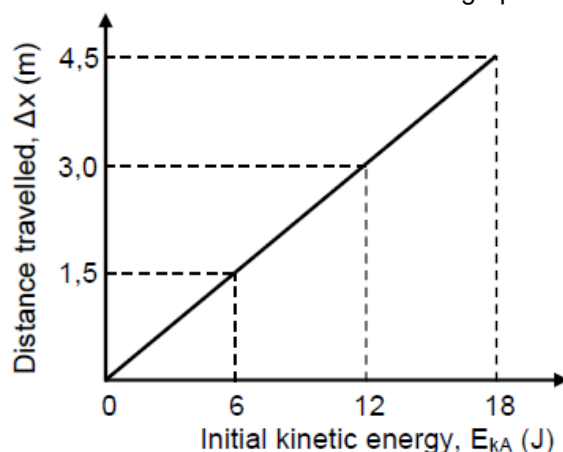
- 16.4 Draw a labelled free-body diagram for the crate as it slides down the plane past point **B**. (3)
 - 16.5 Draw a velocity-time graph for the entire motion of the crate starting from point **A** until it passes point **B** again on its motion down the inclined plane. (4)
- [17]**

QUESTION 17 (November 2023)

Learners conduct an experiment to determine how the initial kinetic energy given to a trolley affects the distance the trolley moves on a rough horizontal surface. A learner pushes a trolley of unknown mass until it reaches point **A** with kinetic energy E_{kA} . The horizontal distance (Δx) travelled by the trolley before it comes to rest is then measured. See the diagram below.



The experiment is repeated with the trolley moving on the same rough horizontal surface but with different initial kinetic energies at point **A**. The results obtained are shown in the graph below.



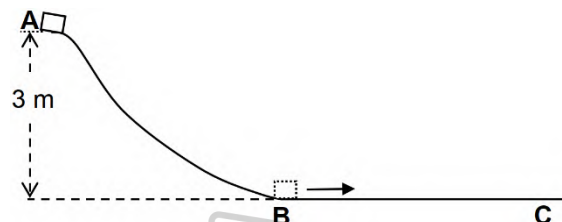
Ignore the rotational effects of the wheels of the trolley.

- 17.1 Draw a labelled free-body diagram showing all the forces acting on the trolley during its motion after passing point **A**. (3)
- 17.2 Name the independent variable in this experiment. (1)
- 17.3 State the work-energy theorem in words. (2)
- 17.4 Calculate the mass of the trolley if the coefficient of kinetic friction between the wheels of the trolley and the rough horizontal surface is 0,18. (6)

[12]

QUESTION 18 (June 2024)

A crate of mass 18 kg, initially at rest, slides down a frictionless slope from point **A** to point **B**. The crate then moves along a rough horizontal surface from point **B** towards point **C**. Point **A** is 3 m above the horizontal surface. See the diagram.



- 18.1 State the *principle of conservation of mechanical energy* in words. (2)
- 18.2 Using ENERGY PRINCIPLES only, calculate the speed of the crate at point **B**. (3)

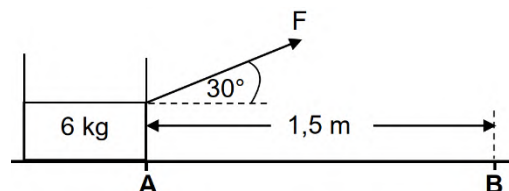
A constant frictional force of 40,6 N acts on the crate as it moves from point **B** towards point **C**. The crate comes to rest at point **C**.

- 18.3 State the *work-energy theorem* in words. (2)
- 18.4 Using ENERGY PRINCIPLES only, calculate the distance that the crate travelled from point **B** to point **C**. (4)
- 18.5 The height of the track is now lowered so that point **A** is at a vertical height less than 3 m. The same crate is again released from point **A**. How will the distance now, travelled by the crate along the horizontal surface before it comes to rest, compare to the distance calculated in QUESTION 18.4? Write only GREATER THAN, SMALLER THAN or EQUAL TO. Explain the answer. (3)

[14]

QUESTION 19 (November 2024)

A constant force F is applied at an angle of 30° to the horizontal on a crate of mass 6 kg that is initially at rest, as shown in the diagram. A constant frictional force of 10 N acts on the crate as it moves from rest at point **A** along a horizontal surface to point **B**. The distance between point **A** and point **B** is $1,5\text{ m}$. The speed of the crate at point **B** is $2\text{ m}\cdot\text{s}^{-1}$.



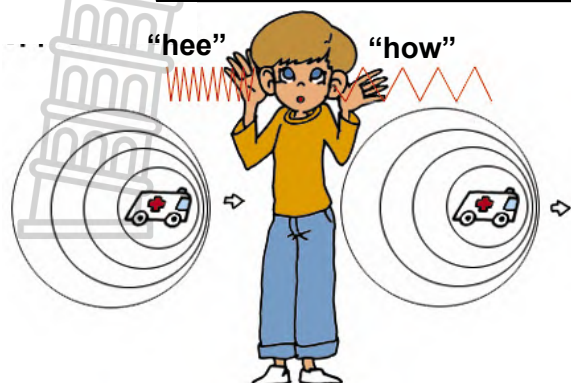
- 19.1 Define the term *work done* by a force. (2)
- 19.2 Draw a labelled free-body diagram showing ALL the forces acting on the crate as it moves. (4)
- 19.3 Using ENERGY PRINCIPLES ONLY, calculate the magnitude of force F . (4)
- 19.4 A 2 kg object is placed in the crate. What effect will this have on the work done by the same force F when the crate is again moved from point **A** to point **B**? Write only INCREASES, DECREASES or REMAINS THE SAME. (2)

[12]



DOPPLER EFFECT

The change in frequency/pitch of the sound detected by a listener due to relative motion between the sound source and the listener.



Doppler Equation

$$f_L = f_s \frac{v \pm v_L}{v \pm v_s}$$

f_L : frequency of sound detected by the listener

f_s : frequency of sound emitted by the source

v : speed of sound in air

v_L : speed of listener

v_s : speed of the source

Stationary Listener, Moving Source

Stationary sound source



No relative motion between listener and sound source – no change in observed frequency.

Sound source moving towards stationary observer.



The waves to the front of the source are compressed resulting in a shorter observed wavelength and thus a **higher observed frequency**.

$$f_L = \frac{v}{v - v_s}$$

Sound source moving away from stationary observer.

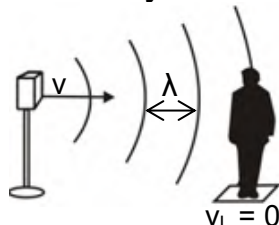


The waves behind the source are further apart due to the motion of the source resulting in a longer observed wavelength and thus a **lower observed frequency**.

$$f_L = \frac{v}{v + v_s}$$

Stationary Source, Moving Listener

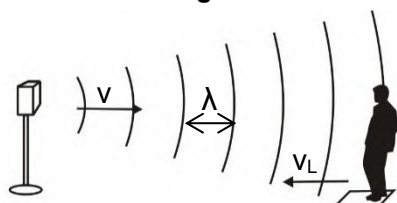
Stationary listener



No relative motion between sound source and listener.

$$f_L = f_s$$

Listener moving towards source



A listener moving at speed v_L towards a stationary source **intercepts more** wave compressions per unit time, than a stationary listener does, and **hears a higher frequency**.

$$f_L = \frac{v + v_L}{v}$$

Listener moving away from source

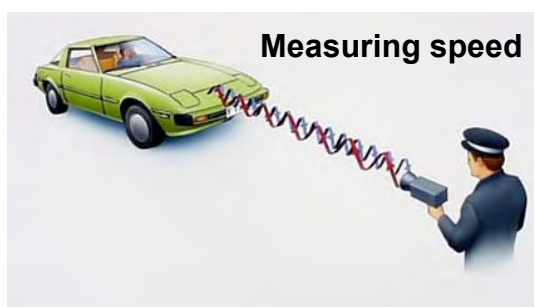


A listener moving at speed v_L away from a stationary source **intercepts fewer** wave compressions per unit time, than a stationary listener does, and **hears a lower frequency**.

$$f_L = \frac{v - v_L}{v}$$

TERMS AND DEFINITIONS	
Doppler Effect	The change in frequency (or pitch) of the sound detected by a listener because the sound source and the listener have different velocities relative to the medium of sound propagation.
Red shift	Observed when light from an object increases in wavelength (decrease in frequency). A red shift occurs when a light source moves away from an observer.
Blue shift	Observed when light from an object decreases in wavelength (increase in frequency). A blue shift occurs when a light source moves towards an observer.
Frequency	The number of vibrations per second. Symbol: f Unit: hertz (Hz) or per second (s^{-1})
Wavelength	The distance between two successive points in phase. Symbol: λ Unit: meter (m)
Wave equation	Speed = frequency x wavelength or $v = \lambda f$

Applications of the Doppler Effect

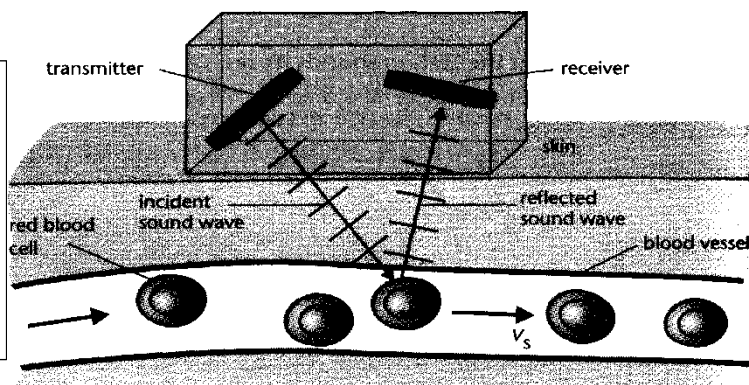


Measuring foetal heartbeat



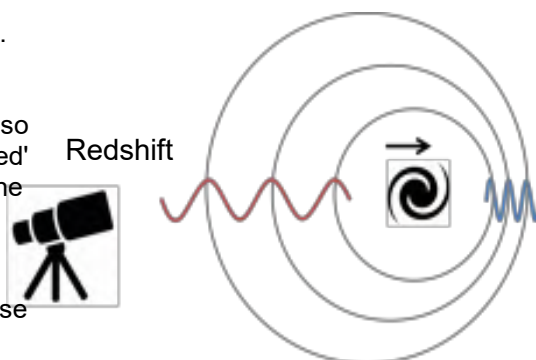
Doppler flow meter

Ultrasound waves, which are sound waves with frequency greater than 20 kHz, are used by medical scientists in **Doppler flow meters to measure the speed of blood flow**. A device consisting of a transmitter and a receiver is placed directly on the skin.



The Doppler Effect and light

'**Red shift**' is a key concept for astronomers. The wavelength of the light observed from a distant star is stretched, so the light is seen as 'shifted' towards the red part of the spectrum. That implies that the star is moving away from the observer. It is evidence that universe is expanding.



Some of the nearest stars are moving towards us and hence show a '**blue shift**' (their light is shifted towards shorter wavelengths). Even some galaxies (for example, the Andromeda Galaxy) are blue shifted.

Typical questions

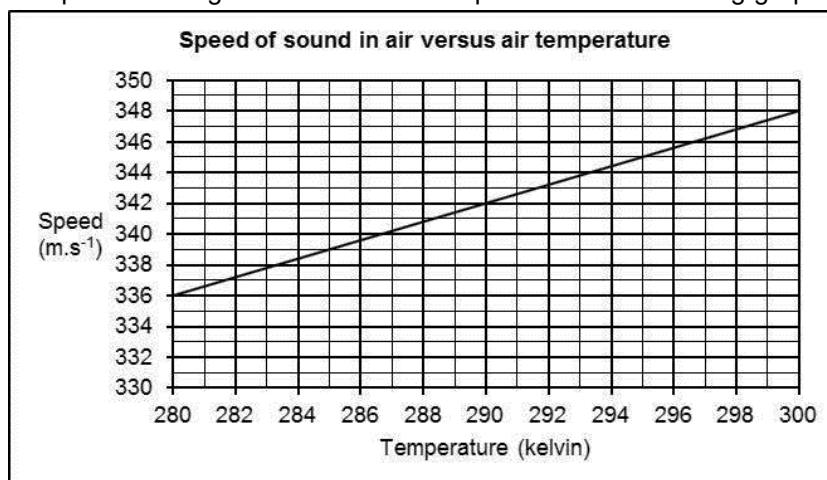
QUESTION 1

- 1.1 An ambulance is moving towards a stationary listener at a constant speed of $30 \text{ m}\cdot\text{s}^{-1}$. The siren of the ambulance emits sound waves having a wavelength of $0,28 \text{ m}$. Take the speed of sound in air as $340 \text{ m}\cdot\text{s}^{-1}$.
- 1.1.1 State the *Doppler effect* in words. (2)
- 1.1.2 Calculate the frequency of the sound waves emitted by the siren as heard by the ambulance driver. (3)
- 1.1.3 Calculate the frequency of the sound waves emitted by the siren as heard by the listener. (5)
- 1.1.4 How would the answer to QUESTION 1.1.3 change if the speed of the ambulance were LESS THAN $30 \text{ m}\cdot\text{s}^{-1}$? Write down only INCREASES, DECREASES or REMAINS THE SAME. (1)
- 1.2 An observation of the spectrum of a distant star shows that it is moving away from the earth. Explain, in terms of the frequencies of the spectral lines, how it is possible to conclude that the star is moving away from the earth. (2)

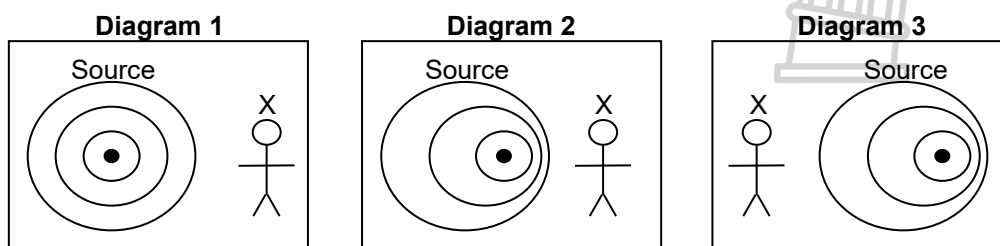
[13]

QUESTION 2

The speed of sound in air depends among others on the air temperature. The following graph shows this relationship.



- 2.1 Which one of temperature or speed is the dependent variable? (1)
- 2.2 The gradient of this graph is equal to $0,6 \text{ m}\cdot\text{s}^{-1}\cdot\text{K}^{-1}$. With how much does the speed, in $\text{m}\cdot\text{s}^{-1}$, increase for every 5 K increase in temperature? (1)
- 2.3 Two experiments are done to verify the Doppler effect. In the first experiment, an object approaches a stationary observer **X** at a constant speed of $57,5 \text{ m}\cdot\text{s}^{-1}$. The object is equipped with a siren that emits sound waves at a fixed frequency of $1\,000 \text{ Hz}$. The motion takes place in still air at a temperature of 295 K .
- 2.3.1 Describe what the Doppler effect is. (2)
- 2.3.2 What is the speed of sound, in $\text{m}\cdot\text{s}^{-1}$, in air at 295 K ? (Use the graph.) (1)
- 2.3.3 Calculate the frequency measured by observer **X**. (4)
- 2.3.4 In the second experiment, the object moves away from observer **X** at the same constant speed as before. What should the air temperature, in kelvin, be to make it a fair test between the two experiments? (1)
- 2.4 Consider the three diagrams below. Each one represents the source (with the siren) and observer **X**. Two of the diagrams are applicable on the above-mentioned experiments.



- 2.4.1 Which diagram is applicable to experiment 2? (1)
- 2.4.2 Which diagram is NOT applicable to any of the experiments? Give a reason for your answer. (2)

[13]

QUESTION 3

- 3.1 A police car is moving at constant velocity on a freeway. The siren of the car emits sound waves with a frequency of 330 Hz. A stationary sound detector measures the frequency of the sound waves of the approaching siren as 365 Hz.
- 3.1.1 State the Doppler Effect in words. (2)
- 3.1.2 Calculate the speed of the car. (Speed of sound in air is $340 \text{ m}\cdot\text{s}^{-1}$.) (5)
- 3.2 The spectrum of a distant star when viewed from an observatory on Earth appears to have undergone a red shift. Explain the term red shift. (3)

[10]

QUESTION 4

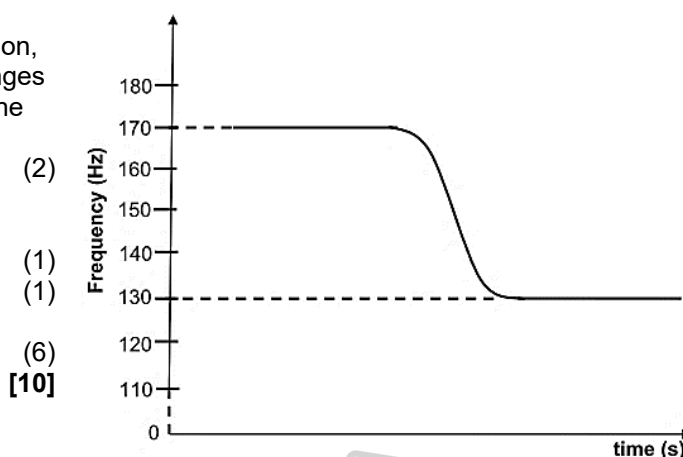
- 4.1 A sound source is moving at constant velocity past a stationary observer. The frequency detected as the source approaches the observer is 2 600 Hz. The frequency detected as the source moves away from the observer is 1 750 Hz. Take the speed of sound in air as $340 \text{ m}\cdot\text{s}^{-1}$.
- 4.1.1 Name the phenomenon that describes the apparent change in frequency detected by the observer. (1)
- 4.1.2 State ONE practical application of the phenomenon in QUESTION 4.1.1 in the field of medicine. (1)
- 4.1.3 Calculate the speed of the moving source. (6)
- 4.1.4 Will the observed frequency INCREASE, DECREASE or REMAIN THE SAME if the velocity of the source increased as it:
- (a) Moves towards the observer (1)
- (b) Moves away from the observer (1)
- 4.2 Spectral lines of star X at an observatory are observed to be red shifted.
- 4.2.1 Explain the term *red shifted* in terms of wavelength. (2)
- 4.2.2 Will the frequency of the light observed from the star INCREASE, DECREASE or REMAIN THE SAME? (1)

[13]

QUESTION 5

A police car moving at a constant velocity with its siren on, passes a stationary listener. The graph shows the changes in the frequency of the sound of the siren detected by the listener.

- 5.1 State the *Doppler Effect* in words. (2)
- 5.2 Write down the frequency of the sound detected by the listener as the police car:
- 5.2.1 Approaches the listener (1)
- 5.2.2 Moves away from the listener (1)
- 5.3 Calculate the speed of the police car. Take the speed of sound in air to be $340 \text{ m}\cdot\text{s}^{-1}$. (6)



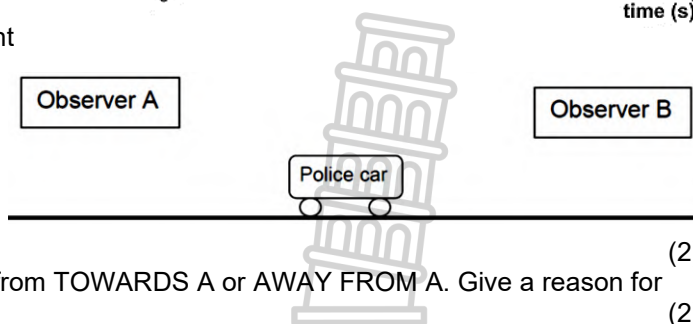
[10]

QUESTION 6

A police car is moving at a constant speed on a straight horizontal road. The siren of the car emits sound of constant frequency. EACH of two observers, **A** and **B**, standing some distance apart on the same side of the road, records the frequency of the detected sound. Observer **A** records a frequency of 690 Hz and observer **B** records a frequency of 610 Hz.

- 6.1 State the *Doppler effect* in words. (2)
- 6.2 In which direction is the car moving? Choose from TOWARDS A or AWAY FROM A. Give a reason for the answer. (2)
- 6.3 Determine the speed of the police car. Take the speed of sound in air as $340 \text{ m}\cdot\text{s}^{-1}$. (6)
- 6.4 Name ONE application of the Doppler effect. (1)

[11]



QUESTION 7

A sound source, moving at a constant speed of $240 \text{ m}\cdot\text{s}^{-1}$ towards a detector, emits sound at a constant frequency. The detector records a frequency of $5\,100 \text{ Hz}$. Take the speed of sound in air to be $340 \text{ m}\cdot\text{s}^{-1}$.

- 7.1 State the *Doppler effect* in words. (2)
- 7.2 Calculate the wavelength of the sound emitted by the source. (7)
- 7.3 Some of the sound waves are reflected from the detector towards the approaching source. Will the frequency of the reflected sound wave detected by the sound source be EQUAL TO, GREATER THAN or SMALLER THAN $5\,100 \text{ Hz}$? (1)

[10]

QUESTION 8

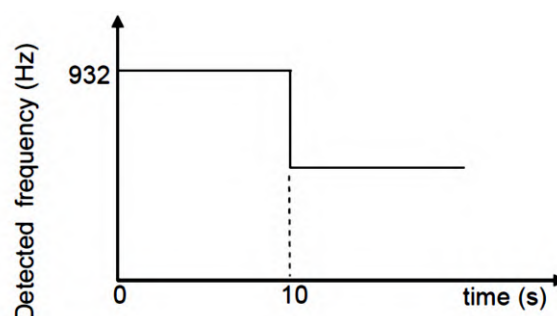
The alarm of a vehicle parked next to a straight horizontal road goes off, emitting sound with a wavelength of $0,34 \text{ m}$. A patrol car is moving at a constant speed on the same road. The driver of the patrol car hears a sound with a frequency of 50 Hz lower than the sound emitted by the alarm. Take the speed of sound in air as $340 \text{ m}\cdot\text{s}^{-1}$.

- 8.1 State the *Doppler effect* in words. (2)
- 8.2 Is the patrol car driving TOWARDS or AWAY FROM the parked vehicle? Give a reason for the answer. (2)
- 8.3 Calculate the frequency of the sound emitted by the alarm. (3)
- 8.4 The patrol car moves a distance of x metres in 10 seconds. Calculate the distance x . (6)

[13]

QUESTION 9 (June 2019)

- 9.1 A patrol car is moving at a constant speed towards a stationary observer. The driver switches on the siren of the car when it is 300 m away from the observer. The observer records the detected frequency of the sound waves of the siren as the patrol car *approaches, passes and moves away* from him. The information obtained is shown in the graph.



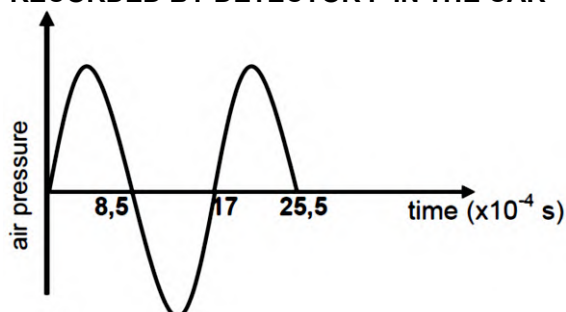
- 9.1.1 Calculate the speed of the patrol car. (2)
- 9.1.2 State the *Doppler effect*. (2)
- 9.1.3 The detected frequency suddenly changes at $t = 10 \text{ s}$. Give a reason for this change. (2)
- 9.1.4 Take the speed of sound in air as $340 \text{ m}\cdot\text{s}^{-1}$. Calculate the frequency of the sound emitted by the siren. (4)
- 9.2 State TWO applications of the Doppler effect. (2)

[12]

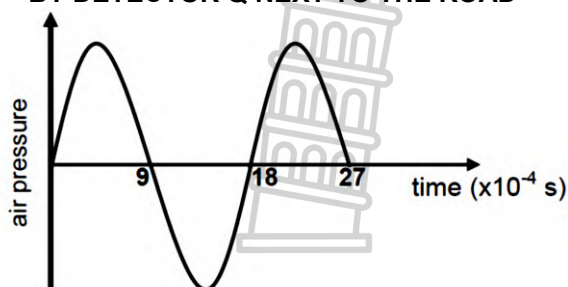
QUESTION 10

The siren of a police car, which is travelling at a constant speed along a straight horizontal road, emits sound waves of constant frequency. Detector **P** is placed inside the police car and detector **Q** is placed next to the road at a certain distance away from the car. The two detectors record the changes in the air pressure readings caused by the sound waves emitted by the siren as a function of time. The graphs below were obtained from the recorded results.

GRAPH A: AIR PRESSURE VS TIME RECORDED BY DETECTOR P IN THE CAR



GRAPH B: AIR PRESSURE VS TIME RECORDED BY DETECTOR Q NEXT TO THE ROAD



- 10.1 Different patterns are shown above for the same sound wave emitted by the siren. What phenomenon is illustrated by the two detectors showing the different patterns? (1)
- The police car is moving AWAY from detector **Q**.
- 10.2 Use the graphs and give a reason why it can be confirmed that the police car is moving away from detector **Q**. (1)
- 10.3 Calculate the frequency of the sound waves recorded by detector **P**. (3)
- 10.4 Use the information in the graphs to calculate the speed of the police car. Take the speed of sound in air as $340 \text{ m}\cdot\text{s}^{-1}$. (6)

[11]

QUESTION 11

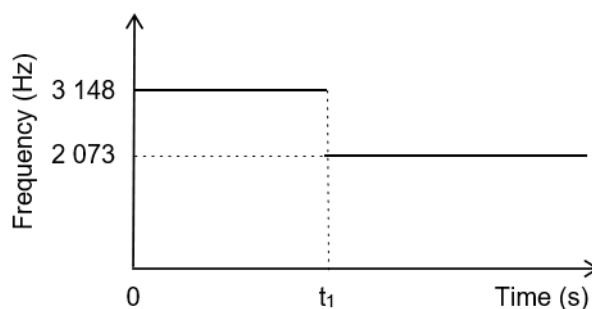
The siren of a train, moving at a constant speed along a straight horizontal track, emits sound with a constant frequency. A detector, placed next to the track, records the frequency of the sound waves. The results obtained are as shown in the graph.

11.1 State the Doppler effect in words. (2)

11.2 Does the detector record the frequency of 3 148 Hz when the train moves TOWARDS the detector or AWAY from the detector? (1)

11.3 Calculate the speed of the train. Take the speed of sound in air as $340 \text{ m}\cdot\text{s}^{-1}$. (6)

11.4 The detector started recording the frequency of the moving train's siren when the train was 350 m away. Calculate time t_1 indicated on the graph above. (2)



[11]

QUESTION 12

A learner in a car, moving at a constant speed of $10 \text{ m}\cdot\text{s}^{-1}$ along a straight horizontal road, records the frequency of sound emitted by a distant stationary source. The learner then repeats the recording of the frequency of the sound when the car travels at a new constant speed of $20 \text{ m}\cdot\text{s}^{-1}$. The graph, not drawn to scale, is obtained from the results.

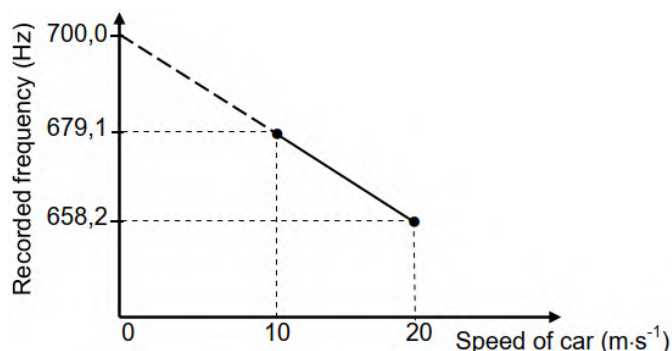
12.1 State the Doppler effect. (2)

12.2 Use the graph to answer the following questions.

12.2.1 Write down the frequency of the sound emitted by the stationary source. Give a reason for the answer. (2)

12.2.2 In which direction is the car moving relative to the source? Choose from TOWARDS or AWAY. Give a reason for the answer. (2)

12.2.3 Calculate the speed of sound in air. (5)



[11]

QUESTION 13

The siren of a stationary ambulance emits sound waves at a constant frequency of 680 Hz. A man is standing with a detector that records the wavelength of the sound emitted by the siren, as shown in the diagram below.



The speed of sound in air is $340 \text{ m}\cdot\text{s}^{-1}$.

13.1 Calculate the wavelength of the detected sound. (3)

The ambulance now moves at a constant speed along the road TOWARDS the man. The detector now records the wavelength of the sound, which differs from the previous reading by 0,05 m. (2)

13.2 State the Doppler effect. (2)

13.3 How would EACH of the following have changed when the ambulance approached the detector compared to when the ambulance was stationary? Choose from INCREASED, DECREASED or NO CHANGE. (1)

13.3.1 Distance between the wave fronts (1)

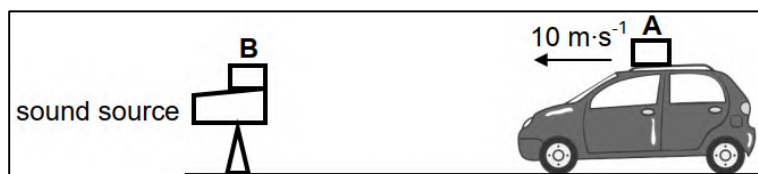
13.3.2 Frequency of the detected waves (1)

13.4 Calculate the speed of the ambulance. (5)

[12]

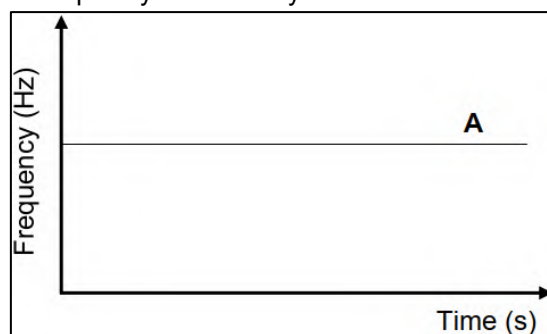
QUESTION 14

A car moves at a constant speed of $10 \text{ m}\cdot\text{s}^{-1}$ TOWARDS a stationary sound source. The sound source emits sound waves of frequency 880 Hz . A sound detector **A** is attached to the car and another sound detector **B** is attached to the sound source. Detector **B** detects the sound waves reflected from the car. The speed of sound in air is $340 \text{ m}\cdot\text{s}^{-1}$.



- 14.1 State the *Doppler effect* in words. (2)
- 14.2 Calculate the wavelength of the sound waves emitted by the source. (3)
- 14.3 Calculate the frequency of the sound waves detected by detector **A**. (4)

The sketch graph below shows the frequency recorded by detector **A**.

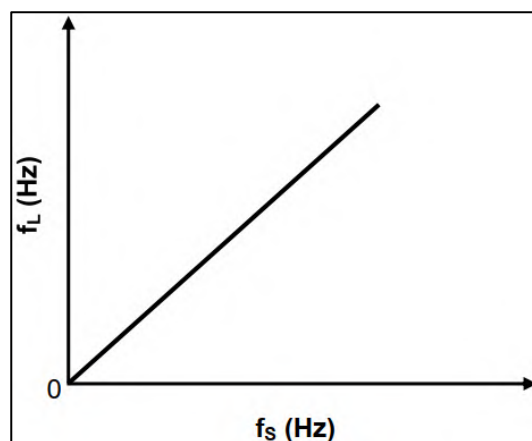


- 14.4 Redraw the graph above for detector **A** in your ANSWER BOOK. On the same set of axes, sketch the graph of the frequency recorded by detector **B**. Label this graph as **B**. (2)

[11]

QUESTION 15

A learner investigates the relationship between the observed frequency and the frequency of sound waves emitted by a stationary source. The learner moves towards the source at a constant velocity and records the observed frequency (f_L) for a given source frequency (f_s). This process is repeated for different frequencies of the source, with the learner moving at the same constant velocity each time. The graph shows how the observed frequency changes as the frequency of sound waves emitted by the source changes.

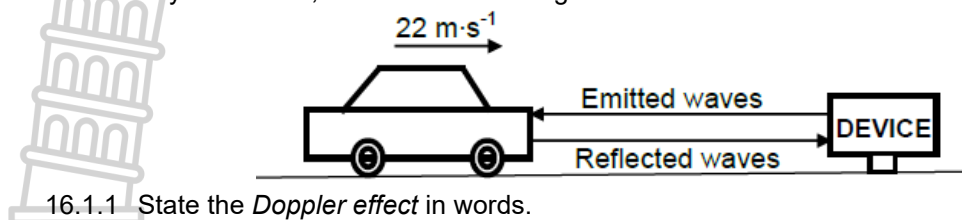


- 15.1 Name the phenomenon illustrated by the graph. (1)
- 15.2 Name ONE application in the medical field of the phenomenon in QUESTION 15.1. (1)
- 15.3 Write down the type of proportionality that exists between f_L and f_s as illustrated by the graph. (1)
- 15.4 The gradient of the graph obtained is found to be 1,06. If the speed of sound in air is $340 \text{ m}\cdot\text{s}^{-1}$, calculate the magnitude of the velocity at which the learner approaches the source. (5)
- 15.5 The investigation is now repeated with the learner moving at a HIGHER constant velocity towards the sound source. Copy the graph in your ANSWER BOOK and label it as **A**. On the same set of axes, sketch the graph that will be obtained when the learner is moving at the HIGHER velocity. Label this graph as **B**. (2)

[10]

QUESTION 16 (June 2023)

- 16.1 A car moves at a constant velocity of $22 \text{ m}\cdot\text{s}^{-1}$ on a straight horizontal road TOWARDS a stationary device, which can both emit and detect sound waves. The device emits sound waves with a frequency of $24\,000 \text{ Hz}$. These sound waves are reflected off the car and the reflected sound waves are then detected by the device, as shown in the diagram below.



- 16.1.1 State the *Doppler effect* in words. (2)
 16.1.2 If the speed of sound in air is $340 \text{ m}\cdot\text{s}^{-1}$, calculate the frequency of the reflected sound waves detected by the device. (6)
 16.2 The spectral lines observed for a distant star show that the star is moving away from Earth. Explain, by referring to frequency, how one can deduce that the star is moving away from Earth. (2)

[10]

QUESTION 17 (November 2023)

- 17.1 An ambulance is moving away from a stationary listener with a constant velocity of $25 \text{ m}\cdot\text{s}^{-1}$. The siren of the ambulance emits sound waves at a frequency of 550 Hz . The listener detects the frequency of these sound waves to be $512,64 \text{ Hz}$. Ignore the effects of wind.
 17.1.1 State the *Doppler effect* in words. (2)
 17.1.2 Use the given information to calculate the speed of sound in air. (5)
 17.1.3 If the ambulance moves away from the stationary listener with a constant velocity which is greater than $25 \text{ m}\cdot\text{s}^{-1}$, how will EACH of the following change? Choose from INCREASES, DECREASES or REMAINS THE SAME.
 (a) The speed of sound in air (1)
 (b) The frequency of the sound waves emitted by the siren (1)
 (c) The frequency of the sound waves detected by the listener (1)
 17.2 The spectrum of a distant star when viewed from the Earth is red shifted.
 17.2.1 Is the star moving AWAY FROM or TOWARDS the Earth? (1)
 17.2.2 Use the Doppler effect to explain the answer to QUESTION 17.2.1. (2)

[13]

QUESTION 18 (June 2024)

A stationary listener records the frequency of the sound emitted by the siren of a police car. When the car, which is travelling at a constant velocity of $26 \text{ m}\cdot\text{s}^{-1}$, approaches the listener, the recorded frequency is 615 Hz . The car passes the listener at time t_1 and then moves away from the listener. The recorded frequency now is 526 Hz . Ignore the effects of wind.

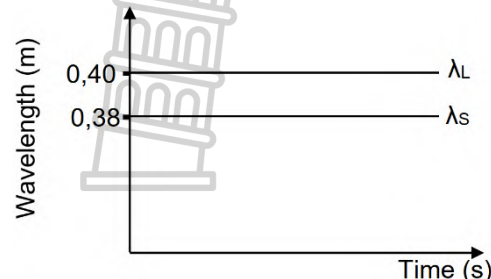
- 18.1 State the *Doppler effect* in words. (2)
 18.2 Use the information given to calculate the speed of sound in air. (5)
 18.3 Calculate the wavelength of the sound emitted by the police siren. (4)
 18.4 Sketch the graph of recorded frequency versus time for the motion of the car as it moved towards the listener, passed the listener and then moved away from the listener. Label time t_1 on the graph. No values need to be indicated on the frequency axis. (3)

[14]

QUESTION 19 (November 2024)

A stationary listener, standing on the roadside, records the wavelength of the sound emitted by the siren of a police car travelling at a constant velocity. In the wavelength-time graph on the right, NOT drawn to scale, λ_L is the wavelength of the sound recorded by the listener and λ_S is the wavelength of the sound emitted by the siren. Take the speed of sound in air to be $343 \text{ m}\cdot\text{s}^{-1}$.

- 19.1 Name the phenomenon that explains why the wavelengths shown in the graph differ. (1)
 19.2 Is the car moving TOWARDS or AWAY FROM the listener? Give a reason for the answer. (2)
 19.3 Calculate the:
 19.3.1 Frequency of the sound emitted by the siren (3)
 19.3.2 Magnitude of the velocity of the car (6)



[12]

ELECTROSTATICS

Study of charges at rest

Neutral (uncharged):

Number of protons = number of electrons

Quantisation of charge

All charges are multiples of the smallest charge i.e. the charge on one electron:

$$q_e = 1,6 \times 10^{-19} \text{ C} \quad (Q = nq_e)$$

Conservation of charge

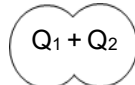
Charge cannot be created or destroyed. It can only be transferred from one object to another.

Charge on each of two identical conductors

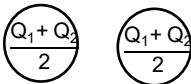
Two identical conductors, each carrying a charge, will during contact share the charges equally and after separation they will have the same charge.



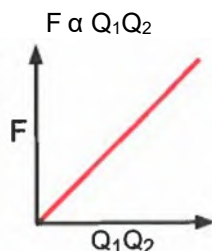
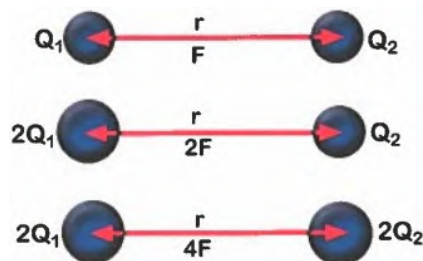
Two identical spheres with charges Q_1 and Q_2 respectively.



The spheres are allowed to touch.



The spheres are separated again.

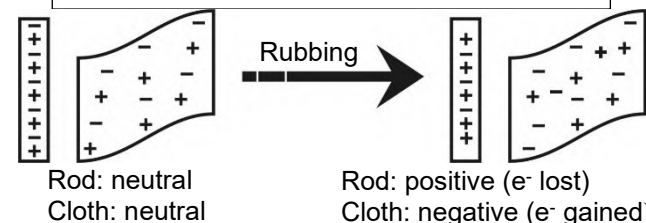


ELECTROSTATICS

Two kinds of charge
Positive: electron deficient
Negative: excess of electrons

Charging of objects

By contact: Electrons transferred from one object to another.



Unit of charge: coulomb (C)

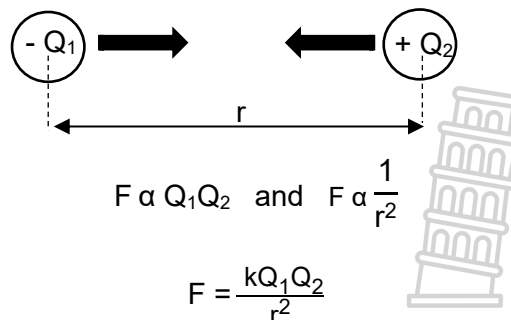
1 C is a huge charge. Charges we encounter are much smaller. Therefore, smaller units of charge are used as given below:

1 C	1 coulomb	
1 mC	1 millicoulomb	$1 \times 10^{-3} \text{ C}$
1 μC	1 microcoulomb	$1 \times 10^{-6} \text{ C}$
1 nC	1 nanocoulomb	$1 \times 10^{-9} \text{ C}$
1 pC	1 picocoulomb	$1 \times 10^{-12} \text{ C}$

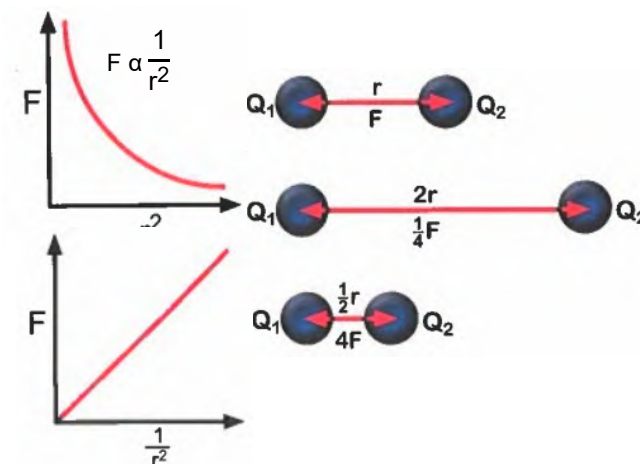
Electrostatic force

Like charges repel; Unlike charges attract

Coulomb's law



F : electrostatic force in newton (N)
 k : Coulomb's constant ($9 \times 10^9 \text{ N} \cdot \text{m}^2 \cdot \text{C}^{-2}$)
 Q_1 : point charge 1 in coulomb (C)
 Q_2 : point charge 2 in coulomb (C)
 r : distance between two charges in meter (m)



Electric field

- Definition:** The electric field at a point is the electrostatic **force experienced per unit positive charge** placed at that point.
- In symbols:** $E = \frac{F}{q}$
- Unit:** $N \cdot C^{-1}$
- Vector quantity** with magnitude and direction
- Direction away from positive towards negative charge**

Electric field

Region in space where an electric charge will experience a force.

At a certain distance from a point charge:

$$E = \frac{kQ}{r^2}$$

E: electric field
 k: Coulomb's constant ($9 \times 10^9 N \cdot m^2 \cdot C^{-2}$)
 Q: charge in coulomb (C)
 r: distance from point charge (m)

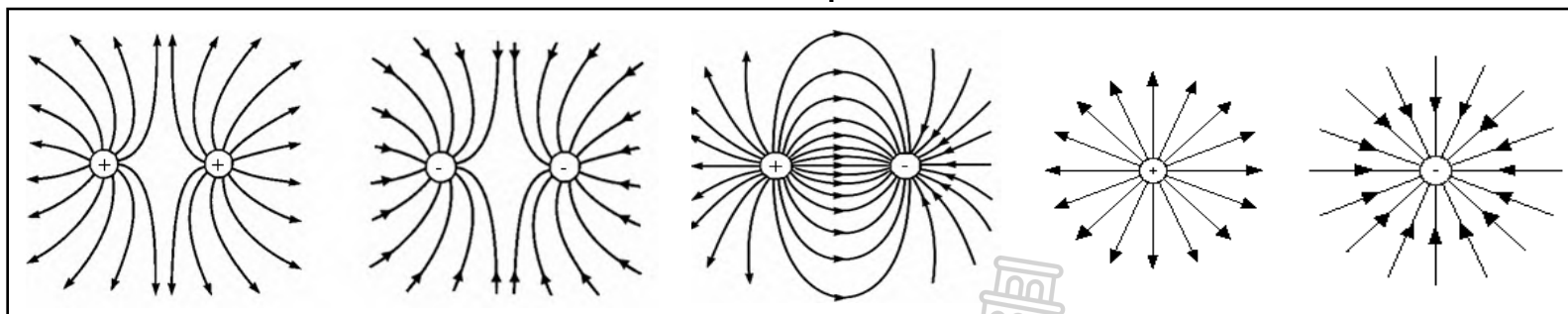
$E \propto Q$

$E \propto \frac{1}{r^2}$

$E \propto \frac{1}{r^2}$

Represented by field lines.

Electric field patterns

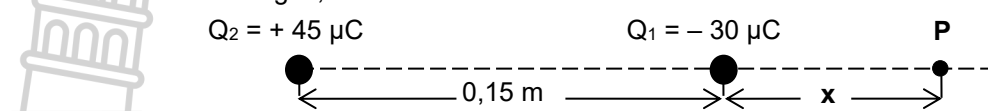


TERMS AND DEFINITIONS	
Coulomb's law	The magnitude of the electrostatic force exerted by one point charge on another point charge is directly proportional to the product of the magnitudes of the charges and inversely proportional to the square of the distance between them. In symbols: $F = \frac{kQ_1Q_2}{r^2}$
Electric field	A region of space in which an electric charge experiences a force.
Electric field at a point	The electric field at a point is the electrostatic force experienced per unit positive charge placed at that point. In symbols: $E = \frac{F}{q}$ Unit: $N \cdot C^{-1}$
Direction of electric field	The direction of the electric field at a point is the direction that a positive test charge would move if placed at that point.

Typical questions

QUESTION 1

- 1.1 Define *electric field at a point* in words. (2)
- 1.2 Draw the electric field pattern for two identical positively charged spheres placed close to each other. (3)
- 1.3 A $-30\ \mu\text{C}$ point charge, Q_1 , is placed at a distance of $0,15\ \text{m}$ from a $+45\ \mu\text{C}$ point charge, Q_2 , in space, as shown in the diagram below. The net electric field at point P , which is on the same line as the two charges, is zero.



Calculate x , the distance of point P from charge Q_1 .

(5)

[10]

QUESTION 2

In the diagram, Q_1 , Q_2 and Q_3 are three stationary point charges placed along a straight line. The distance between Q_1 and Q_2 is $1,5\ \text{m}$ and that between Q_2 and Q_3 is $1\ \text{m}$, as shown in the diagram.



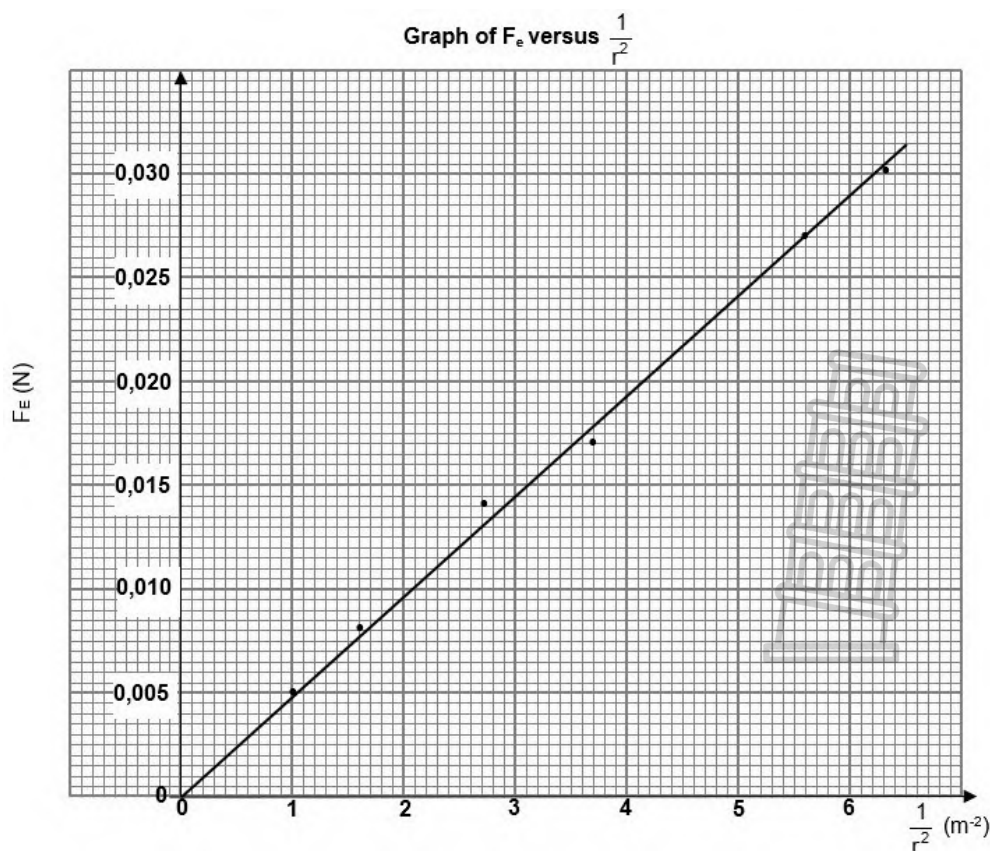
- 2.1 State Coulomb's law in words. (2)
- 2.2 The magnitude of charges Q_1 and Q_2 are unknown. The charge on Q_1 is positive. The charge on Q_3 is $+2 \times 10^{-6}\ \text{C}$ and it experiences a net electrostatic force of $0,3\ \text{N}$ to the left. (2)
- 2.2.1 Write down the sign (POSITIVE or NEGATIVE) of charge Q_2 . (2)
- 2.2.2 Charge Q_2 is now removed. The magnitude of the electrostatic force experienced by charge Q_3 due to Q_1 now becomes $0,012\ \text{N}$. Calculate the magnitudes of the unknown charges Q_1 and Q_2 . (7)

(7)

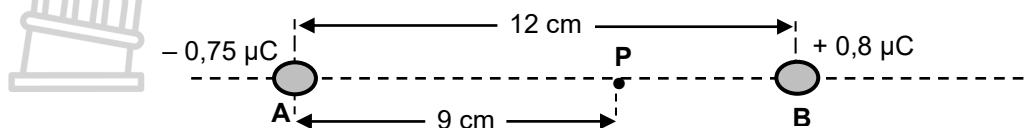
[11]

QUESTION 3

- 3.1 In an experiment to verify the relationship between the electrostatic force, F_E , and distance, r , between two **identical**, positively charged spheres, the graph below was obtained.



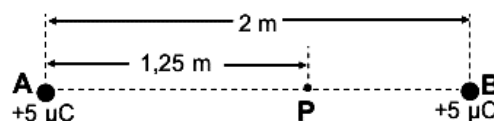
- 3.1.1 State Coulomb's law in words. (2)
- 3.1.2 Write down the dependent variable of the experiment. (1)
- 3.1.3 What relationship between the electrostatic force F_E and the square of the distance, r^2 , between the charged spheres can be deduced from the graph? (1)
- 3.1.4 Use the information in the graph to calculate the charge on each sphere. (6)
- 3.2 A charged sphere, **A**, carries a charge of $-0,75 \mu\text{C}$. (2)
- 3.2.1 Draw a diagram showing the electric field lines surrounding sphere **A**. (2)
- Sphere **A** is placed 12 cm away from another charged sphere, **B**, along a straight line in a vacuum, as shown below. Sphere **B** carries a charge of $+0,8 \mu\text{C}$. Point **P** is located 9 cm to the right of sphere **A**.



- 3.2.2 Calculate the magnitude of the net electric field at point **P**. (5)
- [17]

QUESTION 4

Two small identical spheres, **A** and **B**, each carrying a charge of $+5 \mu\text{C}$, are placed 2 m apart. Point **P** is in the electric field due to the charged spheres and is located 1,25 m from sphere **A**. Study the diagram.



- 4.1 Describe the term *electric field*. (2)
- 4.2 Draw the resultant electric field pattern due to the two charged spheres. (3)
- 4.3 Calculate the magnitude of the net electric field at point **P**. (5)
- [10]

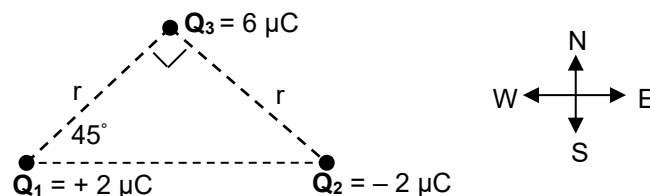
QUESTION 5

- 5.1 A metal sphere **A**, suspended from a wooden beam by means of a non-conducting string, has a charge of $+6 \mu\text{C}$.

5.1.1 Were electrons ADDED TO or REMOVED FROM the sphere to obtain this charge? Assume that the sphere was initially neutral. (1)

5.1.2 Calculate the number of electrons added to or removed from the sphere. (3)

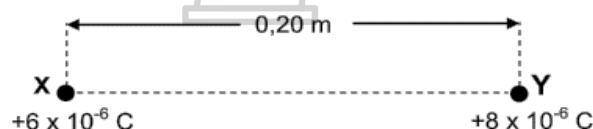
- 5.2 Point charges Q_1 , Q_2 and Q_3 are arranged at the corners of a right-angled triangle, as shown in the diagram. The charges on Q_1 and Q_2 are $+2 \mu\text{C}$ and $-2 \mu\text{C}$ respectively and the magnitude of the charge on Q_3 is $6 \mu\text{C}$. The distance between Q_1 and Q_3 is r . The distance between Q_2 and Q_3 is also r . The charge Q_3 experiences a resultant electrostatic force of $0,12 \text{ N}$ to the west.



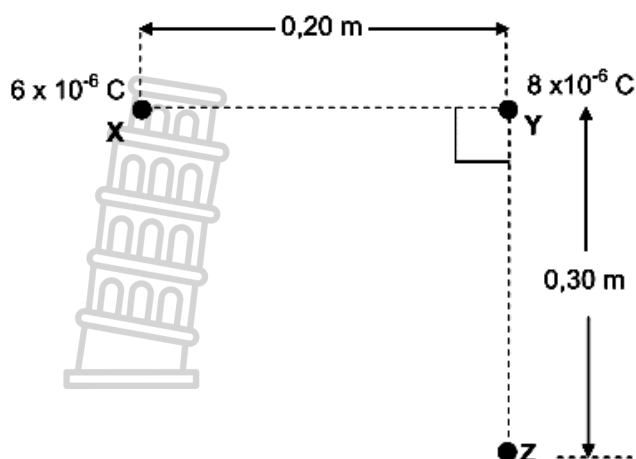
- 5.2.1 Without calculation, identify the sign (positive or negative) on the charge Q_3 . (1)
- 5.2.2 Draw a vector diagram to show the electrostatic forces acting on Q_3 due to charges Q_1 and Q_2 respectively. (2)
- 5.2.3 Write down an expression, in terms of r , for the horizontal component of the electrostatic force exerted on Q_3 by Q_1 . (3)
- 5.2.4 Calculate the distance r . (4)
- 5.3 The magnitude of the electric field is $100 \text{ N}\cdot\text{C}^{-1}$ at a point which is $0,6 \text{ m}$ away from a point charge Q . (2)
- 5.3.1 Define the term *electric field at a point* in words. (2)
- 5.3.2 Calculate the distance from point charge Q at which the magnitude of the electric field is $50 \text{ N}\cdot\text{C}^{-1}$. (5)
- [21]

QUESTION 6

Two small spheres, **X** and **Y**, carrying charges of $+6 \times 10^{-6} \text{ C}$ and $+8 \times 10^{-6} \text{ C}$ respectively, are placed $0,20 \text{ m}$ apart in air.



- 6.1 State *Coulomb's law* in words. (2)
- 6.2 Calculate the magnitude of the electrostatic force experienced by charged sphere **X**. (4)

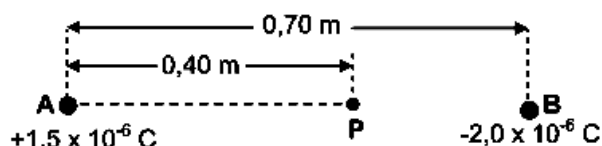


A third sphere, **Z**, of unknown negative charge, is now placed at a distance of 0,30 m below sphere **Y**, in such a way that the line joining the charged spheres **X** and **Y** is perpendicular to the line joining the charged spheres **Y** and **Z**, as shown in the diagram alongside.

- 6.3 Draw a vector diagram showing the directions of the electrostatic forces and the net force experienced by charged sphere **Y** due to the presence of charged spheres **X** and **Z** respectively. (3)
- 6.4 The magnitude of the net electrostatic force experienced by charged sphere **Y** is 15,20 N. Calculate the charge on sphere **Z**. (4)
- [13]

QUESTION 7

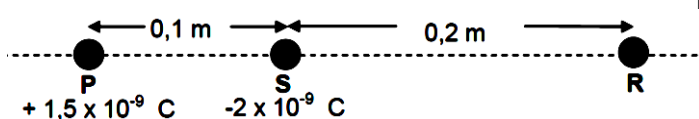
A and **B** are two small spheres separated by a distance of 0,70 m. Sphere **A** carries a charge of $+1,5 \times 10^{-6} \text{ C}$ and sphere **B** carries a charge of $-2,0 \times 10^{-6} \text{ C}$. **P** is a point between spheres **A** and **B** and is 0,40 m from sphere **A**, as shown in the diagram.



- 7.1 Define the term *electric field at a point*. (2)
- 7.2 Calculate the magnitude of the net electric field at point **P**. (4)
- 7.3 A point charge of magnitude $3,0 \times 10^{-9} \text{ C}$ is now placed at point **P**. Calculate the magnitude of the electrostatic force experienced by this charge. (3)
- [9]

QUESTION 8

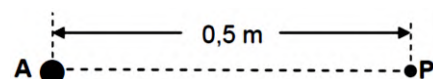
Two point charges, **P** and **S**, are placed a distance 0,1 m apart. The charge on **P** is $+1,5 \times 10^{-9} \text{ C}$ and that on **S** is $-2 \times 10^{-9} \text{ C}$. A third point charge, **R**, with an unknown positive charge, is placed 0,2 m to the right of point charge **S**, as shown in the diagram.



- 8.1 State *Coulomb's law* in words. (2)
- 8.2 Draw a labelled force diagram showing the electrostatic forces acting on **R** due to **P** and **S**. (2)
- 8.3 Calculate the magnitude of the charge on **R**, if it experiences a net electrostatic force of $1,27 \times 10^{-6} \text{ N}$ to the left. Take forces directed to the right as positive. (7)
- [11]

QUESTION 9

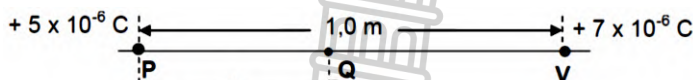
P is a point 0,5 m from charged sphere **A**. The electric field at **P** is $3 \times 10^7 \text{ N} \cdot \text{C}^{-1}$ directed towards **A**. Refer to the diagram.



- 9.1 Draw the electric field pattern due to charged sphere **A**. Indicate the sign of the charge on the sphere in your diagram. (2)
- 9.2 Calculate the magnitude of the charge on sphere **A**. (3)
- 9.3 Another charged sphere, **B**, having an excess of 10^5 electrons, is now placed at point **P**. Calculate the electrostatic force experienced by sphere **B**. (6)
- [11]

QUESTION 10

A particle, **P**, with a charge of $+5 \times 10^{-6} \text{ C}$, is located 1,0 m along a straight line from particle **V**, with a charge of $+7 \times 10^{-6} \text{ C}$. Refer to the diagram. A third charged particle, **Q**, at a point **x** metres away from **P**, as shown above, experiences a net electrostatic force of zero newton.



- 10.1 How do the electrostatic forces experienced by **Q** due to the charges on **P** and **V** respectively, compare with each other? (2)
- 10.2 State *Coulomb's law* in words. (2)
- 10.3 Calculate the distance **x**. (5)
- [9]

QUESTION 11

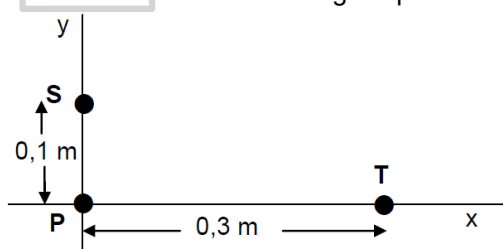
A small metal sphere **Y** carries a charge of $+6 \times 10^{-6} \text{ C}$.

- 11.1 Draw the electric field pattern associated with sphere **Y**. (2)
- 11.2 8×10^{13} electrons are now transferred to sphere **Y**. Calculate the electric field at a point 0,5 m from the sphere. (7)
- [9]

QUESTION 12

Three small identical metal spheres, **P**, **S** and **T**, on insulated stands, are **initially neutral**. They are then charged to carry charges of $-15 \times 10^{-9} \text{ C}$, **Q** and $+2 \times 10^{-9} \text{ C}$ respectively, as shown. The charged spheres are brought together so that all three spheres touch each other at the same time, and are then separated. The charge on each sphere, after separation, is $-3 \times 10^{-9} \text{ C}$.

- 12.1 Determine the value of charge **Q**. (2)
 12.2 Draw the electric field pattern associated with the charged spheres, **S** and **T**, after they are separated and returned to their original positions. (3)



The spheres, each with the **new charge** of $-3 \times 10^{-9} \text{ C}$, are now placed at points on the x-axis and the y-axis, as shown in the diagram, with sphere **P** at the origin.

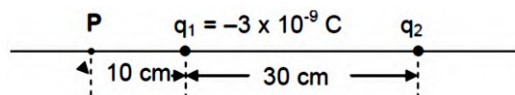
- 12.3 State *Coulomb's law* in words. (2)
 12.4 Calculate the magnitude of the net electrostatic force acting on sphere **P**. (5)
 12.5 Calculate the magnitude of the net electric field at the origin due to charges **S** and **T**. (3)
 12.6 ONE of the charged spheres, **P** and **T**, experienced a very small increase in mass after it was charged initially.

- 12.6.1 Which sphere, **P** or **T**, experienced this very small increase in mass? (1)
 12.6.2 Calculate the increase in mass by the sphere in QUESTION 12.6.1. (3)

[19]

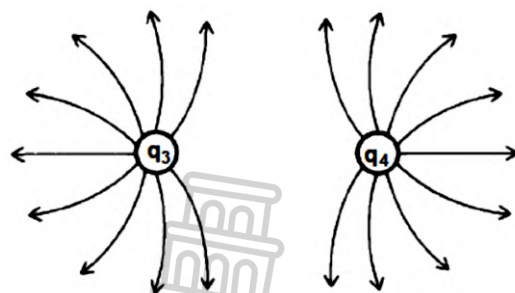
QUESTION 13 (June 2019)

Two point charges, q_1 and q_2 , are placed 30 cm apart along a straight line. Charge $q_1 = -3 \times 10^{-9} \text{ C}$. Point **P** is 10 cm to the left of q_1 , as shown in the diagram below. The **net** electrostatic field at point **P** is **zero**.



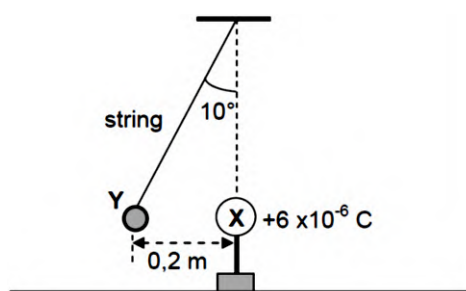
- 13.1 Define the term *electric field at a point*. (2)
 13.2 State, giving reasons, whether point charge q_2 is POSITIVE or NEGATIVE. (3)
 13.3 Calculate the magnitude of charge q_2 . (4)
 13.4 State *Coulomb's law* in words. (2)
 13.5 Calculate the magnitude of the electrostatic force exerted by charge q_1 on charge q_2 . (3)
 13.6 The two charges are now brought into contact with each other and are then separated. A learner draws the electric field pattern for the new charges q_3 and q_4 after contact, as shown on the right. Is the diagram CORRECT? Give a reason for the answer. (2)

[16]



QUESTION 14

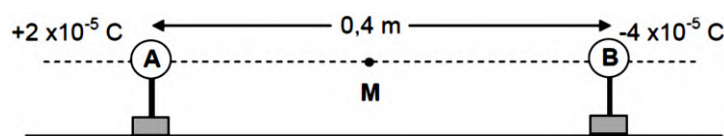
- 14.1 A small sphere, **Y**, carrying an unknown charge, is suspended at the end of a light inextensible string which is attached to a fixed point. Another sphere, **X**, carrying a charge of $+6 \times 10^{-6} \text{ C}$, on an insulated stand is brought close to sphere **Y**.



Sphere **Y** experiences an electrostatic force and comes to rest 0,2 m away from sphere **X**, with the string at an angle of 10° with the vertical, as shown in the diagram.

- 14.1.1 What is the nature of the charge on sphere **Y**? Choose from POSITIVE or NEGATIVE. (1)
 14.1.2 Calculate the magnitude of the charge on sphere **Y** if the magnitude of the electrostatic force acting on it is 3,05 N. (3)
 14.1.3 Draw a labelled free-body diagram for sphere **Y**. (3)
 14.1.4 Calculate the magnitude of the tension in the string. (3)

- 14.2 Two small charged spheres, **A** and **B**, on insulated stands, with charges $+2 \times 10^{-5} \text{ C}$ and $-4 \times 10^{-5} \text{ C}$ respectively, are placed 0,4 m apart, as shown in the diagram below. **M** is the midpoint between spheres **A** and **B**.



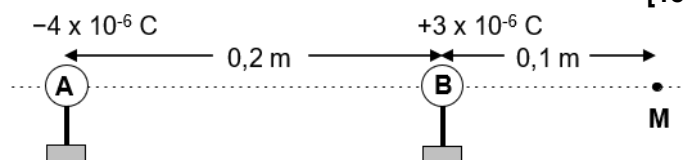
14.2.1 Define the term *electric field at a point*. (2)

14.2.2 Calculate the net electric field at point **M**. (6)

[18]

QUESTION 15

Two small, charged spheres, **A** and **B**, are placed on insulated stands, 0,2 m apart, as shown in the diagram. They carry charges of $-4 \times 10^{-6} \text{ C}$ and $+3 \times 10^{-6} \text{ C}$ respectively.



M is a point that is a distance of 0,1 m to the right of sphere **B**.

15.1 Calculate the number of electrons in excess on sphere **A**. (3)

15.2 Calculate the magnitude of the electrostatic force exerted by sphere **A** on sphere **B**. (3)

15.3 Describe the term *electric field*. (2)

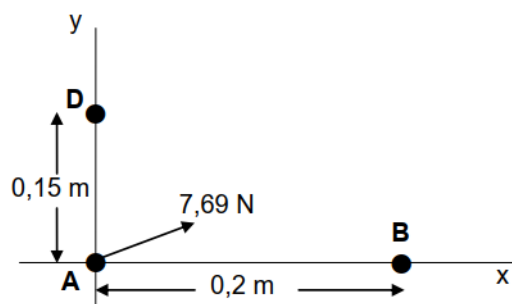
15.4 Calculate the magnitude of the net electric field at point **M**. (5)

Charged spheres **A** and **B** and another charged sphere **D** are now arranged along a rectangular system of axes, as shown in the diagram. The net electrostatic force experienced by sphere **A** is 7,69 N in the direction as shown in the diagram.

15.5 Is the charge on sphere **D** POSITIVE or NEGATIVE? (1)

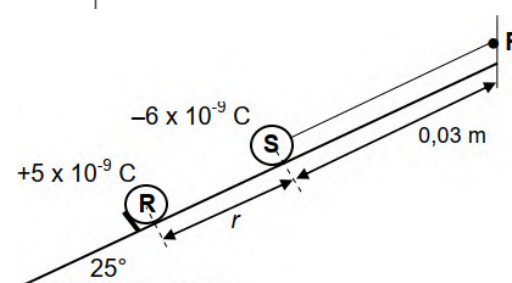
15.6 Calculate the magnitude of the charge on sphere **D**. (3)

[17]



QUESTION 16

Two charged spheres, **R** and **S**, are both stationary on a smooth, insulated surface inclined at an angle of 25° to the horizontal. Sphere **S**, of mass 0,01 kg and carrying a charge of $-6 \times 10^{-9} \text{ C}$, is connected to a 0,03 m long, light inextensible string attached to point **P** at the top of the incline. Sphere **R**, carrying a charge of $+5 \times 10^{-9} \text{ C}$, is held such that the distance between the centres of the spheres is r , as shown in the diagram. Ignore the effects of friction. Sphere **R** exerts an electrostatic force of magnitude $1,2 \times 10^{-3} \text{ N}$ on sphere **S**.



16.1 State *Coulomb's law* in words. (2)

16.2 Calculate the distance r between the spheres. (3)

16.3 Draw a labelled free-body diagram for sphere **S**. (4)

16.4 Calculate the:

16.4.1 Tension in the string (4)

16.4.2 Net electric field at point **P** (5)

[18]

QUESTION 17

17.1 A small neutral sphere acquires a charge of $-1,95 \times 10^{-6} \text{ C}$.

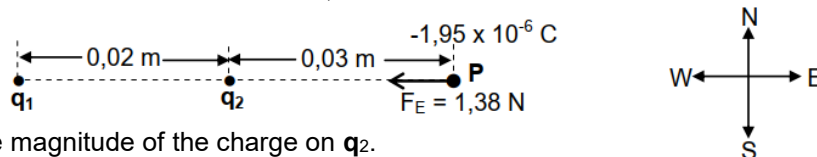
17.1.1 Were electrons ADDED TO or REMOVED FROM the sphere? (1)

17.1.2 Calculate the number of electrons which were added or removed. (3)

17.1.3 Define the term *electric field at a point*. (2)

17.1.4 Calculate the magnitude of the electric field at a point 0,5 m from the centre of the charged sphere. (3)

- 17.2 Two point charges, q_1 and q_2 , are fixed 0,02 m apart. The magnitude of charges q_1 and q_2 is the same and q_1 is NEGATIVELY charged. The small charged sphere with the charge of $-1,95 \times 10^{-6}$ C is placed at point P, 0,03 m east of charge q_2 , as shown in the diagram below. The sphere at point P experiences a net electrostatic force of 1,38 N west.



Calculate the magnitude of the charge on q_2 .

(5)
[14]

QUESTION 18

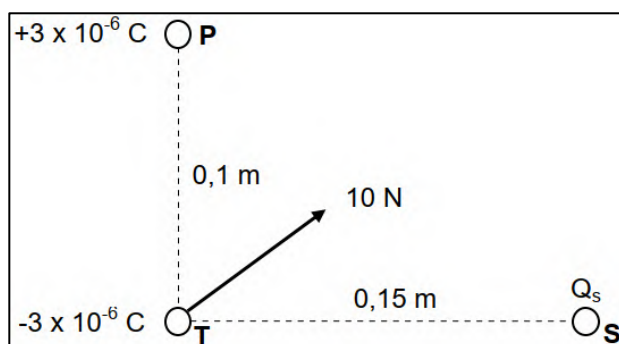
- 18.1 Two small, identical spheres, P and T, are placed a distance of 0,1 m apart, as shown in the diagram on the right. P carries a charge of $+3 \times 10^{-6}$ C and T carries a charge of -3×10^{-6} C.

18.1.1 State *Coulomb's law* in words.

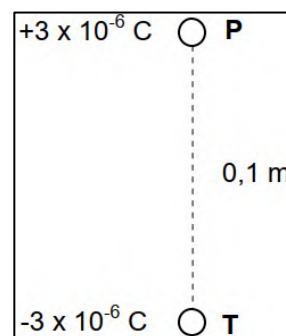
18.1.2 Draw the resultant electric field pattern due to the charges on P and T.

(2)

(3)



A third charged sphere S of unknown charge Q_s is placed a distance of 0,15 m from sphere T such that the three charged spheres are at the vertices of a right-angled triangle. The net electrostatic force on sphere T, due to the other two charged spheres, has a magnitude of 10 N, as shown in the diagram on the left.



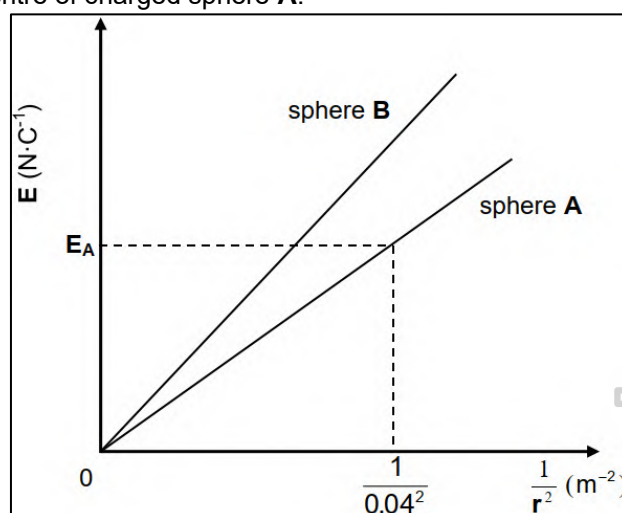
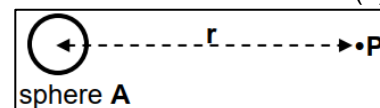
18.1.3 Is charge Q_s POSITIVE or NEGATIVE?

18.1.4 Calculate the number of electrons added to or removed from sphere S to give it a charge of Q_s .

(1)

(6)

- 18.2 P is a variable point in the electric field of charged sphere A and r is the distance between point P and the centre of sphere A. See the diagram on the right. A learner determines the magnitude of the electric field (E) at point P for different values of r . Sphere A is then replaced by another sphere, B, of a different charge. Another set of results are obtained. The graphs below are obtained from the results for sphere A and sphere B. E_A is the magnitude of the electric field at a distance of 0,04 m from the centre of charged sphere A.



Use the graphs to answer the following questions.

18.2.1 State the proportionality between the magnitude of electric field E at a point and $\frac{1}{r^2}$.

18.2.2 Calculate E_A if the numerical value of the gradient of the graph for sphere A is 680.

18.2.3 How does the magnitude of the charge on sphere B compare to the magnitude of the charge on sphere A? Choose from GREATER THAN, SMALLER THAN or EQUAL TO. Give a reason for the answer.

(1)

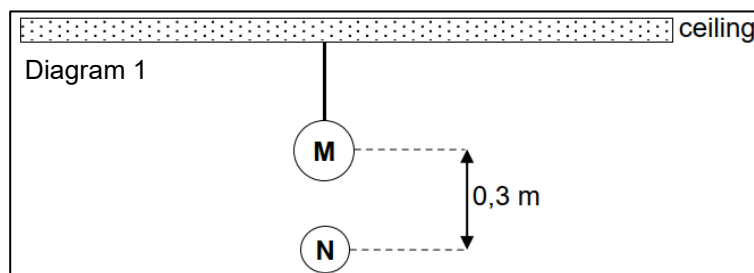
(4)

(3)

[20]

QUESTION 19

A charged sphere **M** is suspended from a ceiling by a light inextensible, insulated string. Another charged sphere **N**, of mass $2,04 \times 10^{-3} \text{ kg}$ and carrying a charge of $+8,6 \times 10^{-8} \text{ C}$, hangs STATIONARY vertically below sphere **M**. The centres of the spheres are $0,3 \text{ m}$ apart, as shown in diagram 1.



19.1 State *Coulomb's law* in words. (2)

19.2 State whether the charge on sphere **M** is POSITIVE or NEGATIVE. (1)

19.3 Draw a labelled free-body diagram for sphere **N**. (2)

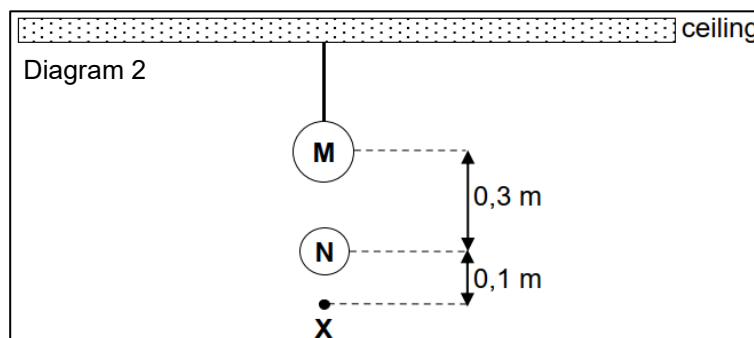
19.4 Calculate the magnitude of the charge on sphere **M**. (5)

19.5 How does the electrostatic force that sphere **M** exerts on sphere **N** compare to that exerted by sphere **N** on sphere **M** with respect to:

19.5.1 Magnitude (1)

19.5.2 Direction

19.6 Point **X** is $0,1 \text{ m}$ vertically below the centre of sphere **N**, as shown in diagram 2. Calculate the net electric field at point **X**. (5)



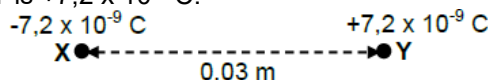
(1)

(5)

[17]

QUESTION 20 (June 2023)

Two point charges, **X** and **Y**, are held $0,03 \text{ m}$ apart, as shown in the diagram below. The charge of **X** is $-7,2 \times 10^{-9} \text{ C}$, while the charge of **Y** is $+7,2 \times 10^{-9} \text{ C}$.

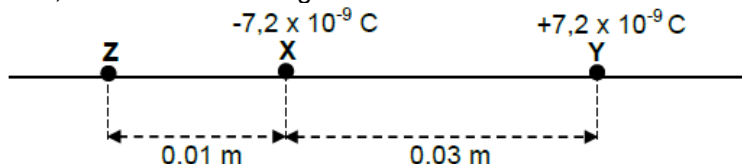


20.1 State *Coulomb's law* in words. (2)

20.2 Draw the net electric field pattern due to the two point charges. (3)

20.3 Calculate the magnitude of the electrostatic force that **Y** exerts on **X**. (3)

A third point charge, **Z**, of unknown positive charge, is positioned $0,01 \text{ m}$ to the left of point charge **X** on the line joining point charges **X** and **Y**, as shown in the diagram below.



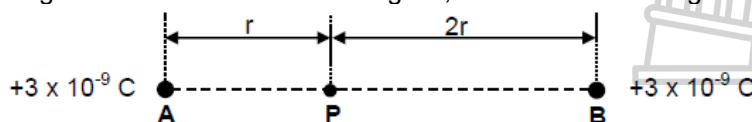
20.4 Draw a labelled vector diagram to show the directions of the electric fields at the point where **X** is positioned. (2)

20.5 The magnitude of the resultant electric field at the point where **X** is positioned is $4,91 \times 10^5 \text{ N}\cdot\text{C}^{-1}$. Calculate the magnitude of charge **Z**. (5)

[15]

QUESTION 21 (November 2023)

Two point charges, **A** and **B**, each with a charge of $+3 \times 10^{-9} \text{ C}$, are stationary on a horizontal surface. Point **P** is r metres from charge **A** and $2r$ metres from charge **B**, as shown in the diagram below.



21.1 Describe an *electric field*. (2)

21.2 Draw the resultant electric field pattern due to charges **A** and **B**. (3)

The magnitude of the net electric field at point **P** is $27 \text{ N}\cdot\text{C}^{-1}$.

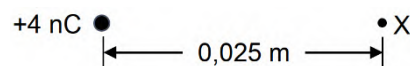
21.3 Calculate the value of r . (5)

21.4 Calculate the magnitude of the net electrostatic force that an electron would experience if placed at point **P**. (3)

[13]

QUESTION 22 (June 2024)

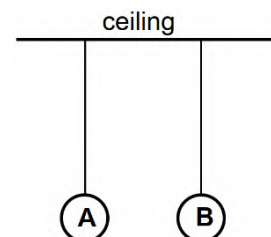
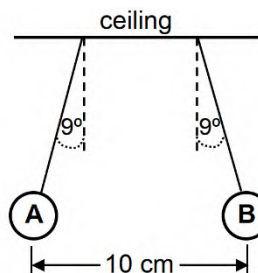
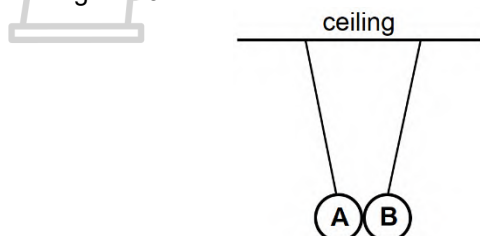
22.1 **X** is a point 0,025 m away from a +4 nC point charge. See the diagram on the right.



22.1.1 Draw the electric field pattern due to the +4 nC charge. (2)

22.1.2 Calculate the magnitude of the electric field at point **X**. (3)

22.2 Two identical neutral polystyrene balls **A** and **B** are suspended from a ceiling by insulated, light inextensible strings of equal length, as shown in the diagram on the right. Ball **B** is then given an initial negative charge, Q_B , of unknown magnitude. The balls attract each other, touch and then repel each other. The balls come to rest with their centres 10 cm apart so that each string makes an angle of 9° with the vertical. See the diagrams below.



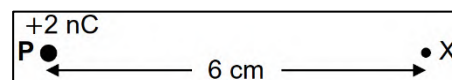
22.2.1 State *Coulomb's Law* in words. (2)

22.2.2 Calculate the magnitude of the initial charge Q_B given to ball **B** if the mass of each ball was 0,012 kg. (6)

[13]

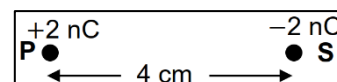
QUESTION 23 (November 2024)

P is a +2 nC point charge. **X** is a point 6 cm away from charge **P**, as shown in the first diagram on the right.

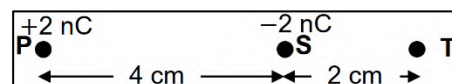


23.1 Calculate the magnitude of the electric field at **X**. (3)

23.2 Point charge **S**, with a charge of -2 nC, is placed 4 cm to the right of charge **P**, as shown in the second diagram on the right. Draw the resultant electric field pattern due to charges **P** and **S**. (3)



23.3 A third point charge **T** is placed 2 cm to the right of **S**, as shown in the third diagram on the right. Point charge **T** experiences a net electrostatic force of $2,5 \times 10^{-4}$ N to the left.



23.3.1 State *Coulomb's law* in words. (2)

23.3.2 What is the polarity of charge **T**? Choose from POSITIVE or NEGATIVE. (1)

23.3.3 Calculate the magnitude of charge **T**. (5)

[14]

ELECTRIC CIRCUITS

TERMS AND DEFINITIONS	
Ohm's law	The potential difference across a conductor is directly proportional to the current in the conductor at constant temperature. In symbols: $R = \frac{V}{I}$ The units: $\Omega = V \cdot A^{-1}$
Emf	Maximum energy provided by a battery per unit charge passing through it.
Terminal potential difference	The energy transferred or the work done per coulomb of charge passing through the battery when the battery delivers a current. (It is the potential difference across the terminals of a battery when there IS a current in the circuit.)
Ohmic conductors	A conductor that obeys Ohm's law, i.e., the ratio of potential difference to current remains constant. (Resistance of the conductor remains constant.)
Non-ohmic conductors	A conductor that does NOT obey Ohm's law, i.e., the ratio of potential difference to current does NOT remain constant. (Resistance of the conductor increases as the current increases, e.g. a bulb.)
Potential difference	Potential difference is the amount of work done (or energy transferred) per coulomb of charge. It is measured in volt (V). In symbols: $V = \frac{W}{Q}$ The units: $V = J \cdot C^{-1}$
Current	Current is the rate of flow of charge. It is measured in ampere (A). In symbols: $I = \frac{Q}{\Delta t}$ The units: $A = C \cdot s^{-1}$
Resistance	Resistance is the opposition to the flow of charge (electric current). It is measured in ohm (Ω) and can be calculated by using the ratio of potential difference (V) to current (I). In symbols: $R = \frac{V}{I}$ The units: $\Omega = V \cdot A^{-1}$
Resistors in series	The total resistance of resistors in series is given by: $R_T = R_1 + R_2 + \dots \text{ OR } R_S = R_1 + R_2 + \dots$
Resistors in parallel	The effective resistance (do NOT use the word "total") of resistors in parallel is given by: $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$
Internal resistance	The resistance within a battery that causes a drop in the potential difference across the battery when there is a current in the circuit.
Power	Power is the rate at which work is done. In symbols: $P = \frac{W}{\Delta t}$ The units: $W = J \cdot s^{-1}$
	Other formulae for power: $P = VI$ $P = I^2 R$ $P = \frac{V^2}{R}$
kilowatt hour (kWh) (This is an energy unit related to the formula $W = P\Delta t$.)	It is the use of 1 kilowatt of electricity for 1 hour.
Other energy formulae (electric circuits)	$W = VQ$ $W = I^2 R \Delta t$ $W = VI \Delta t$ $W = \frac{V^2 \Delta t}{R}$

A simple way of how a circuit (direct current) works

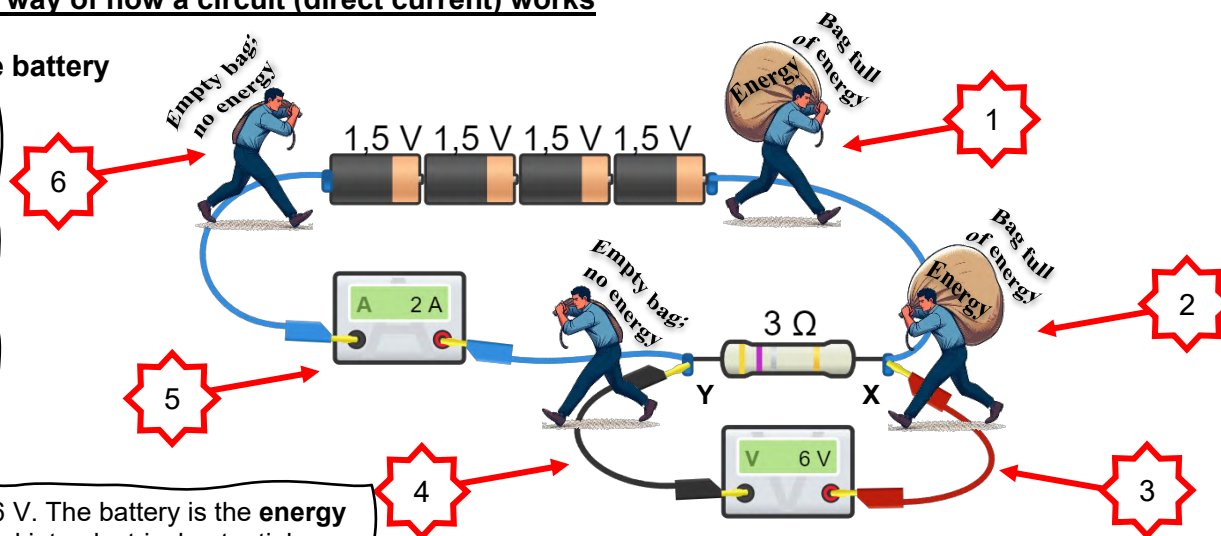
Example 1: One resistor is connected in series with the battery

Read this first to understand the idea of the picture.

In this example, **one coulomb of charge** is represented by the **picture of the man**. He carries a bag initially filled with **energy**. The **same man** is represented at **different positions** in the circuit. **Only ONE coulomb is represented here, but there are millions of charges behind and in front of this one doing the same things described in this example.**

Follow the numbers below and study what happens in the circuit.

Each cell's emf is 1,5 V. For four cells (the battery), the emf is 6 V. The battery is the **energy factory**. Chemical potential energy of the chemicals is converted into electrical potential energy and the **charges are transporting the energy** to the resistor in the circuit.

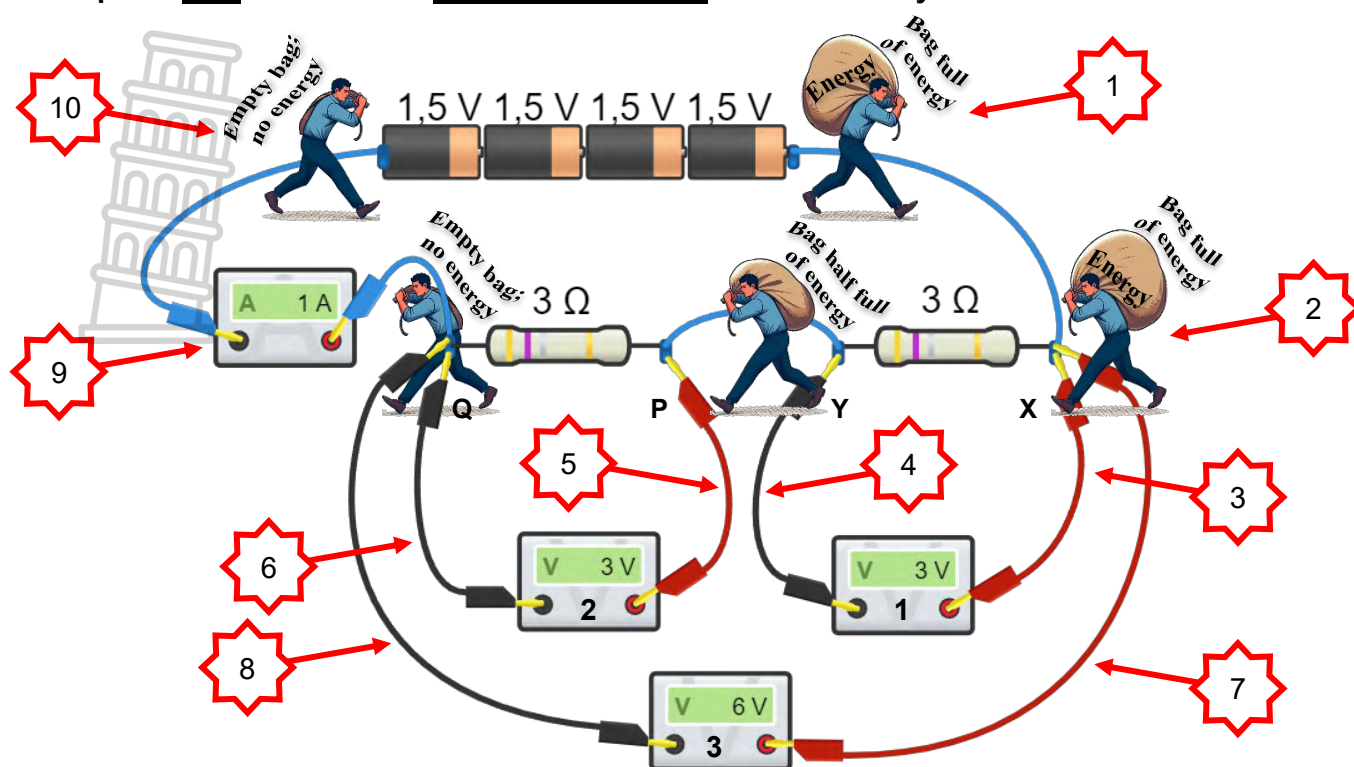


- 1 **One coulomb** of charge leaves the battery with the **maximum amount of energy** (a full bag of energy). In this case it is six joule per coulomb ($6 \text{ J} \cdot \text{C}^{-1}$) because the emf of the battery is 6 V which is equal to $6 \text{ J} \cdot \text{C}^{-1}$ according to $V = \frac{W}{Q}$.
- 2 This coulomb of charge arrives at the resistor with 6 J of energy (assume the conductor offers no resistance; hence, it uses no energy). There is only **one resistor** in the circuit. Therefore, **all the energy** is transferred to this resistor.
- 3 The voltmeter measures the amount of energy in joule per coulomb at **X** before the energy is transferred to the resistor. Therefore, it measures $6 \text{ J} \cdot \text{C}^{-1}$ at **X**.
- 4 The voltmeter measures the amount of energy in joule per coulomb at **Y** after the energy has been transferred to the resistor. Therefore, it measures $0 \text{ J} \cdot \text{C}^{-1}$ at **Y**.
- 3&4 The voltmeter reading is equal to the **difference** between the readings at **X** and **Y**.
 $\therefore \text{Voltmeter reading} = \text{reading at X} - \text{reading at Y}$
 $= 6 \text{ J} \cdot \text{C}^{-1} - 0 \text{ J} \cdot \text{C}^{-1} = 6 \text{ J} \cdot \text{C}^{-1} = 6 \text{ V}$

Note a few things about the voltmeter and its reading:

- The reading represents the energy per coulomb of charge transferred to the resistor that is **connected between its wires**.
 - The voltmeter **measures the energy per coulomb of charge, also called potential difference, at TWO points**. Therefore, the **voltmeter** must be connected in **parallel** with the resistor. To prevent the charges from flowing through the voltmeter, the latter is manufactured with a **very high resistance**.
 - **There is no change in the charges** itself. It is **only the energy that is transferred** to the resistor.
- 5 The **charges flow THROUGH** the ammeter. Therefore, the **ammeter** must have a very **low resistance** and it is connected in **series** with the resistor. The **current, which is measured at ONE point**, is then given by $I = \frac{Q}{\Delta t}$. If two coulomb of charge flows through the ammeter in one second, the current is:
 $I = \frac{Q}{\Delta t} = \frac{2}{1} = 2 \text{ C} \cdot \text{s}^{-1} = 2 \text{ A}$
 - 6 The coulomb of charge returns to the battery where the bag is again filled with energy and the process is repeated for this coulomb of charge until the battery is flat.

Example 2: Two resistors are connected in series with the battery



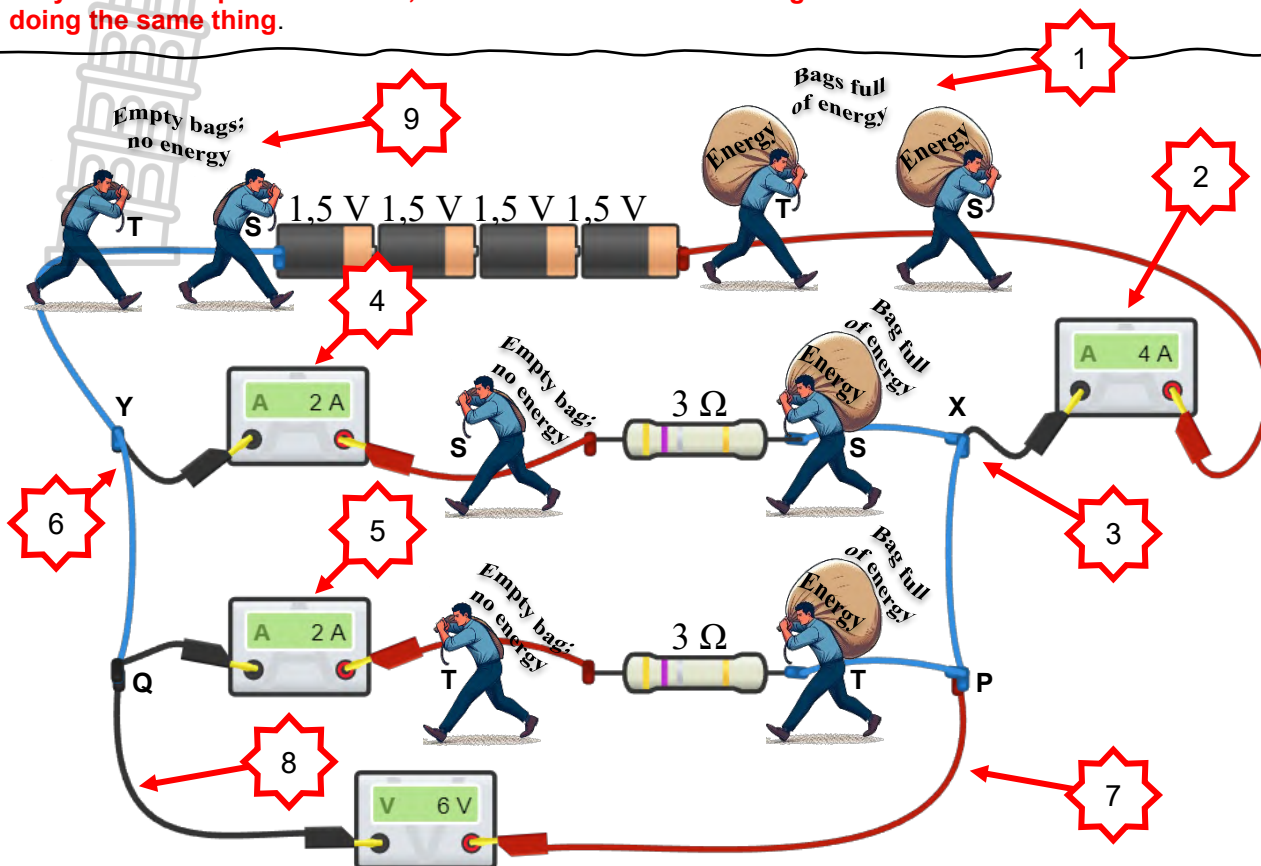
- 1 **One coulomb** of charge leaves the battery with the **maximum amount of energy**. In this case it is $6 \text{ J} \cdot \text{C}^{-1}$ because the emf of the battery is 6 V .
- 2 This coulomb of charge arrives at the first resistor with 6 J of energy. If there are **more than one resistor in series**, the charges must **transfer energy to EACH of the resistors**. The amount of energy transferred depends on each resistor's resistance. In this example, the resistances are the same; hence, each one gets half of the energy.
- 3 Voltmeter 1 measures $6 \text{ J} \cdot \text{C}^{-1}$ at **X before** the energy is transferred to the resistor. Half of this energy is transferred to the first resistor. Hence, there is $3 \text{ J} \cdot \text{C}^{-1}$ of energy left.
- 4 Voltmeter 1 measures $3 \text{ J} \cdot \text{C}^{-1}$ at **Y after** the energy was transferred.
The reading on voltmeter 1 = reading at X - reading at Y = $6 \text{ J} \cdot \text{C}^{-1} - 3 \text{ J} \cdot \text{C}^{-1} = 3 \text{ J} \cdot \text{C}^{-1} = 3 \text{ V}$
- 5 The coulomb of charge flows further and reaches the second resistor where the remaining energy is transferred to the resistor. Voltmeter 2 measures $3 \text{ J} \cdot \text{C}^{-1}$ at **P before** the energy is transferred to the resistor. It is the same reading as at Y because no energy is transferred to the conductors.
- 6 Voltmeter 2 measures $0 \text{ J} \cdot \text{C}^{-1}$ at **Q after** the energy was transferred.
The reading on voltmeter 2 = reading at P - reading at Q = $3 \text{ J} \cdot \text{C}^{-1} - 0 \text{ J} \cdot \text{C}^{-1} = 3 \text{ J} \cdot \text{C}^{-1} = 3 \text{ V}$
- 7 Voltmeter 3 is connected **across BOTH resistors**. It therefore measures the amount of energy at X and Q. At X it measures $6 \text{ J} \cdot \text{C}^{-1}$. All the energy is transferred to both resistors.
- 8 Voltmeter 3 measures $0 \text{ J} \cdot \text{C}^{-1}$ at **Q after** the energy was transferred.
The reading on voltmeter 3 = reading at X - reading at Q = $6 \text{ J} \cdot \text{C}^{-1} - 0 \text{ J} \cdot \text{C}^{-1} = 6 \text{ J} \cdot \text{C}^{-1} = 6 \text{ V}$
Note that the reading on voltmeter 3 is equal to the sum of the readings on voltmeters 1 and 2.
- 9 The charges flow **through** the ammeter. If one coulomb of charge flows through the ammeter in one second, the current is:
$$I = \frac{Q}{\Delta t} = \frac{1}{1} = 1 \text{ C} \cdot \text{s}^{-1} = 1 \text{ A}.$$
- 10 The coulomb of charge returns to the battery where the bag is again filled with energy and the process is then repeated for this coulomb of charge until the battery is flat.

Note once again that:

- Each voltmeter measures the **potential difference at TWO points**.
- The voltmeter reading represents the energy per coulomb of charge transferred to the resistor that is **connected between its wires**. This is important. **The reading on any voltmeter is only applicable to the resistor(s) connected between its wires while charges are flowing.**
- All three **voltmeters** are connected in **parallel** with the resistors.
- **There is no change in the charges** itself. It is **only the energy that is transferred** to the resistor.
- The charges flow **through** the ammeter and the latter measures the current **at the point where it is connected**.

Example 3: Two resistors are connected in parallel with the battery

In this example, we look at **two coulomb of charge**, each one represented by the **picture of the man**. **Only TWO are represented here, but there are millions of charges behind and in front of those doing the same thing.**



- 1 Each coulomb of charge, labelled S and T, leaves the battery with the **maximum amount of energy**. In each case it is $6 \text{ J} \cdot \text{C}^{-1}$ because the emf of the battery is 6 V.
- 2 Both coulombs of charge flow **through** this ammeter because it is the **only path** for them to follow. This is what is called the **main current** of the circuit. If, for example, four coulomb of charge flows through this ammeter in one second, the **main current** is: $I = \frac{Q}{\Delta t} = \frac{4}{1} = 4 \text{ C} \cdot \text{s}^{-1} = 4 \text{ A}$.
- 3 At point X, the main current splits into **two branch currents**. One branch is represented by XY and the other branch by XPQY. Some of the coulombs of charge (labelled S) flow through branch XY and the others (labelled T) flow through branch XPQY. **The ratio in which the main current splits into two branch currents depend on the ratio of the resistors in the branches.** In this example the **resistances are equal**; hence the main current splits into **two equal branch currents**.
- 4 This ammeter measures the current in branch XY only. Hence, it measures 2 A if the main current is 4 A with equal branch resistances.
- 5 This ammeter measures 2 A, which is the current in branch XPQY.
- 2,4&5 **Very important: The sum of the two branch currents is equal to the main current.**
- 6 At point Y, the two branch currents combine again to form the main current.
- Note that the voltmeter is connected across both resistors because they are connected in parallel.**
- 7 The wire of the voltmeter is connected at P, but the reading on the voltmeter is also valid for X because there is just another wire between P and X. The coulomb of charge labelled S arrives at the resistor in branch XY with 6 J of energy. Hence, the voltmeter measures $6 \text{ J} \cdot \text{C}^{-1}$ at X **before** the energy is transferred to the resistor.
- 8 The voltmeter measures $0 \text{ J} \cdot \text{C}^{-1}$ at Y **after** the energy was transferred to the single resistor in branch XY.
- 7 The coulomb of charge labelled T arrives at the resistor in branch XPQY with 6 J of energy. Hence, the voltmeter also measures $6 \text{ J} \cdot \text{C}^{-1}$ at P **before** the energy is transferred to the resistor.
- 8 For branch XPQY, the voltmeter measures $0 \text{ J} \cdot \text{C}^{-1}$ at Q **after** the energy was transferred to the single resistor in branch XPQY.
 The reading on the voltmeter = reading at P (or X) - reading at Q (or Y)
 = $6 \text{ J} \cdot \text{C}^{-1} - 0 \text{ J} \cdot \text{C}^{-1} = 6 \text{ J} \cdot \text{C}^{-1} = 6 \text{ V}$
- Very important: The reading on the voltmeter is the same for both resistors.**
- 9 The two coulombs of charge return to the battery where new energy is obtained from the battery and the process is then repeated until the battery is flat.

Let's confirm the readings on the ammeters and voltmeters in the three examples and make some important conclusions about the use of formulae.

Example 1

Known data is: emf = 6 V; external resistor = 3 Ω; internal resistance = 0 Ω

To calculate the reading on the ammeter, which is the total (main) current in the circuit:

$$R_{total} = \frac{V_{emf}}{I_{total}}$$

$$3 = \frac{6}{I_{total}}$$

$$I_{total} = 2 \text{ A}$$

The three variables deal with the **same situation**. The **total** resistance, the **total** current and the **emf**, which is the "maximum" potential difference.

To calculate the reading on the voltmeter, which is the potential difference across the specific resistor:

$$R = \frac{V}{I_{total}}$$

$$3 = \frac{V}{2}$$

$$V = 6 \text{ V}$$

Once again, the three variables deal with the **same situation**. The **specific** resistance, the current in **that** resistor and the potential difference across the **specific** resistor.

Example 2

Known data is: emf = 6 V; each external resistor = 3 Ω and they are connected in series; internal resistance = 0 Ω

To calculate the reading on the ammeter, which is the total (main) current in the circuit:

$$R_T = R_1 + R_2$$

$$= 3 + 3$$

$$= 6 \text{ Ω}$$

$$R_T = \frac{V_{emf}}{I_{total}}$$

$$6 = \frac{6}{I_{total}}$$

$$I_{total} = 1 \text{ A}$$

The three variables in $R = \frac{V}{I}$ deal with the **same situation**. The **total** resistance, the **total** current and the **emf**.

To calculate the reading on voltmeter 1, which is the potential difference across **one** of the resistors:

$$R = \frac{V_1}{I_{total}}$$

$$3 = \frac{V_1}{1}$$

$$V_1 = 3 \text{ V}$$

The three variables deal with the **same situation**. The **specific** resistance, the current in **that** resistor and the potential difference across the **specific** resistor.

To calculate the reading on voltmeter 2, which is the potential difference across **one** of the resistors:

$$R = \frac{V_2}{I_{total}}$$

$$3 = \frac{V_2}{1}$$

$$V_2 = 3 \text{ V}$$

The three variables deal with the **same situation**. The **specific** resistance, the current in **that** resistor and the potential difference across the **specific** resistor.

To calculate the reading on voltmeter 3, which is the potential difference across **both** resistors:

$$R_T = \frac{V_3}{I_{total}}$$

$$6 = \frac{V_3}{1}$$

$$V_3 = 6 \text{ V}$$

The three variables deal with the **same situation**. The **total** resistance is used, the current in **both** resistors and the potential difference across **both** resistors.

Example 3

Known data is: $\text{emf} = 6 \text{ V}$; each external resistor $= 3 \Omega$ and they are connected in parallel;
internal resistance $= 0 \Omega$

To calculate the reading on the ammeter that measures the **main** current in the circuit:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$= \frac{1}{3} + \frac{1}{3}$$

$$R_p = 1,5 \Omega$$

$$R_p = \frac{V_{\text{emf}}}{I_{\text{total}}}$$

$$1,5 = \frac{6}{I_{\text{total}}}$$

$$I_{\text{total}} = 4 \text{ A}$$

The three variables in $R = \frac{V}{I}$ deal with the **same situation**. The **effective** resistance, the **total** current and the **emf**.

To calculate the reading on the voltmeter, which is the potential difference across each resistor: (*)

$$R_p = \frac{V}{I_{\text{total}}}$$

$$1,5 = \frac{V}{4}$$

$$V = 6 \text{ V}$$

The three variables deal with the **same situation**. The **effective** resistance, the **total** current in both resistors and the potential difference across **one or both** resistors.

To calculate the reading on the ammeter in branch **XY**, which is **one** of the branch currents:

$$R = \frac{V}{I_{XY}}$$

$$3 = \frac{6}{I_{XY}}$$

$$I_{XY} = 2 \text{ A}$$

The three variables deal with the **same situation**. The **specific** resistance, the current in **that** resistor and the potential difference across the **specific** resistor.

To calculate the reading on the ammeter in branch **XPQY**, which is the **other** branch current:

$$R = \frac{V}{I_{XPQY}}$$

$$3 = \frac{6}{I_{XPQY}}$$

$$I_{XPQY} = 2 \text{ A}$$

The three variables deal with the **same situation**. The **specific** resistance, the current in **that** resistor and the potential difference across the **specific** resistor.

- (*) In this solution, the voltmeter reading was calculated by using the main current, followed by the calculation of the two branch currents by using the voltmeter reading. If a branch current is available, it can also be used to calculate the voltmeter reading.

The most important conclusion to make from these examples is that you must always ensure that **variables** in an equation **address the same situation!**

That is true for any equation!

A few notes about internal resistance

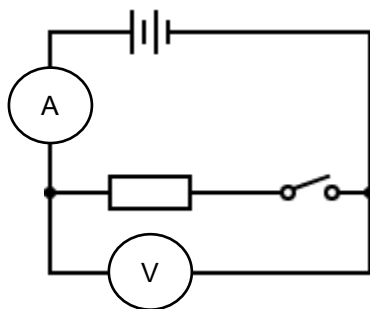
Questions usually indicate whether the cell or battery has internal resistance and therefore it is important to know how to deal with internal resistance if it must be taken into consideration.

The following are important aspects of internal resistance:

- Cells consists of chemicals and other materials and in real life it resists the flow of charge (the current) like an ordinary resistor. This resistance of a cell (or battery) is called "internal resistance".
- Cells are connected in series with the external resistors.** Hence, the internal resistance must be seen as **connected in series** with the external resistors, irrespective if the external resistors are connected in series or parallel.

Voltmeter readings with or without internal resistance

Consider the following circuit and study the summary below to see how a voltmeter reading differs when internal resistance is present or not.



No internal resistance

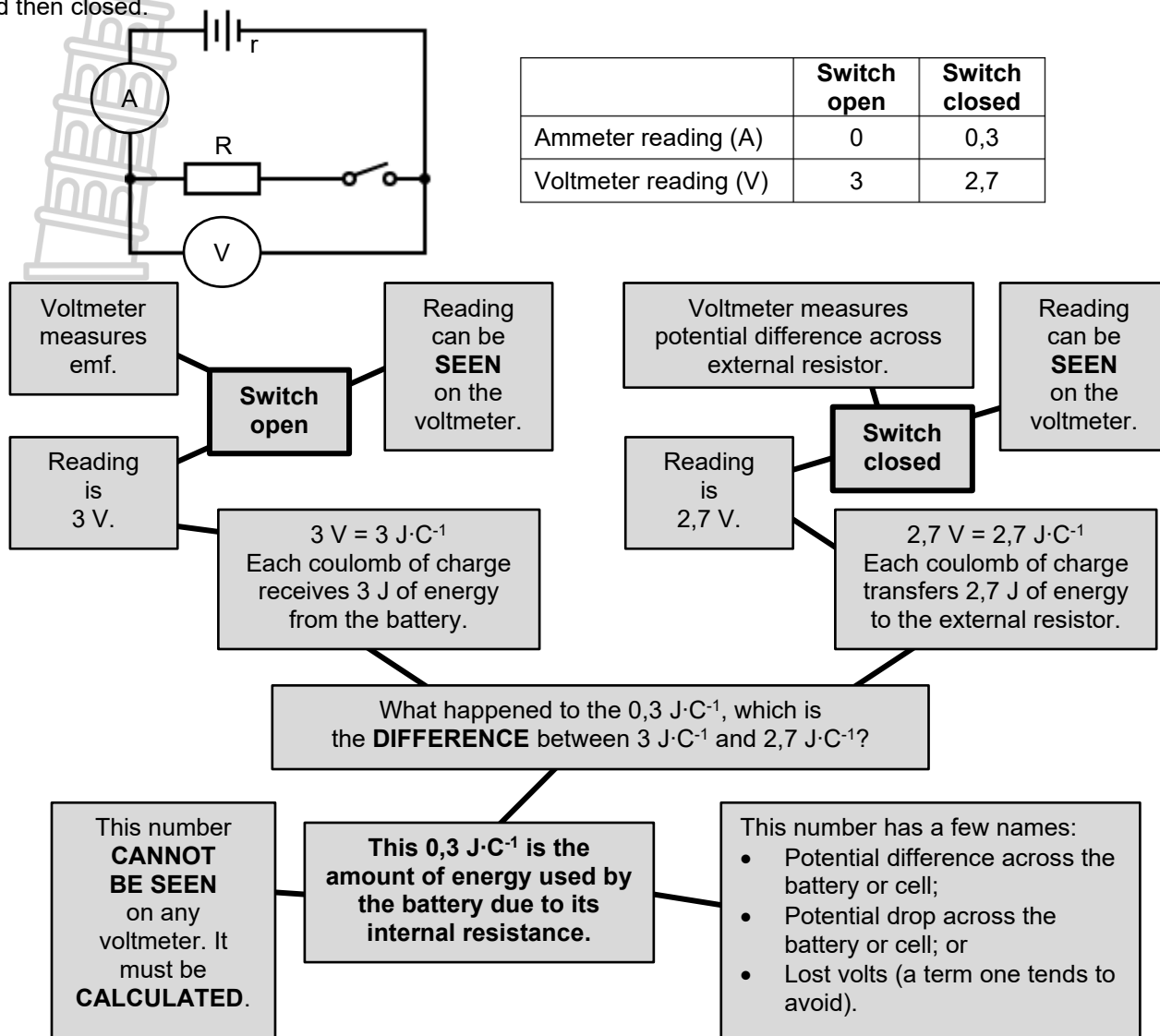
Switch open	Switch closed
Ammeter reading is zero.	Ammeter measures the current. In this case the main current.
Voltmeter measures the emf.	The voltmeter measures the potential difference across the resistor, and it is the SAME as the emf.

With internal resistance

Switch open	Switch closed
Ammeter reading is zero.	Ammeter measures the current. In this case the main current.
Voltmeter measures the emf.	The voltmeter measures the potential difference across the resistor, and it is LESS THAN the emf.



A mind experiment may further clarify the situation of internal resistance. Consider the circuit below. The battery has an internal resistance represented by r . Ammeter and voltmeter readings are taken; first with the switch open and then closed.



Look at the following applications of the formula $R = \frac{V}{I}$ to see how it should be used correctly.

To calculate the external resistance R :

$$R = \frac{V}{I_{\text{total}}}$$

$$R = \frac{2,7}{0,3}$$

$$= 9 \Omega$$

The three variables deal with the **same situation**. The **external** resistor, the current in **that** resistor and the potential difference across **that** resistor.

To calculate the total resistance R_T :

$$R_T = \frac{V_{\text{emf}}}{I_{\text{total}}}$$

$$= \frac{3}{0,3}$$

$$= 10 \Omega$$

The three variables deal with the **same situation**. The **total** resistance ($R+r$), the current in **those** resistors and the **emf**.

To calculate the internal resistance r :

(**)

$$r = \frac{V_{\text{cell}}}{I_{\text{total}}}$$

$$= \frac{3 - 2,7}{0,3}$$

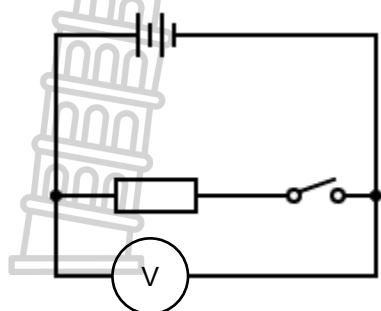
$$= 1 \Omega$$

The three variables deal with the **same situation**. The **internal** resistance, the current in **the battery** and the potential drop across **the battery**.

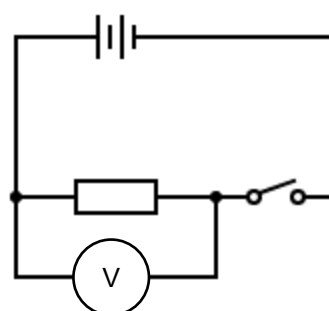
(**) The internal resistance is also the total resistance minus the external resistance, because $R_T = R + r$.

General useful hints about electric circuits

1. Check the connections of the voltmeters when emf is considered.

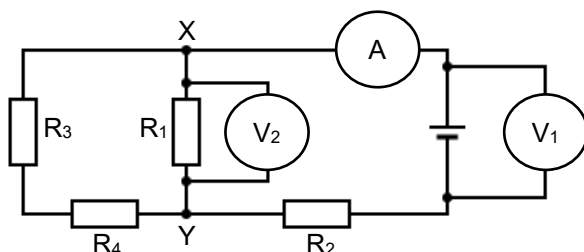


This voltmeter measures the emf. The switch is open and both wires are in contact with the battery.

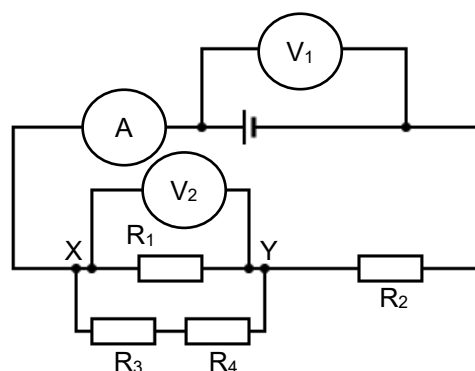


This voltmeter **does not** measure the emf although the switch is open. One of the wires is not in contact with the battery.

2. From examples 1 to 3 above you must remember that a voltmeter measures potential difference across the resistor(s) between its wires when there is a current in that resistor / those resistors.
3. Simplify a circuit diagram if it is complicated.



Circuit 1



Circuit 2

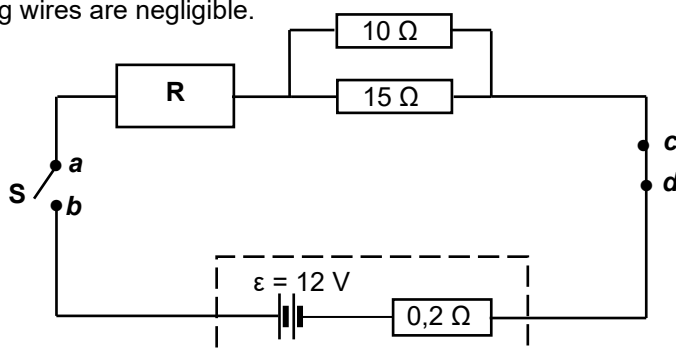
Circuit 1 can be simplified to look like circuit 2 where all the resistors are put on one side of the circuit diagram. One way of doing it is as follows:

- Do the voltmeters after the battery, wires, resistors, ammeters and switches have been connected.
- Follow the direction of the conventional current from the positive terminal of the cell.
- The ammeter is reached first, and it measures the main current.
- The main current splits into two branch currents at **X** and combine again at **Y**.
- In one of the branches resistors **R₃** and **R₄** are connected in series.
- Resistor **R₁** is in the other branch, and **R₁** is connected in parallel with **R₃** and **R₄**.
- From **Y** back to the negative terminal of the cell one has the main current again.
- Resistor **R₂** is between **Y** and the cell. In circuit 2 it is easy to see that **R₂** is connected in series with the parallel combination of resistors.
- Finally, consider the position of the voltmeters:
 - One of the wires of **V₁** is connected between the positive terminal of the cell and the ammeter. The other wire is connected between the negative terminal and **R₂**. Looking at circuit 2 it is easy to see that **V₁** is actually connected across all four resistors. When no current exists, it measures the emf, and with current in the circuit, it measures the potential difference across all four external resistors (terminal potential difference).
 - One of the wires of **V₂** is connected between **X** and **R₁**. The other wire is connected between **R₁** and **Y**. Looking at circuit 2 it is easy to see that **V₂** is connected across the parallel set of resistors. It therefore measures the potential difference across **R₁**, but also the potential difference across **R₃** and **R₄**. It has nothing to do with **R₂**.

Typical questions

QUESTION 1

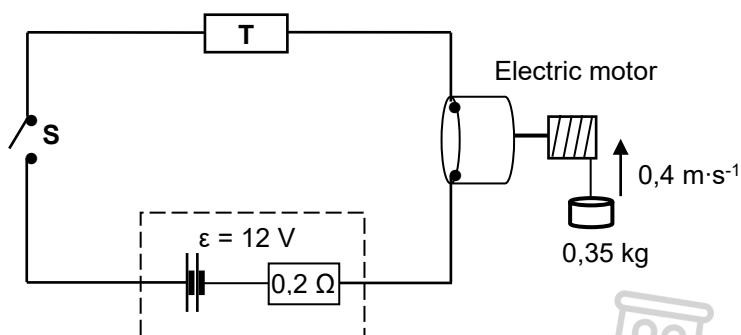
- 1.1 In the circuit below the battery has an emf (ϵ) of 12 V and an internal resistance of $0,2 \Omega$. The resistances of the connecting wires are negligible.



- 1.1.1 Define the term *emf of a battery*. (2)
 1.1.2 Switch **S** is open. A high-resistance voltmeter is connected across points **a** and **b**. What will the reading on the voltmeter be? (1)
 1.1.3 Switch **S** is now closed. The same voltmeter is now connected across points **c** and **d**. What will the reading on the voltmeter be? (1)

When switch **S** is closed, the potential difference across the terminals of the battery is 11,7 V. Calculate the:

- 1.1.4 Current in the battery (3)
 1.1.5 Effective resistance of the parallel branch (2)
 1.1.6 Resistance of resistor **R** (4)
- 1.2 A battery with an emf of 12 V and an internal resistance of $0,2 \Omega$ are connected in series to a very small electric motor and a resistor, **T**, of unknown resistance, as shown in the circuit below. The motor is rated **X** watts, 3 volts, and operates at optimal conditions. When switch **S** is closed, the motor lifts a $0,35 \text{ kg}$ mass vertically upwards at a constant speed of $0,4 \text{ m}\cdot\text{s}^{-1}$. Assume that there is no energy conversion into heat and sound.



- 1.2.1 Calculate the value of **X**. (3)
 1.2.2 Calculate the resistance of resistor **T**. (5)

[21]

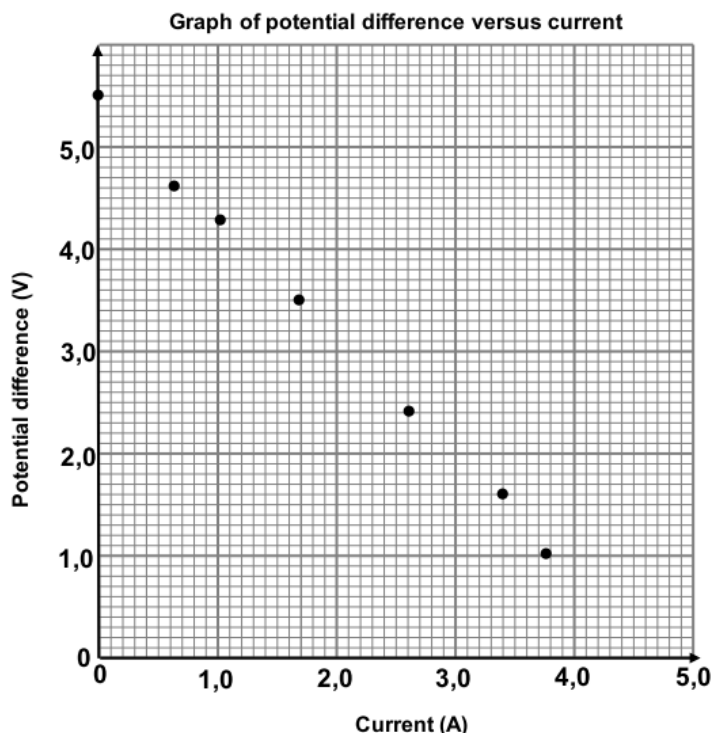
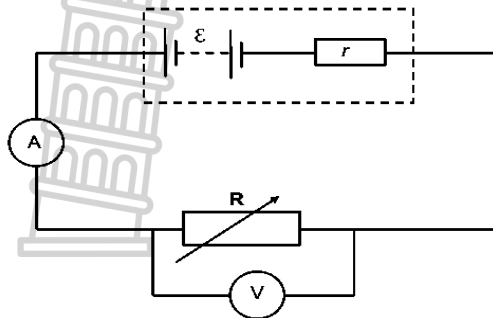


QUESTION 2

- 2.1 The emf and internal resistance of a certain battery were determined experimentally. The circuit used for the experiment is shown in the diagram below.

2.1.1 State Ohm's law in words.

(2)



The data obtained from the experiment is plotted on the graph sheet alongside.

2.1.2 Draw the line of best fit through the plotted points. Ensure that the line cuts both axes. (2)

Use information in the graph to answer QUESTIONS 2.1.3 and 2.1.4.

2.1.3 Write down the value of the emf (ϵ) of the battery. (1)

2.1.4 Determine the internal resistance of the battery. (3)

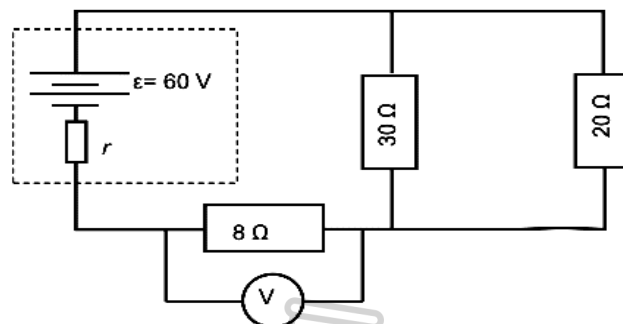
- 2.2 The circuit diagram shows a battery with an emf (ϵ) of 60 V and an unknown internal resistance r , connected to three resistors. A voltmeter connected across the 8 Ω resistor reads 21,84 V. Calculate the:

2.2.1 Current in the 8 Ω resistor (3)

2.2.2 Equivalent resistance of the resistors in parallel (2)

2.2.3 Internal resistance r of the battery (4)

2.2.4 Heat dissipated in the external circuit in 0,2 seconds (3)



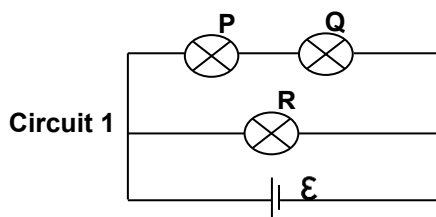
[20]

QUESTION 3

- 3.1 In **circuit 1**, three identical light bulbs, **P**, **Q** and **R**, with the same resistance, are connected to a battery with emf ϵ and negligible internal resistance.

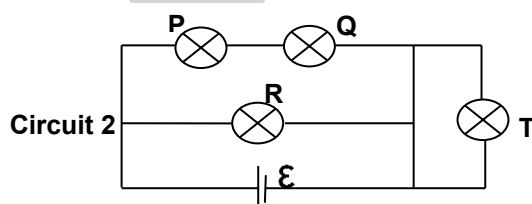
3.1.1 How does the brightness of bulb **P** compare with that of bulb **Q**? Give a reason. (2)

3.1.2 How does the brightness of bulb **P** compare with that of bulb **R**? Give a reason. (2)



A fourth, identical bulb **T**, with the same resistance as the other three, is connected to the circuit by means of an ordinary wire of negligible resistance, as shown in **circuit 2**.

3.1.3 How does the brightness of bulb **T** compare with that of bulb **R**? Give a reason for the answer. (2)



- 3.2 A battery with an emf of 20 V and an internal resistance of $1\ \Omega$ is connected to three resistors, as shown in the circuit.

Calculate the:

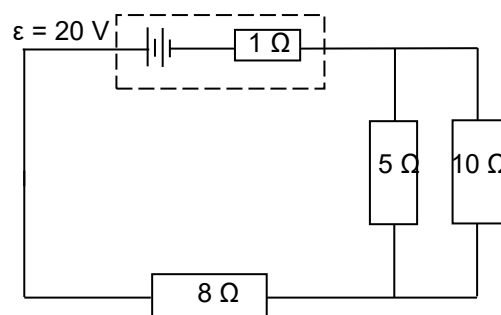
- 3.2.1 Current in the $8\ \Omega$ resistor
 3.2.2 Potential difference across the $5\ \Omega$ resistor
 3.2.3 Total power supplied by the battery

(6)

(4)

(3)

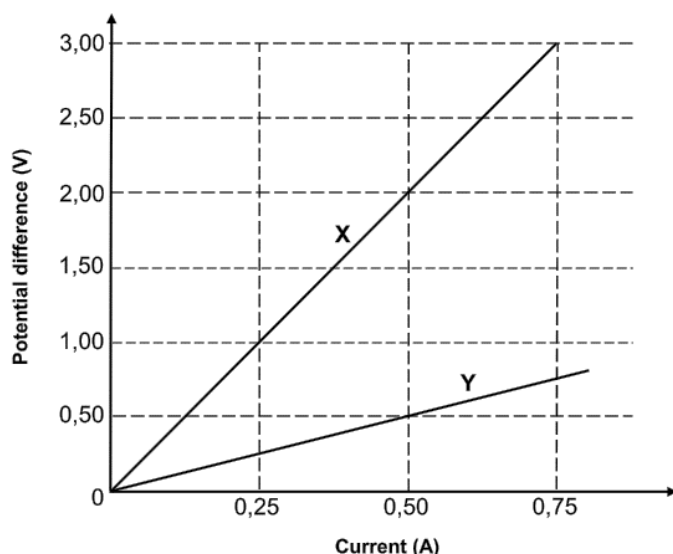
[19]



QUESTION 4

- 4.1 Learners investigated the relationship between potential difference (V) and current (I) for the combination of two resistors, R_1 and R_2 .

GRAPHS OF POTENTIAL DIFFERENCE VERSUS CURRENT FOR THE COMBINATION OF TWO RESISTORS IN SERIES AND IN PARALLEL



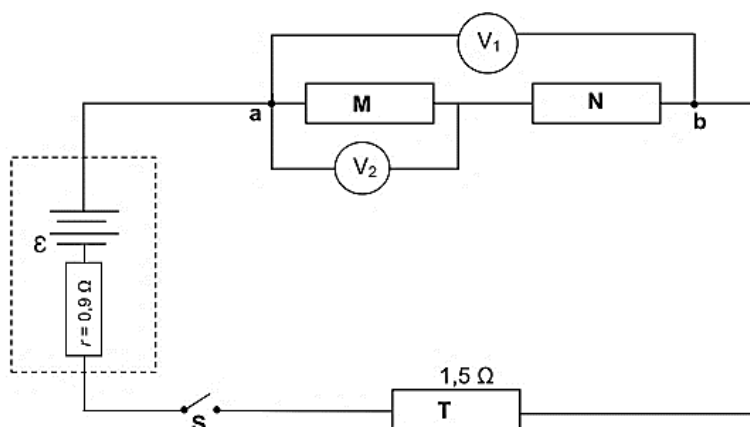
In one experiment, resistors R_1 and R_2 were connected in parallel.

In a second experiment, resistors R_1 and R_2 were connected in series.

The learners then plotted graph **X**, the results of one of the experiments, and graph **Y**, the results of the other experiment, as shown.

- 4.1.1 State Ohm's law in words. (2)
 4.1.2 What physical quantity does the gradient (slope) of the V-I graph represent? (1)
 4.1.3 Calculate the gradient (slope) of graph **X**. (2)
 4.1.4 Determine the resistance of resistor R_1 . (4)

- 4.2 The circuit below consists of three resistors, **M**, **N** and **T**, a battery with emf $\mathcal{E}$ and an internal resistance of $0,9\ \Omega$. The effective resistance between points **a** and **b** in the circuit is $6\ \Omega$. The resistance of resistor **T** is $1,5\ \Omega$. When switch **S** is closed, a high-resistance voltmeter, V_1 , across **a** and **b** reads 5 V.



Calculate the

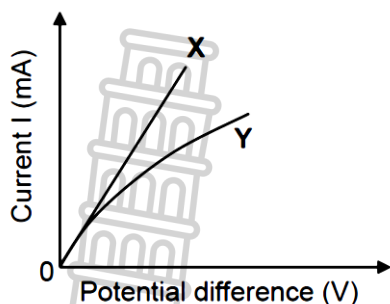
- 4.2.1 Current delivered by the battery (3)
 4.2.2 Emf ($\mathcal{E}$) of the battery (4)

V_2 reads 2,5 V when the switch is closed.

- 4.2.3 Write down the resistance of **N**. (No calculations required.)
 Give a reason for the answer. (2)

[18]

QUESTION 5

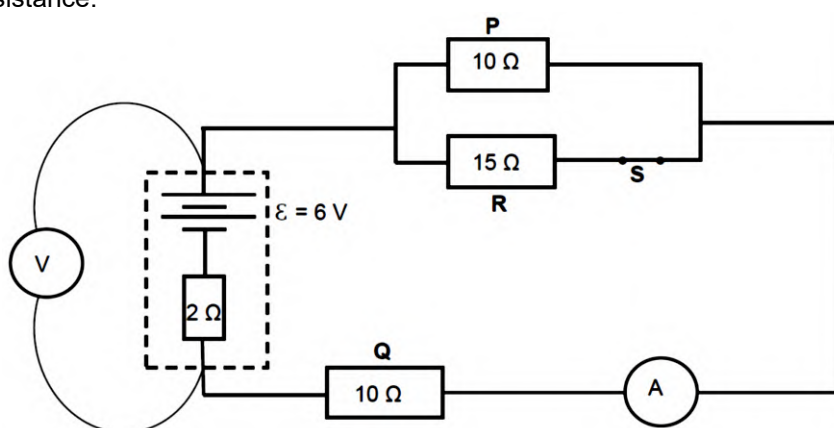


5.1 The two graphs alongside show the relationship between current and potential difference for two different conductors, **X** and **Y**.

5.1.1 State *Ohm's law* in words. (2)

5.1.2 Which ONE of the two conductors, **X** or **Y**, is ohmic? Refer to the graph and give a reason for the answer. (2)

5.2 In the diagram below, a battery with an emf of 6 V and an internal resistance of 2 Ω , is connected to three resistors **P**, **Q** and **R**. A voltmeter **V** is connected across the battery. The ammeter **A** has a negligible resistance.



5.2.1 Calculate the ammeter reading when switch **S** is closed. (5)

The switch **S** is now open.

5.2.2 Will the ammeter reading in QUESTION 5.2.1 INCREASE, DECREASE or REMAIN THE SAME? Give a reason for the answer. (2)

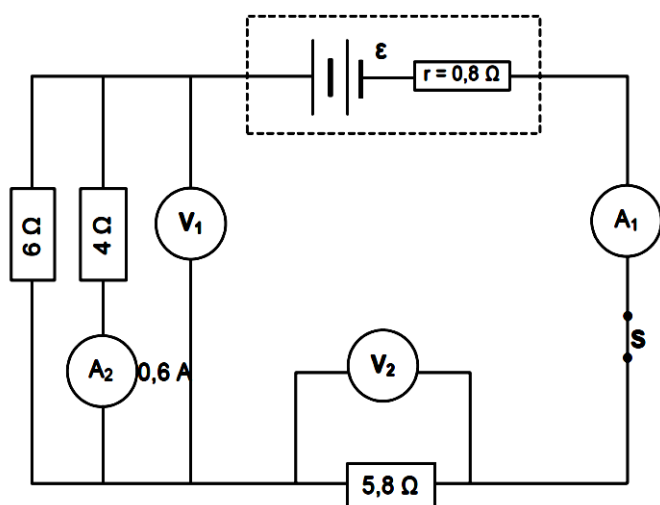
5.2.3 How will the voltmeter reading now compare with the voltmeter reading when the switch is closed? Choose from INCREASE, DECREASE or REMAIN THE SAME. (1)

5.2.4 Explain the answer to QUESTION 5.2.3. (3)

[15]

QUESTION 6

6.1 In the circuit diagram below the battery has an unknown emf (ϵ) and an internal resistance (r) of 0,8 Ω .



6.1.1 State *Ohm's law* in words. (2)

The reading on ammeter **A₂** is 0,6 A when switch **S** is closed. Calculate the:

6.1.2 Reading on voltmeter **V₁** (3)

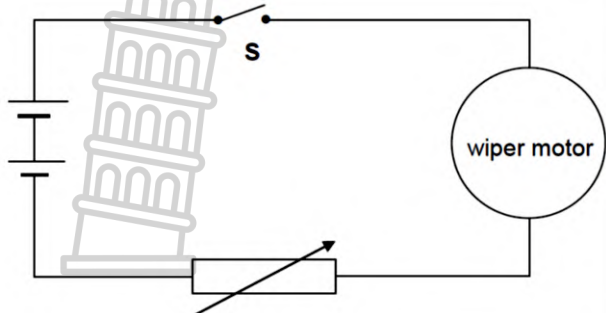
6.1.3 Current through the 6 Ω resistor (2)

6.1.4 Reading on voltmeter **V₂** (2)

6.1.5 Emf (ϵ) of the battery (3)

6.1.6 Energy dissipated as heat inside the battery if the current flows in the circuit for 15 s (3)

- 6.2 A simplified circuit diagram for the windscreen wiper of a car consists of a variable resistor and a wiper motor connected to a 12 volt battery. When switch **S** is closed, the potential difference across the variable resistor is 2,8 V and the current passing through it is 0,7 A.



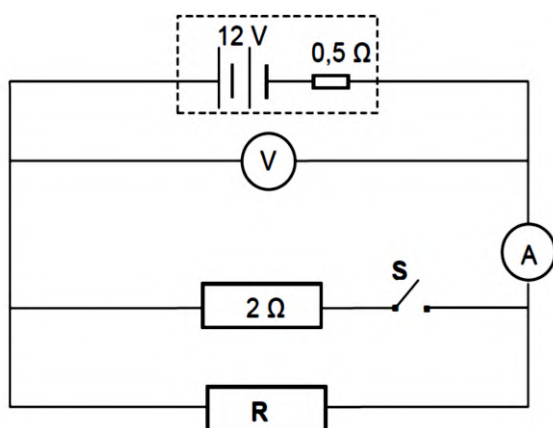
- 6.2.1 Calculate the resistance of the variable resistor. (2)

The resistance of the variable resistor is now decreased.

- 6.2.2 State whether the speed at which the wiper turns will INCREASE, DECREASE or REMAIN THE SAME. Give a reason for the answer. (3)
[20]

QUESTION 7

The battery in the circuit diagram below has an emf of 12 V and an internal resistance of 0,5 Ω . Resistor **R** has an unknown resistance.



- 7.1 What is the meaning of the following statement?
The emf of the battery is 12 V. (2)

The reading on the ammeter is 2 A when switch **S** is OPEN. Calculate the:

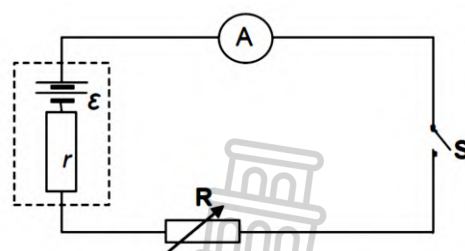
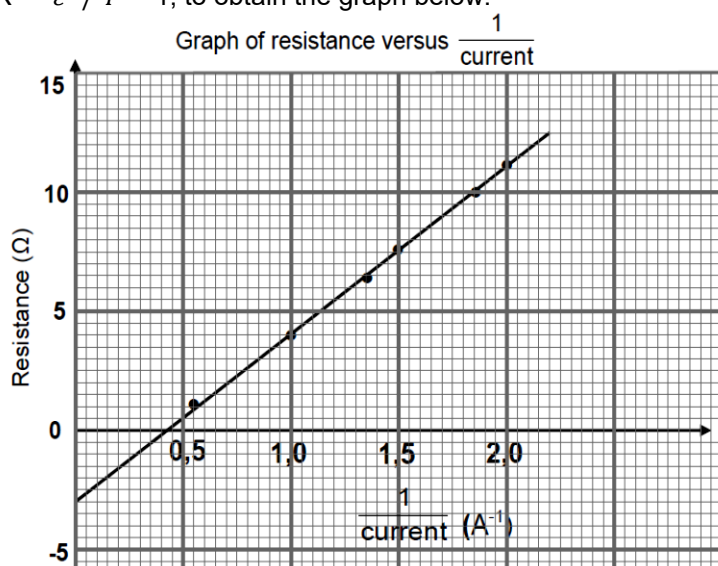
- 7.2 Reading on the voltmeter (3)
7.3 Resistance of resistor **R** (2)

Switch **S** is now CLOSED.

- 7.4 How does this change affect the reading on the voltmeter? Choose from: INCREASES, DECREASES or REMAINS THE SAME. Explain the answer. (4)
[11]

QUESTION 8

Learners perform an experiment to determine the emf (ϵ) and the internal resistance (r) of a battery using the circuit below. The learners use their recorded readings of current and resistance, together with the equation $R = \frac{\epsilon}{I} - r$, to obtain the graph below.



- 8.1 Which variable has to be kept constant in the experiment? (1)

Refer to the graph.

- 8.2 Write down the value of the internal resistance of the cell. (2)
8.3 Calculate the emf of the battery. (3)
[6]

QUESTION 9 (June 2019)

9.1 Three identical light bulbs, **A**, **B** and **C**, are each rated at 6 W, 12 V.

9.1.1 Define the term *power*. (2)

9.1.2 Calculate the resistance of EACH bulb when used as rated. (3)

The light bulbs are connected in a circuit with a battery having an emf (ϵ) of 12 V and internal resistance (r) of $2\ \Omega$. Refer to the diagram.

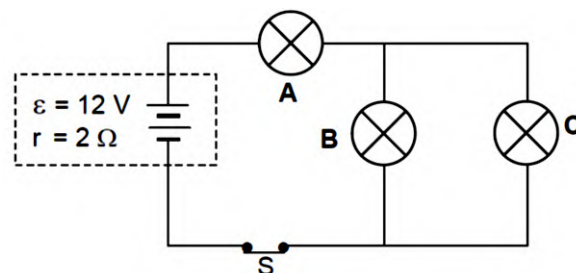
Assume that the resistance of each light bulb is the same as that calculated in QUESTION 9.1.2.

Switch **S** is closed.

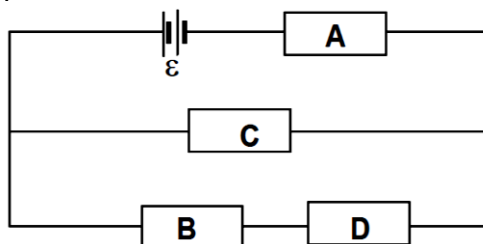
9.1.3 Calculate the total current in the circuit. (5)

9.1.4 Calculate the potential difference across light bulb **C**. (3)

9.1.5 Explain why light bulb **C** in the circuit will NOT burn at its maximum brightness. (3)



9.2 Resistors **A**, **B**, **C** and **D** are connected to a battery having emf (ϵ) and negligible internal resistance, as shown in the diagram below.



9.2.1 Give a reason why the current in resistor **A** is greater than that in resistor **C**. (2)

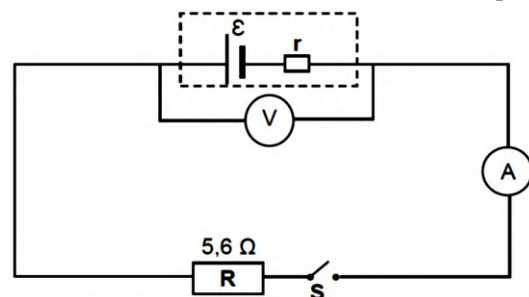
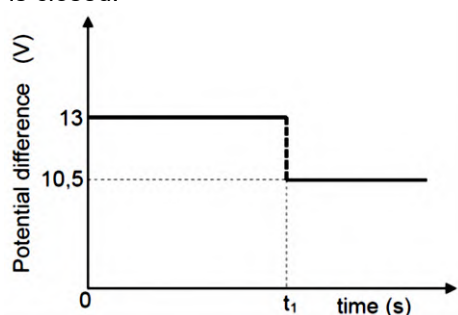
9.2.2 Resistor **C** is removed. How will the current in resistor **B** compare to the current in **A**? Give a reason for the answer. (2)

[20]

QUESTION 10

In the circuit diagram, resistor **R**, with a resistance of $5,6\ \Omega$, is connected, together with a switch, an ammeter and a high-resistance voltmeter, to a battery with an unknown internal resistance, r . The resistance of the connecting wires and the ammeter may be ignored.

The graph below shows the potential difference across the terminals of the battery as a function of time. At time t_1 , switch **S** is closed.



10.1 Define the term *emf of a battery*. (2)

10.2 Write down the value of the emf of the battery. (1)

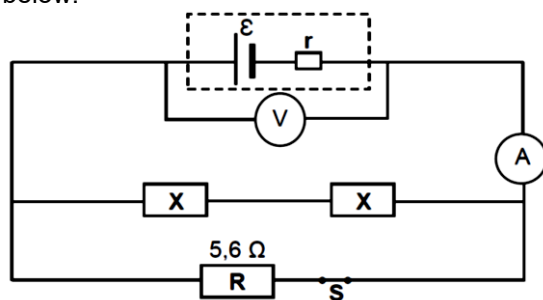
10.3 When switch **S** is CLOSED, calculate the:

10.3.1 Current through resistor **R** (3)

10.3.2 Power dissipated in resistor **R** (3)

10.3.3 Internal resistance, r , of the battery (3)

- 10.4 Two IDENTICAL resistors, each with resistance X , are now connected in the same circuit with switch S closed, as shown below.



The ammeter reading now increases to 4 A.

- 10.4.1 How would the voltmeter reading change? Choose from INCREASES, DECREASES or REMAINS THE SAME. Give a reason for the answer by referring to $V_{\text{internal resistance}}$.

- 10.4.2 Calculate resistance X .

(2)

(5)

[19]

QUESTION 11

A battery with an internal resistance of $0,5 \Omega$ and an unknown emf (ϵ) is connected to three resistors, a high resistance voltmeter and an ammeter of negligible resistance, as shown in the circuit diagram. The resistance of the connecting wires must be ignored.

- 11.1 Define the term *emf* of a battery. (2)

The reading on the voltmeter DECREASES by $1,5 \text{ V}$ when switch S is closed.

- 11.2 Give a reason why the voltmeter reading decreases. (2)

- 11.3 Calculate the following when switch S is closed:

- 11.3.1 Reading on the ammeter (3)

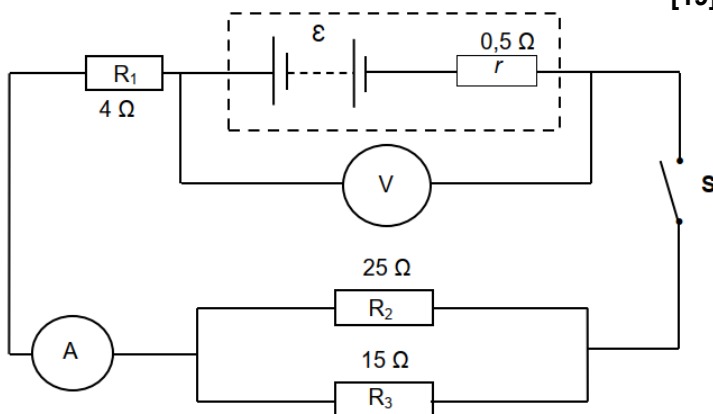
- 11.3.2 Total external resistance of the circuit (4)

- 11.3.3 Emf of the battery (3)

- 11.4 A learner makes the following statement: *The current through resistor R_3 is larger than the current through resistor R_2 .* Is this statement CORRECT? Choose from YES or NO. Explain the answer. (3)

- 11.5 The 4Ω resistor is now removed from the circuit. How will this affect the emf of the battery? Choose from INCREASES, DECREASES or REMAINS THE SAME. (1)

[18]



QUESTION 12

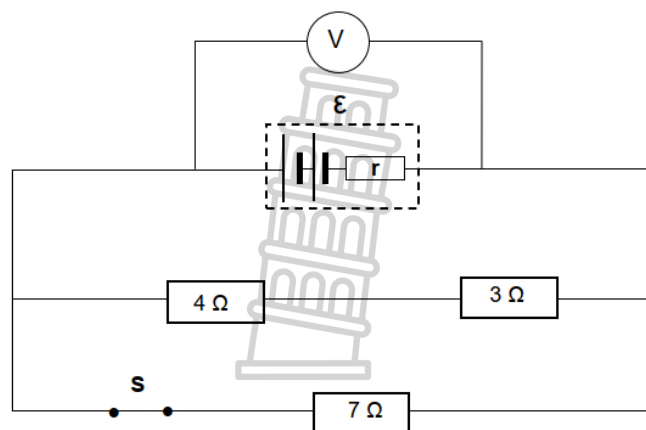
Three resistors are connected to a battery with an unknown emf and unknown internal resistance r , as shown. Ignore the resistance of the connecting wires.

- 12.1 In the definition of the emf of a battery given below, (a) and (b) represent missing words or phrases.

The emf of the battery is the maximum (a) ... supplied by a battery per (b) passing through it.

Write down (a) and (b) in your ANSWER BOOK and next to each the missing word or phrase.

(2)



With switch S CLOSED, the voltmeter reads $2,63 \text{ V}$.

- 12.2 Calculate the equivalent external resistance of the circuit. (4)

Switch S is now OPENED and the voltmeter reads $2,8 \text{ V}$.

- 12.3 Calculate:

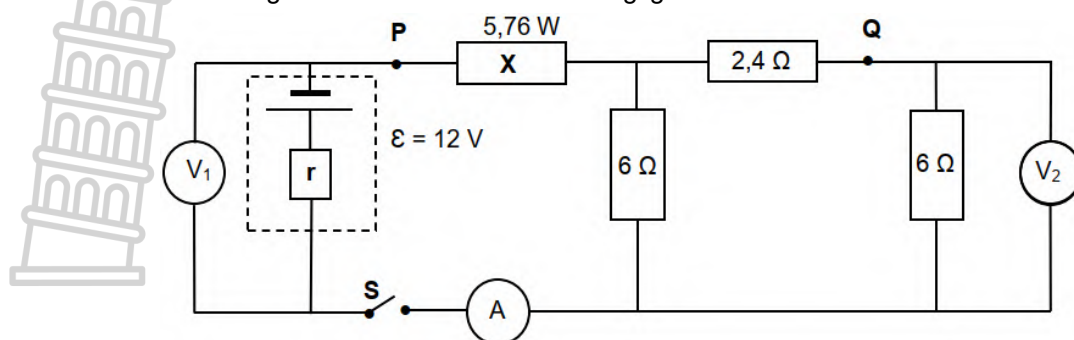
- 12.3.1 The internal resistance of the battery (8)

- 12.3.2 The emf of the battery (2)

[16]

QUESTION 13

The battery in the circuit shown below has an emf of 12 V and an unknown internal resistance r . The resistance of the connecting wires and the ammeter is negligible.



Switch **S** is OPEN.

13.1 Write down the reading on:

13.1.1 Voltmeter V_1

(1)

13.1.2 Voltmeter V_2

(1)

Switch **S** is now CLOSED. The reading on the ammeter is 1,2 A and the power dissipated in resistor **X** is 5,76 W.

13.2 Define the term *power*.

(2)

Calculate the:

13.3 Resistance of resistor **X**

(3)

13.4 Total EXTERNAL resistance of the circuit

(3)

13.5 Reading on voltmeter V_2

(5)

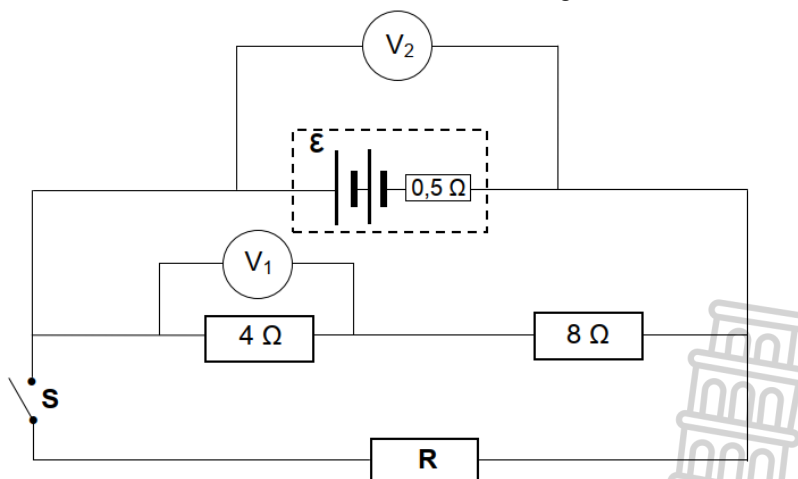
13.6 A length of wire of negligible resistance is used to connect point **P** to point **Q** in the circuit. How will the reading on voltmeter V_1 be affected? Choose from INCREASES, DECREASES or NO EFFECT. Explain the answer.

(4)

[19]

QUESTION 14

In the circuit below a battery of UNKNOWN emf and an internal resistance of $0,5 \Omega$ is connected to two resistors of 4Ω and 8Ω each, and a resistor **R** of unknown resistance. Ignore the resistance of the connecting wires.



14.1 The three external resistors are ohmic conductors. Explain the meaning of the term *ohmic conductor*.

(2)

14.2 When switch **S** is OPEN, voltmeter V_1 reads 3,2 V. Calculate the:

14.2.1 Current through the battery

(3)

14.2.2 Emf of the battery

(4)

14.3 When switch **S** is CLOSED, voltmeter V_2 reads 8,8 V.

14.3.1 Calculate the resistance of resistor **R**.

(5)

14.3.2 The battery becomes heated when voltmeter V_2 is replaced by a connecting wire. Explain this observation.

(3)

[17]

QUESTION 15

The circuit diagram shows four resistors connected to a battery of emf $\mathcal{E}$ and internal resistance r . The resistances of the ammeter and the connecting wires are negligible, while the voltmeters have very high resistances. Switch **S** is CLOSED.

15.1 State *Ohm's law* in words. (2)

15.2 The reading on the ammeter is 3,5 A. (1)

15.2.1 Calculate the total external resistance of the circuit. (4)

15.2.2 Calculate the reading on voltmeter V_1 . (3)

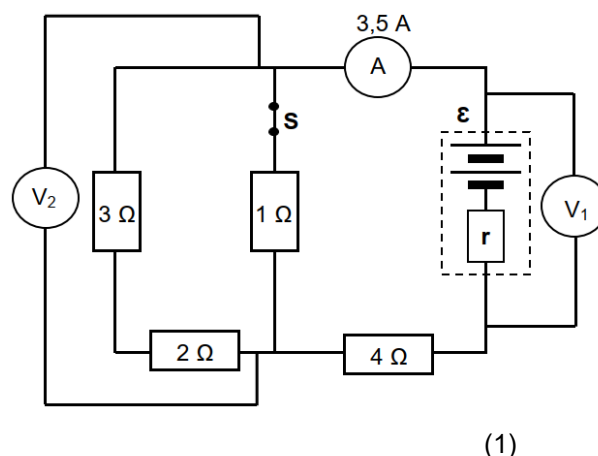
15.2.3 How does the reading on voltmeter V_2 compare to the reading on voltmeter V_1 ? Choose from SMALLER THAN, EQUAL TO or GREATER THAN. (1)

15.3 A learner concludes that the emf of the battery is equal to the reading on voltmeter V_1 . (2)

15.3.1 Define the term *emf*. (2)

15.3.2 Is the learner's conclusion CORRECT? Choose from YES or NO. (1)

15.3.3 Give a reason for the answer to QUESTION 15.3.2. (1)



Switch **S** is now removed and replaced by voltmeter V_2 , as shown in the circuit diagram on the right.

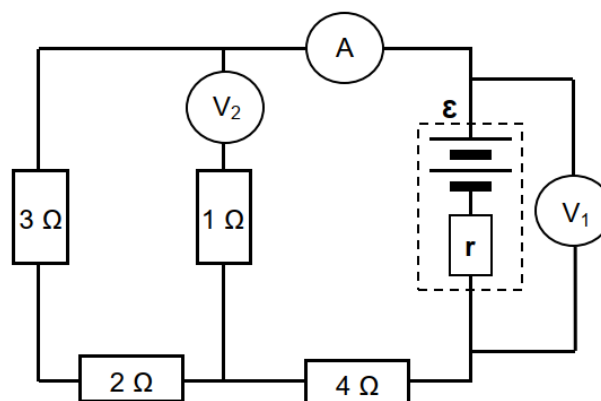
15.4 How will EACH of the following change? (Choose from INCREASES, DECREASES or REMAINS THE SAME.)

15.4.1 The power dissipated by the 4 Ω resistor (1)

15.4.2 The reading on voltmeter V_1 (1)

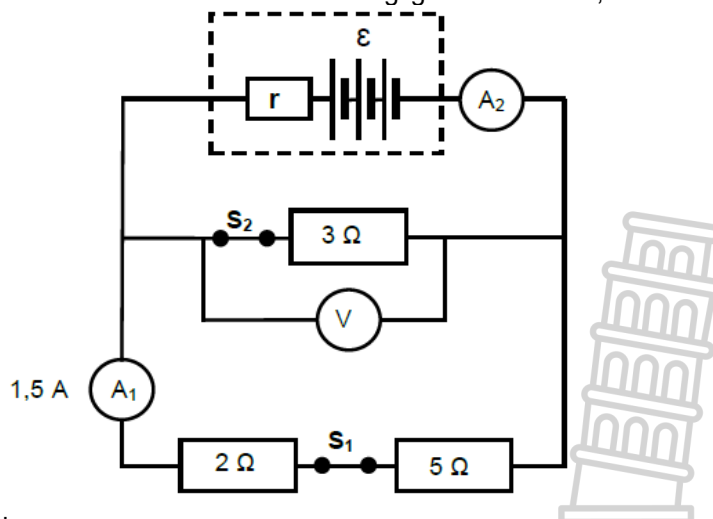
15.5 Explain the answer to QUESTION 15.4.2. (4)

[20]



QUESTION 16 (June 2023)

A battery with unknown emf ($\mathcal{E}$) and unknown internal resistance (r) is connected to three resistors, a high-resistance voltmeter, two switches and two ammeters of negligible resistance, as shown below.



16.1 State *Ohm's law* in words. (2)

16.2 Both switch **S**₁ and switch **S**₂ are CLOSED. The reading on ammeter **A**₁ is 1,5 A. Calculate the: (2)

16.2.1 Reading on the voltmeter (3)

16.2.2 Reading on ammeter **A**₂ (4)

16.2.3 Power dissipated in the 3 Ω resistor (3)

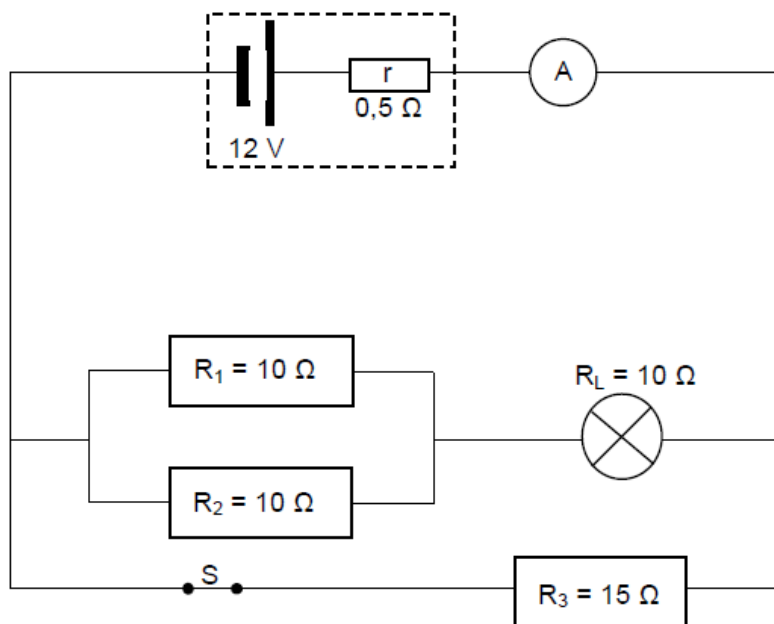
16.3 Switch **S**₁ is now OPENED, while switch **S**₂ remains CLOSED. The reading on ammeter **A**₂ is now 3,64 A. Calculate the emf of the battery. (5)

16.4 Switch **S**₂ is now OPENED, while switch **S**₁ is CLOSED. How does the voltmeter reading change? Choose from INCREASES, DECREASES or REMAINS THE SAME. Explain the answer. (4)

[21]

QUESTION 17 (November 2023)

The circuit diagram below shows a battery with an emf of 12 V and an internal resistance of $0,5 \Omega$ connected to three resistors, a light bulb, a switch, an ammeter and connecting wires. The ammeter and connecting wires have negligible resistance.



- 17.1 State Ohm's law in words. (2)
- Switch **S** is initially CLOSED.
- 17.2 Calculate the: (5)
- 17.2.1 Total external resistance of the circuit (3)
- 17.2.2 Reading on the ammeter (4)
- 17.2.3 Power dissipated by resistor R_3 (4)
- 17.3 Switch **S** is now OPENED.
- 17.3.1 If the resistance of the light bulb remains constant, how will the brightness of the light bulb be affected? Choose from INCREASES, DECREASES or REMAINS THE SAME. (1)
- 17.3.2 Explain the answer to QUESTION 17.3.1. (3)

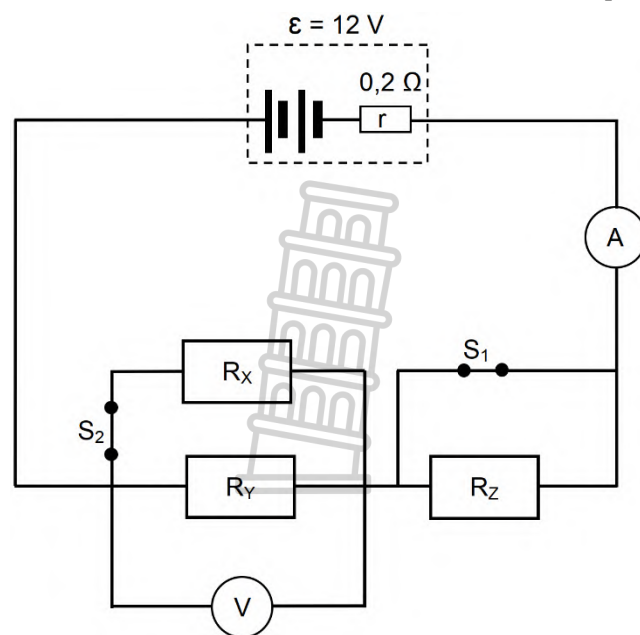
[18]

QUESTION 18 (June 2024)

A battery of emf 12 V and internal resistance $0,2 \Omega$ is connected to three resistors, a high-resistance voltmeter and two switches, an ammeter and connecting wires of negligible resistance, as shown in the circuit diagram. The three resistors have different and unknown resistances.

The resistance of R_Y is TWICE the resistance of R_X . When both switches are CLOSED, the reading on the ammeter is 5,5 A.

- 18.1 Give a reason why there is no current through resistor R_Z . (1)
- 18.2 Calculate the resistance of resistor R_Y . (5)
- 18.3 Calculate the power dissipated by resistor R_X . (4)
- 18.4 When both switches are now OPENED, the reading on the ammeter is 1,3 A. Calculate the reading on the voltmeter. (3)
- 18.5 Switch S_1 remains OPEN while switch S_2 is now CLOSED. Calculate the reading on the ammeter. (6)



[19]

QUESTION 19 (November 2024)

L_1 and L_2 are two light bulbs that have the following ratings:

L_1 : 36 W; 20 V and L_2 : 48 W; 32 V

The two bulbs are connected as shown in the circuit diagram. The battery has an internal resistance of $0,6 \Omega$ while the conducting wires and the ammeters have negligible resistance. R_1 and R_2 are resistors.

19.1 Define the term *power*.

19.2 If both light bulbs operate as RATED, calculate the:

19.2.1 Reading on ammeter A_2

19.2.2 Reading on ammeter A_1

19.2.3 Resistance of resistor R_1

19.2.4 Emf of the battery

19.3 Bulb L_1 burns out after a while. Assume that the resistance of bulb L_2 remains constant. Will bulb L_2 continue to glow after bulb L_1 burns out? Choose from YES or NO. Support your answer with a suitable calculation.

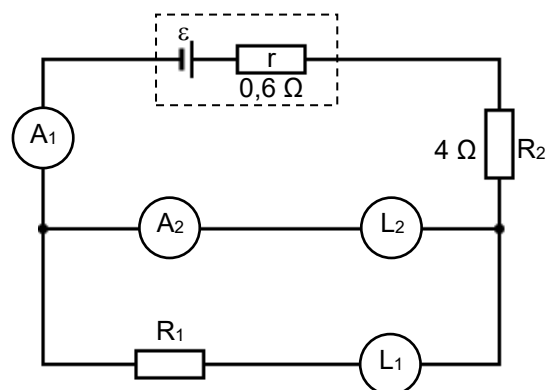
(2)

(3)

(3)

(4)

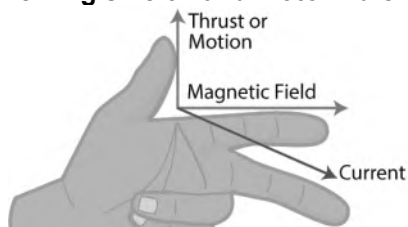
(4)



(5)
[21]

ELECTRICAL MACHINES

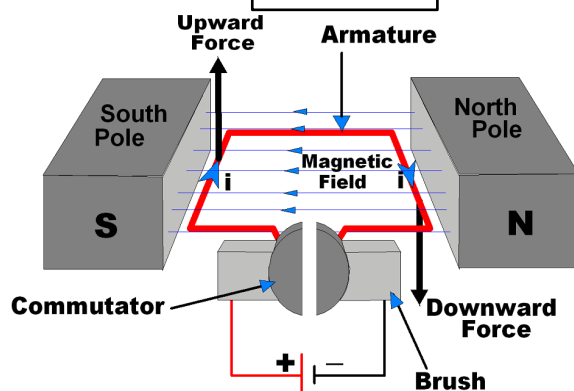
Fleming's Left Hand Motor Rule



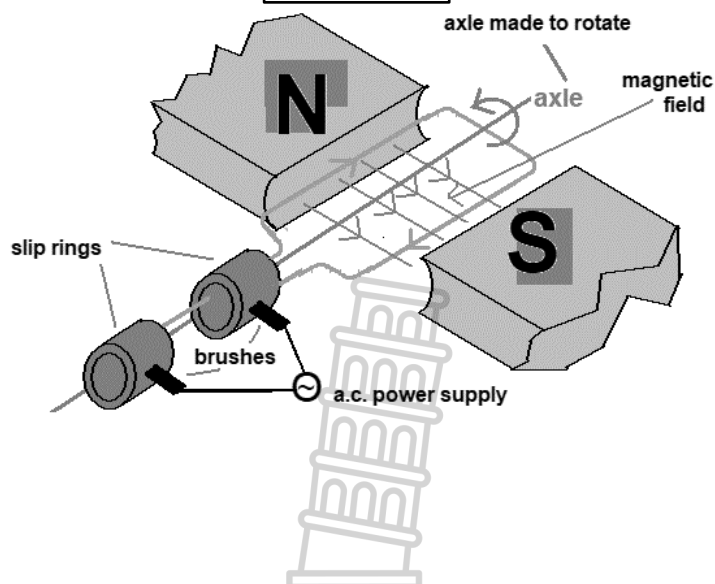
MOTORS

Converts electrical energy to mechanical energy.

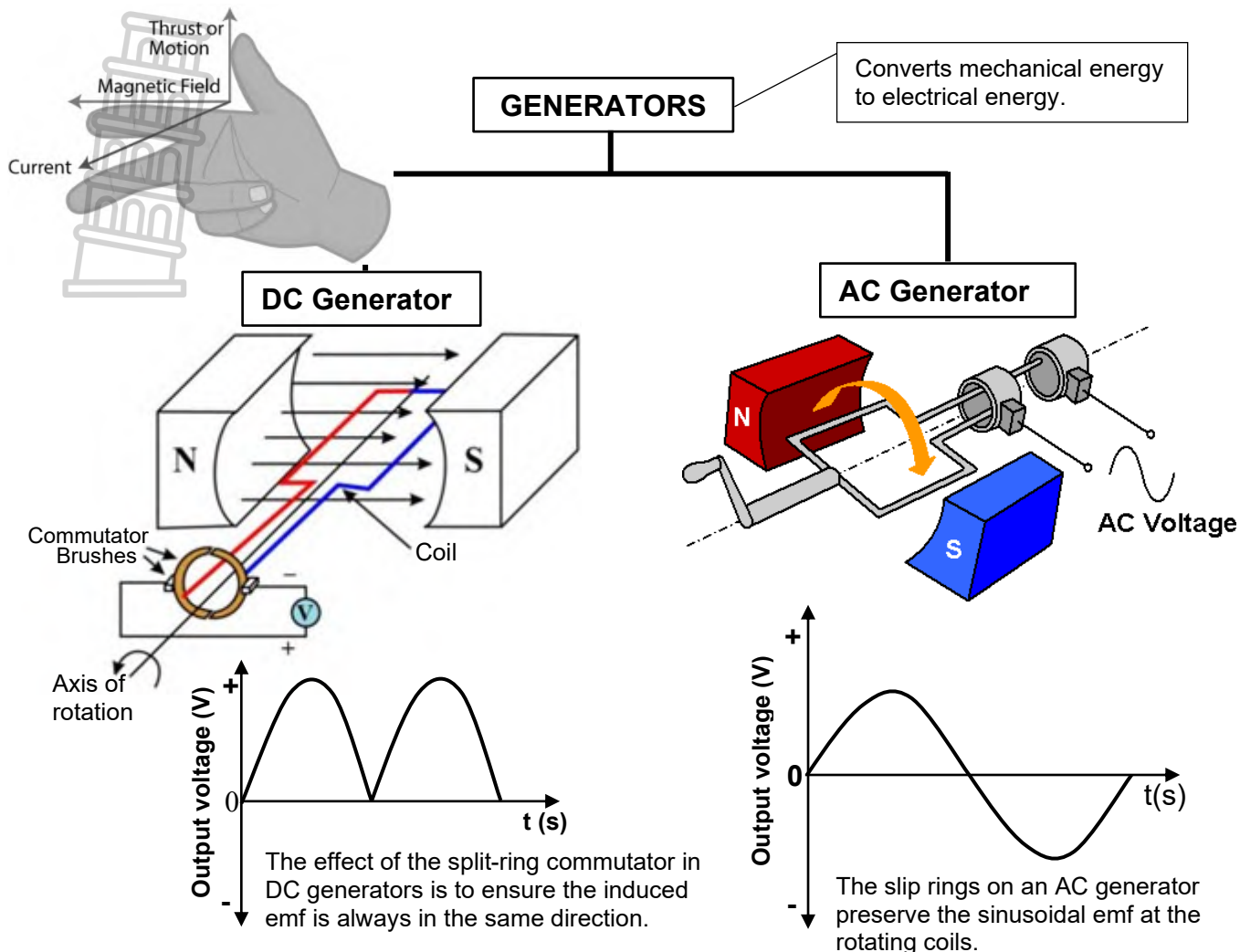
DC Motor



AC Motor



Fleming's Right Hand Dynamo Rule



TERMS AND DEFINITIONS

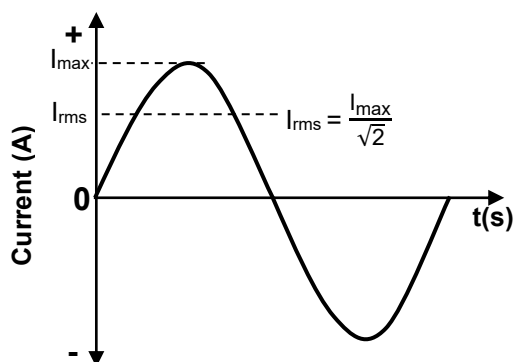
Generator	A device that transfers mechanical energy into electrical energy.
Faraday's law of electromagnetic induction	The magnitude of the induced emf across the ends of a conductor is directly proportional to the rate of change in the magnetic flux linkage with the conductor. (When a conductor is moved in a magnetic field, a potential difference is induced across the conductor.)
Fleming's Right Hand Rule for generators	Hold the thumb, forefinger and second finger of the RIGHT hand at right angles to each other. If the forefinger points in the direction of the magnetic field (N to S) and the thumb points in the direction of the force (movement), then the second finger points in the direction of the induced current.
Electric motor	A device that transfers electrical energy into mechanical energy.
Fleming's Left Hand Rule for electric motors	Hold the thumb, forefinger and second finger of the LEFT hand at right angles to each other. If the forefinger points in the direction of the magnetic field (N to S) and the second finger points in the direction of the conventional current, then the thumb will point in the direction of the force (movement).
Conventional current	Flow of electric charge from positive to negative.
AC	Alternating current The direction of the current changes each half cycle.
DC	Direct current The direction of the current remains constant. (The direction of <u>conventional</u> current is from the <u>positive to the negative</u> pole of a battery. The direction of electron current is from the <u>negative to the positive</u> pole of the battery.)
Root-mean-square potential difference (V_{rms})	The AC potential difference which dissipates/produces the same amount of energy as an equivalent direct current (DC).

Peak potential difference (V_{max})	The maximum potential difference reached by the alternating current as it fluctuates, i.e. the peak of the sine wave representing an AC potential difference.
Root-mean-square current (I_{rms})	The alternating current which dissipates/produces the same amount of energy as an equivalent direct current (DC).
Peak current (I_{max})	The maximum current reached by the alternating current as it fluctuates, i.e. the peak of the sine wave representing an AC current.

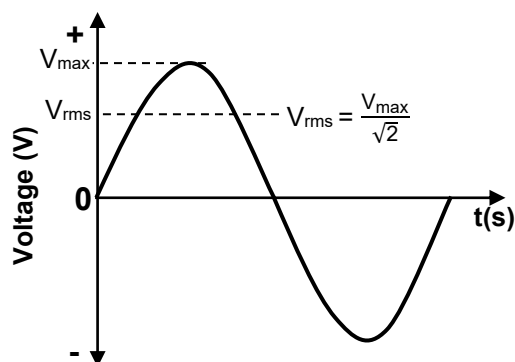
ALTERNATING CURRENT AC

The direction of the current changes each half cycle.

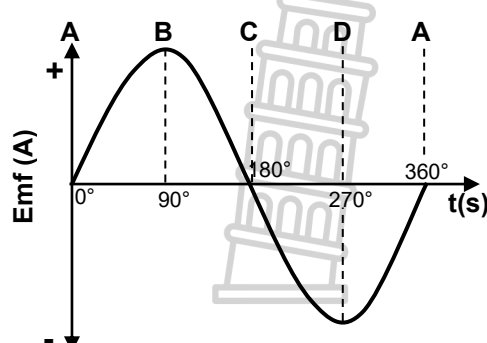
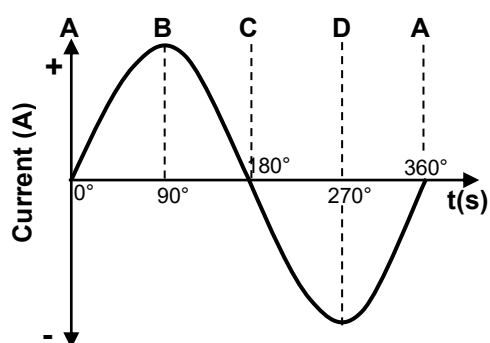
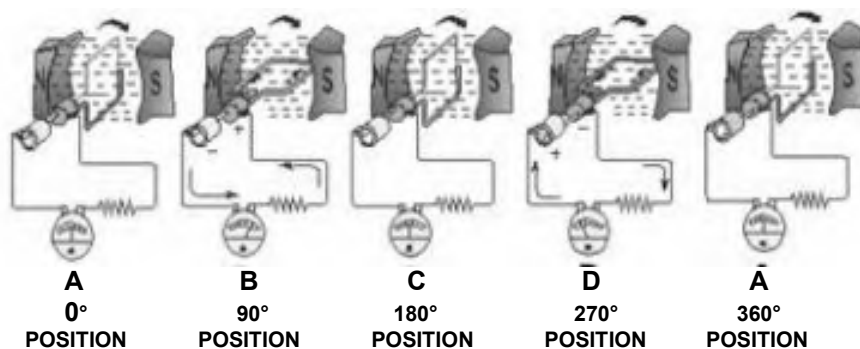
I versus t graph



V versus t graph



Change in potential difference and current in an AC circuit



Electrical power

When voltage and current values are specified for AC circuits, they are rms values and the power values are average power:

$$P_{\text{ave}} = V_{\text{rms}} I_{\text{rms}}$$

$$P_{\text{ave}} = \frac{V_{\text{rms}}^2}{R}$$

$$P_{\text{ave}} = I_{\text{rms}}^2 R$$

P_{ave} : average value of the power in watt (W)

V_{rms} : rms potential difference in volt (V)

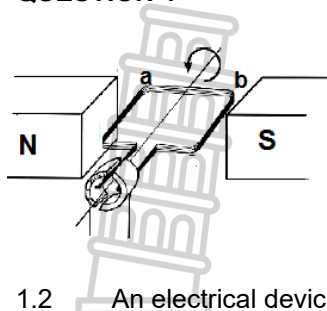
I_{rms} : rms current in ampere (A)

R : resistance in ohm (Ω)

The mains supply in SA is 220 V. This is the rms voltage. V_{max} varies between + 311 V and – 311 V.

Typical questions

QUESTION 1



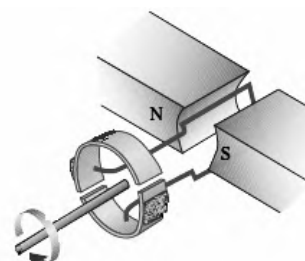
- 1.1 A part of a simplified DC motor is shown in the sketch.
- 1.1.1 In which direction (**a to b** OR **b to a**) is the current flowing through the coil if the coil rotates anticlockwise as indicated in the diagram? (1)
- 1.1.2 Name the rule you used to answer QUESTION 1.1.1. (1)
- 1.1.3 Which component in the diagram must be replaced in order for the device to operate as an AC generator? (1)
- 1.2 An electrical device of resistance $400\ \Omega$ is connected across an AC generator that produces a maximum emf of 430 V . The resistance of the coils of the generator can be ignored.
- 1.2.1 State the energy conversion that takes place when the AC generator is in operation. (2)
- 1.2.2 Calculate the root mean square value of the current passing through the resistor. (5)

[10]

QUESTION 2

- 2.1 A generator is shown here. Assume that the coil is in a vertical position.

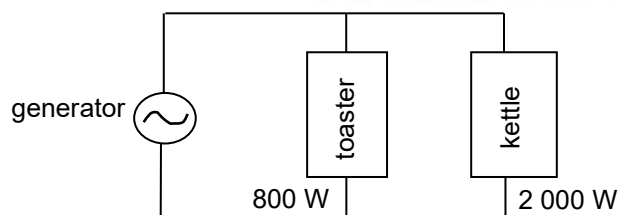
- 2.1.1 Is the generator above AC or DC? Give a reason for the answer. (2)
- 2.1.2 Sketch an induced emf versus time graph for ONE complete rotation of the coil. (The coil starts turning from the vertical position.) (2)



- 2.2 An AC generator is operating at a maximum emf of 340 V . It is connected across a toaster and a kettle, as shown in the diagram. The toaster is rated at 800 W , while the kettle is rated at $2\ 000\text{ W}$. Both are working under optimal conditions.

- 2.2.1 Calculate the rms current passing through the toaster (3)
- 2.2.2 Calculate the total rms current delivered by the generator. (4)

[11]



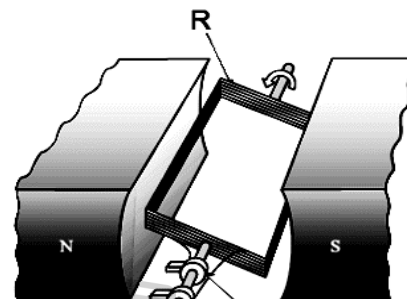
QUESTION 3

- 3.1 The diagram shows a simplified version of a generator.

- 3.1.1 Write down the name of EACH part, **R**, **T** and **X**. (3)
- 3.1.2 Give the NAME of the law upon which the operation of the generator is based. (1)

- 3.2 An AC supply is connected to a light bulb. The light bulb lights up with the same brightness as it does when connected to a 15 V battery.

- 3.2.1 Write down the rms value of the potential difference of the AC supply. (1)
- 3.2.2 If the resistance of the light bulb is $45\ \Omega$, calculate the maximum current delivered to the light bulb. (4)



[9]

QUESTION 4

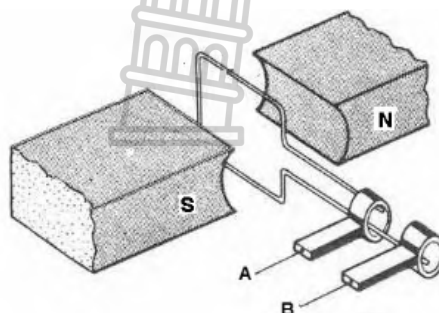
The diagram shows a simplified version of an AC generator.

- 4.1 Name the component in this arrangement that makes it different from a DC generator. (1)
- 4.2 Sketch a graph of induced emf versus time for TWO complete rotations of the coil. (2)

A practical version of the generator has a large number of turns of the coil and it produces an rms potential difference of 240 V .

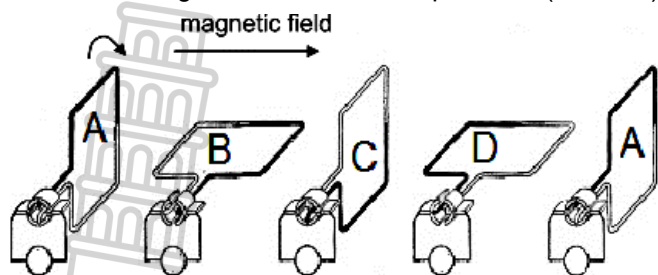
- 4.3 State TWO ways in which the induced emf can be increased. (2)
- 4.4 Define the term root mean square (rms) value of an AC potential difference. (2)
- 4.5 The practical version of the generator is connected across an appliance rated at $1\ 500\text{ W}$. Calculate the rms current passing through the appliance. (3)

[10]



QUESTION 5

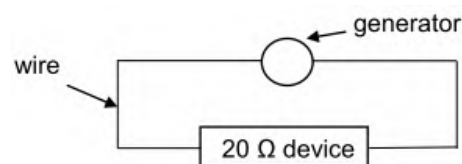
- 5.1 The diagram shows different positions (**ABCD**) of the coil in a DC generator for a complete revolution. The coil is rotated clockwise at a constant speed in a uniform magnetic field. The direction of the magnetic field is shown in the diagram.



- 5.1.1 Write down the energy conversion that takes place during the operation of the DC generator. (1)
- 5.1.2 Sketch a graph to show how the induced emf of the generator varies with time. Clearly indicate positions **A**, **B**, **C**, **D** and **A** on the graph. (2)

- 5.2 A small AC generator, providing an rms voltage of 25 V, is connected across a device with a resistance of 20 Ω . The wires connecting the generator to the device have a total resistance of 0,5 Ω . Refer to the diagram.

- 5.2.1 Write down the total resistance of the circuit. (1)
- 5.2.2 Calculate the average power delivered to the device. (5)



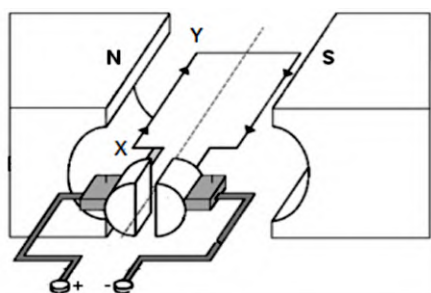
[9]

QUESTION 6

- 6.1 Learners want to build a small DC motor as a project. Write down THREE essential components that are needed for the building of the motor. (3)
- 6.2 An electrical device with a resistance of 11 Ω is connected to an AC source with an rms voltage of 240 V.
- 6.2.1 Define the term rms voltage. (2)
- 6.2.2 Calculate the maximum (peak) current passing through the device. (4)

[9]

QUESTION 7



- 7.1 The diagram is a simplified representation of a DC motor. The current in the coil is in the direction **XY**.
- 7.1.1 Name the component that ensures that the coil rotates continuously in ONE DIRECTION. (1)
- 7.1.2 In which direction will the coil rotate? Write down only **CLOCKWISE** or **ANTICLOCKWISE**. (2)
- 7.1.3 Write down the energy conversion which takes place while the motor is working. (2)

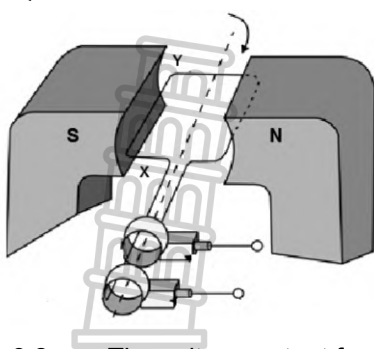
- 7.2 An AC generator, producing a maximum voltage of 320 V, is connected to a heater of resistance 35 Ω .

- 7.2.1 Write down the structural difference between an AC generator and a DC generator. (1)
- 7.2.2 Calculate the root mean square (rms) value of the voltage. (3)
- 7.2.3 Calculate the root mean square (rms) value of the current in the heater. (4)

[13]

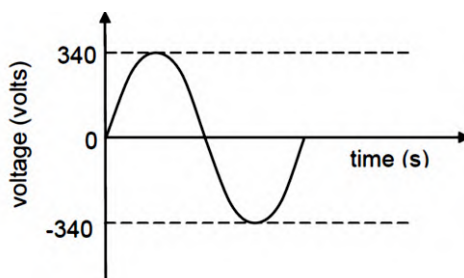


QUESTION 8



- 8.1 In the simplified AC generator, the coil is rotated clockwise
- 8.1.1 In which direction does the induced current flow in the coil?
Choose from: **X to Y** or **Y to X** (1)
- 8.1.2 On which principle or law is the working of the generator based? (1)
- 8.1.3 State the energy conversion that takes place while the generator is in operation. (2)

8.2 The voltage output for an AC generator is shown below.



- 8.2.1 Write down the maximum (peak) output voltage of the generator. (1)

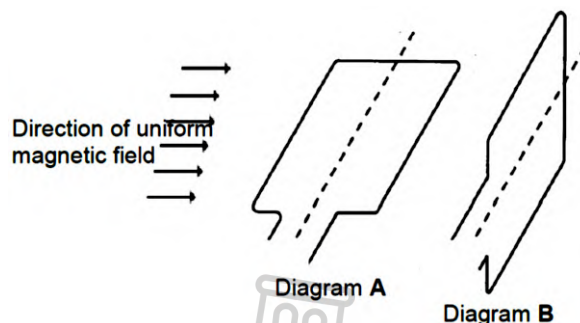
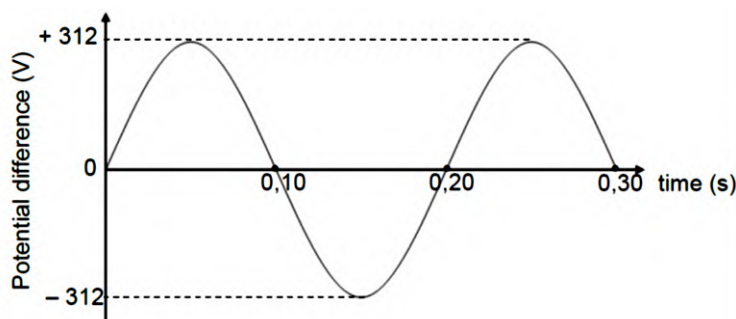
A stove is connected to the generator above, and delivers an average power of 1 600 W.

- 8.2.2 Calculate the rms voltage delivered to the stove. (3)
- 8.2.3 Calculate the resistance of the stove. (3)

[11]

QUESTION 9 (June 2019)

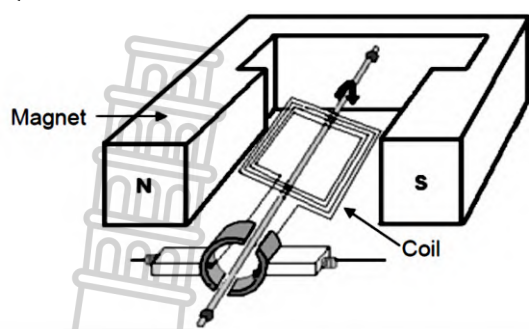
The graph shows the voltage output of a generator. Diagrams **A** and **B** show the position of the generator at different times.



- 9.1 Does this generator have split rings or slip rings? (1)
- 9.2 Which ONE of the diagrams below, **A** or **B**, shows the position of the generator's coil at time = 0,10 s? (1)
- 9.3 Calculate the root mean square (rms) voltage for this generator. (3)
- 9.4 A device with a resistance of $40\ \Omega$ is connected to this generator. Calculate the:
- 9.4.1 Average power delivered by the generator to the device (3)
- 9.4.2 Maximum current delivered by the generator to the device (4)

[12]

QUESTION 10



- 10.1 A simplified diagram of an electric generator is shown. When the coil is rotated with a constant speed, an emf is induced in the coil.
- 10.1.1 Is this an AC generator or a DC generator? (1)
- 10.1.2 Briefly explain how an emf is generated in the coil when the coil is rotated by referring to the principle of electromagnetic induction. (2)
- 10.1.3 Draw a sketch graph of the output voltage versus time for this generator. Show ONE complete cycle. (2)

- 10.2 A $200\ \Omega$ resistor is connected to a DC voltage supply, as shown in diagram A. The energy dissipated in the resistor in 10 s is 500 J. The same resistor is now connected to an AC source (diagram B) and 500 J of energy is also dissipated in the resistor in 10 s.

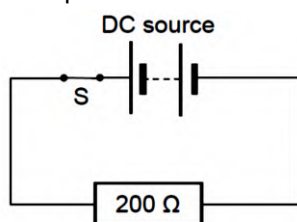


Diagram A

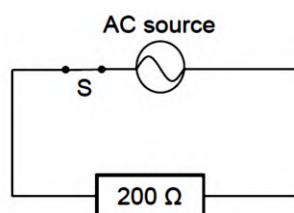


Diagram B

- 10.2.1 Define the term *rms voltage* of an AC source. (2)
- 10.2.2 Calculate the maximum (peak) voltage of the AC source. (5)

[12]

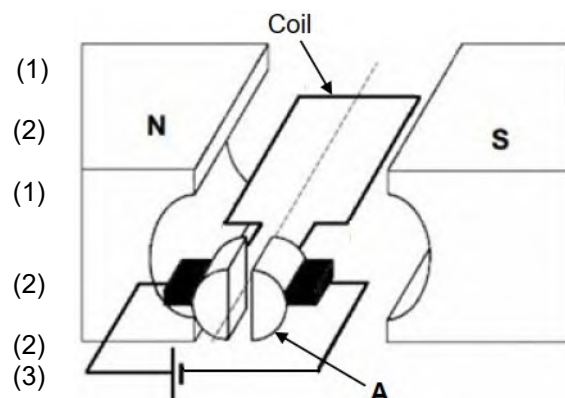
QUESTION 11

- 11.1 A simplified diagram of an electrical machine is shown.

- 11.1.1 Is this machine a DC motor or a DC generator? (1)
- 11.1.2 Write down the energy conversion that takes place while this machine is in operation. (2)
- 11.1.3 Write down the name of component A in the diagram. (1)
- 11.1.4 In which direction will the coil, shown in the diagram, rotate? Choose from CLOCKWISE or ANTICLOCKWISE. (2)

- 11.2 An electrical device is marked 200 W; 220 V.

- 11.2.1 Define the term *rms voltage*. (2)
- 11.2.2 Calculate the resistance of the device. (2)
- 11.2.3 This device is now connected to a 150 V AC source. Calculate the energy dissipated by the device in 10 minutes. (3)

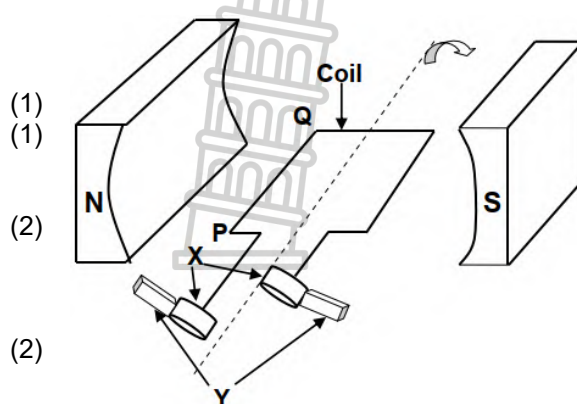


(5)
[16]

QUESTION 12

The diagram is a simplified representation of an AC generator. The coil is rotated in a clockwise direction in the magnetic field.

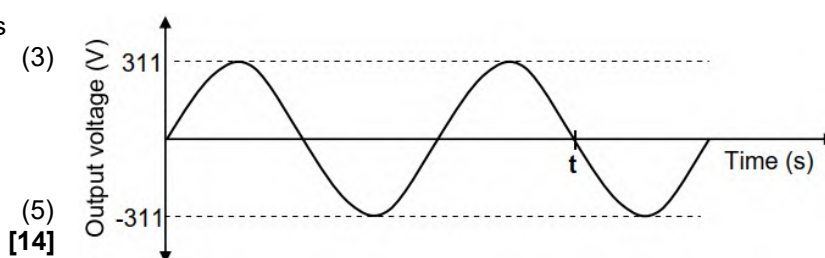
- 12.1 Write down the name of component X. (1)
- 12.2 Write down the function of component Y. (1)
- 12.3 Use the relevant principle to explain why an emf is induced in the coil when the coil is rotated in the magnetic field. (2)
- 12.4 The coil rotates CLOCKWISE from the position shown in the diagram. In which direction will current be induced in segment PQ of the coil? Choose from 'P to Q' or 'Q to P'. (2)



The output voltage versus time graph was obtained for the above generator. The output voltage is generated at a frequency of 50 Hz.

12.5 Calculate the time t indicated in this graph.

12.6 The generator is now connected to an appliance with a resistance of $100\ \Omega$. Calculate the energy dissipated when the appliance is in operation for ONE minute.



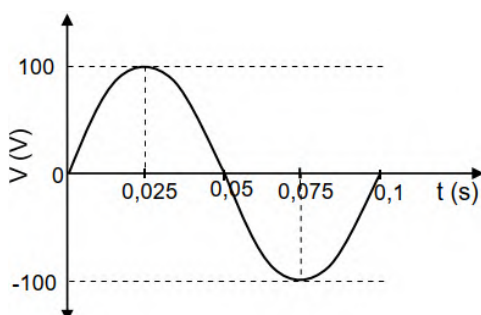
QUESTION 13

A simplified diagram of an AC generator connected to a $25\ \Omega$ resistor is shown. The coil rotates anticlockwise.

13.1 Name the component that distinguishes this generator from a DC generator.

13.2 In which direction will the induced current flow in section XY of the coil? Choose from X to Y OR Y to X .

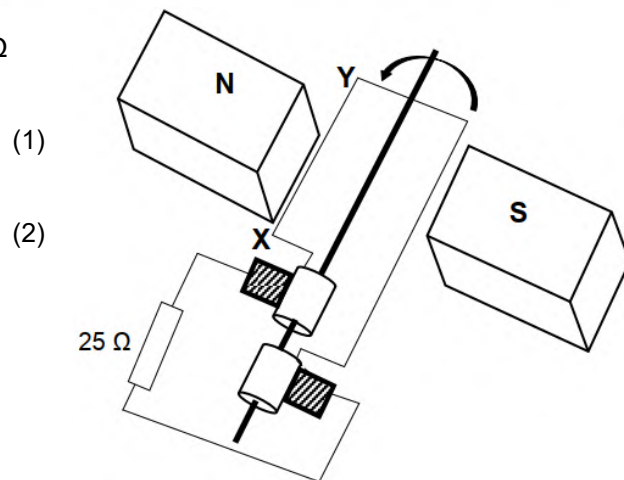
The graph below shows the output voltage of the generator for one cycle of rotation of the coil.



13.3 Define the term rms potential difference.

13.4 Calculate the rms current in the circuit.

13.5 Calculate the average power dissipated in the $25\ \Omega$ resistor.



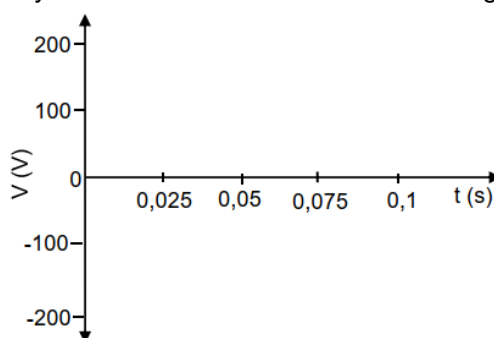
(2)

(4)

(3)

The speed of rotation of the coil in the generator is now DOUBLED.

13.6 Copy the set of axes below in your ANSWER BOOK and sketch the graph of output voltage versus time for 0,1 s.



(3)

[15]

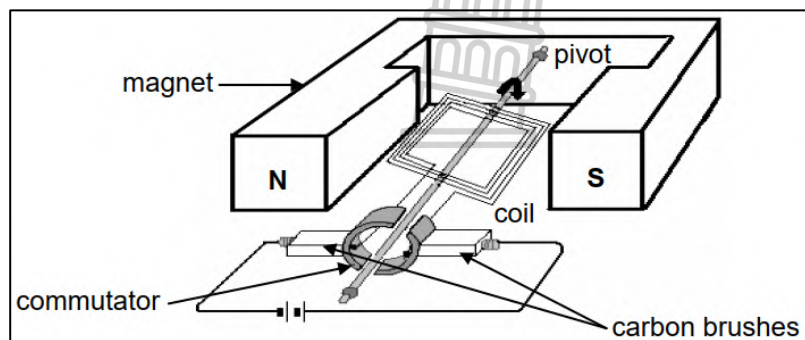
QUESTION 14

14.1 The simplified sketch of an electric motor is shown on the right.

14.1.1 Write down the energy conversion that takes place in this motor.

14.1.2 Is this motor an AC motor or a DC motor?

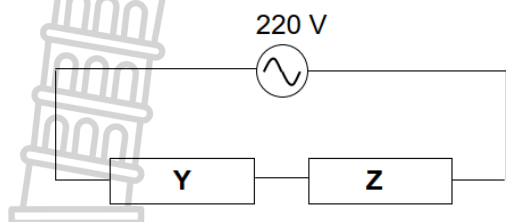
14.1.3 What is the function of the commutator in this motor?



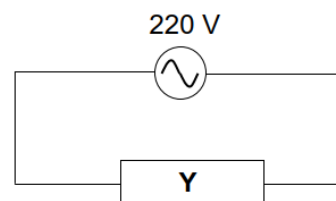
- 14.2 A resistor **Y** is rated 220 V, 100 W and is connected to a 220 V AC source, as shown in the circuit on the right.

14.2.1 Calculate the resistance of resistor **Y**.

Another resistor **Z** with a rating 220 V, X W, is now connected in series to resistor **Y** and to the same AC source. See the diagram below.



(3)



The power dissipated by resistor **Y** changes to 80 W, while its resistance remains constant.

14.2.2 Calculate the power rating **X** of resistor **Z**, if resistor **Z** has constant resistance.

(6)

[12]

QUESTION 15

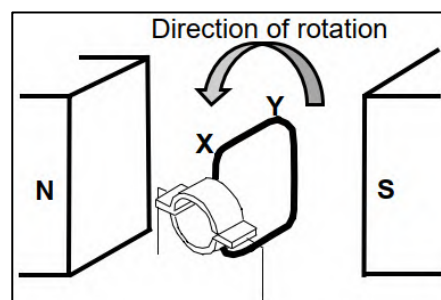
- 15.1 The diagram shows the initial position of the coil in a simple DC generator. The coil is rotated in an anticlockwise direction, as shown.

15.1.1 Name the component in this generator that ensures that the induced current in the external circuit is in one direction only.

15.1.2 Is the direction of the induced current from **X** to **Y** or from **Y** to **X**?

(1)

(1)



A maximum voltage of 90 V is generated when the coil is rotating at a frequency of 20 Hz.

15.1.3 Write down the time taken for the coil to complete ONE rotation.

(1)

15.1.4 The coil starts rotating from the initial position, as shown in the diagram. Sketch a graph of output voltage versus time for one complete rotation of the coil. Indicate the maximum voltage and the relevant time values on the graph.

(4)

- 15.2 Wall sockets supply rms voltage and current. A 220 V AC voltage is supplied from a wall socket to an electric kettle having a resistance of 32 Ω . Calculate the average energy dissipated by the kettle in TWO minutes.

(4)

[11]

QUESTION 16 (June 2023)

- 16.1 The simplified sketch represents an AC generator with the coil initially horizontal between the poles of a magnet. **X** and **Y** are two points on the coil, while **A** is one of the poles of the magnet.

When the coil of the generator rotates clockwise between the two poles of the magnet, the direction of the induced current is from **X** to **Y**, as shown.

16.1.1 Is **A** the NORTH POLE or the SOUTH POLE of the magnet?

(1)

16.1.2 The coil is now rotated through 180°. Will the direction of the current be from **X** to **Y** or from **Y** to **X**?

(1)

16.1.3 Sketch an emf-time graph for TWO complete rotations of the coil, starting from the position of the coil as shown in the diagram above.

(3)

- 16.2 An electrical device is connected to an AC generator. The rms potential difference across the device is 200 V and the maximum current passing through the device is 6 A. Calculate the:

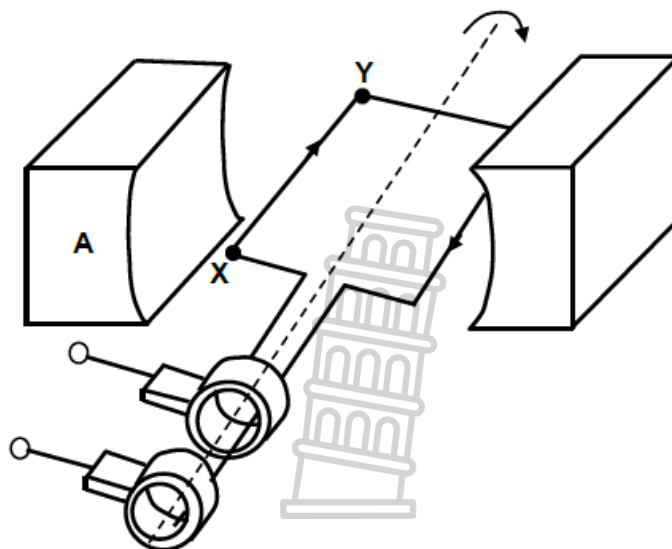
16.2.1 Resistance of the device

(4)

16.2.2 Energy consumed by the device in two hours

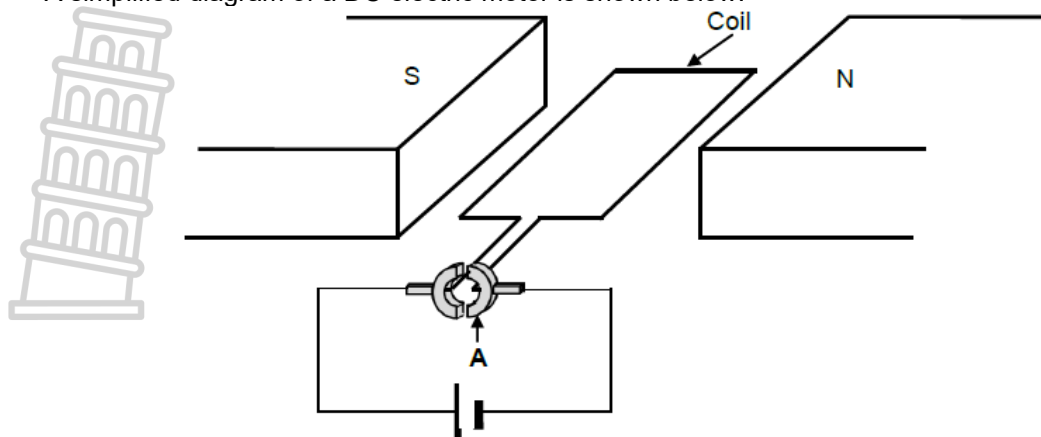
(4)

[13]



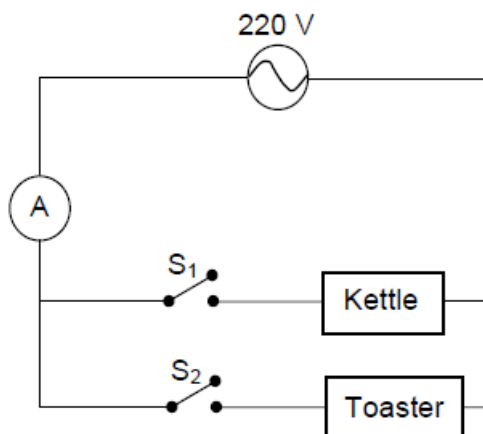
QUESTION 17 (November 2023)

17.1 A simplified diagram of a DC electric motor is shown below.



- 17.1.1 Write down the name of component **A**. (1)
 17.1.2 State the energy conversion that takes place in the motor. (1)
 17.1.3 In which direction will the coil rotate? Choose from CLOCKWISE or ANTICLOCKWISE. (2)
 17.1.4 State TWO changes that can be made to the motor for the coil to rotate faster. (2)

17.2 The circuit diagram below shows an electric kettle, and a toaster connected to an AC source with an rms voltage of 220 V. The ammeter, connecting wires and switches S_1 and S_2 have negligible resistance.

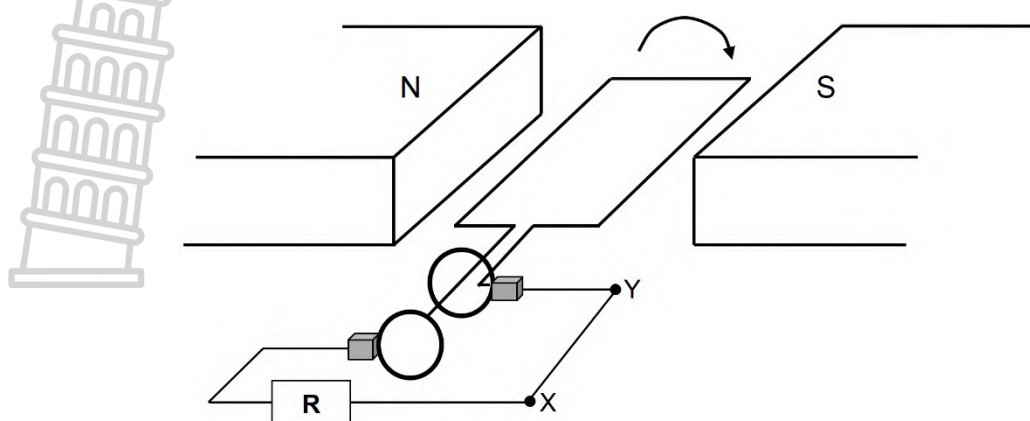


- 17.2.1 Define the term *root mean square current*. (2)
 17.2.2 When switch S_1 is CLOSED and switch S_2 is OPEN, the maximum current through the circuit is 3,6 A. Calculate the root mean square current in the circuit. (3)
 17.2.3 When switch S_1 is OPEN and switch S_2 is CLOSED, the root mean square current in the circuit is 2,62 A. Calculate the energy consumed by the toaster in two minutes. (3)

[14]

QUESTION 18 (June 2024)

The simplified diagram below represents an AC generator with a coil rotating clockwise. **X** and **Y** are two points in the external circuit.

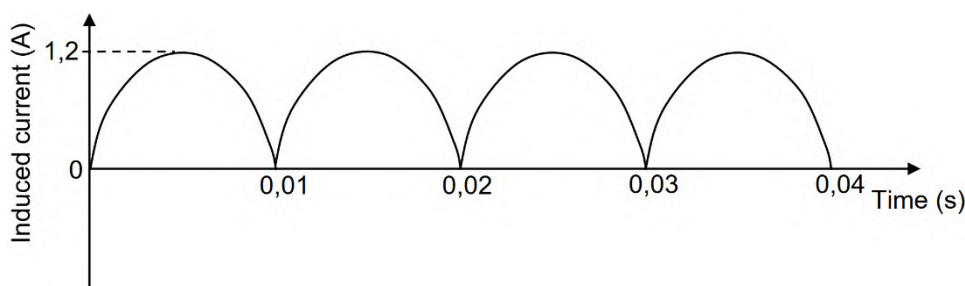


- 18.1 What is the direction of the current in the external circuit? Write either **X to Y** or **Y to X**. (2)
 - 18.2 State the energy conversion that takes place in this generator. (1)
- The maximum voltage produced by the generator is 125 V.
- 18.3 Define the term *root mean square voltage*. (2)
 - 18.4 Calculate the root mean square voltage of the generator. (3)
 - 18.5 The total resistance in the external circuit is $42,4 \, \Omega$. Calculate the maximum current induced. (3)
 - 18.6 The generator induced current at a frequency of 20 Hz. The coil started rotating from the initial position, as shown in the diagram above. Sketch a graph of induced current versus time for two complete rotations of the coil. Indicate the following on the graph: (4)
- The time taken for two rotations
 - The maximum current induced by the generator

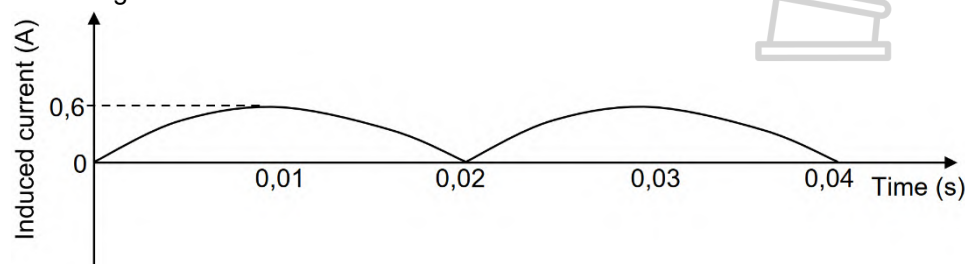
[15]

QUESTION 19 (November 2024)

The graph below shows how the induced current in a generator varies with time.



- 19.1 Name the type of generator. Choose from AC or DC. (1)
- 19.2 State the energy conversion that takes place in this generator while it is in operation. (2)
- 19.3 Give a reason why this generator is NOT suitable for the transmission of electricity over long distances. (1)
- 19.4 Calculate the frequency at which the coil rotates in the generator. (2)
- 19.5 Define the term *root mean square (rms) current*. (2)
- 19.6 Calculate the root mean square current delivered by the generator. (3)
- 19.7 The graph below shows how the induced current varies with time after a change was made to the operation of the generator.



Fully describe the change that was made.

(2)

[13]

OPTICAL PHENOMENA AND PROPERTIES OF MATERIALS

Photo-electric effect

The process whereby electrons are ejected from a metal surface when light of suitable frequency shines on the surface.

$E = hf$

$E_{k(max)} = \frac{1}{2}mv_{max}^2$

$W_0 = hf_0$

Each incident light photon has energy:
 The metal needs energy (work function) to release an electron:
 A photoelectron moves away at kinetic energy:

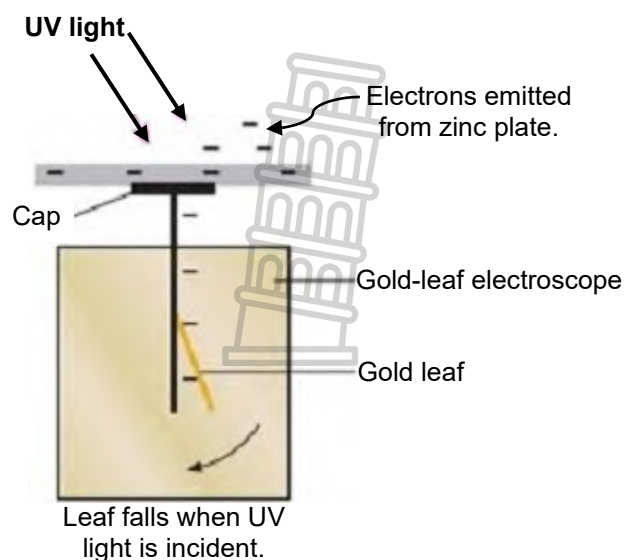
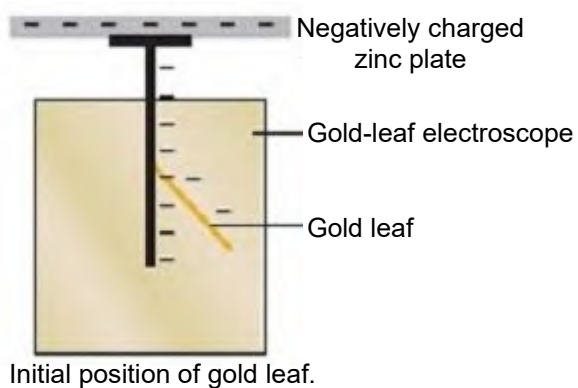
$E = hf$
 $W_0 = hf_0$
 $E_{k(max)} = \frac{1}{2}mv_{max}^2$

$E_{k(max)} = hf - W_0$ OR $hf = W_0 + E_{k(max)}$

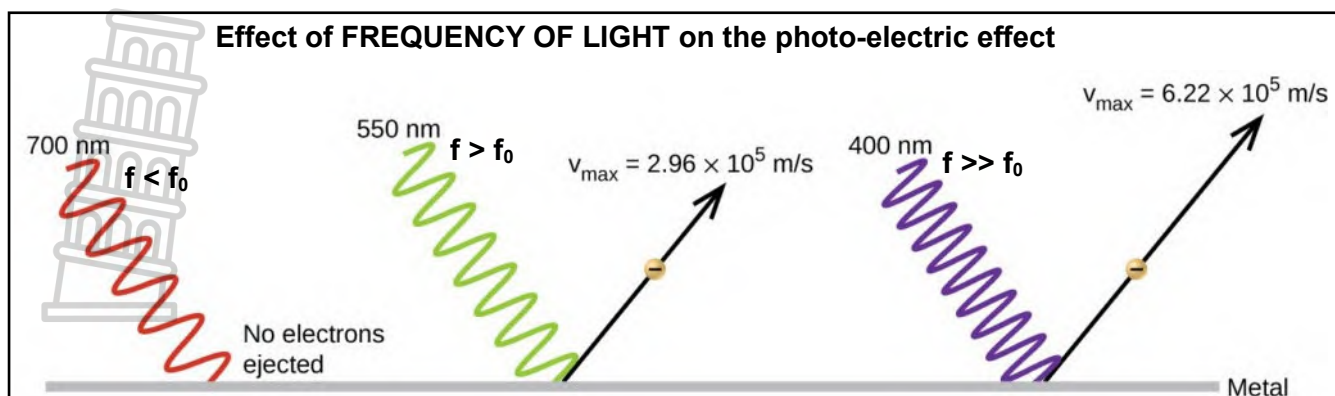
h :	Planck's constant $6,63 \times 10^{-34} \text{ J}\cdot\text{s}$
f :	Frequency of incident light in hertz (Hz)
W_0 :	Work function of the metal in joule (J)
$E_{k(max)}$:	Maximum kinetic energy of photoelectrons in joule (J)

Demonstration of the photo-electric effect

- A zinc plate is placed on the cap of an electroscope.
- The electroscope and zinc plate are negatively charged.
- UV light is shone on the zinc plate.

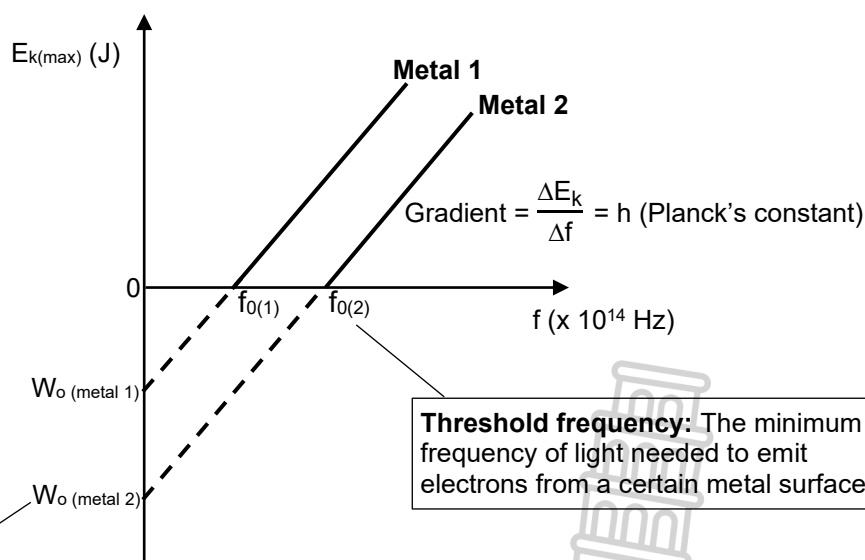


FACTORS INFLUENCING THE PHOTO-ELECTRIC EFFECT



- The frequency of incident light determines whether electrons will be emitted or not.
- A metal has a minimum frequency needed to release electrons from the surface of the metal. It is called the **threshold frequency**, f_0 .
- If the frequency of the incident light is smaller than the threshold frequency for the metal, no electrons will be emitted.
- If the frequency of the incident light is equal to the threshold frequency for the metal, electrons will be emitted with zero kinetic energy.
- If the frequency of the incident light is greater than the threshold frequency for the metal, electrons will be emitted with a certain kinetic energy i.e. $E_{k(\max)} = \frac{1}{2}mv_{\max}^2 = hf - hf_0$.
- Work function** of a metal, $W_0 = hf_0$, is the minimum energy that an electron in the metal needs to be emitted from the metal surface.

Graph of $E_{k(\max)}$ versus frequency



Work function: The minimum energy that an electron in the metal needs to be emitted from the metal surface.

Threshold frequency: The minimum frequency of light needed to emit electrons from a certain metal surface.

Effect of INTENSITY OF LIGHT on the photo-electric effect

Light ejects electrons from the metal.

More light photons (light of higher intensity) ejects more electrons with the same kinetic energy.

Metal

If the frequency of incident light is high enough to emit electrons from a metal surface, a **higher intensity** of this light will **emit more electrons per unit time with the same kinetic energy**.

Cathode

Photocell

Electrons

Evacuated quartz tube

V

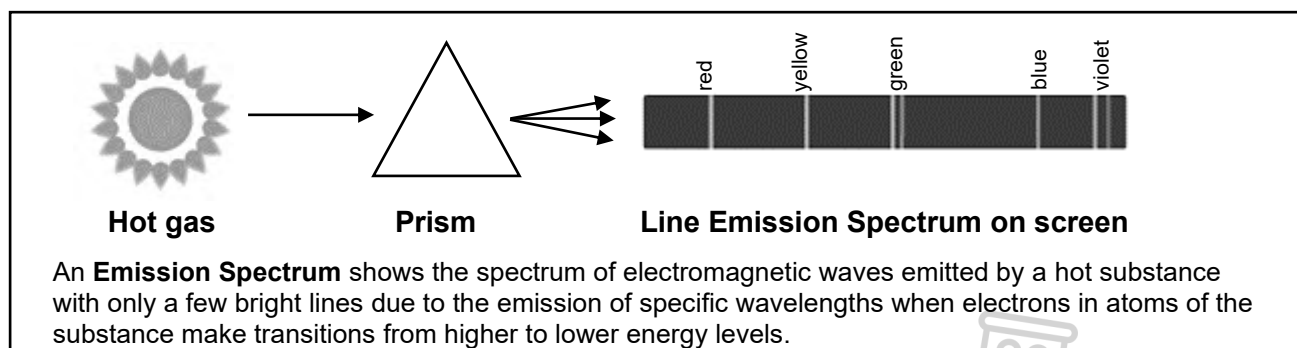
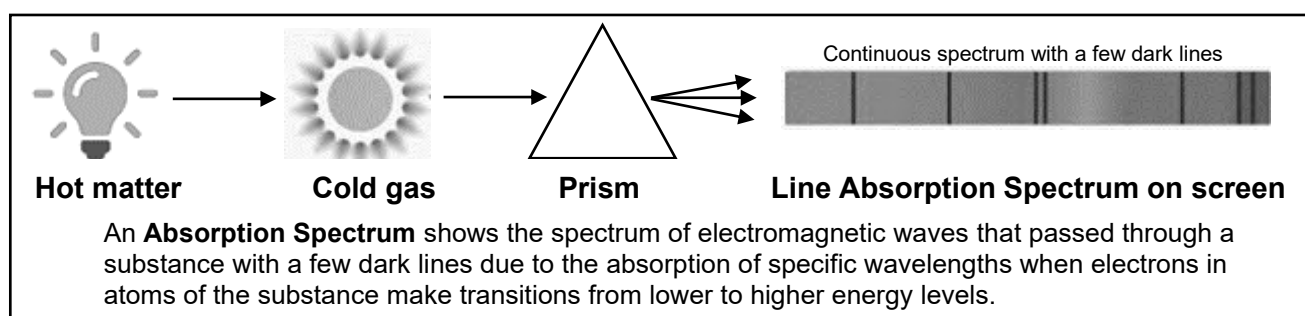
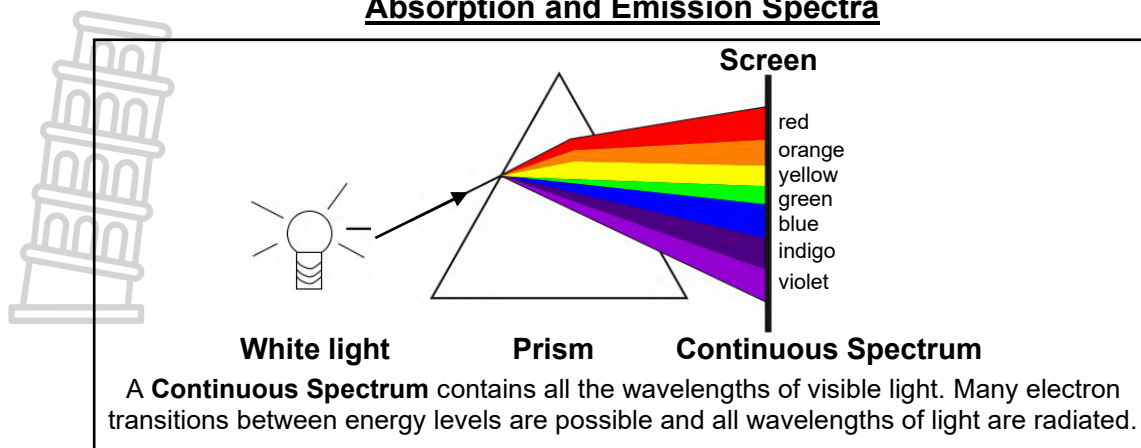
R

A

- Incident light emits electrons from the cathode.
- The photoelectrons are then attracted by the positive anode and a current flow in the circuit.
- The ammeter registers a reading.
- If light of higher intensity is used, the ammeter will register a higher reading because more photons strike the metal cathode per unit time and therefore more electrons are emitted from the cathode per unit time.

TERMS AND DEFINITIONS	
Photo-electric effect	The process whereby electrons are ejected from a metal surface when light of suitable frequency is incident on the surface.
Threshold frequency (f_o)	The minimum frequency of light needed to emit electrons from a certain metal surface.
Work function (W_o)	The minimum energy that an electron in the metal needs to be emitted from the metal surface.
Photo-electric equation	$E = W_o + K_{max}$, waar $E = hf$, $W_o = hf_o$ en $K_{max} = \frac{1}{2}mv_{max}^2$
Atomic absorption spectrum	An atomic absorption spectrum is formed when certain frequencies of electromagnetic radiation passing through a substance is absorbed.
Atomic emission spectrum	An atomic emission spectrum is formed when certain frequencies of electromagnetic radiation are emitted due to an atom making a transition from a higher energy state to a lower energy state.

Absorption and Emission Spectra



Typical questions

QUESTION 1

1.1 In an experiment on the photoelectric effect, light is incident on the surface of a metal and electrons are ejected.

1.1.1 What does the photoelectric effect indicate about the nature of light? (1)

1.1.2 The intensity of the light is increased. Will the maximum speed of the ejected electrons INCREASE, DECREASE or REMAIN THE SAME? Give a reason for the answer. (2)

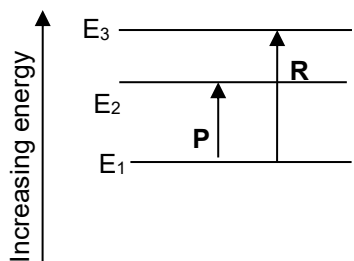
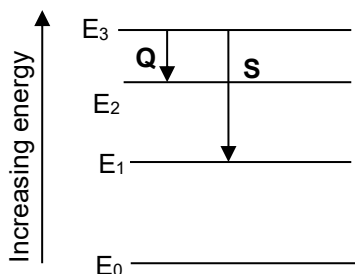
The wavelength corresponding with the threshold frequency is referred to as threshold wavelength.

The table below gives the values of threshold wavelengths for three different metals.

METAL	THRESHOLD WAVELENGTH (λ_0) IN METRES
Silver	$2,88 \times 10^{-7}$
Calcium	$4,32 \times 10^{-7}$
Sodium	$5,37 \times 10^{-7}$

In the experiment using one of the metals above, the maximum speed of the ejected electrons was recorded as $4,76 \times 10^5 \text{ m} \cdot \text{s}^{-1}$ for light of wavelength 420 nm.

1.1.3 Identify the metal used in the experiment by means of suitable calculations. (5)

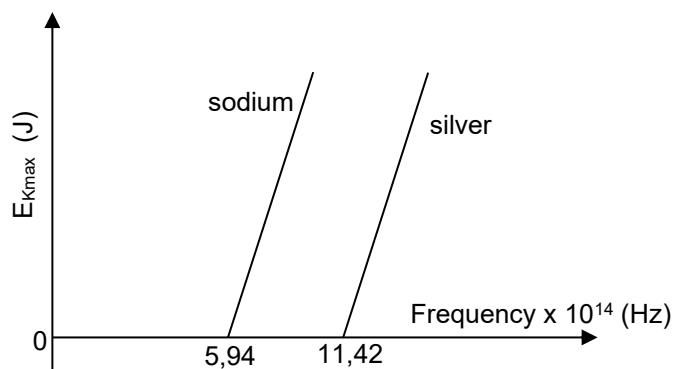


1.2 The simplified energy diagrams showing the possible electron transitions in an atom are shown alongside. Using the letters **P**, **Q**, **R** and **S**, identify the lines that CORRECTLY show transitions that will result in the atom giving off an EMISSION SPECTRUM. Give a reason for the answer. (4)

[12]

QUESTION 2

2.1 A learner is investigating the photoelectric effect for two different metals, silver and sodium, using light of different frequencies. The maximum kinetic energy of the emitted photoelectrons is plotted against the frequency of the light for each of the metals, as shown in the graphs below.



2.1.1 Define the term *threshold frequency*. (2)

2.1.2 Which metal, sodium, or silver, has the larger work function? Explain the answer. (3)

2.1.3 Name the physical constant represented by the slopes of the graphs. (1)

2.1.4 If light of the same frequency is shone on each of the metals, in which metal will the ejected photoelectrons have a larger maximum kinetic energy? (1)

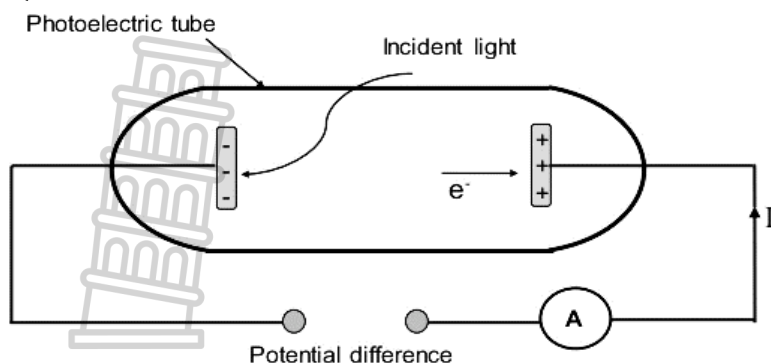
2.2 In a different photoelectric experiment blue light obtained from a light bulb is shone onto a metal plate and electrons are released. The wavelength of the blue light is $470 \times 10^{-9} \text{ m}$ and the bulb is rated at 60 mW. The bulb is only 5% efficient.

2.2.1 Calculate the number of photons that will be incident on the metal plate per second, assuming all the light from the bulb is incident on the metal plate. (5)

2.2.2 Without any further calculation, write down the number of electrons emitted per second from the metal. (1)

[13]

QUESTION 3



A simplified diagram of an apparatus for an experiment to investigate the photoelectric effect is shown alongside. Light of fixed frequency is incident on the cathode of a photoelectric tube. During the experiment the ammeter (A) registers the photocurrent.

3.1 Define the term *photoelectric effect*. (2)

The intensity of the incident light is now increased.

3.2 State how this increase in intensity will affect the reading on the ammeter. Choose from INCREASE, DECREASE or REMAIN THE SAME. Give a reason for the answer. (3)

When the frequency of the incident light is $5,9 \times 10^{14}$ Hz, the maximum recorded kinetic energy of photoelectrons is $2,9 \times 10^{-19}$ J.

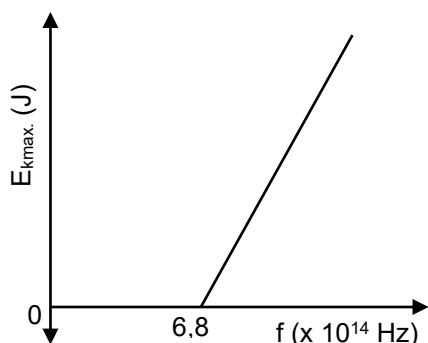
3.3 Calculate the maximum wavelength (threshold wavelength) of the incident light that will emit an electron from the cathode of the photo-electric tube. (5)

The maximum kinetic energy of the photoelectrons ejected increases when light of a higher frequency is used.

3.4 Use the photoelectric equation to explain this observation. (2)

[12]

QUESTION 4



The graph is obtained for an experiment on the photoelectric effect using different frequencies of light and a given metal plate. The threshold frequency for the metal is $6,8 \times 10^{14}$ Hz.

4.1 Define the term *threshold frequency*. (2)

In the experiment, the brightness of the light incident on the metal surface is increased.

4.2 State how this change will influence the speed of the photoelectrons emitted. Choose from INCREASES, DECREASES or REMAINS UNCHANGED. (1)

4.3 Show by means of a calculation whether the photoelectric effect will be OBSERVED or NOT OBSERVED, if monochromatic light with a wavelength of 6×10^{-7} m is used in this experiment. (5)

One of the radiations used in this experiment has a frequency of $7,8 \times 10^{14}$ Hz.

4.4 Calculate the maximum speed of an ejected photoelectron. (5)

[13]

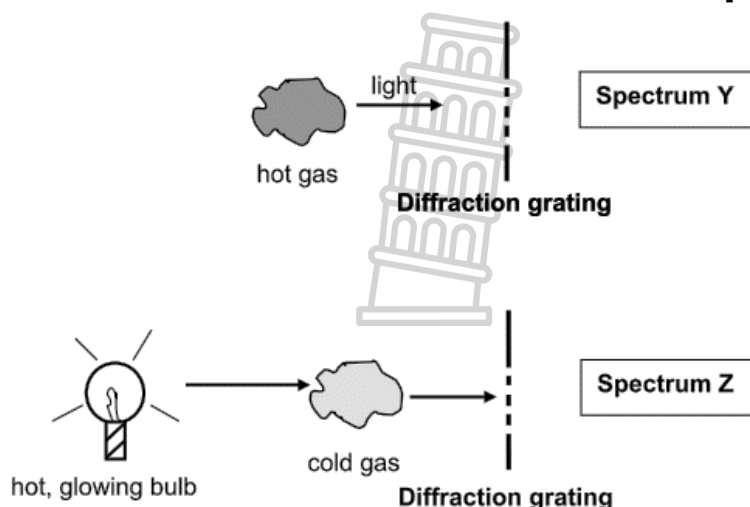
QUESTION 5

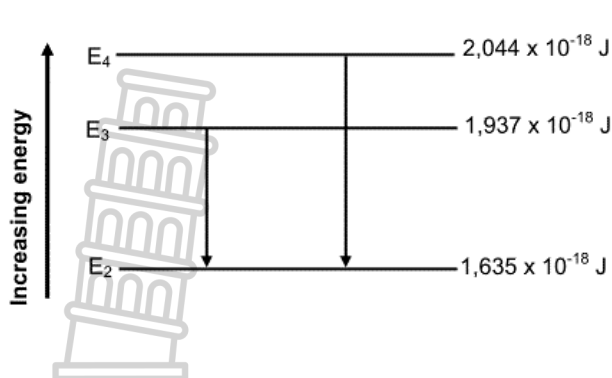
5.1 A teacher in a science class explains how different types of spectra are obtained. The teacher uses the simplified diagrams shown for the explanation. Name the type of spectrum of:

5.1.1 **Y** (1)

5.1.2 **Z** (1)

5.2 In an excited atom, electrons can 'jump' from lower energy levels to higher energy levels. They can also 'drop' from higher energy levels to lower energy levels. The diagram (not drawn to scale) shows some of the transitions for electrons in an excited atom.





5.2.1 Do the transitions indicated in the diagram lead to ABSORPTION or EMISSION spectra? (1)

5.2.2 Calculate the frequency of the photon produced when an electron in an excited atom makes a transition from E_4 to E_2 , as shown in the diagram. (4)

The threshold frequency of a metal, **Q**, is $4,4 \times 10^{14}$ Hz.

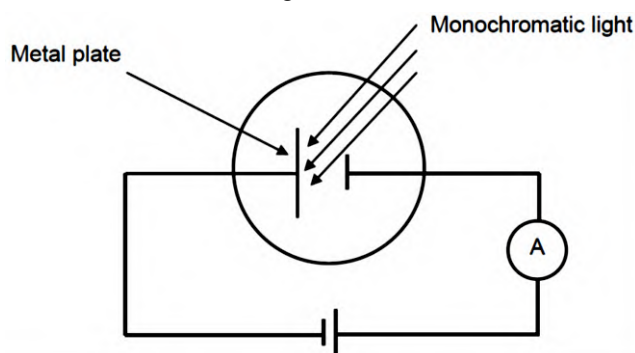
5.2.3 Calculate the kinetic energy of the most energetic electron ejected when the photon produced in QUESTION 5.2.2 is incident on the surface of metal **Q**. (4)

Another metal, **R**, has a threshold frequency of $7,5 \times 10^{14}$ Hz.

5.2.4 Will the photon produced in QUESTION 5.2.2 be able to eject electrons from the surface of metal **R**? Write down only YES or NO. Give a reason for the answer. (2)
[13]

QUESTION 6

6.1 In the diagram, monochromatic light is incident on the metal plate of a photocell. A sensitive ammeter shows a reading.



6.1.1 How does the energy of the photons of the incident light compare to the work function of the metal plate? Choose from GREATER THAN, LESS THAN or EQUAL TO. Give a reason for the answer. (2)

6.1.2 When a change is made to the monochromatic light, the reading on the ammeter increases. A learner makes the following statement with regard to this change:
The increase in the ammeter reading is due to an increase in the energy of the incident photons. Give a reason why this statement is INCORRECT. (2)
(1)

6.1.3 What does the photoelectric effect tell us about the nature of light?

6.2 Ultraviolet radiation is incident on the surface of sodium metal. The threshold frequency (f_0) for sodium is $5,73 \times 10^{14}$ Hz. The maximum speed of an electron emitted from the metal surface is $4,19 \times 10^5$ m·s⁻¹.

6.2.1 Define or explain the term *threshold frequency*. (2)

6.2.2 Calculate the work function of sodium. (3)

6.2.3 Calculate the frequency of the incident photon. (3)
[13]

QUESTION 7

A group of students investigates the relationship between the work function of different metals and the maximum kinetic energy of the ejected electrons when the metals are irradiated with light of suitable frequency.

7.1 Define the term *work function*. (2)

During the investigation ultraviolet rays of wavelength 2×10^{-8} m are allowed to fall on different metal plates. The corresponding maximum kinetic energies of ejected electrons are measured. The data obtained is displayed in the table below.

METAL PLATE USED	MAXIMUM KINETIC ENERGY ($E_{k(max)}$) ($\times 10^{-18}$ J)
Lead	9,28
Potassium	9,58
Silver	9,19

7.2 Write down the dependent variable for this investigation. (1)

7.3 Write down ONE control variable for this investigation. (1)

7.4 Using the information in the table, and without any calculation, identify the metal with the largest work function. Explain the answer. (3)

7.5 Use information in the table to calculate the work function of potassium. (4)

7.6 State how an increase in the intensity of the ultraviolet light affects the maximum kinetic energy of the photoelectrons. Choose from INCREASES, DECREASES, REMAINS THE SAME. Explain the answer. (3)
[14]

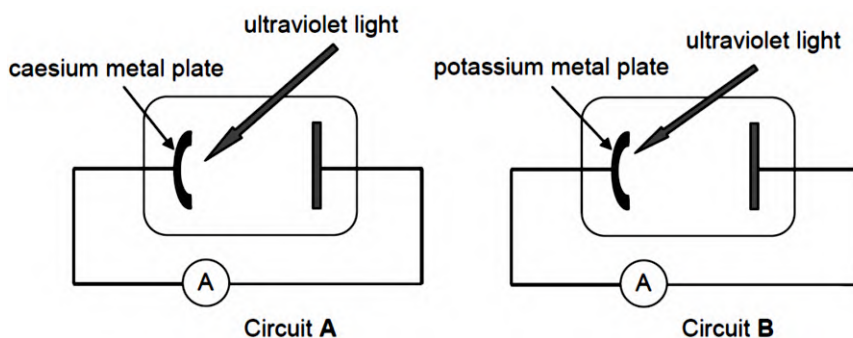
QUESTION 8

The threshold frequencies of caesium and potassium metals are given in the table below.

METAL	THRESHOLD FREQUENCY
Caesium	$5,07 \times 10^{14} \text{ Hz}$
Potassium	$5,55 \times 10^{14} \text{ Hz}$

- 8.1 Define the term *work function* in words. (2)
- 8.2 Which ONE of the two metals in the table has the higher work function? Give a reason for the answer by referring to the information in the table. (2)

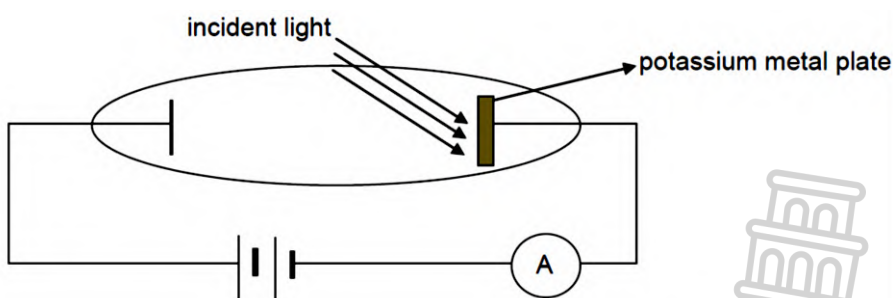
The simplified diagrams below show two circuits, **A** and **B**, containing photocells. The photocell in circuit **A** contains a caesium metal plate, while the photocell in circuit **B** contains a potassium metal plate. Ultraviolet light with the same intensity and wavelength of $5,5 \times 10^{-7} \text{ m}$ is incident on the metal plate in EACH of the photocells and the ammeter in circuit **A** registers a current.



- 8.3 By means of a calculation, determine whether the ammeter in circuit **B** will also register a current. (3)
- 8.4 Calculate the maximum kinetic energy of an ejected electron in circuit **A**. (5)
- 8.5 How will the maximum kinetic energy of the ejected electron, calculated in QUESTION 8.4, change when the intensity of the incident light increases? Choose from: INCREASES, DECREASES or REMAINS THE SAME (1)
- [13]**

QUESTION 9 (June 2019)

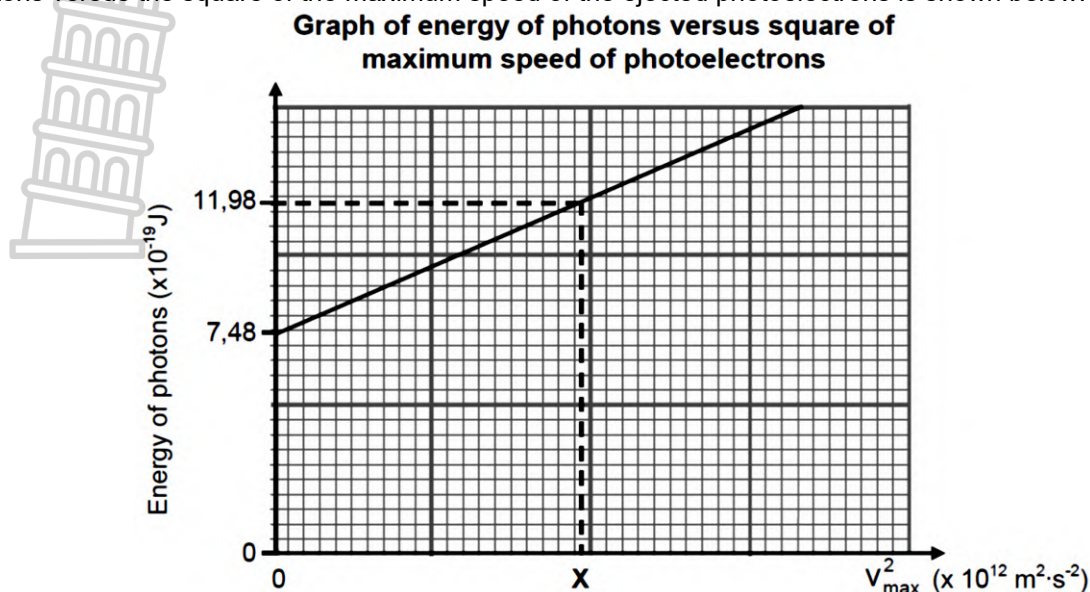
A potassium metal plate is irradiated with light of wavelength $5 \times 10^{-7} \text{ m}$ in an arrangement, as shown below. The threshold frequency of potassium is $5,55 \times 10^{14} \text{ Hz}$.



- 9.1 Define the term threshold frequency. (2)
- 9.2 Calculate the energy of a photon incident on the metal plate. (3)
- 9.3 Using a suitable calculation, prove that the ammeter will show a reading. (4)
- 9.4 The intensity of the light is now increased. Explain why this change causes an increase in the ammeter reading. (3)
- [12]**

QUESTION 10

During an experiment, light of different frequencies is radiated onto a silver cathode of a photocell and the corresponding maximum speed of the ejected photoelectrons are measured. A graph of the energy of the incident photons versus the square of the maximum speed of the ejected photoelectrons is shown below.



10.1 Define the term *photoelectric effect*. (2)

Use the graph to answer the following questions.

10.2 Write down the value of the work function of silver. Use a relevant equation to justify the answer. (3)

10.3 Which physical quantity can be determined from the gradient of the graph? (1)

10.4 Calculate the value of X as shown on the graph. (5)

The experiment above is now repeated using light of higher intensity.

10.5 How will EACH of the following be affected? Choose from INCREASES, DECREASES or REMAINS THE SAME.

10.5.1 The gradient of the graph (1)

10.5.2 The number of photoelectrons emitted per unit time (1)

[13]

QUESTION 11

An experiment is conducted to investigate the relationship between the frequency of light incident on a metal and the maximum kinetic energy of the emitted electrons from the surface of the metal. This experiment is conducted for three different metals. The graph represents the results obtained.

11.1 Name the phenomenon on which this experiment is based. (1)

11.2 Name the physical quantity represented by X on the graph. (1)

11.3 Which ONE of the three metals needs incident light with the *largest wavelength* for the emission of electrons? Give a reason for the answer. (2)

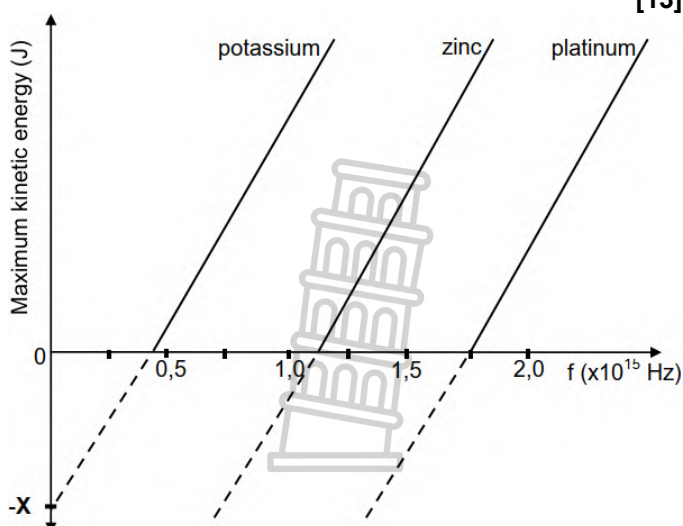
11.4 Define the term *work function* in words. (2)

11.5 Calculate the:

11.5.1 Work function of **platinum** (3)

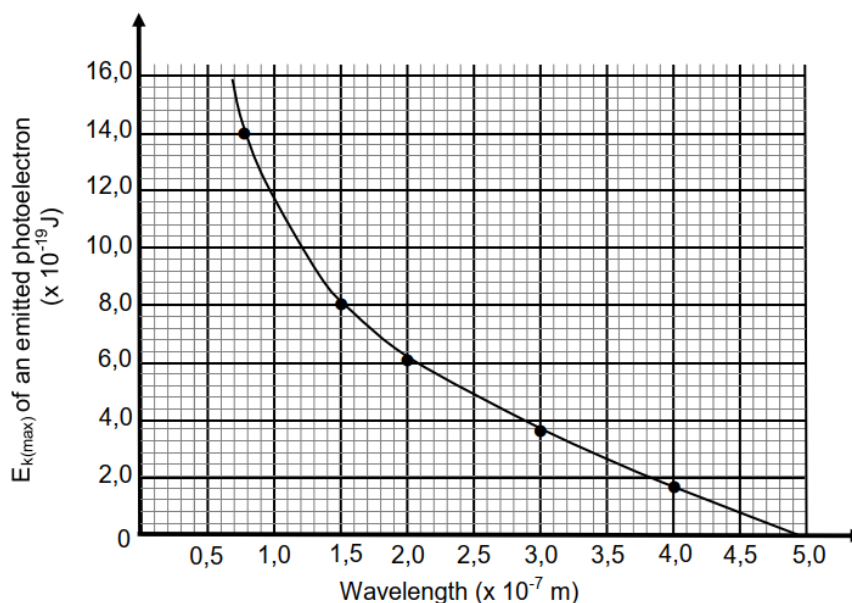
11.5.2 Frequency of the incident light that will emit electrons from the surface of **platinum** with a maximum velocity of $5,60 \times 10^5 \text{ m} \cdot \text{s}^{-1}$ (4)

[13]



QUESTION 12

When light of various frequencies is incident on the metal cathode of a photocell, photoelectrons are emitted from the surface of the cathode. The graph below shows the relationship between the maximum kinetic energy ($E_{k(max)}$) of an emitted photoelectron and the wavelength of the incident light.



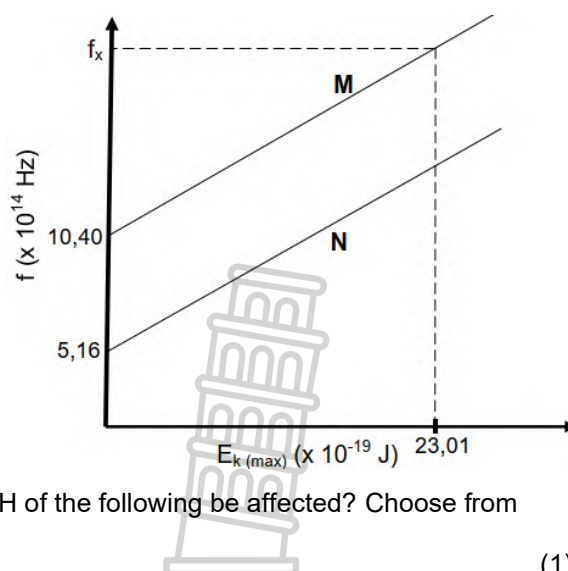
- 12.1 Use the graph to determine the maximum kinetic energy of the emitted photoelectron when the wavelength of the incident light is $1,0 \times 10^{-7} \text{ m}$. (1)
- 12.2 What relationship between the maximum kinetic energy of the emitted photoelectron and the wavelength of the incident light can be deduced from the graph? (2)
- 12.3 Define the term *work function* in words. (2)
- 12.4 Use the graph to calculate the work function of the metal used as cathode of this photocell. (4)
- 12.5 Calculate the maximum kinetic energy of the emitted photoelectron when the wavelength of the incident light is $0,5 \times 10^{-7} \text{ m}$. (4)

[13]

QUESTION 13

The relationship between frequency (f) and maximum kinetic energy ($E_{k(max)}$) of photoelectrons emitted from two cathodes, **M** and **N**, of different photoelectric cells is investigated. The graphs have been obtained from the results.

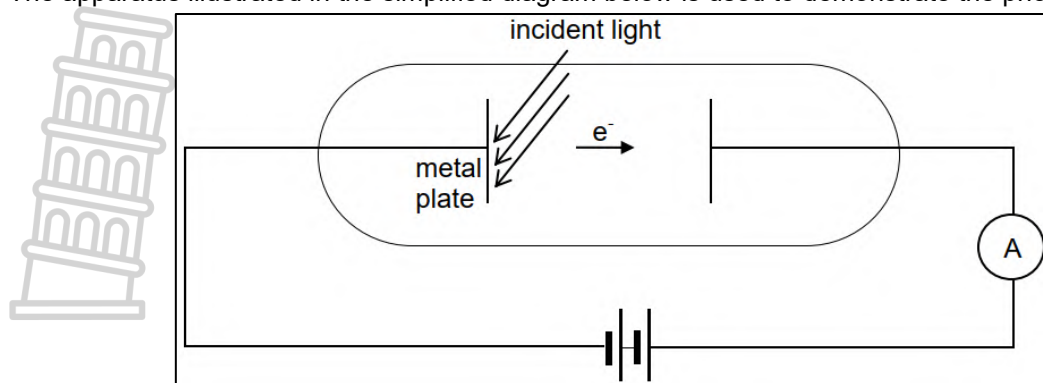
- 13.1 Define the term *threshold frequency*. (2)
- 13.2 How does the maximum kinetic energy of photoelectrons emitted from cathode **N** compare to the maximum kinetic energy of those emitted from cathode **M** when light of a frequency greater than $10,40 \times 10^{14} \text{ Hz}$ is shone on each of the cathodes? Choose from GREATER THAN, SMALLER THAN or EQUAL TO. (2)
- 13.3 Calculate the value of frequency f_x indicated on the graph. (5)
- 13.4 The experiment is now repeated for cathode **M** using light of frequency f_x , but of higher intensity. How will EACH of the following be affected? Choose from INCREASES, DECREASES or NO EFFECT.
 - 13.4.1 The y-intercept of the graph (1)
 - 13.4.2 The number of photoelectrons emitted per unit time (1)
 - 13.4.3 The maximum kinetic energy of the emitted photoelectrons (1)



[12]

QUESTION 14

14.1 The apparatus illustrated in the simplified diagram below is used to demonstrate the photoelectric effect.



14.1.1 Define the term *photoelectric effect*.

(2)

Incident light of frequency $1,2 \times 10^{15}$ Hz is shone onto the metal plate and electrons are emitted. Calculate the:

14.1.2 Number of photoelectrons emitted in one second if the total energy transferred by the light to the metal plate per second is $1,75 \times 10^{-9}$ J

(4)

14.1.3 Maximum speed of a photoelectron if the threshold frequency of the metal plate is $9,09 \times 10^{14}$ Hz

(5)

14.2 Briefly explain how an emission spectrum is formed in terms of energy transitions.

(2)

[13]

QUESTION 15

Light is incident on the cathode of a photoelectric cell connected to a battery and a sensitive ammeter, as shown. The cathode has a work function of $3,42 \times 10^{-19}$ J. Light of frequency $5,96 \times 10^{14}$ Hz is shone onto the cathode.

15.1 What conclusive evidence about the nature of light is provided by the photoelectric effect?

(1)

15.2 Define the term *work function*.

(2)

15.3 Calculate the maximum kinetic energy of an electron ejected from the cathode.

(4)

15.4 The ammeter registers a constant current of 0,012 A.

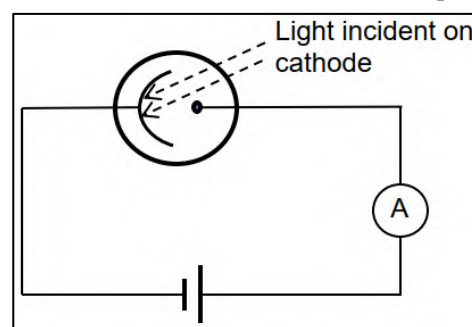
Calculate the minimum number of photons of light that strike the cathode in a 10 s period.

(4)

15.5 The intensity of the incident light is now INCREASED. How will this change affect the reading on the ammeter? Choose from INCREASES, DECREASES or REMAINS THE SAME. Explain the answer.

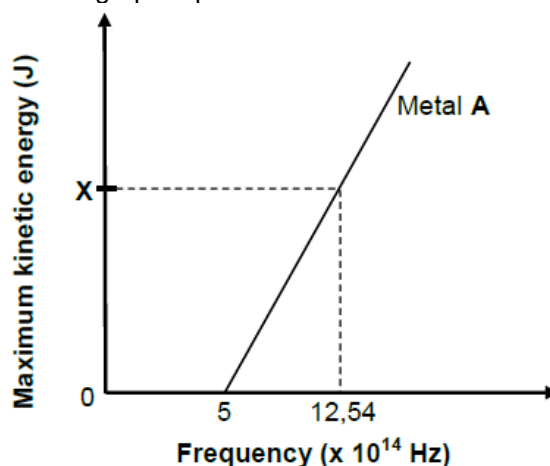
(3)

[14]



QUESTION 16 (June 2023)

In a photoelectric investigation, light of different frequencies was radiated on each of two metals, **A** and **B**. The graph of maximum kinetic energy of the ejected electrons from metal **A** and the frequency of the incident photons is shown below. Point **X** on the graph represents an unknown maximum kinetic energy.



16.1 Write down the numerical value of the gradient of the graph.

(1)

16.2 Define the term *work function*.

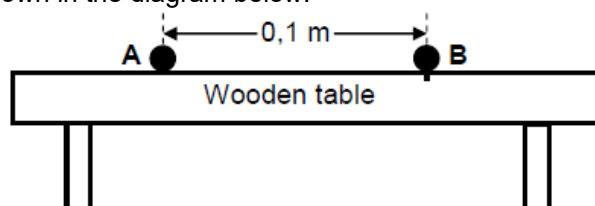
(2)

- 16.3 Calculate the:
- 16.3.1 Work function of metal **A** (3)
 - 16.3.2 Value of **X** shown on the graph (4)
- 16.4 How will EACH of the following be affected if light of frequency $12,54 \times 10^{14}$ Hz, but of higher intensity, is used? Choose from INCREASES, DECREASES or NO EFFECT.
- 16.4.1 The value of **X** (1)
 - 16.4.2 The number of photoelectrons emitted per unit time (1)
- Metal **B** has a larger work function than metal **A**.
- 16.5 Redraw the graph above in your ANSWER BOOK. (Do NOT include values on the axes.) Label this graph as **A**. On the SAME set of axes, sketch the graph for metal **B**. Label this graph as **B**. (2)

[14]

QUESTION 17 (November 2023)

- 17.1 Two small spheres, **A** and **B**, made of pure zinc are at rest 0,1 m apart on a wooden table. Sphere **A** is negatively charged and is free to move on the table, while sphere **B** is uncharged and fixed to the table, as shown in the diagram below.



High-intensity ultraviolet light of frequency $2,8 \times 10^{16}$ Hz is now shone continuously onto sphere **B**. The work function of zinc is $6,63 \times 10^{-19}$ J.

- 17.1.1 Define the term *work function of a metal*. (2)
 - 17.1.2 Explain, using a suitable calculation, why the ultraviolet light shone on sphere **B** will eject electrons from its surface. (4)
 - 17.1.3 Sphere **A** carries a charge of $-5,4 \times 10^{-6}$ C and requires a minimum force of 0,027 N to move from rest. Calculate the minimum number of photons of ultraviolet light that must strike sphere **B** which will cause sphere **A** to move from its rest position. (6)
- 17.2 A beam of white light is shone through a cold gas. The emerging light is dispersed and a line spectrum is observed on a screen.
- 17.2.1 Name the type of line spectrum observed. (1)
 - 17.2.2 Describe the spectrum referred to in QUESTION 17.2.1. (2)
 - 17.2.3 The diagrams below indicate some possible energy transitions made by atoms. Which ONE of the diagrams could result in the type of spectrum observed in QUESTION 17.2.1? Choose from DIAGRAM A or DIAGRAM B.

DIAGRAM A

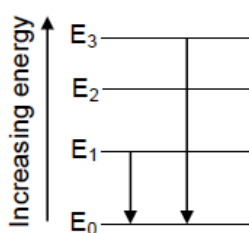
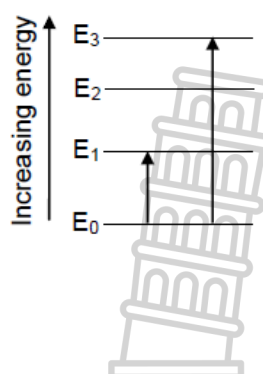


DIAGRAM B



(2)
[17]

QUESTION 18 (June 2024)

- 18.1 Define the term *photoelectric effect*. (2)
- 18.2 Light of wavelength $4,7 \times 10^{-7} \text{ m}$ is shone onto the surface of a piece of caesium metal. If the threshold frequency of caesium is $4,37 \times 10^{14} \text{ Hz}$, calculate the maximum speed of an electron ejected from the surface of the metal. (5)
- 18.3 A simple electroscope consists of a zinc disc, a metal stem and a thin length of gold foil. When the electroscope is neutral, the foil hangs vertically, as shown in DIAGRAM 1 below. When the electroscope is negatively charged, the foil is repelled from the stem, as shown in DIAGRAM 2 below.

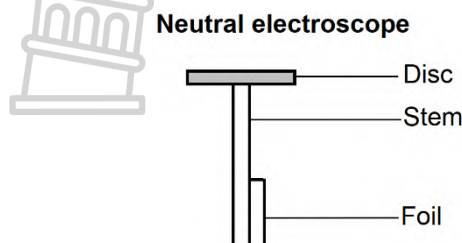


DIAGRAM 1

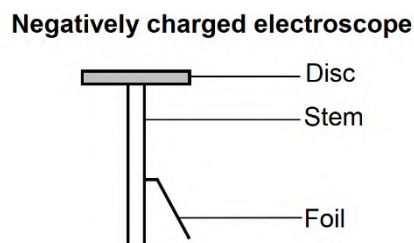


DIAGRAM 2

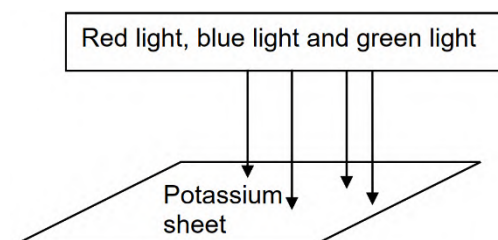
When ultraviolet light is shone on the disc of the negatively charged zinc electroscope, the foil collapses towards the stem (hangs vertically).

- 18.3.1 How does the frequency of the ultraviolet light compare to the threshold frequency of zinc? Write only HIGHER THAN, LOWER THAN or EQUAL TO. (1)
- 18.3.2 Explain why the foil of the electroscope collapses. (3)
- 18.3.3 Green light is now shone on another negatively charged zinc electroscope. The foil does not collapse. Will the foil collapse if the intensity of the green light is increased? Write either YES or NO. Give a reason for the answer. (2)

[13]

QUESTION 19 (November 2024)

- 19.1 Define the term *photoelectric effect*. (2)
- 19.2 Red light, blue light and green light are shone simultaneously on a sheet of potassium, as shown in the diagram. Each colour of light consists of a single frequency. Two maximum kinetic energies of the ejected electrons are possible, namely $6,96 \times 10^{-20} \text{ J}$ and $2,65 \times 10^{-20} \text{ J}$. Each ejected electron has only one of these maximum kinetic energies.
- 19.2.1 Which colour of light is responsible for ejecting electrons that have a maximum kinetic energy equal to $2,65 \times 10^{-20} \text{ J}$? (1)
- 19.2.2 Explain the answer to QUESTION 19.2.1. (2)
- 19.2.3 The electrons with a maximum kinetic energy of $2,65 \times 10^{-20} \text{ J}$ are ejected by light that has a frequency of $5,85 \times 10^{14} \text{ Hz}$. Calculate the frequency of the light that ejected electrons with a maximum kinetic energy of $6,96 \times 10^{-20} \text{ J}$. (5)
- 19.2.4 The intensity of the red light is increased, while the intensities of the blue light and green light remain the same. What effect will this change have on the rate at which electrons are ejected? Choose from INCREASES, DECREASES or REMAINS THE SAME. (2)
- 19.3 Some of the atoms of a hot gas, made up of a single element, are in an excited state. The spectrum formed by the hot gas is observed on a screen in a darkened room. The spectrum consists of specific coloured lines on a black background.
- 19.3.1 Name the type of spectrum formed. (1)
- 19.3.2 Explain the presence of the coloured lines in the spectrum. (2)



[15]

**DATA FOR PHYSICAL SCIENCES GRADE 12
PAPER 1 (PHYSICS)**

TABLE 1: PHYSICAL CONSTANTS / TABEL 1: FISIESE KONSTANTES

NAME/NAAM	SYMBOL/SIMBOOL	VALUE/WAARDE
Acceleration due to gravity <i>Swaartekragversnelling</i>	g	$9,8 \text{ m}\cdot\text{s}^{-2}$
Universal gravitational constant <i>Universele gravitasiekonstante</i>	G	$6,67 \times 10^{-11} \text{ N}\cdot\text{m}^2\cdot\text{kg}^{-2}$
Radius of the earth <i>Radius van die aarde</i>	R_E	$6,38 \times 10^6 \text{ m}$
Mass of the earth <i>Massa van die aarde</i>	M_E	$5,98 \times 10^{24} \text{ kg}$
Speed of light in a vacuum <i>Speed van lig in 'n vakuum</i>	c	$3,0 \times 10^8 \text{ m}\cdot\text{s}^{-1}$
Planck's constant <i>Planck se konstante</i>	h	$6,63 \times 10^{-34} \text{ J}\cdot\text{s}$
Coulomb's constant <i>Coulomb se konstante</i>	k	$9,0 \times 10^9 \text{ N}\cdot\text{m}^2\cdot\text{C}^{-2}$
Charge of electron <i>Lading op elektron</i>	e	$-1,6 \times 10^{-19} \text{ C}$
Electron mass <i>Elektronmassa</i>	m_e	$9,11 \times 10^{-31} \text{ kg}$



TABLE 2: FORMULAE / TABEL 2: FORMULES

MOTION/BEWEGING

$v_f = v_i + a\Delta t$	$\Delta x = v_i\Delta t + \frac{1}{2}a\Delta t^2$ or/of $\Delta y = v_i\Delta t + \frac{1}{2}a\Delta t^2$
$v_f^2 = v_i^2 + 2a\Delta x$ or/of $v_f^2 = v_i^2 + 2a\Delta y$	$\Delta x = \left(\frac{v_i+v_f}{2}\right)\Delta t$ or/of $\Delta y = \left(\frac{v_i+v_f}{2}\right)\Delta t$

FORCE/KRAG

$F_{net} = ma$	$p = mv$
$f_s^{max} = \mu_s N$	$f_k = \mu_k N$
$F_{net}\Delta t = \Delta p$ $\Delta p = mv_f - mv_i$	$w = mg$
$F = G \frac{m_1 m_2}{d^2}$ or/of $F = G \frac{m_1 m_2}{r^2}$	$g = G \frac{M}{d^2}$ or/of $g = G \frac{M}{r^2}$

WORK, ENERGY AND POWER / ARBEID, ENERGIE EN DRYWING

$W = F\Delta x \cos\theta$	$U = mgh$ or/of $E_p = mgh$
$K = \frac{1}{2}mv^2$ or/of $E_k = \frac{1}{2}mv^2$	$W_{net} = \Delta K$ or/of $W_{net} = \Delta E_k$ $\Delta K = K_f - K_i$ or/of $\Delta E_k = E_{kf} - E_{ki}$
$W_{nc} = \Delta K + \Delta U$ or/of $W_{nc} = \Delta E_k + \Delta E_p$	$P = \frac{W}{\Delta t}$
$P_{ave} = Fv_{ave}$ / $P_{gemid} = Fv_{gemid}$	

WAVES, SOUND AND LIGHT / GOLWE, KLANK EN LIG

$v = f\lambda$	$T = \frac{1}{f}$
$f_L = \frac{v \pm v_L}{v \pm v_s} f_s$ / $f_L = \frac{v \pm v_L}{v \pm v_b} f_b$	$E = hf$ or/of $E = \frac{hc}{\lambda}$
$E = W_0 + E_{k(max)}$ or/of $E = W_0 + K_{max}$ where/waar	
$E = hf$ and/en $W_0 = hf_0$ and/en $E_{k(max)} = \frac{1}{2}mv_{max}^2$ or/of $K_{max} = \frac{1}{2}mv_{max}^2$	

ELECTROSTATICS/ELEKTROSTATIKA

$F = \frac{kQ_1Q_2}{r^2}$	$E = \frac{kQ}{r^2}$
$V = \frac{W}{q}$	$E = \frac{F}{q}$
$n = \frac{Q}{e}$ or/of $n = \frac{Q}{q_e}$	

ELECTRIC CIRCUITS / ELEKTRIESE STROOMBANE

$R = \frac{V}{I}$	$emf(\varepsilon) = I(R + r)$ $emk(\varepsilon) = I(R + r)$
$R_s = R_1 + R_2 + \dots$ $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$	$q = I\Delta t$
$W = Vq$ $W = VI\Delta t$ $W = I^2R\Delta t$ $W = \frac{V^2\Delta t}{R}$	$P = \frac{W}{\Delta t}$ $P = VI$ $P = I^2R$ $P = \frac{V^2}{R}$

ALTERNATING CURRENT / WISSELSTROOM

$I_{rms} = \frac{I_{max}}{\sqrt{2}}$ / $I_{wgk} = \frac{I_{maks}}{\sqrt{2}}$ $V_{rms} = \frac{V_{max}}{\sqrt{2}}$ / $V_{wgk} = \frac{V_{maks}}{\sqrt{2}}$	$P_{ave} = V_{rms}I_{rms}$ / $P_{gemid} = V_{wgk}I_{wgk}$ $P_{ave} = I_{rms}^2R$ / $P_{gemid} = I_{wgk}^2R$ $P_{ave} = \frac{V_{rms}^2}{R}$ / $P_{gemid} = \frac{V_{wgk}^2}{R}$
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NUMERICAL ANSWERS

NEWTON'S LAWS

1.2	0 N	1.4	0,21	2.2	4,41 N	2.4.1	4,08 N
2.4.2	13,19 N	3.1.2	15 N	3.1.3	13,45 N	3.1.4	4,82 m·s ⁻²
4.1	0 N	4.3.1	2,39 N	4.3.2	0,12	4.3.3	0,96 m
5.1.3	59,57 N	5.1.4	7,45 m·s ⁻²	5.2.2	4,41 x 10 ²² kg	6.3	9,24 N
6.4	3 s	7.2	0,11 m·s ⁻²	7.3	29,07 N	7.5	18,82 N
8.3.2	16,96 N	8.3.3	5,82 N	9.1.3	68,94 N	9.1.4	101,11 N
9.2	1,08 x 10 ²² kg	10.3	1,85 m·s ⁻²	11.3	3,25 kg	12.3	19,6 N
12.4.1	2,04 m·s ⁻²	12.4.2	23,68 N	13.3	116 N	13.5	0,34 m
14.3	466,72 N	14.5	679,20 N	15.3.1	1,925 N	15.3.2	54,55° ~ 54,78°
16.4	3,25 kg	16.5	2,09 m	17.3.1	36,36 N	17.3.2	118,18 N
18.3.1	0,92 N ~ 0,93 N	18.3.2	1,25 m·s ⁻² ~ 1,26 m·s ⁻²			19.3.1	0,49
19.3.2	2,47 m·s ⁻²						

VERTICAL MOTION

1.2	6,12 s	1.3	25,08 m	1.4	8,05 m·s ⁻¹	2.2	19,08 m·s ⁻¹
2.3	2,02 s	3.2.1	14,10 m	3.2.2	19,41 m·s ⁻¹	3.2.3	12,52 m·s ⁻¹
4.3	2 s	4.4	17,4 m	5.1	1,22 s	5.2	6,00 m·s ⁻¹
5.3	43,76 m	6.1	10 m·s ⁻¹	6.3	39,6 m	6.4	10,55 s
6.5.1	0,2 s	6.5.2	4,96 s	6.5.3	-27 m·s ⁻¹	7.2	9,8 m
7.3	13,86 m·s ⁻¹	7.4	47,32 N	8.2	0,77 s	9.2.1	14,99 m·s ⁻¹
9.2.2	11,47 m	10.2	45,10 m	10.4	1,34 s	11.2.1	0,03 s
11.2.2	0,62 s	11.2.3	6,08 m·s ⁻¹	11.2.4	4,85 m·s ⁻¹	12.2	9,8 m·s ⁻²
12.3	3 m	12.4.1	10 m·s ⁻¹	12.4.2	8,1 m	12.5	-1,28 J
13.2.1	2,96 m·s ⁻¹	13.2.2	6,70 s	13.2.3	14,16 m	14.2.1	1,53 s
14.2.2	41,48 m	14.3	1,71 s	15.2.1	1,22 s - 1,23 s		
15.2.2	25,03 m·s ⁻¹ ~ 25,59 m·s ⁻¹			15.2.3	0,07 s ~ 0,08 s	16.2.1	1,76 s
16.2.2	4,66 m·s ⁻¹	17.2	10,74 m·s ⁻¹	17.3.1	67,91 ~ 69,34 J		
17.3.2	1,22 s	18.3.1	1,44 s	18.3.2	10,1 m	18.4	0,73 s
19.1.1	4 (s)	19.1.2	30,4 (m)	19.1.3	37,72 ~ 37,75 (m)		

MOMENTUM AND IMPULSE

1.2.1	2,5 m·s ⁻¹	1.2.2	66,67 N	2.2.1	20 m·s ⁻¹	2.2.2	2,78 m·s ⁻¹
2.2.3	160 000 N	3.3	2,21 N	4.2.1	21 kg·m·s ⁻¹	4.2.2	77 N
4.3	77 N	5.2	306,67 m·s ⁻¹	5.3	0,71 m	6.2	2,23 m·s ⁻¹
6.3	4,48 N·s	7.1	6,26 m·s ⁻¹	7.3	2,68 m·s ⁻¹	7.4	8,04 m
8.3	51,33 kg	8.4	32 N·s	8.5	32 kg·m·s ⁻¹	9.2	6,23 m·s ⁻¹
10.2.1	0,05 kg	10.2.2	2 000 N	11.2.1	3 m·s ⁻¹	11.2.2	2,4 N·s
12.2	3v	13.2	4 m·s ⁻¹	13.3	200 N	14.2.2	1,38 kg
15.2.1	9,6 m·s ⁻¹	15.2.2	48 N	16.2.1	0,23 m·s ⁻¹	16.2.2	61,056 N ~ 61,2 N
17.2	394 m·s ⁻¹	17.4	1,36 m·s ⁻¹	18.2	0,64 m	18.3	0,3 s
19.2	5,42 m·s ⁻¹	19.3.1	10,84 kg·m·s ⁻¹	19.3.2	10,84 kg·m·s ⁻¹	19.4	7,23 m·s ⁻¹

WORK, ENERGY AND POWER

1.1	24,50 J	1.2	0,67 m	1.4	-10,7 J	2.1	2,39 m·s ⁻¹
2.3	838,89 m·s ⁻¹	3.3	9,95 m·s ⁻¹	3.4	-623,75 W	4.2.1	-646 800 J
4.2.2	512 050 J	4.3	748,61 W	5.3	94,08 J	5.4	3,71 m·s ⁻¹
6.2	7,67 m·s ⁻¹	6.4	33,95 N	6.5	10,05 m·s ⁻¹	7.3	-8 820 J
7.5	3,95 m·s ⁻¹	8.3	5,33 x 10 ⁴ W	8.5	-1,39 x 10 ⁶ J	9.4	-712,88 J
9.5	4,52 m·s ⁻¹	9.6	-1 425,76 J	10.4	26,46 J	10.5	-12,87 J
11.2	-1 200 J	11.3	8,88 m	11.4	1 814,35 W	12.2	1 184,17 W
12.6	284 089 N	13.2	9,90 m·s ⁻¹	13.3	8,2 N	13.4	7 J
14.3	96,64 m	15.3	559,575 J	15.4	13,32 m	16.2	5,96 m·s ⁻¹
16.3	286,52 W ~ 288,73 W			17.4	2,27 kg	18.2	7,67 m·s ⁻¹
18.4	13,03 m ~ 13,04 m			19.3	20,78 N		

DOPPLER EFFECT

1.1.2	1 214,29 Hz	1.1.3	1 331,80 Hz	2.2	3 m·s ⁻¹	2.3.2	345 m·s ⁻¹
2.3.3	1 200 Hz	2.3.4	295 K	3.1.2	32,60 m·s ⁻¹	4.1.3	66,44 m·s ⁻¹
5.2.1	170 Hz	5.2.2	130 Hz	5.3	45,33 m·s ⁻¹	6.3	20,90 m·s ⁻¹
7.2	0,23 m	8.3	1 000 Hz	8.4	170 m	9.1.1	30 m·s ⁻¹
9.1.4	849,76 Hz	10.3	588,24 Hz	10.4	20 m·s ⁻¹	11.3	70 m·s ⁻¹
11.4	5 s	12.2.1	700 Hz	12.2.3	334,93 m·s ⁻¹	13.1	0,5 m
13.4	34 m·s ⁻¹	14.2	0,39 m	14.3	897,44 Hz / 905,88 Hz		
15.4	20,4 m·s ⁻¹	16.1.2	27 320,75 Hz	17.1.2	343,04 m·s ⁻¹	18.2	331,88 ~ 333,33 m·s ⁻¹
18.3	0,59 m	19.3.1	902,63 Hz	19.3.2	18,05 m·s ⁻¹ ~ 18,45 m·s ⁻¹		

ELECTROSTATICS

1.3	0,67 m	2.2.2	1,6 x 10 ⁻⁵ C	3.1.4	7,32 x 10 ⁻⁷ C	3.2.2	8,83 x 10 ⁶ N·C ⁻¹
4.3	5,12 x 10 ⁴ N·C ⁻¹	5.1.2	3,75 x 10 ¹³	5.2.4	1,128 m	5.3.2	0,85 m
6.2	10,8 N	6.4	1,34 x 10 ⁻⁵ C	7.2	2,84 x 10 ⁵ N·C ⁻¹	7.3	8,52 x 10 ⁻⁴ N
8.3	4,23 x 10 ⁻⁹ C	9.2	8,33 x 10 ⁻⁴ C	9.3	4,80 x 10 ⁻⁷ N	10.3	0,46 m
11.2	2,45 x 10 ⁵ N·C ⁻¹	12.1	4 x 10 ⁻⁹ C	12.4	8,15 x 10 ⁻⁶ N	12.5	2,72 x 10 ³ N·C ⁻¹
12.6.2	2,85 x 10 ⁻²⁰ kg	13.3	4,80 x 10 ⁻⁸ C	13.5	1,44 x 10 ⁻⁵ N	14.1.2	2,26 x 10 ⁻⁶ C
14.1.4	17,56 N	14.2.2	1,35 x 10 ⁷ N·C ⁻¹	15.1	2,5 x 10 ¹³	15.2	2,7 N
15.4	2,3 x 10 ⁶ N·C ⁻¹	15.6	4,5 x 10 ⁻⁶ C	16.2	0,015 m	16.4.1	0,04 N
16.4.2	3,78 x 10 ⁴ N·C ⁻¹	17.1.2	1,22 x 10 ¹³	17.1.4	7,02 x 10 ⁴ N·C ⁻¹	17.2	1,11 x 10 ⁻⁷ C
18.1.4	3,05 x 10 ¹³	18.2.2	4,25 x 10 ⁵ N·C ⁻¹	19.4	2,32 x 10 ⁻⁶ C ~ 2,33 x 10 ⁻⁶ C		
19.6	5,31 x 10 ⁴ N·C ⁻¹ ~ 5,37 x 10 ⁴ N·C ⁻¹			20.3	5,18 x 10 ⁻⁴ N		
20.5	6,25 x 10 ⁻⁹ C ~ 6,26 x 10 ⁻⁹ C			21.3	0,87 (m)	21.4	4,28 x 10 ⁻¹⁸ N
22.1.2	57 600 N·C ⁻¹	22.2.2	2,88 x 10 ⁻⁷ C	23.1	5 000 N·C ⁻¹	23.3.3	6,25 x 10 ⁻⁹ C



ELECTRIC CIRCUITS

1.1.2	12 V	1.1.3	0 V	1.1.4	1,5 A	1.1.5	6 Ω
1.1.6	1,8 Ω	1.2.1	1,37 W	1.2.2	19,37 Ω	2.1.3	5,5 V
2.1.4	1,2 Ω	2.2.1	2,73 A	2.2.2	12 Ω	2.2.3	1,98 Ω
2.2.4	29,81 J	3.2.1	1,62 A	3.2.2	5,42 V	3.2.3	32,4 W
4.1.3	4 Ω	4.1.4	2 Ω	4.2.1	0,83 A	4.2.2	7 V
4.2.3	3 Ω	5.2.1	0,33 A	6.1.2	2,4 V	6.1.3	0,4 A
6.1.4	5,8 V	6.1.5	9 V	6.1.6	12 J	6.2.1	4 Ω
7.2	11 V	7.3	5,5 Ω	8.2	3 Ω	8.3	7 V
9.1.2	24 Ω	9.1.3	0,32 A	9.1.4	3,78 V	10.2	13 V
10.3.1	1,88 A	10.3.2	19,69 W	10.3.3	1,31 Ω	10.4.2	1,49 Ω
11.3.1	3 A	11.3.2	13,38 Ω	11.3.3	41,64 V	12.2	3,5 Ω
12.3.1	0,49 Ω	12.3.2	3 V	13.1.1	12 V	13.1.2	0 V
13.3	4 Ω	13.4	7,5 Ω	13.5	3 V	14.2.1	0,8 A
14.2.2	10 V	14.3.1	5,28 Ω	15.2.1	4,83 Ω		
15.2.2	16,91 V ~ 16,92 V	16.2.1	10,5 V	16.2.2	5 A	16.2.3	36,75 W
16.3	12,05 V	17.2.1	7,5 Ω	17.2.2	1,5 A	17.2.3	8,44 W
18.2	5,98 Ω	18.3	39,84 W ~ 40 W	18.4	7,70 V ~ 7,75 V	18.5	2,28 A
19.2.1	1,5 A	19.2.2	3,3 A	19.2.3	6,67 Ω	19.2.4	47,16 ~ 47,19 V

ELECTRICAL MACHINES

1.2.2	0,76 A	2.2.1	3,33 A	2.2.2	11,65 A	3.2.1	15 V
3.2.2	0,47 A	4.5	6,25 A	5.2.1	20,5 Ω	5.2.2	29,75 W
6.2.2	30,86 A	7.2.2	226,27 V	7.2.3	6,46 A	8.2.1	340 V
8.2.2	240,42 V	8.2.3	36,13 Ω	9.3	220,62 V	9.4.1	1 216,83 W
9.4.2	7,8 A	10.2.2	141,42 V	11.2.2	242 Ω	11.2.3	5,58 x 10 ⁴ J
12.5	0,03 s	12.6	2,90 x 10 ⁴ J	13.4	2,83 A	13.5	200 W
14.2.1	484 Ω ~ 494 Ω	14.2.2	846 W ~ 848 W	15.1.3	0,05 s	15.2	181 500 J ~ 181 764 J
16.2.1	47,14 Ω	16.2.2	6,11 x 10 ⁶ J	17.2.2	2,55 A		
17.2.3	69 167,56 J ~ 69 168,44 J			18.4	88,39 V	18.5	2,94 A ~ 2,95 A
19.4	50 Hz	19.6	0,85 A				

OPTICAL PHENOMENA AND PROPERTIES OF MATERIALS

2.2.1	7,09 x 10 ¹⁵	2.2.2	7,09 x 10 ¹⁵	3.3	1,97 x 10 ⁻⁶ m	4.4	3,82 x 10 ⁵ m·s ⁻¹
5.2.2	6,17 x 10 ¹⁴ Hz	5.2.3	1,17 x 10 ⁻¹⁹ J	6.2.2	3,8 x 10 ⁻¹⁹ J	6.2.3	9,94 x 10 ¹⁴ Hz
7.5	3,65 x 10 ⁻¹⁹ J	8.4	2,55 x 10 ⁻²⁰ J	9.2	3,98 x 10 ⁻¹⁹ J	10.4	0,9868
11.5.1	1,16 x 10 ⁻¹⁸ J	11.5.2	1,97 x 10 ¹⁵ Hz	12.1	11,6 x 10 ⁻¹⁹ J	12.4	4,06 x 10 ⁻¹⁹ J
12.5	3,57 x 10 ⁻¹⁸ J	13.3	4,51 x 10 ¹⁵ Hz	14.1.2	2,198 x 10 ⁹	14.1.3	6,51 x 10 ⁵ m·s ⁻¹
15.3	5,30 x 10 ⁻²⁰ J ~ 5,32 x 10 ⁻²⁰ J			15.4	7,5 x 10 ¹⁷	16.3.1	3,32 x 10 ⁻¹⁹ J
16.3.2	5 x 10 ⁻¹⁹	17.1.3	3,47 x 10 ¹⁰	18.2	541 289,67 m·s ⁻¹ ~ 541 292,69 m·s ⁻¹		
19.2.3	6,5 x 10 ¹⁴ Hz						

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